Lab 07: Introduction to Optimization

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Objective: By the end of this lab, students can successfully reformulate a linear program, formulate an optimization model from the given description, and solve them using the Pyomo framework.

Instructions: Below are some instructions. Please go through them carefully:

- Along with the .ipynb file, you must submit a report (.pdf) file that answers all the questions from lab 07.
- Also, explicitly mention the assumptions used throughout your modeling technique.
- Use Pyomo, Numpy, and other library documentation if you need help.
- An introductory ready-to-use Pyomo framework along with instructions is available as an Introduction to Pyomo to solve optimization problem along with a few exercises. Use this .ipynb file, save it to your iitb Gdrive, and perform all given tasks carefully.
- Your task is to submit the questions asked below and the task performed in the above-shared file.
- Use the traditional approach to name your files for submission:
  - **ROLLNUMBER_IE507_Lab07_InClass.ipynb** ;
  - **ROLLNUMBER_IE507_Lab07_Submission.ipynb**
  - **ROLLNUMBER_IE507_Lab07_Report.pdf**

Question 1: LP- Reformulation

Consider the following optimization problem:

$$\underset{x, y}{\text{minimize}} \quad 0.043x_1 + 0.027x_2 + 0.025x_3 + 0.022x_4 + 0.045x_5 - 0.0275y \tag{1a}$$

subject to

$$x_1 + 3x_2 + x_3 + 7x_4 + x_5 - y \leq 100, \tag{1b}$$

$$x_2 + 2x_3 + 4x_4 \geq 60, \tag{1c}$$

$$\frac{2x_1 + 2x_2 + x_3 + x_4 + 5x_5}{x_1 - x_2 + x_3 - x_4 + x_5} \leq 2.8, \tag{1d}$$

$$\frac{2x_1 + 15x_2 + 4x_3 + 3x_4 - 8x_5}{x_1 + x_2 + x_3 - x_5} \leq 3, \tag{1e}$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0, \tag{1f}$$

$$y \in [0, 2.5] \tag{1g}$$

1. Rewrite this optimization problem as a linear program and include it in your report. Did you make any assumptions when you constructed the linear program? If so, explain those assumptions.
2. Solve the optimization problem.
3. Report the optimal solution value, the values of variables at the optimal solution, and the activities of all constraints of the LP model.

4. Explain how you would modify the model if no assumptions are made during the construction of the linear program. **Note**: You need not solve the modified model.

5. In case no assumption is made, how sensitive is the optimal solution against the denominator? (**Hint:** Plot your objective values (all four cases from the denominators) for various $\epsilon = 10^{-N}$ on $\log_{10}$-scale in (4). You can restrict $N \in \{1, 2, 3, 4, 5, 6\}$).

Question 2: Optimal Mix

ABC Industries Ltd. has two refineries situated in India (say, Domestic (D) and Export (E)). Each refinery makes two products: #1 and #2. Standard profit contributions towards revenue are Rs. 10 per unit and Rs. 15 per unit, respectively. Each factory uses a two-stage process: Distillation and Treatment. The following table summarises the processing capacity (in hours per week) of both D and E refineries:

	Refinery D	**Refinery E**
Distillation	80	60
Treatment	60	75

ABC Industries Ltd. has 120 units of raw crude oil available each week, which is processed into two products (#1 and #2) as the finished product. Below is the time required (in hours) for each type of product (#1 and #2) in the Distillation and Treatment process.

	Refinery D		**Refinery E**	
Product	**#1**	**#2**	**#1**	**#2**
Distillation	4	2	5	3
Treatment	2	5	5	6

1. **Decentralized System**: Formulate the optimization model of Refinery D and E at the decentralized level and describe your formulation along with the variables, objective function, and constraints. Solve your formulated model of product mix to maximize profit using *Pyomo* and report your solution. Assume that 50-50 % is the raw crude oil distribution among D and E refineries.

2. **Centralized System**: Formulate the optimization model of Refinery D and E at a centralized level. Solve your formulated product mix model to maximize profit for *ABC Industries Ltd.* Report your solution and describe your formulation.

3. **Best Allocation**: Consider the decentralized system above. Is there a fraction of allocation to domestic plant, r, such that the decentralized profit is more than the centralized setup of Industry? To answer this, plot the sum of profits against various values of r. Interpret the plot obtained with context to your optimization formulation. (**Hint**: Try solving your formulation for various values of $r \in [0, 1]$). Argue in the context of the decentralized system, how a change in the allocation of raw crude oil to the Domestic-Export refinery impacts profit for *ABC Industries Ltd.*

4. **Managerial Insights**: Recall Primal(P)-Dual(D) formulation from Problem 3 in the shared framework model. Formulate Dual for Refinery D in the decentralized system and solve your dual LP. Write some managerial insights. For example, comment on the sensitivity of how an infinitesmal change in the bound of each resource capacity for Refinery D impacts its optimal product mix. Next, try an intuitive explanation why some optimal dual variables are zeros.