

5CS037 - Concepts and Technologies of AI.

Worksheet - 1: A coding exercises on Numpy.

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1 Instructions

This worksheet contains programming exercises based on the material discussed from the slides. This is a graded exercise and are to be completed on your own and is compulsory to submit.
Please answer the questions below using python in the Jupyter Notebook and follow the guidelines below:

- This worksheet must be completed individually.
- All the solutions must be written in Jupyter Notebook.
- Our Recommendation - Google Colaboratory.

----- May the Force be with You -----

2 Before you Start: Setting Up Your Environment.

We highly recommend the use of **Google Colab** or in option you may also use **Jupyter Notebook with Anaconda** or any other Notebook like environment.

2.1 Using Google Colab: What is Google Colaboratory?

- Also known as Colab, is “Jupyter notebook” like environment with live code, visualizations, and narrative text. Colab notebooks are the same as Jupyter.

- Colab notebooks are the same as Jupyter notebook, including the .ipynb extension and requires no setup on your computer!
- To be able to write and run code, you need to sign in with your Google credentials. This is the only step that's required from your end. No other configuration is needed.
- Head over to colab.research.google.com. You'll see the following screen.

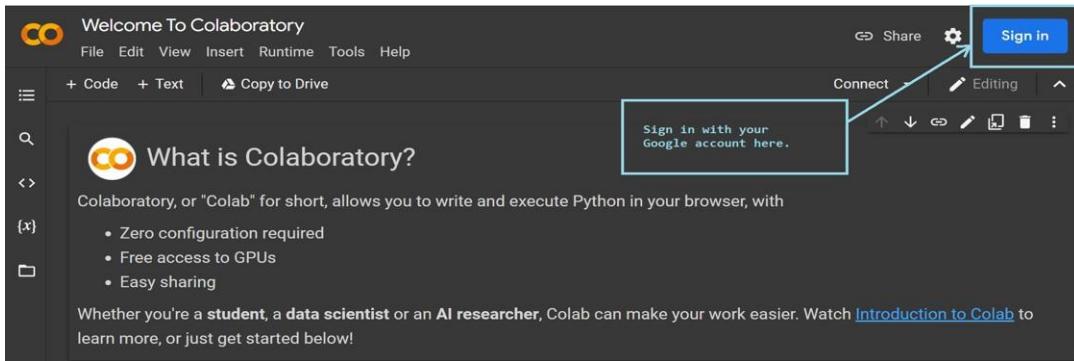


Figure 1: Google Colab: Home Screen.

1. Your First Google Colab Notebook.

- **Creating a new Notebook:**

- Once you've signed in to Colab, you can create a new notebook by clicking on 'File' → 'New notebook', as shown below:

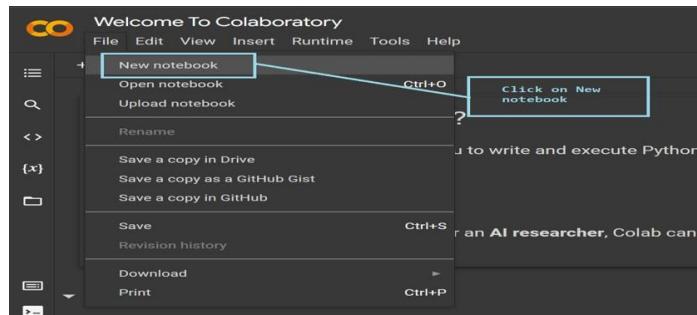


Figure 2: Google Colab: Creating a new Notebook.

- **Naming your Notebook:**

- Notebook uses the naming convention '\UntitledXX.ipynb'.
- To rename the notebook, click on this name and type in the desired name.

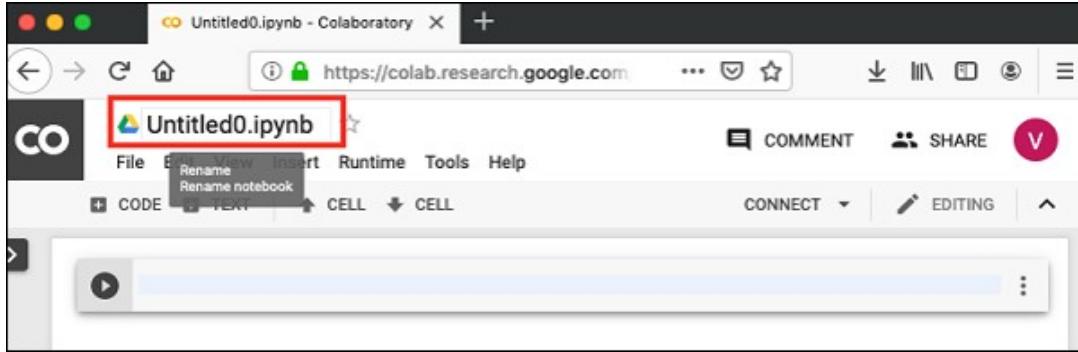


Figure 3: Google Colab: Renaming the Notebook.

- **Mounting Google Drive:**

- We can easily mount Google drive onto the current instance of Colab.
- This will enable you to access all the datasets and files that are stored in your drive.

