Chapter 1

Number Sets

1.1 Set Identities

Sets: A, B, C
Universal set: I
Complement : A'
Proper subset: $A \subset B$
Empty set: $\varnothing$
Union of sets: $A \cup B$
Intersection of sets: $A \cap B$
Difference of sets: $A \setminus B$

1. $A \subset I$

2. $A \subset A$

3. $A = B$ if $A \subset B$ and $B \subset A$.

4. Empty Set
$\varnothing \subset A$

5. Union of Sets
$C = A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

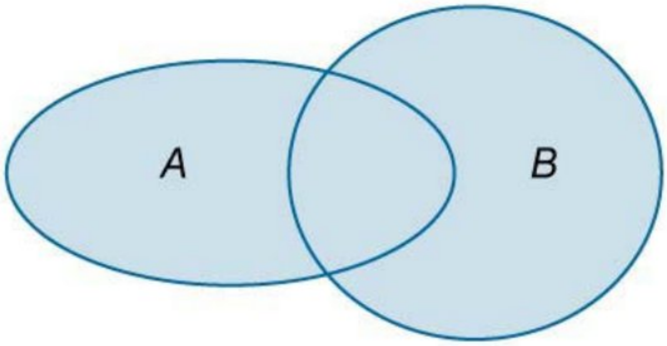

Figure 1.

6. Commutativity
$$A \cup B = B \cup A$$

7. Associativity
$$A \cup (B \cup C) = (A \cup B) \cup C$$

8. Intersection of Sets
$$C = A \cup B = \{x \mid x \in A \text{ and } x \in B\}$$

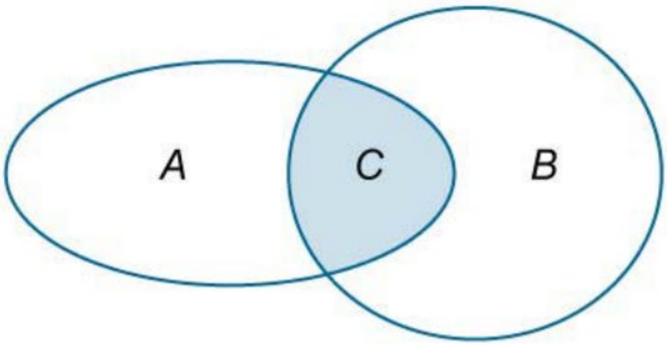

Figure 2.

9. Commutativity
$$A \cap B = B \cap A$$

10. Associativity
$$A \cap (B \cap C) = (A \cap B) \cap C$$

11. Distributivity
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C),$$
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C).$$

12. Idempotency
$$A \cap A = A,$$
$$A \cup A = A$$

13. Domination
$$A \cap \varnothing = \varnothing,$$
$$A \cup I = I$$

14. Identity
$$A \cup \varnothing = A,$$
$$A \cap I = A$$

15. Complement
$$A' = \{x \in I \mid x \notin A\}$$

16. Complement of Intersection and Union
$$A \cup A' = I,$$
$$A \cap A' = \varnothing$$

17. De Morgan's Laws
$$(A \cup B)' = A' \cap B',$$
$$(A \cap B)' = A' \cup B'$$

18. Difference of Sets
$$C = B \setminus A = \{x \mid x \in B \text{ and } x \notin A\}$$

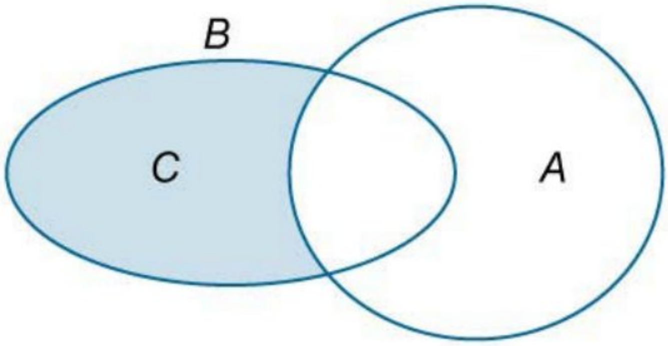

Figure 3.

