Chapter 3

Geometry

3.1 Right Triangle

Legs of a right triangle: a, b
Hypotenuse: c
Altitude: h
Medians: m_a, m_b, m_c
Angles: α, β
Radius of circumscribed circle: R
Radius of inscribed circle: r
Area: S

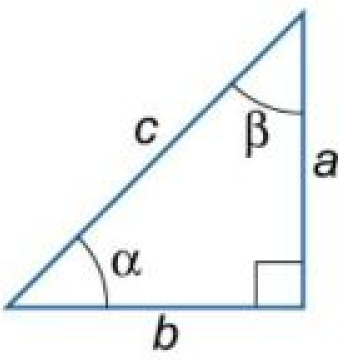


Figure 8.

156. $\alpha + \beta = 90^\circ$

157. $\sin\alpha = \frac{a}{c} = \cos\beta$

158. $\cos\alpha = \frac{b}{c} = \sin\beta$

159. $\tan\alpha = \frac{a}{b} = \cot\beta$

160. $\cot\alpha = \frac{b}{a} = \tan\beta$

161. $\sec\alpha = \frac{c}{b} = \operatorname{cosec}\beta$

162. $\operatorname{cosec}\alpha = \frac{c}{a} = \sec\beta$

163. Pythagorean Theorem
$a^2 + b^2 = c^2$

164. $a^2 = fc$, $b^2 = gc$,
where f and c are projections of the legs a and b, respectively, onto the hypotenuse c.

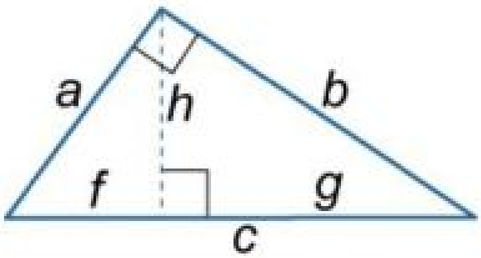


Figure 9.

165. $h^2 = fg$,
where h is the altitude from the right angle.

166. $m_a^2 = b^2 - \frac{a^2}{4}$, $m_b^2 = a^2 - \frac{b^2}{4}$,
where m_a and m_b are the medians to the legs a and b.

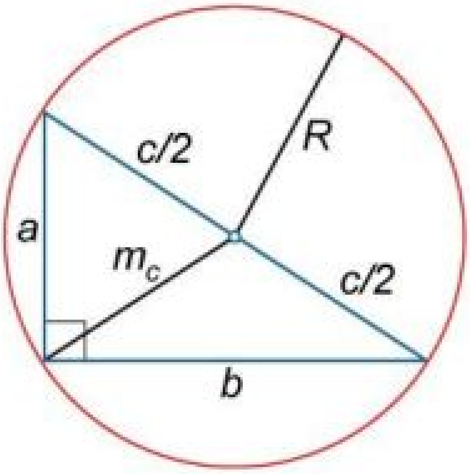


Figure 10.

167. $m_c = \frac{c}{2}$,
where m_c is the median to the hypotenuse c.

168. $R = \frac{c}{2} = m_c$

169. $r = \frac{a+b-c}{2} = \frac{ab}{a+b+c}$

170. $ab = ch$

171. $S = \frac{ab}{2} = \frac{ch}{2}$

3.2 Isosceles Triangle

Base: a
Legs: b
Base angle: β
Vertex angle: α
Altitude to the base: h
Perimeter: L
Area: S

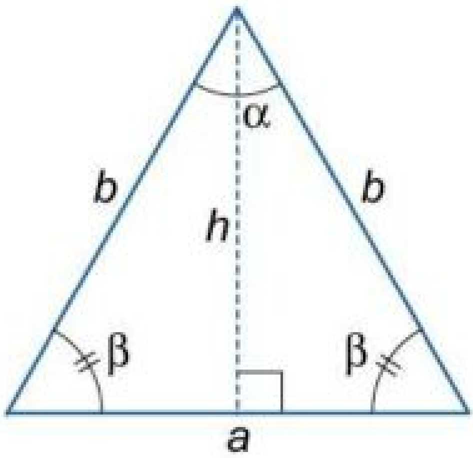


Figure 11.

172. $\beta = 90^\circ - \frac{\alpha}{2}$

173. $h^2 = b^2 - \frac{a^2}{4}$

174. $L = a + 2b$

175. $S = \frac{ah}{2} = \frac{b^2}{2} \sin\alpha$

3.3 Equilateral Triangle

Side of a equilateral triangle: a
Altitude: h
Radius of circumscribed circle: R
Radius of inscribed circle: r
Perimeter: L
Area: S

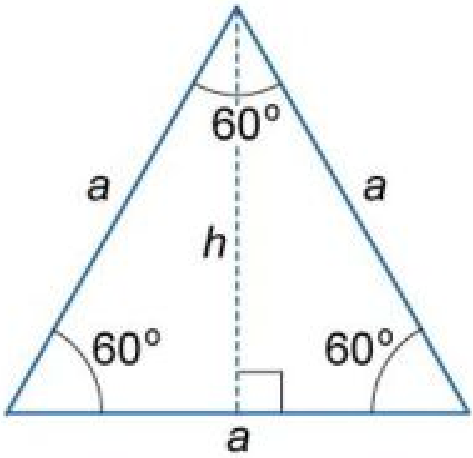


Figure 12.

176. $h = \frac{a\sqrt{3}}{2}$

177. $R = \frac{2}{3}h = \frac{a\sqrt{3}}{3}$

178. $r = \frac{1}{3}h = \frac{a\sqrt{3}}{6} = \frac{R}{2}$

179. $L = 3a$

180. $S = \frac{ah}{2} = \frac{a^2\sqrt{3}}{4}$

3.4 Scalene Triangle

(A triangle with no two sides equal)

Sides of a triangle: a, b, c
Semiperimeter: $p = \frac{a+b+c}{2}$
Angles of a triangle: α, β, γ
Altitudes to the sides a, b, c: h_a, h_b, h_c
Medians to the sides a, b, c: m_a, m_b, m_c
Bisectors of the angles α, β, γ: t_a, t_b, t_c
Radius of circumscribed circle: R
Radius of inscribed circle: r
Area: S

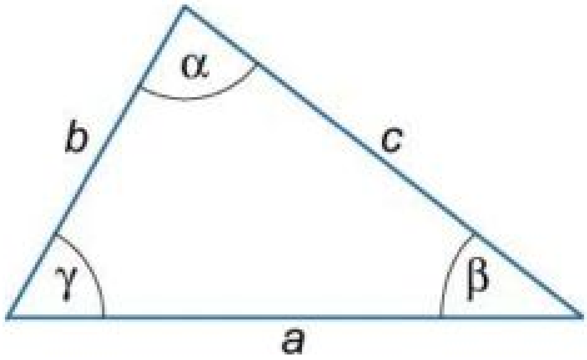


Figure 13.

