Lab 10: Discrete Optimization: Introduction to MILP

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Objective: By the end of this lab, students can successfully formulate a mixed integer linear program from the given description and solve it using the Pyomo framework.

Instructions: Below are some instructions. Please go through them carefully:

- Use Lab10_Practice_I and Lab10_Practice_II as practice files to get familiarized with modeling Mixed Integer Linear Programs (MILP).
- Along with the .ipynb file, you must submit a report (as a **.pdf** only, **not a photo-captured image** of a handwritten mathematical model as .pdf file) file that answers all the questions from lab 10.
- Also, explicitly mention the assumptions used throughout your modeling technique.
- Use Pyomo, Numpy, and other library documentation if you need help.
- Your task is to submit the questions asked below and the task performed in the above-shared file.
- Use the traditional approach to name your files for submission:
  - **ROLLNUMBER_IE507_Lab10_Q*i*.ipynb;** for i^{th} question
  - **ROLLNUMBER_IE507_Lab10_Report.pdf**

Question 1: Course Registration

Congratulations on almost completing your first semester!! Assume that you will be starting your second semester in IEOR soon. You must register for various courses IEOR offers as part of this semester's requirements. Suppose the upcoming semester IEOR offers in total 11 courses as **C0, C1, ..., C10** with credits in the following table:

Course	Credit
C0	3
C1	2
C2	6
C3	5
C4	5
C5	4
C6	6
C7	8
C8	8
C9	5
C10	4

Table 1: Courses offered by IEOR

Suppose it is mandatory to register for at least 30 credits in this semester, along with the following constraints:

- In total, the least number of courses can be registered is 5, with a maximum of 9 courses each semester.
- C2 and C7 run in the same course slot. Further, C4 and C9 are offered in the same slot as well.

- Given you registered in C2, you are not allowed to register for C8.
- Also, it is necessary to register for C0 if you register for C10.
- C0, C1, and C2 are meant for new entrants. So, it is mandatory to register for at least one course out of these three courses, and at most two can only be registered.
- Among C5 and C6, you can only register at most one course.
- Lastly, you can only have at most two courses registered from C3, C6, and C9.

1. Write a mathematical model so that you can maximize your credits to be accumulated this semester. Explain how to design the model variables and indicate which variables are integers.
2. Using the Pyomo framework and *cbc* solver in Python, solve your MILP formulation and report optimal registration.
3. Relax the above-constructed formulation to a linear program and solve for optimality. Report the solution for the relaxed program and compare it with the solution MILP formulation. Comment on about "can the solution of MILP be obtained by merely rounding the LP solution?". Explain.
4. A faculty in IEOR lately proposed an advanced course as C11 with 9 credits. But this course clashes slot with C7, C8 and C9. Modify the above MILP formulation by adding this additional constraint. Solve your modified MILP for the optimal solution.
5. Relax this modified MILP to a simple linear program and check if the modified MILP can be obtained by merely rounding the solution of the corresponding modified linear program.

Question 2: Linear Programming Vs MILP

The simplest application of MILP is when the variables model discrete objects that can only take whole-number values (e.g., the number of drones to manufacture). The Indian Air Force (IAF) has planned to purchase drones from Hindustan Aeronautics Limited (HAL) for surveillance activities and during crises like floods, fires, and earthquakes. The drones are expected to carry cameras and freight. Five different types of drones are available with HAL. Their carrying capacities, costs, and parking space requirements are listed below:

Drone Model	Carrying Capacity (Kgs)	Cost (in Crores of rupees)	Parking Space (sq. m.)
MANAS	12	6	4.25
JAY	14	7.5	3.25
VEER	10	9.5	2.5
SPASHT	15	10	3.45
DHAKSH	14	12.5	2.25

Table 2: Types of Drones manufactured by HAL

The IAF wants to have the maximum possible carrying capacity. The budget of IAF is limited to 59 crores of rupees. In addition, parking space for drones is limited to 73 sq. m.

1. Write and report a mathematical formulation to find the number of drones of each type that can be bought to maximize the goal. Mention which variables must be integers.
2. Create a model using the Pyomo framework to solve your proposed formulation. In your code, remember to specify which variables are integers. Use *cbc* solver to solve and report the optimal solution.

3. Let us compare the above results to a linear program. Suppose you relax variables in the above problem from being integers. Solve this relaxed problem to report optimal value.

4. Can the solution of the MILP be obtained by merely rounding the solution of the LP? Why or why not?

5. Suppose now we are interested in finding how the solution changes when the right-hand sides in our constraints change. The government approves to increase budget allocation for new order to 71 crores of rupees, and parking space is increased to 87 units. How additional resource constraints change optimal values in both LP and MILP?

6. Now try decreasing the limit on a budget from 71 units to 69, 67, 65, 63, and 61 units. Correspondingly, increase the parking space to 88, 89, 90 and 91 units. Solve the LP and MILP for each of these four possibilities and comment on the pattern seen in optimal objective function values of the LP and MILP. Also, comment on your observations about the optimal solutions obtained.

Question 3: Transportation Problem

A customer contacted TCI Ltd. to transport stock-keeping units (SKUs) from warehouses to markets. There are eight warehouses, each with some truck-loads of SKUs in Inventory. These SKUs must be transported to eight markets to satisfy the demand. As an intern at TCI Ltd., you are tasked with determining how many SKUs must be transported from each warehouse to each market.

	Market								
Warehouse	M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8	**Inventory**
W_1	627	617	1270	2072	1415	2943	1064	4265	*185*
W_2	159	1432	351	3372	237	3373	1456	5059	*350*
W_3	609	1943	77	3365	298	3021	1484	4803	*405*
W_4	2143	671	3876	537	1902	1941	1682	3348	*280*
W_5	2742	2089	1931	2199	1226	739	691	1128	*105*
W_6	2053	860	2995	68	2069	2065	993	3342	*75*
W_7	656	817	1245	2141	1534	3351	1137	3887	*425*
W_8	195	1862	1331	2921	402	2825	563	4441	*270*
Demand	*175*	*400*	*550*	*70*	*650*	*100*	*65*	*85*	*2095*

Table 3: Cost of transportation, Demand for market and Inventory level at Warehouse for SKUs

1. Let $C_{W,M}$ be the cost of transporting one SKUs from W to M. Write a general optimization problem to minimize the total transportation cost. Use appropriate notations and define appropriate sets for the optimization model.

2. Create a model using the Pyomo framework to solve your formulation. Report the number of SKUs transported (report only those nonzero values) from warehouses W to markets M. Also, report the total cost for transportation of SKUs.

3. Suppose the $W_2 - M_1$ link is disrupted, and no transportation is possible on the $W_2 - M_1$ link. Without changing the Pyomo model, how will you solve this problem? Report the new solution value and the nonzero flows in the network. Comment on your observations.

4. Further in addition to $W_2 - M_1$ link, the $W_6 - M_4$ link and $W_5 - M_8$ links are disrupted and no transportation is possible along these links as well. Report the new solution value and the nonzero flows in the network. Explain your observations.