19. $B \setminus A = B \setminus (A \cap B)$

20. $B \setminus A = B \cap A'$

21. $A \setminus A = \varnothing$

22. $A \setminus B = A$ if $A \cap B = \varnothing$.

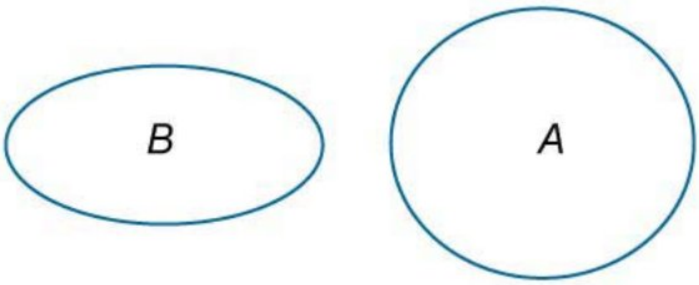

Figure 4.

23. $(A \setminus B) \cap C = (A \cap C) \setminus (B \cap C)$

24. $A' = I \setminus A$

25. Cartesian Product
$C = A \times B = \{(x, y) \mid x \in A \text{ and } y \in B\}$

1.2 Sets of Numbers

Natural numbers: N
Whole numbers: N_0
Integers: Z
Positive integers: Z^+
Negative integers: Z^-
Rational numbers: Q
Real numbers: R
Complex numbers: C

26. Natural Numbers
Counting numbers: $N = \{1, 2, 3, \ldots\}$.

27. Whole Numbers
Counting numbers and zero: $N_0 = \{0, 1, 2, 3, \ldots\}$.

28. Integers
Whole numbers and their opposites and zero:
$Z^+ = N = \{1, 2, 3, \ldots\}$,
$Z^- = \{\ldots, -3, -2, -1\}$,
$Z = Z^- \cup \{0\} \cup Z^+ = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$.

29. Rational Numbers
Repeating or terminating decimals:
$$Q = \left\{ x \mid x = \frac{a}{b} \text{ and } a \in Z \text{ and } b \in Z \text{ and } b \neq 0 \right\}.$$