181. $\alpha+\beta+\gamma=180^\circ$

182. $a+b>c$,
$b+c>a$,
$a+c>b$.

183. $|a-b|<c$,
$|b-c|<a$,
$|a-c|<b$.

184. Midline

$q=\dfrac{a}{2},\ q\,||\,a$.

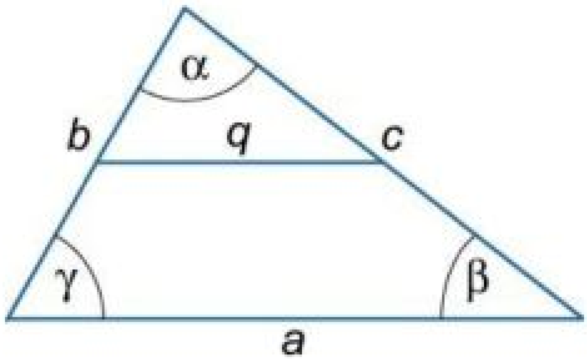


Figure 14.

185. Law of Cosines

$a^2 = b^2 + c^2 - 2bc\cos\alpha$,

$b^2 = a^2 + c^2 - 2ac\cos\beta$,

$c^2 = a^2 + b^2 - 2ab\cos\gamma$.

186. Law of Sines

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma} = 2R,$$

where R is the radius of the circumscribed circle.

187. $$R = \frac{a}{2\sin\alpha} = \frac{b}{2\sin\beta} = \frac{c}{2\sin\gamma} = \frac{bc}{2h_a} = \frac{ac}{2h_b} = \frac{ab}{2h_c} = \frac{abc}{4S}$$

188. $$r^2 = \frac{(p-a)(p-b)(p-c)}{p},$$

$$\frac{1}{r} = \frac{1}{h_a} + \frac{1}{h_b} + \frac{1}{h_c}.$$

189. $$\sin\frac{\alpha}{2} = \sqrt{\frac{(p-b)(p-c)}{bc}},$$

$$\cos\frac{\alpha}{2} = \sqrt{\frac{p(p-a)}{bc}},$$

$$\tan\frac{\alpha}{2} = \sqrt{\frac{(p-b)(p-c)}{p(p-a)}}.$$

190. $$h_a = \frac{2}{a}\sqrt{p(p-a)(p-b)(p-c)},$$

$$h_b = \frac{2}{b}\sqrt{p(p-a)(p-b)(p-c)},$$

$$h_c = \frac{2}{c}\sqrt{p(p-a)(p-b)(p-c)}.$$

191. $h_a = b\sin\gamma = c\sin\beta$,

$h_b = a\sin\gamma = c\sin\alpha$,

$h_c = a\sin\beta = b\sin\alpha$.

192. $m_a^2 = \dfrac{b^2+c^2}{2} - \dfrac{a^2}{4}$,

$m_b^2 = \dfrac{a^2+c^2}{2} - \dfrac{b^2}{4}$,

$m_c^2 = \dfrac{a^2+b^2}{2} - \dfrac{c^2}{4}$.

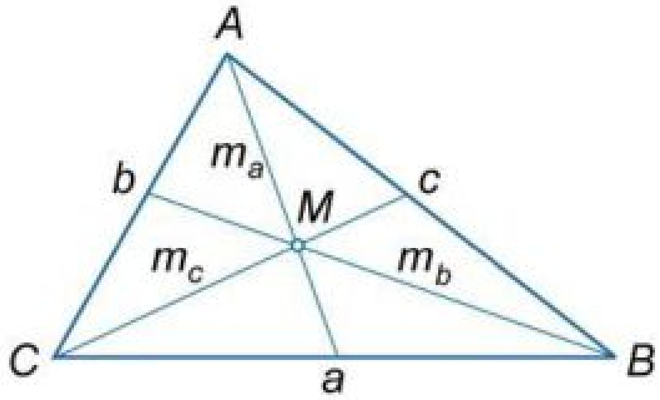


Figure 15.

193. $AM = \dfrac{2}{3}m_a$, $BM = \dfrac{2}{3}m_b$, $CM = \dfrac{2}{3}m_c$ (Fig.15).

194. $t_a^2 = \dfrac{4bcp(p-a)}{(b+c)^2}$,

$t_b^2 = \dfrac{4acp(p-b)}{(a+c)^2}$,

$t_c^2 = \dfrac{4abp(p-c)}{(a+b)^2}$.

195. $S = \frac{ah_a}{2} = \frac{bh_b}{2} = \frac{ch_c}{2}$,

$$S = \frac{ab\sin\gamma}{2} = \frac{ac\sin\beta}{2} = \frac{bc\sin\alpha}{2},$$

$$S = \sqrt{p(p-a)(p-b)(p-c)} \text{ (Heron's Formula)},$$

$$S = pr,$$

$$S = \frac{abc}{4R},$$

$$S = 2R^2 \sin\alpha \sin\beta \sin\gamma,$$

$$S = p^2 \tan\frac{\alpha}{2} \tan\frac{\beta}{2} \tan\frac{\gamma}{2}.$$

3.5 Square

Side of a square: a
Diagonal: d
Radius of circumscribed circle: R
Radius of inscribed circle: r
Perimeter: L
Area: S

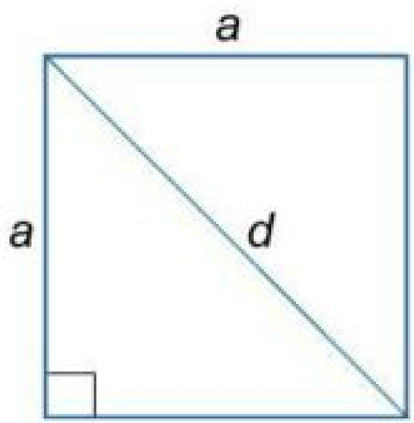


Figure 16.

196. $d = a\sqrt{2}$

197. $R = \frac{d}{2} = \frac{a\sqrt{2}}{2}$

198. $r = \frac{a}{2}$

199. $L = 4a$

200. $S = a^2$

3.6 Rectangle

Sides of a rectangle: a, b
Diagonal: d
Radius of circumscribed circle: R
Perimeter: L
Area: S

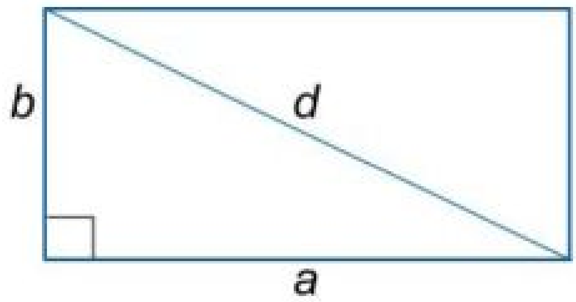


Figure 17.