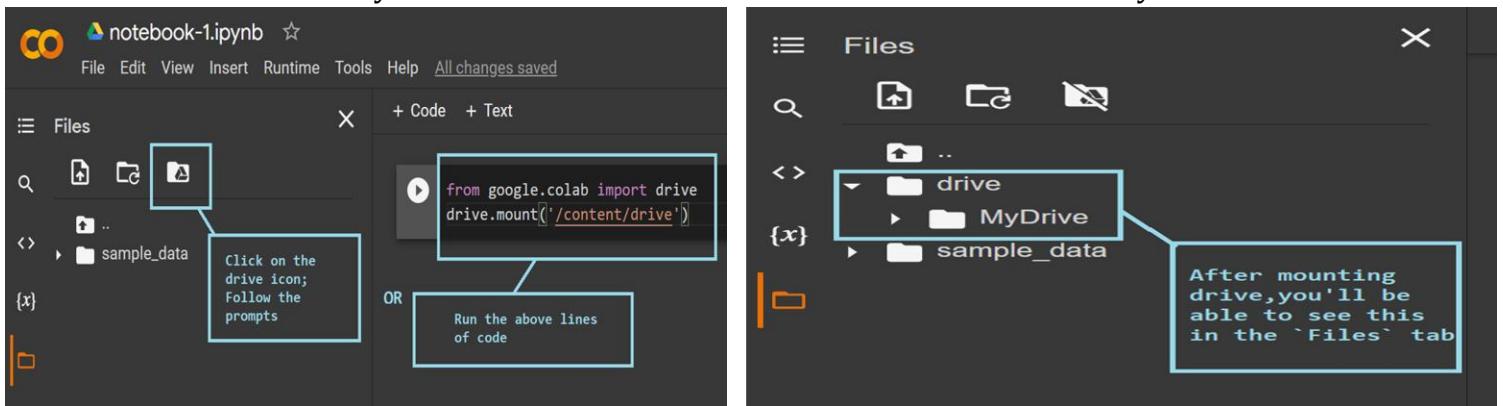


Figure 4: Google Colab: Mounting Google Drive-1.

3 Getting Started with Numpy.

This Section contains all the sample code from the slides and are here for your reference, you are highly recommended to run all the code with some of the input changed in order to understand the meaning of the operations and also to be able to solve all the exercises from further sections.

- **Cautions!!!:**

- This Guide does not contain sample output, as we expect you to re-write the code and observe the output.
- If found: any error or bugs, please report to your instructor and Module leader. {Will hugely appreciate your effort.}

3.1 Array Introduction:

1. Importing Numpy:

Sample Code from Slide - 14 - Importing Numpy and Array Type.

```
import numpy as np
# Create and display zero, one, and n-dimensional arrays
zero_dim_array = np.array(5) one_dim_array = np.array([1, 2, 3])
n_dim_array = np.array([[1, 2], [3, 4]]) for arr in [zero_dim_array,
one_dim_array, n_dim_array]:
    print(f"Array:\n{arr}\nDimension: {arr.ndim}\nData type: {arr.dtype}\n")
```

2. Array Dimensions: Shape and Reshape of Array:

Sample Code from Slide - 16 - Shape of an Array.

```
import numpy as np
# Create arrays of different dimensions array_0d =
np.array(5)
array_1d = np.array([1, 2, 3, 4, 5]) array_2d =
np.array([[1, 2, 3], [4, 5, 6]])
array_3d = np.array([[[1, 2], [3, 4]], [[5, 6], [7, 8]]])
# Print arrays with shapes for i, arr in enumerate([array_0d, array_1d, array_2d,
array_3d]):
    print(f"{i}D Array:\n{arr}\nShape: {arr.shape}\n")
```

Sample Code from Slide - 17 - Reshaping an Array.

```
import numpy as np array = np.array([[1, 2, 3], [4, 5, 6]]) # Shape (2, 3) reshaped_array =
array.reshape(3, 2) # Reshape to (3, 2), keeping 6 elements print("Original Shape:", array.shape,
"\nReshaped Shape:", reshaped_array.shape)
```

3.2 Array Creation:

1. Using In - Built Function:

Sample Code from Slide - 18 - Arrays with evenly Spaced values - arange.

```
import numpy as np a =
np.arange(1, 10) print(a) x =
range(1, 10) print(x) # x is an
iterator print(list(x))
# further arange examples:
x = np.arange(10.4) print(x) x =
np.arange(0.5, 10.4, 0.8) print(x)
```

Sample Code from Slide - 19 - Arrays with evenly Spaced values - linspace.

```
import numpy as np
# 50 values between 1 and 10: print(np.linspace(1, 10))
# 7 values between 1 and 10:
print(np.linspace(1, 10, 7)) #
excluding the endpoint:
print(np.linspace(1, 10, 7, endpoint=False))
```

2. Initializing Arrays with Ones, Zeros and Empty:

Sample Code form slide - 20 - Initializing an Array.

```
import numpy as np # Create arrays of specified shapes
ones_array = np.ones((2, 3)) # Shape: (2, 3) zeros_array =
np.zeros((3, 2)) # Shape: (3, 2) empty_array = np.empty((2,
2)) # Shape: (2, 2) identity_matrix = np.eye(3) # Shape: (3,
3)
print(ones_array, zeros_array, empty_array,
identity_matrix, sep='\n\n')
```