30. Irrational Numbers
Nonrepeating and nonterminating decimals.

31. Real Numbers
Union of rational and irrational numbers: R.

32. Complex Numbers
$C = \{x + iy \mid x \in R \text{ and } y \in R\}$,
where i is the imaginary unit.

33. $N \subset Z \subset Q \subset R \subset C$

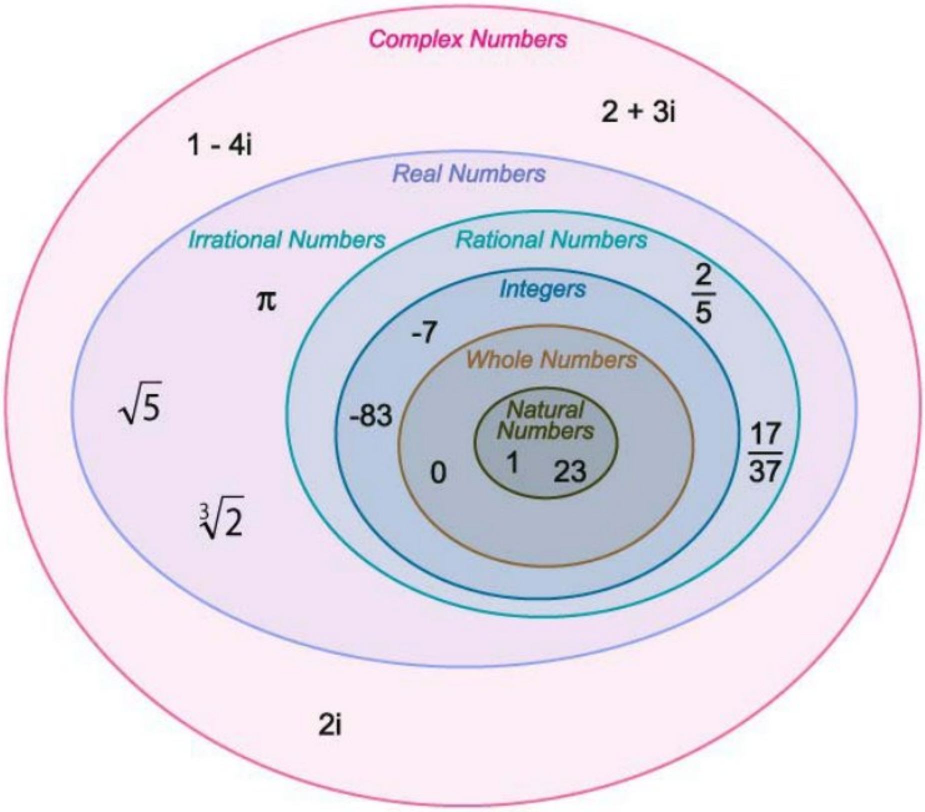

Figure 5.

1.3 Basic Identities

Real numbers: a, b, c

34. Additive Identity
$a + 0 = a$

35. Additive Inverse
$a + (-a) = 0$

36. Commutative of Addition
$a + b = b + a$

37. Associative of Addition
$(a + b) + c = a + (b + c)$

38. Definition of Subtraction
$a - b = a + (-b)$

39. Multiplicative Identity
$a \cdot 1 = a$

40. Multiplicative Inverse
$a \cdot \frac{1}{a} = 1, \ a \neq 0$

41. Multiplication Times 0
$a \cdot 0 = 0$

42. Commutative of Multiplication
$a \cdot b = b \cdot a$

43. Associative of Multiplication
$(a \cdot b) \cdot c = a \cdot (b \cdot c)$

44. Distributive Law
$a(b + c) = ab + ac$

45. Definition of Division
$\frac{a}{b} = a \cdot \frac{1}{b}$

1.4 Complex Numbers

Natural number: n
Imaginary unit: i
Complex number: z
Real part: a, c
Imaginary part: bi, di
Modulus of a complex number: r, r_1, r_2
Argument of a complex number: φ, φ_1, φ_2

46.

$i^1 = i$	$i^5 = i$	$i^{4n+1} = i$
$i^2 = -1$	$i^6 = -1$	$i^{4n+2} = -1$
$i^3 = -i$	$i^7 = -i$	$i^{4n+3} = -i$
$i^4 = 1$	$i^8 = 1$	$i^{4n} = 1$

47. $z = a + bi$

48. Complex Plane

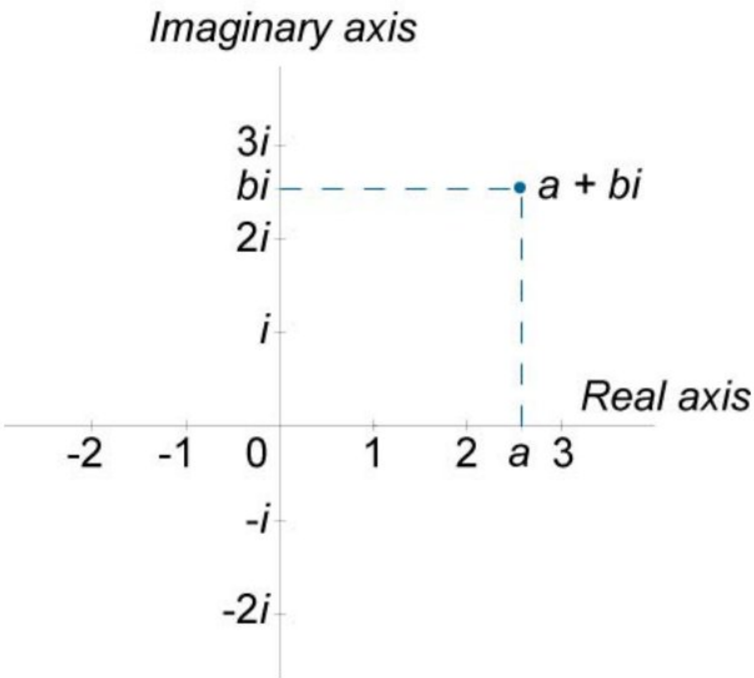

Figure 6.