201. $d = \sqrt{a^2 + b^2}$

202. $R = \dfrac{d}{2}$

203. $L = 2(a + b)$

204. $S = ab$

3.7 Parallelogram

Sides of a parallelogram: a, b
Diagonals: d_1, d_2
Consecutive angles: α, β
Angle between the diagonals: φ
Altitude: h
Perimeter: L
Area: S

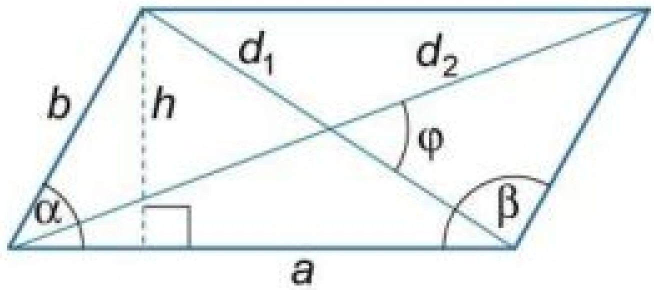


Figure 18.

205. $\alpha + \beta = 180°$

206. $d_1^2 + d_2^2 = 2(a^2 + b^2)$

207. $h = b\sin\alpha = b\sin\beta$

208. $L = 2(a+b)$

209. $S = ah = ab\sin\alpha$,

$$S = \frac{1}{2}d_1 d_2 \sin\varphi.$$

3.8 Rhombus

Side of a rhombus: a
Diagonals: d_1, d_2
Consecutive angles: α, β
Altitude: H
Radius of inscribed circle: r
Perimeter: L
Area: S

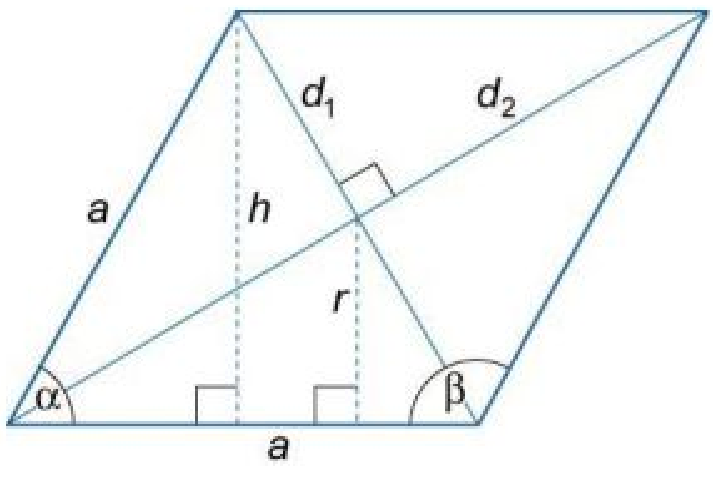


Figure 19.

210. $\alpha + \beta = 180°$

211. $d_1^2 + d_2^2 = 4a^2$

212. $h = a \sin \alpha = \dfrac{d_1 d_2}{2a}$

213. $r = \dfrac{h}{2} = \dfrac{d_1 d_2}{4a} = \dfrac{a \sin \alpha}{2}$

214. $L = 4a$

215. $S = ah = a^2 \sin \alpha$,

$S = \dfrac{1}{2} d_1 d_2$.

3.9 Trapezoid

Bases of a trapezoid: a, b
Midline: q
Altitude: h
Area: S

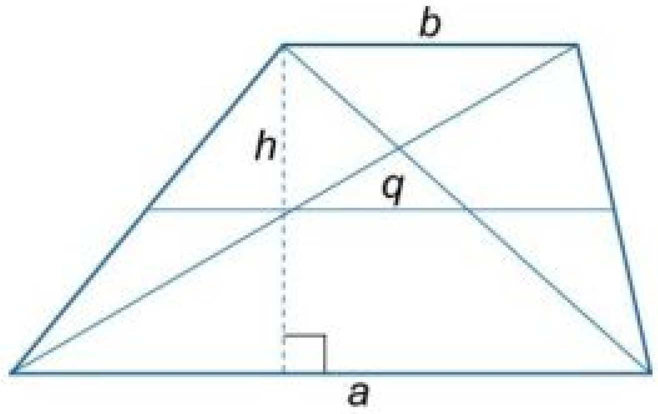


Figure 20.

216. $q = \dfrac{a+b}{2}$

217. $S = \dfrac{a+b}{2} \cdot h = qh$

3.10 Isosceles Trapezoid

Bases of a trapezoid: a, b
Leg: c
Midline: q
Altitude: h
Diagonal: d
Radius of circumscribed circle: R
Area: S

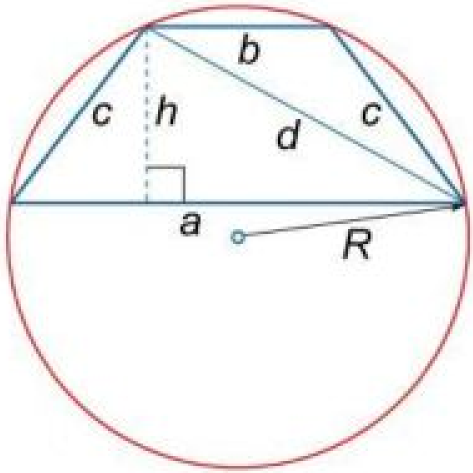


Figure 21.

218. $q = \frac{a+b}{2}$

219. $d = \sqrt{ab + c^2}$

220. $h = \sqrt{c^2 - \frac{1}{4}(b-a)^2}$

221. $R = \frac{c\sqrt{ab + c^2}}{\sqrt{(2c - a + b)(2c + a - b)}}$

222. $S = \frac{a+b}{2} \cdot h = qh$

3.11 Isosceles Trapezoid with Inscribed Circle

Bases of a trapezoid: a, b
Leg: c
Midline: q
Altitude: h
Diagonal: d
Radius of inscribed circle: R
Radius of circumscribed circle: r
Perimeter: L
Area: S

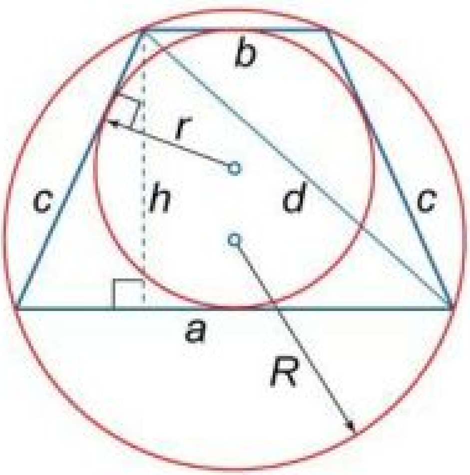


Figure 22.