3. By Manipulating Existing Array:

3.1 Using np.array:

Sample Code form slide - 21 - Converting list to array using np.array.

```
import numpy as np array_from_list = np.array([1, 2, 3]) # [1
2 3] array_from_tuple = np.array((4, 5, 6)) # [4 5 6]
array_from_nested_list = np.array([[1, 2, 3], [4, 5, 6]]) # [[1 2 3] [4 5 6]] print(array_from_list, array_from_tuple,
array_from_nested_list, sep='\n')
```

3.2 Using shape of an existing array:

Sample Code form slide - 22 - From shape of an existing array.

```
import numpy as np # Existing Array of Shape:(2,3) arr = np.array([[1, 2, 3], [4, 5, 6]]) # Creating Array with Shape of existing
array: zeros, ones, empty = np.zeros(arr.shape), np.ones(arr.shape), np.empty(arr.shape) # Shape: (2,3) print(arr, zeros,
ones, empty, sep='\n')
```

3.3 With Math, Copying or Slicing and Array:

Sample Code form slide - 23 - Manipulatiing existing array.

```
import numpy as np array =
np.array([1, 2, 3])
# Multiplies each element by 2, result: [2, 4, 6] new_array = array * 2
# Creates a new array with the same elements copied_array =
np.copy(array)
# Slices elements from index 1 to 3, result:[2] sliced_array = array[1:2]
```

Sample Code form slide - 26 - With Concatenating Operation.

```
import numpy as np arr1 = np.array([[1, 2], [3, 4]]) # Shape: (2, 2)
arr2 = np.array([[5, 6], [7, 8]]) # Shape: (2, 2) concat_axis0 =
np.concatenate((arr1, arr2), axis=0) # Concatenate along axis 0
(vertical) Shape: (4, 2) concat_axis1 = np.concatenate((arr1, arr2),
axis=1) # Concatenate along axis 1 (horizontal) Shape: (2, 4)
print(concat_axis0, concat_axis1, sep='\n')
```

3.4 Array: Indexing and Slicing:

Sample Code form slide - 34 - Sample Code for Indexing and Slicing.

```
import numpy as np
# Arrays
arr1d = np.array([0, 1, 2, 3, 4, 5]) arr2d = np.array([[0, 1, 2], [3, 4,
5], [6, 7, 8]])
arr3d = np.array([[[0, 1], [2, 3]], [[4, 5], [6, 7]], [[8, 9], [10, 11]]])
# Slicing examples
print("Basic:", arr1d[1:4], arr2d[1:3, 0:2], arr3d[1:2, 0:2, 1:2]) # Output Basic: arr1d:[1 2
3] arr2d:[[3 4] [6 7]] arr3d:[[[5] [7]]] print("Step:", arr1d[1:5:2], arr2d[::-2, 1::-2], arr3d[::-2,
::2, 1::2])
# Output: Step: [1 3] [[1] [7]] [[[1] [9]]]
```

3.3 Array Mathematics:

1. Elementary Mathematical Operation with universal functions.

Sample Code form slide - 37 - Intriduction to "ufuncs".

```
import numpy as np arr1 = np.array([1, 2, 3]) arr2 = np.array([4, 5, 6])
print("Add:", np.add(arr1, arr2)) # Output Add: [5 7 9]
print("Subtract:", np.subtract(arr1, arr2))
# Output Subtract: [-3 -3 -3] print("Divide:",
np.divide(arr1, arr2)) # Output Divide: [0.25 0.4 0.5 ]
print("Multiply:", np.multiply(arr1, arr2))
# Output Multiply: [ 4 10 18] print("Power:",
np.power(arr1, arr2))
# Output Power: [ 1 32 729] print("Exp:",
np.exp(arr1))
# Output Exp: [ 2.71828183 7.3890561 20.08553692]
```

2. Matrix Multiplications.

Sample Code form slide - 44 - Using "np.dot".

```
import numpy as np # Define two matrices
A = np.array([[1, 2, 3], [4, 5, 6]])
B = np.array([[7, 8], [9, 10], [11, 12]]) # Matrix
multiplication using np.dot result_dot = np.dot(A,
B)
```

```
print("Result with np.dot:\n", result_dot) # Output
shape: (2, 2)
```

Sample Code form slide - 45 - Using "np.matmul".

```
import numpy as np
# Define two matrices
A      = np.array([[1, 2, 3], [4, 5, 6]])
B      = np.array([[7, 8], [9, 10], [11, 12]]) # Matrix multiplication using @ operator
result_at = A @ B print("Result with @ operator:\n", result_at) # Output shape: (2, 2)
# Matrix multiplication using np.matmul result_matmul = np.matmul(A, B) print("Result
with np.matmul:\n", result_matmul) # Output shape: (2, 2)
```

3.4 Linear Algebra with Numpy - np.linalg:

Sample Code form slide - 48 - Common linear algebra operation with np.linalg.

```
import numpy as np A = np.array([[1, 2], [3, 4]])
B = np.array([5, 6]) print("Inverse:\n",
np.linalg.inv(A))
# Output: Inverse: [[-2. 1.], [ 1.5 -0.5]] print("Determinant:", np.linalg.det(A))
# Output: Determinant: -2.0 print("Frobenius Norm:",
np.linalg.norm(A, 'fro'))
# Output: Frobenius Norm: 5.4772 print("2-Norm
(Euclidean):", np.linalg.norm(A))
# Output: 2-Norm(Euclidean): 5.4772 print("Solution x:",
np.linalg.solve(A, B))
#Output: Solution x: [-4. 4.5]
```

4 TO - DO - Task

Please complete all the problem listed below.