49. $(a + bi) + (c + di) = (a + c) + (b + d)i$

50. $(a + bi) - (c + di) = (a - c) + (b - d)i$

51. $(a + bi)(c + di) = (ac - bd) + (ad + bc)i$

52. $\dfrac{a + bi}{c + di} = \dfrac{ac + bd}{c^2 + d^2} + \dfrac{bc - ad}{c^2 + d^2} \cdot i$

53. Conjugate Complex Numbers

$\overline{a + bi} = a - bi$

54. $a = r\cos\varphi, \; b = r\sin\varphi$

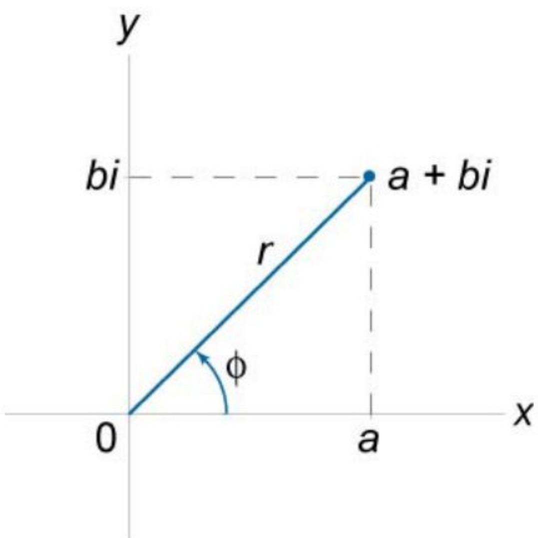

Figure 7.

55. Polar Presentation of Complex Numbers

$$a + bi = r(\cos\varphi + i\sin\varphi)$$

56. Modulus and Argument of a Complex Number

If $a + bi$ is a complex number, then

$r = \sqrt{a^2 + b^2}$ (modulus),

$\varphi = \arctan\frac{b}{a}$ (argument).

57. Product in Polar Representation

$$\begin{aligned} z_1 \cdot z_2 &= r_1(\cos\varphi_1 + i\sin\varphi_1)\cdot r_2(\cos\varphi_2 + i\sin\varphi_2) \\ &= r_1 r_2[\cos(\varphi_1 + \varphi_2) + i\sin(\varphi_1 + \varphi_2)] \end{aligned}$$

58. Conjugate Numbers in Polar Representation

$$\overline{r(\cos\varphi + i\sin\varphi)} = r[\cos(-\varphi) + i\sin(-\varphi)]$$

59. Inverse of a Complex Number in Polar Representation

$$\frac{1}{r(\cos\varphi + i\sin\varphi)} = \frac{1}{r}[\cos(-\varphi) + i\sin(-\varphi)]$$

60. Quotient in Polar Representation

$$\frac{z_1}{z_2}=\frac{r_1(\cos\varphi_1+i\sin\varphi_1)}{r_2(\cos\varphi_2+i\sin\varphi_2)}=\frac{r_1}{r_2}[\cos(\varphi_1-\varphi_2)+i\sin(\varphi_1-\varphi_2)]$$

61. Power of a Complex Number

$$z^n=[r(\cos\varphi+i\sin\varphi)]^n=r^n[\cos(n\varphi)+i\sin(n\varphi)]$$

62. Formula “De Moivre”

$$(\cos\varphi+i\sin\varphi)^n=\cos(n\varphi)+i\sin(n\varphi)$$

63. Nth Root of a Complex Number

$$\sqrt[n]{z}=\sqrt[n]{r(\cos\varphi+i\sin\varphi)}=\sqrt[n]{r}\left(\cos\frac{\varphi+2\pi k}{n}+i\sin\frac{\varphi+2\pi k}{n}\right),$$

where

$k=0,1,2,\ldots,n-1$.

64. Euler’s Formula

$$e^{ix}=\cos x+i\sin x$$