223. $a + b = 2c$

224. $q = \dfrac{a+b}{2} = c$

225. $d^2 = h^2 + c^2$

226. $r = \frac{h}{2} = \frac{\sqrt{ab}}{2}$

227. $R = \frac{cd}{2h} = \frac{cd}{4r} = \frac{c}{2}\sqrt{1 + \frac{c^2}{ab}} = \frac{c}{2h}\sqrt{h^2 + c^2} = \frac{a+b}{8}\sqrt{\frac{a}{b} + 6 + \frac{b}{a}}$

228. $L = 2(a+b) = 4c$

229. $S = \frac{a+b}{2} \cdot h = \frac{(a+b)\sqrt{ab}}{2} = qh = ch = \frac{Lr}{2}$

3.12 Trapezoid with Inscribed Circle

Bases of a trapezoid: a, b
Lateral sides: c, d
Midline: q
Altitude: h
Diagonals: d_1, d_2
Angle between the diagonals: φ
Radius of inscribed circle: r
Radius of circumscribed circle: R
Perimeter: L
Area: S

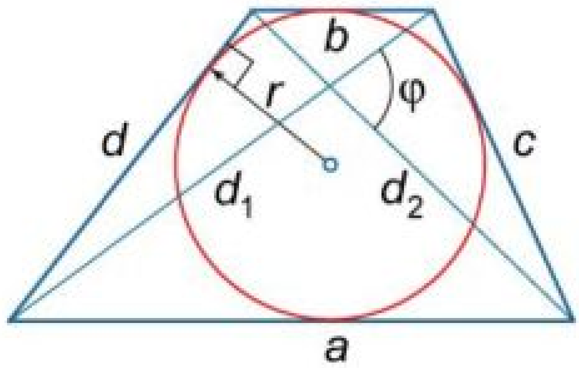


Figure 23.

230. $a + b = c + d$

231. $q = \dfrac{a+b}{2} = \dfrac{c+d}{2}$

232. $L = 2(a+b) = 2(c+d)$

233. $S = \dfrac{a+b}{2} \cdot h = \dfrac{c+d}{2} \cdot h = qh$,

$S = \dfrac{1}{2} d_1 d_2 \sin\varphi$.

3.13 Kite

Sides of a kite: a, b
Diagonals: d_1, d_2
Angles: α, β, γ
Perimeter: L
Area: S

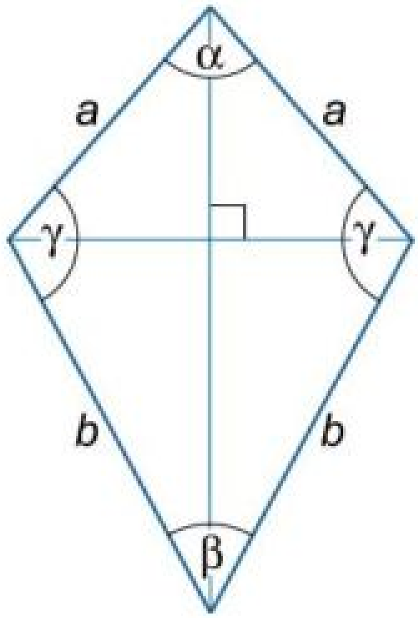


Figure 24.

234. $\alpha + \beta + 2\gamma = 360^\circ$

235. $L = 2(a + b)$

236. $S = \frac{d_1 d_2}{2}$

3.14 Cyclic Quadrilateral

Sides of a quadrilateral: a, b, c, d
Diagonals: d_1, d_2
Angle between the diagonals: φ
Internal angles: $\alpha, \beta, \gamma, \delta$
Radius of circumscribed circle: R
Perimeter: L
Semiperimeter: p
Area: S

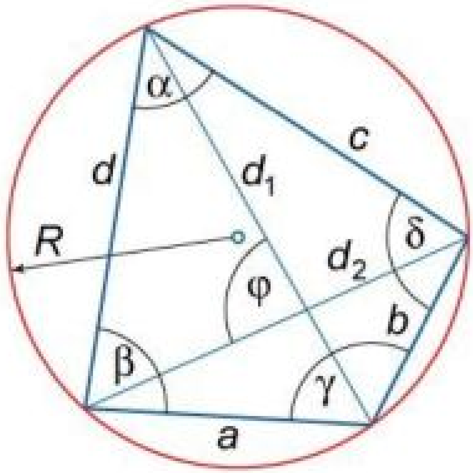


Figure 25.

237. $\alpha+\gamma=\beta+\delta=180^\circ$

238. Ptolemy's Theorem
$ac+bd=d_1d_2$

239. $L=a+b+c+d$

240. $R=\frac{1}{4}\sqrt{\frac{(ac+bd)(ad+bc)(ab+cd)}{(p-a)(p-b)(p-c)(p-d)}}$,
where $p=\frac{L}{2}$.

241. $S=\frac{1}{2}d_1d_2\sin\varphi$,
$S=\sqrt{(p-a)(p-b)(p-c)(p-d)}$,
where $p=\frac{L}{2}$.

3.15 Tangential Quadrilateral

Sides of a quadrilateral: a, b, c, d
Diagonals: d_1, d_2
Angle between the diagonals: φ
Radius of inscribed circle: r
Perimeter: L
Semiperimeter: p
Area: S

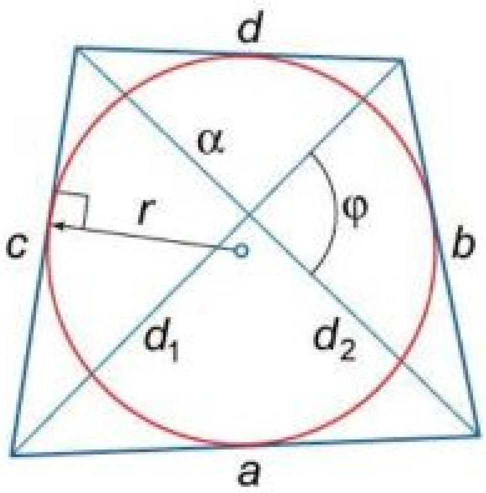


Figure 26.

242. $a + c = b + d$

243. $L = a + b + c + d = 2(a + c) = 2(b + d)$

244. $r = \dfrac{\sqrt{d_1^2 d_2^2 - (a - b)^2 (a + b - p)^2}}{2p}$,

where $p = \dfrac{L}{2}$.