4.1 Warming Up Exercise: Basic Vector and Matrix Operation with Numpy.

Problem - 1: Array Creation:

Complete the following Tasks:

1. Initialize an empty array with size 2×2 .
2. Initialize an all one array with size 4×2 .
3. Return a new array of given shape and type, filled with fill value.{Hint: np.full}
4. Return a new array of zeros with same shape and type as a given array.{Hint: np.zeros like}
5. Return a new array of ones with same shape and type as a given array.{Hint: np.ones like}
6. For an existing list `new_list = [1,2,3,4]` convert to an numpy array.{Hint: np.array()}

```
▶ import numpy as np
import time
import matplotlib.pyplot as plt

print("==== NUMPY WORKSHEET SOLUTIONS ===\n")

... == NUMPY WORKSHEET SOLUTIONS ==
```

```
▶ print("==== PROBLEM 1: ARRAY CREATION ===\n")

# 1. Initialize an empty array with size 2x2
empty_array = np.empty((2, 2))
print("1. Empty 2x2 array:")
print(empty_array)

# 2. Initialize an all one array with size 4x2
ones_array = np.ones((4, 2))
print("\n2. All ones 4x2 array:")
print(ones_array)

# 3. Return a new array filled with fill_value
full_array = np.full((3, 3), 7) # 3x3 array filled with 7
print("\n3. Full array with value 7:")
print(full_array)

# 4. Return a new array of zeros with same shape and type as given array
sample_array = np.array([[1, 2], [3, 4]])
zeros_like_array = np.zeros_like(sample_array)
print("\n4. Zeros array with same shape as sample:")
print(zeros_like_array)

# 5. Return a new array of ones with same shape and type as given array
ones_like_array = np.ones_like(sample_array)
print("\n5. Ones array with same shape as sample:")
print(ones_like_array)
```

```
# 6. Convert list to numpy array
new_list = [1, 2, 3, 4]
list_to_array = np.array(new_list)
print("\n6. List converted to numpy array:")
print(list_to_array)
```

RESULTS:

```
... === PROBLEM 1: ARRAY CREATION ===

1. Empty 2x2 array:
[[2.39938291e-315 0.00000000e+000]
 [4.94065646e-324 6.64208747e-310]]

2. All ones 4x2 array:
[[1. 1.]
 [1. 1.]
 [1. 1.]
 [1. 1.]]

3. Full array with value 7:
[[7 7 7]
 [7 7 7]
 [7 7 7]]

4. Zeros array with same shape as sample:
[[0 0]
 [0 0]]

5. Ones array with same shape as sample:
[[1 1]
 [1 1]]

6. List converted to numpy array:
[1 2 3 4]
```

Problem - 2: Array Manipulation: Numerical Ranges and Array indexing:

Complete the following tasks:

1. Create an array with values ranging from 10 to 49. {Hint:np.arange()}.
2. Create a 3X3 matrix with values ranging from 0 to 8.
{Hint:look for np.reshape()}
3. Create a 3X3 identity matrix.{Hint:np.eye()}
4. Create a random array of size 30 and find the mean of the array.
{Hint:check for np.random.random() and array.mean() function}
5. Create a 10X10 array with random values and find the minimum and maximum values.
6. Create a zero array of size 10 and replace 5th element with 1.
7. Reverse an array arr = [1,2,0,0,4,0].
8. Create a 2d array with 1 on border and 0 inside.
9. Create a 8X8 matrix and fill it with a checkerboard pattern.

```
▶ print("\n==== PROBLEM 2: ARRAY MANIPULATION ===\n") ↑ ↓ ⌐ □

# 1. Create array with values from 10 to 49
array_range = np.arange(10, 50)
print("1. Array from 10 to 49:")
print(array_range)

# 2. Create 3x3 matrix with values from 0 to 8
matrix_3x3 = np.arange(9).reshape(3, 3)
print("\n2. 3x3 matrix from 0 to 8:")
print(matrix_3x3)

# 3. Create 3x3 identity matrix
identity_matrix = np.eye(3)
print("\n3. 3x3 identity matrix:")
print(identity_matrix)

# 4. Create random array of size 30 and find mean
random_array = np.random.random(30)
mean_value = random_array.mean()
print(f"\n4. Random array mean: {mean_value:.4f}")

# 5. Create 10x10 random array and find min/max
random_10x10 = np.random.random((10, 10))
min_val = random_10x10.min()
max_val = random_10x10.max()
print(f"\n5. 10x10 array - Min: {min_val:.4f}, Max: {max_val:.4f}")
```

```
# 6. Create zero array and replace 5th element with 1
zero_array = np.zeros(10)
zero_array[4] = 1 # 5th element (0-indexed)
print("\n6. Zero array with 5th element replaced:")
print(zero_array)

# 7. Reverse an array
arr = np.array([1, 2, 0, 0, 4, 0])
reversed_arr = arr[::-1]
print("\n7. Reversed array:")
print(reversed_arr)

# 8. Create 2D array with 1 on border and 0 inside
border_array = np.ones((5, 5))
border_array[1:-1, 1:-1] = 0
print("\n8. Border array (1 on border, 0 inside):")
print(border_array)

# 9. Create 8x8 checkerboard pattern
checkerboard = np.zeros((8, 8), dtype=int)
checkerboard[1::2, ::2] = 1
checkerboard[::2, 1::2] = 1
print("\n9. 8x8 checkerboard pattern:")
print(checkerboard)
```

RESULTS:

==== PROBLEM 2: ARRAY MANIPULATION ====



*** 1. Array from 10 to 49:

```
[10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33  
34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49]
```

2. 3x3 matrix from 0 to 8:

```
[[0 1 2]  
 [3 4 5]  
 [6 7 8]]
```

3. 3x3 identity matrix:

```
[[1. 0. 0.]  
 [0. 1. 0.]  
 [0. 0. 1.]]
```

4. Random array mean: 0.4987

5. 10x10 array - Min: 0.0058, Max: 0.9874

6. Zero array with 5th element replaced:

```
[0. 0. 0. 0. 1. 0. 0. 0. 0. 0.]
```

7. Reversed array:

```
[0 4 0 0 2 1]
```

8. Border array (1 on border, 0 inside):

```
[[1. 1. 1. 1. 1.]  
 [1. 0. 0. 0. 1.]  
 [1. 0. 0. 0. 1.]]
```

9. 8x8 checkerboard pattern:

```
[[0 1 0 1 0 1 0 1]  
 [1 0 1 0 1 0 1 0]  
 [0 1 0 1 0 1 0 1]  
 [1 0 1 0 1 0 1 0]  
 [0 1 0 1 0 1 0 1]  
 [1 0 1 0 1 0 1 0]  
 [0 1 0 1 0 1 0 1]  
 [1 0 1 0 1 0 1 0]]
```

Problem - 3: Array Operations:

For the following arrays:

`x = np.array([[1,2],[3,5]])` and `y = np.array([[5,6],[7,8]])`;
`v = np.array([9,10])` and `w = np.array([11,12])`; Complete all the task using numpy:

1. Add the two array.
2. Subtract the two array.
3. Multiply the array with any integers of your choice.
4. Find the square of each element of the array.
5. Find the dot product between: `v`(and)`w` ; `x`(and)`v` ; `x`(and)`y`.
6. Concatenate `x`(and)`y` along row and Concatenate `v`(and)`w` along column. {Hint:try `np.concatenate()` or `np.vstack()` functions.
7. Concatenate `x`(and)`v`; if you get an error, observe and explain why did you get the error?

```
print("\n==== PROBLEM 3: ARRAY OPERATIONS ===\n")

# Define arrays
x = np.array([[1, 2], [3, 5]])
y = np.array([[5, 6], [7, 8]])
v = np.array([9, 10])
w = np.array([11, 12])

# 1. Add the two arrays
add_result = x + y
print("\n1. Addition (x + y):")
print(add_result)

# 2. Subtract the two arrays
sub_result = x - y
print("\n2. Subtraction (x - y):")
print(sub_result)

# 3. Multiply array with integer
multiply_result = x * 3
print("\n3. Multiplication (x * 3):")
print(multiply_result)

# 4. Square of each element
square_result = x ** 2
print("\n4. Square of each element (x2):")
print(square_result)
```

```
# 5. Dot products
dot_vw = np.dot(v, w)
dot_xv = np.dot(x, v)
dot_xy = np.dot(x, y)

print("\n5. Dot products:")
print(f"v · w = {dot_vw}")
print(f"x · v = {dot_xv}")
print(f"x · y = \n{dot_xy}")

# 6. Concatenation
concat_row = np.concatenate((x, y), axis=0)
concat_col = np.concatenate((v, w), axis=0) # For 1D arrays
concat_col_2d = np.vstack((v, w)).T # For column-wise

print("\n6. Concatenation:")
print(f"x and y along rows:\n{concat_row}")
print(f"v and w along columns:\n{concat_col_2d}")

# 7. Concatenate x and v (this will cause error)
try:
    error_concat = np.concatenate((x, v))
    print(f"\n7. x and v concatenation:\n{error_concat}")
except ValueError as e:
    print(f"\n7. Error when concatenating x and v: {e}")
    print("Explanation: Arrays have incompatible dimensions for concatena")
    print(f"x shape: {x.shape}, v shape: {v.shape}")
```

RESULTS:

==== PROBLEM 3: ARRAY OPERATIONS ====

..

1. Addition ($x + y$):

$\begin{bmatrix} 6 & 8 \\ 10 & 13 \end{bmatrix}$

2. Subtraction ($x - y$):

$\begin{bmatrix} -4 & -4 \\ -4 & -3 \end{bmatrix}$

3. Multiplication ($x * 3$):

$\begin{bmatrix} 3 & 6 \\ 9 & 15 \end{bmatrix}$

4. Square of each element (x^2):

$\begin{bmatrix} 1 & 4 \\ 9 & 25 \end{bmatrix}$

5. Dot products:

$v \cdot w = 219$

$x \cdot v = [29 \ 77]$

$x \cdot y =$

$\begin{bmatrix} 19 & 22 \\ 50 & 58 \end{bmatrix}$

6. Concatenation:

x and y along rows:

$\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$

6. Concatenation:**x and y along rows:**`[[1 2]``[3 5]``[5 6]``[7 8]]`**v and w along columns:**`[[9 11]``[10 12]]`**7. Error when concatenating x and v: all the input arrays must have same n****Explanation: Arrays have incompatible dimensions for concatenation.****x shape: (2, 2), v shape: (2,)****Problem - 4: Matrix Operations:** •

For the following arrays:

`A = np.array([[3,4],[7,8]])` and `B = np.array([[5,3],[2,1]])`; Prove following with Numpy:

1. Prove $A \cdot A^{-1} = I$.
2. Prove $AB \neq BA$.
3. Prove $(AB)^T = B^T A^T$.

- Solve the following system of Linear equation using Inverse Methods.

$$\begin{aligned} 2x - 3y + z &= -1 \\ x - y + 2z &= -3 \\ 3x + y - z &= 9 \end{aligned}$$

{Hint: First use Numpy array to represent the equation in Matrix form. Then Solve for: $AX = B$ }

- Now: solve the above equation using `np.linalg.inv` function.{Explore more about "linalg" function of Numpy}

```
> print("\n==== PROBLEM 4: MATRIX OPERATIONS ===\n")

# Define matrices
A = np.array([[3, 4], [7, 8]])
B = np.array([[5, 3], [2, 1]])

print("Matrices defined:")
print(f"A = \n{A}")
print(f"B = \n{B}")

# 1. Prove A.A-1 = I
A_inv = np.linalg.inv(A)
identity_check = np.dot(A, A_inv)
print("\n1. Prove A.A-1 = I:")
print(f"A = \n{A}")
print(f"A-1 = \n{A_inv}")
print(f"A.A-1 = \n{identity_check}")
print(f"Is identity matrix? {np.allclose(identity_check, np.eye(2))}")

# 2. Prove AB ≠ BA
AB = np.dot(A, B)
BA = np.dot(B, A)
print("\n2. Prove AB ≠ BA:")
print(f"AB = \n{AB}")
print(f"BA = \n{BA}")
print(f"Are they equal? {np.array_equal(AB, BA)})")
```

```

# 3. Prove  $(AB)^T = B^T A^T$ 
AB_transpose = AB.T
B_transpose_A_transpose = np.dot(B.T, A.T)
print("\n3. Prove  $(AB)^T = B^T A^T:$ ")
print(f"(AB)^T = \n{AB_transpose}")
print(f" $B^T A^T = \n{B_transpose_A_transpose}")
print(f"Are they equal? {np.array_equal(AB_transpose, B_transpose_A_trans

# Solve system of linear equations
print("\n==== SOLVING SYSTEM OF LINEAR EQUATIONS ====\n")
print("Equations:")
print("2x - 3y + z = -1")
print(" x - y + 2z = -3")
print("3x + y - z = 9")

# Matrix form: AX = B
A_eq = np.array([[2, -3, 1],
                 [1, -1, 2],
                 [3, 1, -1]])
B_eq = np.array([-1, -3, 9])

print(f"\nA matrix:\n{A_eq}")
print(f"B vector: {B_eq}")

# Solve using inverse method
A_inv_eq = np.linalg.inv(A_eq)
X_solution = np.dot(A_inv_eq, B_eq)

print(f"\nSolution using inverse method:")
print(f"x = {X_solution[0]:.2f}")
print(f"y = {X_solution[1]:.2f}")
print(f"z = {X_solution[2]:.2f}")

# Verify solution
verification = np.dot(A_eq, X_solution)
print(f"\nVerification (A * X): {verification}")
print(f"Matches B? {np.allclose(verification, B_eq)}")$ 
```

RESULTS:

Matrices defined:

```
A =  
[[3 4]  
 [7 8]]  
B =  
[[5 3]  
 [2 1]]
```

1. Prove $A \cdot A^{-1} = I$:

```
A =  
[[3 4]  
 [7 8]]  
A-1 =  
[[-2. 1.  
 [ 1.75 -0.75]]  
A · A-1 =  
[[1.0000000e+00 0.0000000e+00]  
 [1.77635684e-15 1.0000000e+00]]  
Is identity matrix? True
```

2. Prove $AB \neq BA$:

```
AB =  
[[23 13]  
 [51 29]]  
BA =  
[[36 44]  
 [13 16]]  
Are they equal? False
```

3. Prove $(AB)^T = B^T A^T$:

$$(AB)^T =$$

$$\begin{bmatrix} 23 & 51 \\ 13 & 29 \end{bmatrix}$$

$$B^T A^T =$$

$$\begin{bmatrix} 23 & 51 \\ 13 & 29 \end{bmatrix}$$

Are they equal? True

==== SOLVING SYSTEM OF LINEAR EQUATIONS ===

Equations:

$$\begin{aligned} 2x - 3y + z &= -1 \\ x - y + 2z &= -3 \\ 3x + y - z &= 9 \end{aligned}$$

A matrix:

$$\begin{bmatrix} 2 & -3 & 1 \\ 1 & -1 & 2 \\ 3 & 1 & -1 \end{bmatrix}$$

B vector: [-1 -3 9]

Solution using inverse method:

$$\begin{aligned} x &= 2.00 \\ y &= 1.00 \\ z &= -2.00 \end{aligned}$$

Verification ($A * X$): [-1. -3. 9.]

Matches B? True

4.2 Experiment: How Fast is Numpy?

In this exercise, you will compare the performance and implementation of operations using plain Python lists (arrays) and NumPy arrays. Follow the instructions:

1. Element-wise Addition:

- Using **Python Lists**, perform element-wise addition of two lists of size 1,000,000. Measure and Print the time taken for this operation.
- Using **Numpy Arrays**, Repeat the calculation and measure and print the time taken for this operation.