245. $S = pr = \frac{1}{2}d_1 d_2 \sin\varphi$

3.16 General Quadrilateral

Sides of a quadrilateral: a, b, c, d
Diagonals: d_1, d_2
Angle between the diagonals: φ
Internal angles: $\alpha, \beta, \gamma, \delta$
Perimeter: L
Area: S

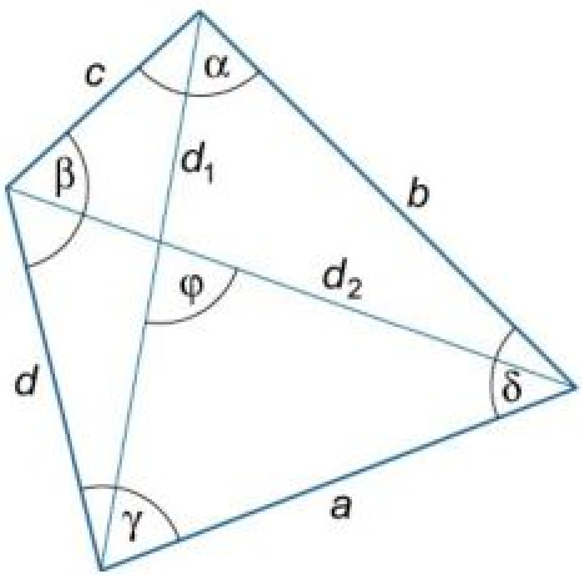


Figure 27.

246. $\alpha + \beta + \gamma + \delta = 360°$

247. $L = a + b + c + d$

248. $S = \frac{1}{2} d_1 d_2 \sin \varphi$

3.17 Regular Hexagon

Side: a
Internal angle: α
Slant height: m
Radius of inscribed circle: r
Radius of circumscribed circle: R
Perimeter: L
Semiperimeter: p
Area: S

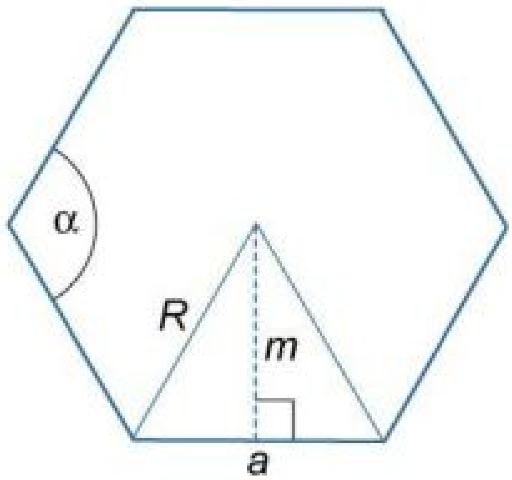


Figure 28.

249. $\alpha = 120^\circ$

250. $r = m = \frac{a\sqrt{3}}{2}$

251. $R = a$

252. $L = 6a$

253. $S = pr = \dfrac{a^2 3\sqrt{3}}{2}$,

where $p = \dfrac{L}{2}$.

3.18 Regular Polygon

Side: a
Number of sides: n
Internal angle: α
Slant height: m
Radius of inscribed circle: r
Radius of circumscribed circle: R
Perimeter: L
Semiperimeter: p
Area: S

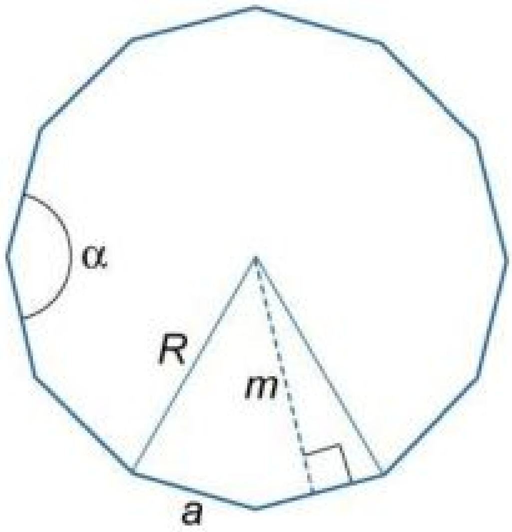


Figure 29.

254. $\alpha = \frac{n-2}{2} \cdot 180°$

255. $\alpha = \frac{n-2}{2} \cdot 180°$

256. $R = \frac{a}{2\sin\frac{\pi}{n}}$

257. $r = m = \frac{a}{2\tan\frac{\pi}{n}} = \sqrt{R^2 - \frac{a^2}{4}}$

258. $L = na$

259. $S = \frac{nR^2}{2}\sin\frac{2\pi}{n}$,

$S = pr = p\sqrt{R^2 - \frac{a^2}{4}}$,

where $p = \frac{L}{2}$.

3.19 Circle

Radius: R
Diameter: d
Chord: a
Secant segments: e, f
Tangent segment: g
Central angle: α
Inscribed angle: β
Perimeter: L
Area: S

260. $a = 2R\sin\frac{\alpha}{2}$

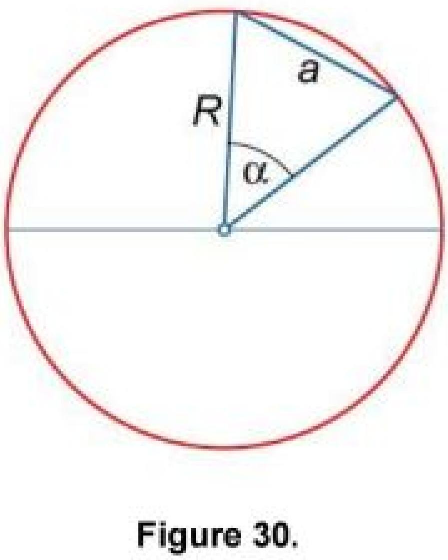


Figure 30.

261. $a_1 a_2 = b_1 b_2$

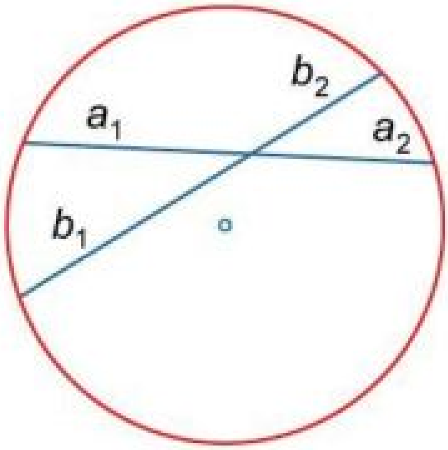


Figure 31.