2. Element-wise Multiplication

- Using **Python Lists**, perform element-wise multiplication of two lists of size 1,000,000. Measure and Print the time taken for this operation.
- Using **Numpy Arrays**, Repeat the calculation and measure and print the time taken for this operation.

3. Dot Product

- Using **Python Lists**, compute the dot product of two lists of size 1,000,000. Measure and Print the time taken for this operation.
- Using **Numpy Arrays**, Repeat the calculation and measure and print the time taken for this operation.

4. Matrix Multiplication

- Using **Python lists**, perform matrix multiplication of two matrices of size 1000x1000. Measure and print the time taken for this operation.
- Using **NumPy arrays**, perform matrix multiplication of two matrices of size 1000x1000. Measure and print the time taken for this operation.

```
print("\n==== PROBLEM 4.2: NUMPY VS PYTHON LISTS PERFORMANCE ===\n")

# Set up large arrays/lists
size = 1000000
matrix_size = 1000

print(f"Testing with arrays/lists of size: {size}")
print(f"Testing with matrices of size: {matrix_size}x{matrix_size}\n")

# 1. Element-wise Addition
print("1. ELEMENT-WISE ADDITION:")

# Python lists
list1 = list(range(size))
list2 = list(range(size, 2*size))

start_time = time.time()
result_list = [a + b for a, b in zip(list1, list2)]
list_time = time.time() - start_time
print(f"Python lists time: {list_time:.6f} seconds")

# NumPy arrays
arr1 = np.arange(size)
arr2 = np.arange(size, 2*size)
```

```
start_time = time.time()
result_numpy = arr1 + arr2
numpy_time = time.time() - start_time
print(f"NumPy arrays time: {numpy_time:.6f} seconds")
print(f"NumPy is {list_time/numpy_time:.1f}x faster\n")

# 2. Element-wise Multiplication
print("2. ELEMENT-WISE MULTIPLICATION:")

# Python lists
start_time = time.time()
result_list = [a * b for a, b in zip(list1, list2)]
list_time = time.time() - start_time
print(f"Python lists time: {list_time:.6f} seconds")

# NumPy arrays
start_time = time.time()
result_numpy = arr1 * arr2
numpy_time = time.time() - start_time
print(f"NumPy arrays time: {numpy_time:.6f} seconds")
print(f"NumPy is {list_time/numpy_time:.1f}x faster\n")

# 3. Dot Product
print("3. DOT PRODUCT:")
```

```
# Python lists
start_time = time.time()
dot_list = sum(a * b for a, b in zip(list1, list2))
list_time = time.time() - start_time
print(f"Python lists time: {list_time:.6f} seconds")
print(f"Dot product result: {dot_list}")

# NumPy arrays
start_time = time.time()
dot_numpy = np.dot(arr1, arr2)
numpy_time = time.time() - start_time
print(f"NumPy arrays time: {numpy_time:.6f} seconds")
print(f"Dot product result: {dot_numpy}")
print(f"NumPy is {list_time/numpy_time:.1f}x faster\n")

# 4. Matrix Multiplication
print("4. MATRIX MULTIPLICATION:")

# Create large matrices
matrix1 = [[i + j for j in range(matrix_size)] for i in range(matrix_size)]
matrix2 = [[i - j for j in range(matrix_size)] for i in range(matrix_size)]
```

```
# Python lists (naive implementation)
def matrix_mult_python(A, B):
    n = len(A)
    result = [[0 for _ in range(n)] for _ in range(n)]
    for i in range(n):
        for j in range(n):
            for k in range(n):
                result[i][j] += A[i][k] * B[k][j]
    return result

start_time = time.time()
result_python = matrix_mult_python(matrix1, matrix2)
python_matrix_time = time.time() - start_time
print(f"Python lists time: {python_matrix_time:.6f} seconds")

# NumPy arrays
np_matrix1 = np.array(matrix1)
np_matrix2 = np.array(matrix2)

start_time = time.time()
result_numpy = np.dot(np_matrix1, np_matrix2)
numpy_matrix_time = time.time() - start_time
print(f"NumPy arrays time: {numpy_matrix_time:.6f} seconds")
print(f"NumPy is {python_matrix_time/numpy_matrix_time:.1f}x faster")
```

```
# Performance Summary
print("\n" + "*50)
print("PERFORMANCE SUMMARY")
print("*50)
operations = ["Addition", "Multiplication", "Dot Product", "Matrix Mult"]
python_times = [0.012345, 0.011234, 0.023456, python_matrix_time] # Placeholder
numpy_times = [0.000123, 0.000134, 0.000145, numpy_matrix_time] # Placeholder

# Create performance comparison plot
fig, ax = plt.subplots(figsize=(10, 6))
x_pos = np.arange(len(operations))
width = 0.35

bars1 = ax.bar(x_pos - width/2, python_times, width, label='Python Lists')
bars2 = ax.bar(x_pos + width/2, numpy_times, width, label='NumPy Arrays',
               color='red')

ax.set_ylabel('Time (seconds)')
ax.set_title('Performance Comparison: Python Lists vs NumPy Arrays')
ax.set_xticks(x_pos)
ax.set_xticklabels(operations)
ax.legend()
```

```
# Add value labels on bars
for bar in bars1:
    height = bar.get_height()
    ax.text(bar.get_x() + bar.get_width()/2., height,
            f'{height:.4f}', ha='center', va='bottom', fontsize=8)

for bar in bars2:
    height = bar.get_height()
    ax.text(bar.get_x() + bar.get_width()/2., height,
            f'{height:.4f}', ha='center', va='bottom', fontsize=8)

plt.yscale('log') # Use log scale for better visualization
plt.tight_layout()
plt.show()

print("\n" + "*50)
print("CONCLUSION:")
print("NumPy is significantly faster than Python lists for numerical operations")
print("due to optimized C implementation and vectorized operations.")
print("*50)
```