262. $ee_1 = ff_1$

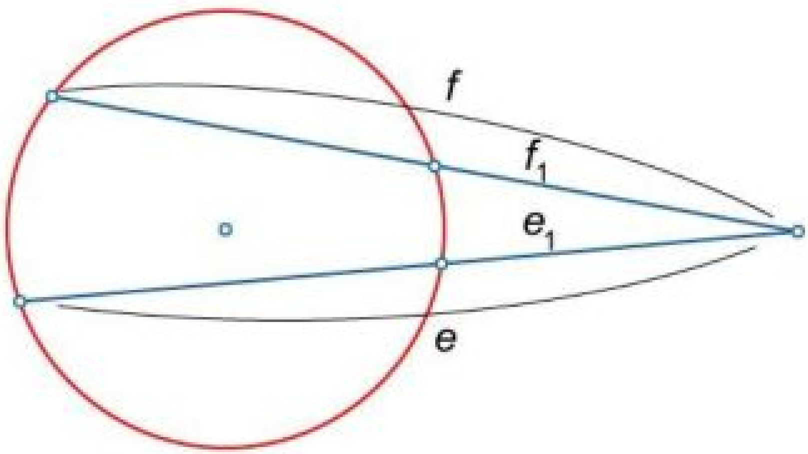


Figure 32.

263. $g^2 = ff_1$

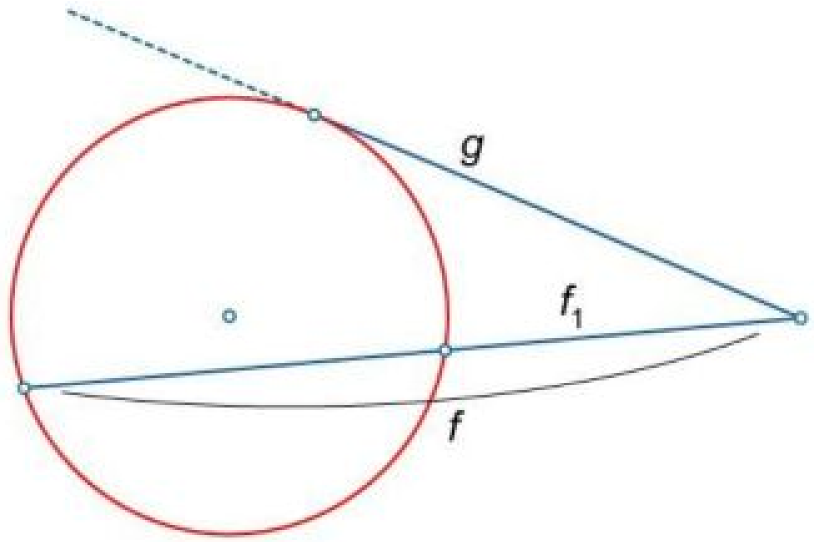


Figure 33.

264. $\beta = \frac{\alpha}{2}$

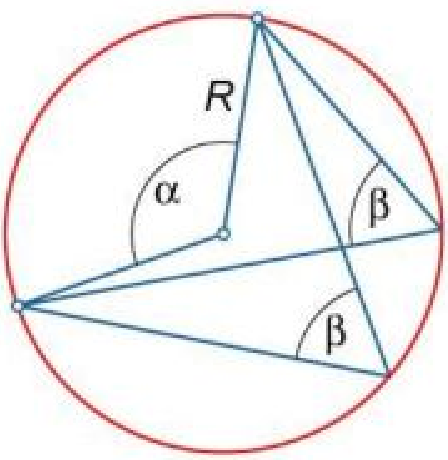


Figure 34.

265. $L = 2\pi R = \pi d$

266. $S = \pi R^2 = \frac{\pi d^2}{4} = \frac{LR}{2}$

3.20 Sector of a Circle

Radius of a circle: R
Arc length: s
Central angle (in radians): x
Central angle (in degrees): α
Perimeter: L
Area: S

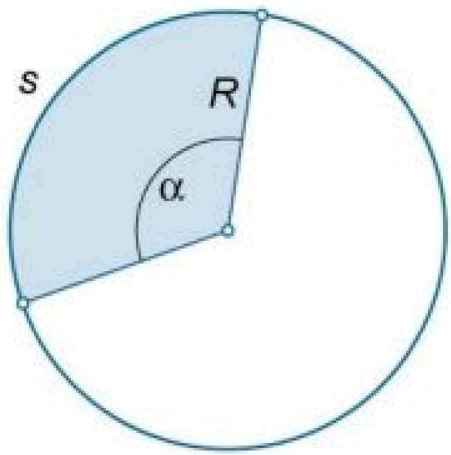


Figure 35.

267. $s = Rx$

268. $s = \dfrac{\pi R \alpha}{180°}$

269. $L = s + 2R$

270. $S = \dfrac{Rs}{2} = \dfrac{R^2 x}{2} = \dfrac{\pi R^2 \alpha}{360°}$

3.21 Segment of a Circle

Radius of a circle: R
Arc length: s
Chord: a
Central angle (in radians): x
Central angle (in degrees): α
Height of the segment: h
Perimeter: L
Area: S

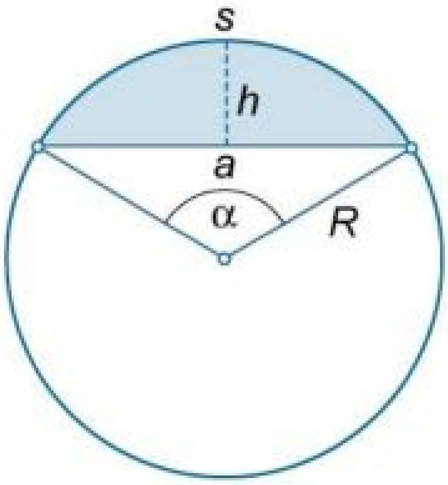


Figure 36.

271. $a = 2\sqrt{2hR - h^2}$

272. $h = R - \frac{1}{2}\sqrt{4R^2 - a^2}$, $h < R$

273. $L = s + a$

274. $S=\frac{1}{2}\left[sR-a(R-h)\right]=\frac{R^2}{2}\left(\frac{\alpha\pi}{180^\circ}-\sin\alpha\right)=\frac{R^2}{2}(x-\sin x)$,

$S\approx\frac{2}{3}ha$.

3.22 Cube

Edge: a
Diagonal: d
Radius of inscribed sphere: r
Radius of circumscribed sphere: r
Surface area: S
Volume: V

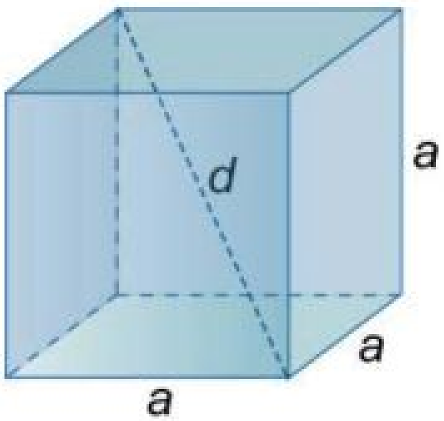


Figure 37.

275. $d=a\sqrt{3}$

276. $r=\frac{a}{2}$

277. $R = \frac{a\sqrt{3}}{2}$

278. $S = 6a^2$

279. $V = a^3$

3.23 Rectangular Parallelepiped

Edges: a, b, c
Diagonal: d
Surface area: S
Volume: V

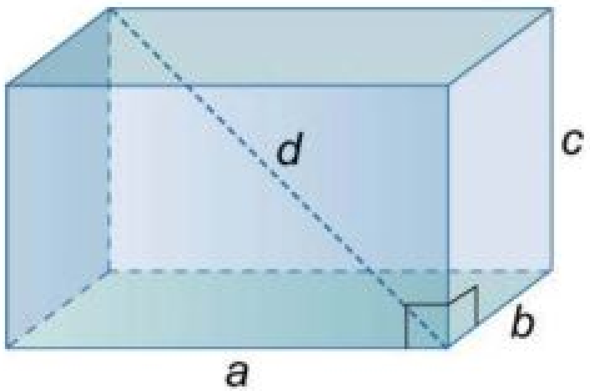


Figure 38.

280. $d = \sqrt{a^2 + b^2 + c^2}$

281. $S = 2(ab + ac + bc)$

282. $V = abc$

3.24 Prism

Lateral edge: l
Height: h
Lateral area: S_L
Area of base: S_B
Total surface area: S
Volume: V

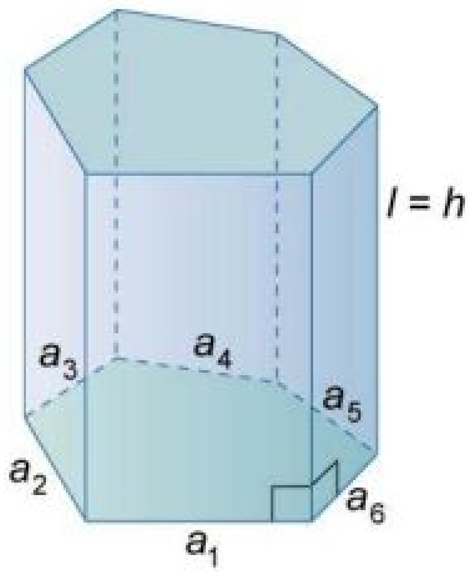


Figure 39.

283. $S = S_L + 2S_B$.

284. Lateral Area of a Right Prism
$S_L = (a_1 + a_2 + a_3 + \ldots + a_n)l$

285. Lateral Area of an Oblique Prism
$S_L = pl$,
where p is the perimeter of the cross section.

286. $V = S_B h$

287. Cavalieri's Principle
Given two solids included between parallel planes. If every plane cross section parallel to the given planes has the same area in both solids, then the volumes of the solids are equal.

3.25 Regular Tetrahedron

Triangle side length: a
Height: h
Area of base: S_B
Surface area: S
Volume: V

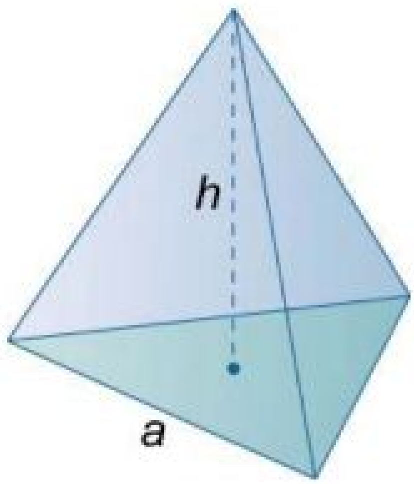


Figure 40.

288. $h = \sqrt{\frac{2}{3}}\,a$

289. $S_B = \dfrac{\sqrt{3}a^2}{4}$

290. $S = \sqrt{3}a^2$

291. $V = \dfrac{1}{3}S_B h = \dfrac{a^3}{6\sqrt{2}}$.

3.26 Regular Pyramid

Side of base: a
Lateral edge: b
Height: h
Slant height: m
Number of sides: n
Semiperimeter of base: p
Radius of inscribed sphere of base: r
Area of base: S_B
Lateral surface area: S_L
Total surface area: S
Volume: V

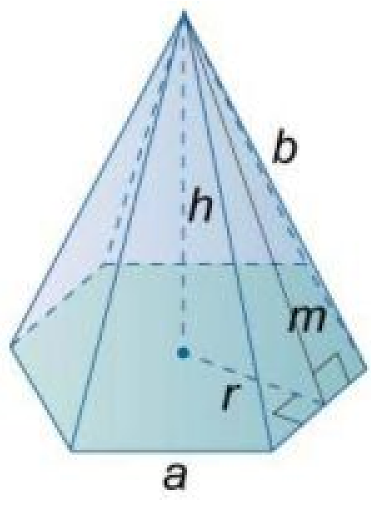


Figure 41.

292. $m=\sqrt{b^2-\dfrac{a^2}{4}}$

293. $h=\dfrac{\sqrt{4b^2\sin^2\dfrac{\pi}{n}-a^2}}{2\sin\dfrac{\pi}{n}}$

294. $S_L=\dfrac{1}{2}nam=\dfrac{1}{4}na\sqrt{4b^2-a^2}=pm$

295. $S_B=pr$

296. $S=S_B+S_L$

297. $V=\dfrac{1}{3}S_Bh=\dfrac{1}{3}prh$

3.27 Frustum of a Regular Pyramid

Base and top side lengths: $\begin{cases} a_1, a_2, a_3, \ldots, a_n \\ b_1, b_2, b_3, \ldots, b_n \end{cases}$

Height: h
Slant height: m
Area of bases: S_1, S_2
Lateral surface area: S_L
Perimeter of bases: P_1, P_2
Scale factor: k
Total surface area: S
Volume: V

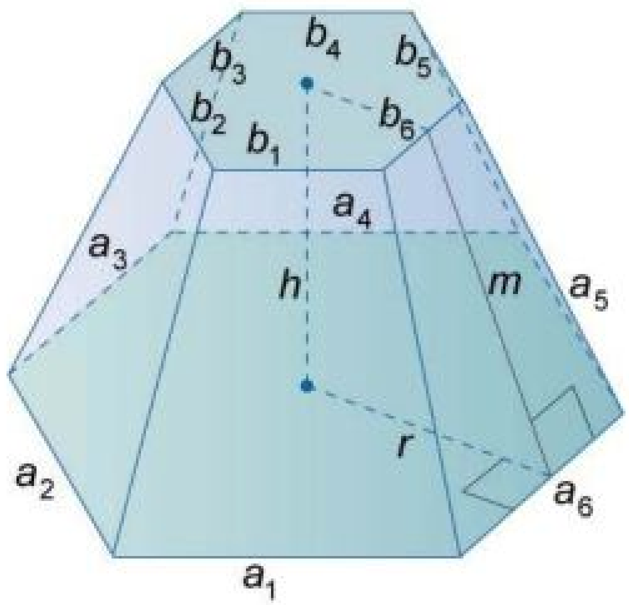


Figure 42.