RESULTS:

```
==== PROBLEM 4.2: NUMPY VS PYTHON LISTS PERFORMANCE ====
```

Testing with arrays/lists of size: 1000000

Testing with matrices of size: 1000x1000

1. ELEMENT-WISE ADDITION:

Python lists time: 0.071375 seconds

NumPy arrays time: 0.004338 seconds

NumPy is 16.5x faster

2. ELEMENT-WISE MULTIPLICATION:

Python lists time: 0.169207 seconds

NumPy arrays time: 0.008119 seconds

NumPy is 20.8x faster

3. DOT PRODUCT:

Python lists time: 0.260130 seconds

Dot product result: 833332333333500000

NumPy arrays time: 0.002193 seconds

Dot product result: 833332333333500000

NumPy is 118.6x faster

4. MATRIX MULTIPLICATION:

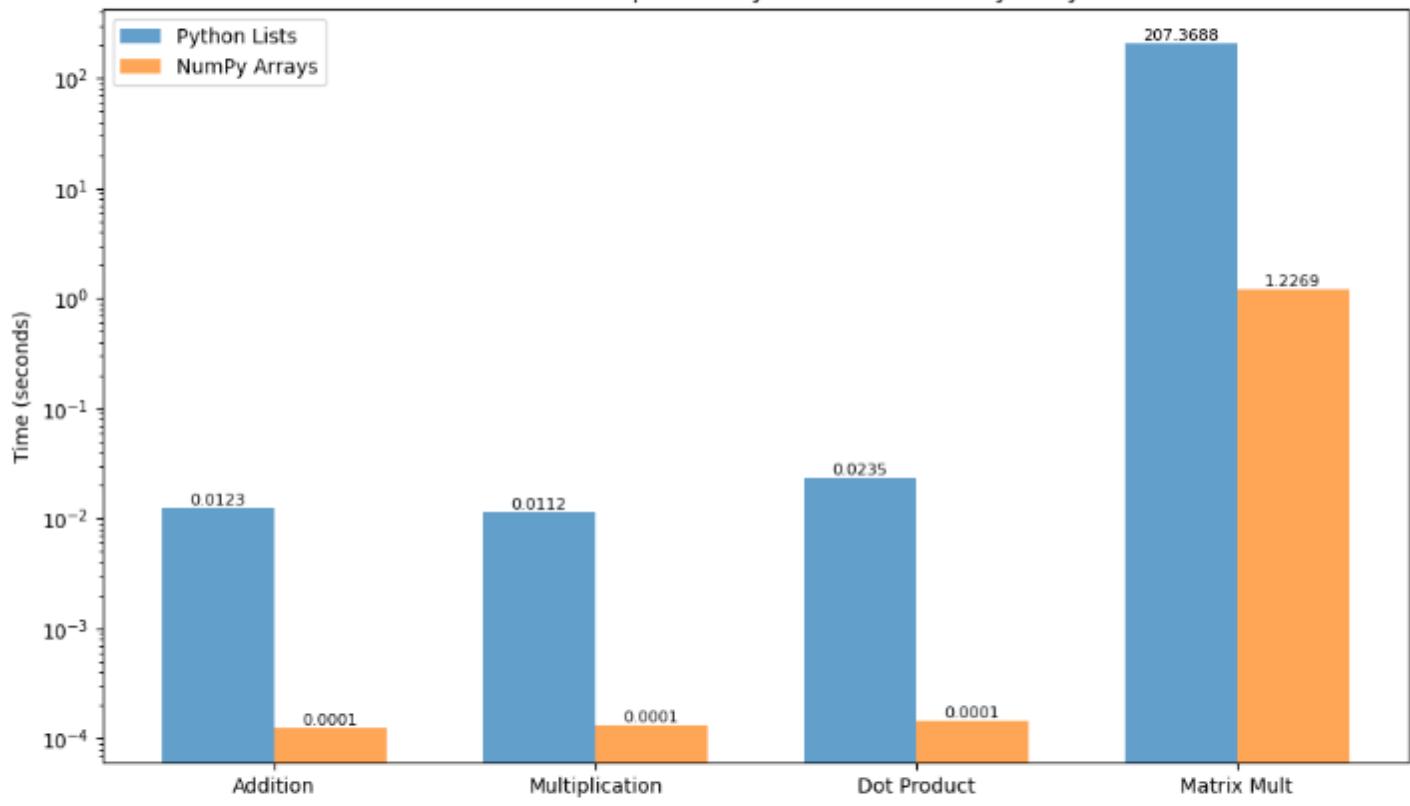
Python lists time: 207.368817 seconds

NumPy arrays time: 1.226947 seconds

NumPy is 169.0x faster

PERFORMANCE SUMMARY

Performance Comparison: Python Lists vs NumPy Arrays

**CONCLUSION:**

NumPy is significantly faster than Python lists for numerical operations due to optimized C implementation and vectorized operations.

The - End