298. $\dfrac{b_1}{a_1} = \dfrac{b_2}{a_2} = \dfrac{b_3}{a_3} = \ldots = \dfrac{b_n}{a_n} = \dfrac{b}{a} = k$

299. $\frac{S_2}{S_1} = k^2$

300. $S_L = \frac{m(P_1 + P_2)}{2}$

301. $S = S_L + S_1 + S_2$

302. $V = \frac{h}{3}\left(S_1 + \sqrt{S_1 S_2} + S_2\right)$

303. $V = \frac{hS_1}{3}\left[1 + \frac{b}{a} + \left(\frac{b}{a}\right)^2\right] = \frac{hS_1}{3}\left[1 + k + k^2\right]$

3.28 Rectangular Right Wedge

Sides of base: a, b
Top edge: c
Height: h
Lateral surface area: S_L
Area of base: S_B
Total surface area: S
Volume: V

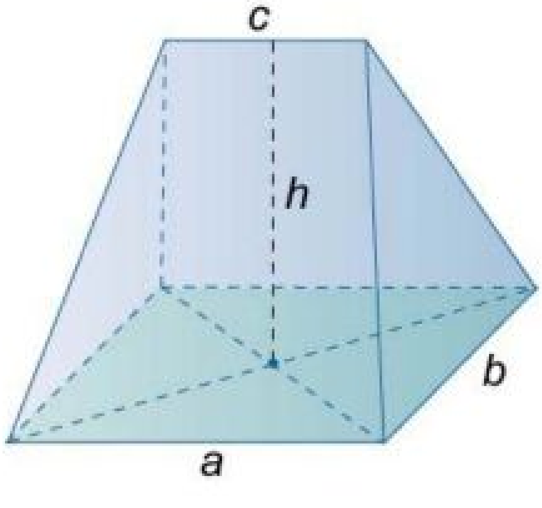


Figure 43.

304. $S_L = \frac{1}{2}(a+c)\sqrt{4h^2+b^2} + b\sqrt{h^2+(a-c)^2}$

305. $S_B = ab$

306. $S = S_B + S_L$

307. $V = \frac{bh}{6}(2a+c)$

3.29 Platonic Solids

Edge: a
Radius of inscribed circle: r
Radius of circumscribed circle: R
Surface area: S
Volume: V

308. Five Platonic Solids
The platonic solids are convex polyhedra with equivalent faces composed of congruent convex regular polygons.

Solid	Number of Vertices	Number of Edges	Number of Faces	Section
Tetrahedron	4	6	4	3.25
Cube	8	12	6	3.22
Octahedron	6	12	8	3.27
Icosahedron	12	30	20	3.27
Dodecahedron	20	30	12	3.27

Octahedron

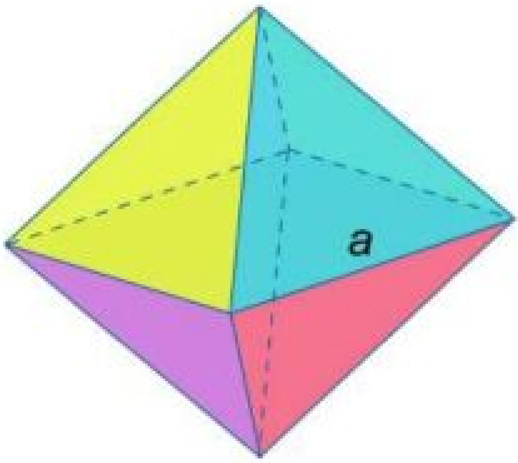


Figure 44.

309. $r = \frac{a\sqrt{6}}{6}$

310. $R = \frac{a\sqrt{2}}{2}$

311. $S = 2a^2\sqrt{3}$

312. $V = \dfrac{a^3\sqrt{2}}{3}$

Icosahedron

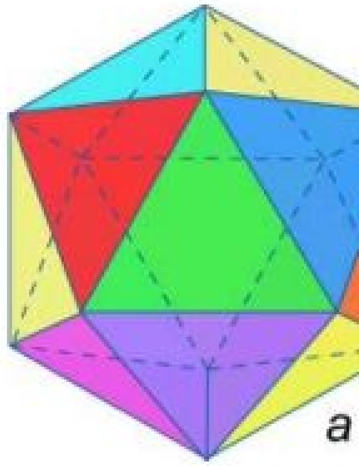


Figure 45.

313. $r = \dfrac{a\sqrt{3}\left(3+\sqrt{5}\right)}{12}$

314. $R = \dfrac{a}{4}\sqrt{2\left(5+\sqrt{5}\right)}$

315. $S = 5a^2\sqrt{3}$

316. $V = \dfrac{5a^3\left(3+\sqrt{5}\right)}{12}$

Dodecahedron

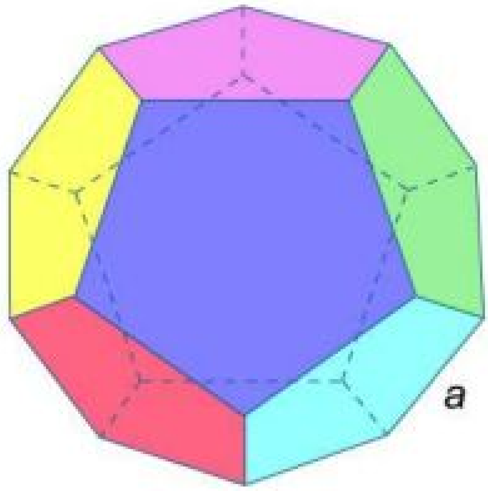


Figure 46.

317. $r = \dfrac{a\sqrt{10\left(25 + 11\sqrt{5}\right)}}{2}$

318. $R = \dfrac{a\sqrt{3}\left(1 + \sqrt{5}\right)}{4}$

319. $S = 3a^2\sqrt{5\left(5 + 2\sqrt{5}\right)}$

320. $V = \dfrac{a^3\left(15 + 7\sqrt{5}\right)}{4}$

3.30 Right Circular Cylinder

Radius of base: R
Diameter of base: d

Height: H
Lateral surface area: S_L
Area of base: S_B
Total surface area: S
Volume: V

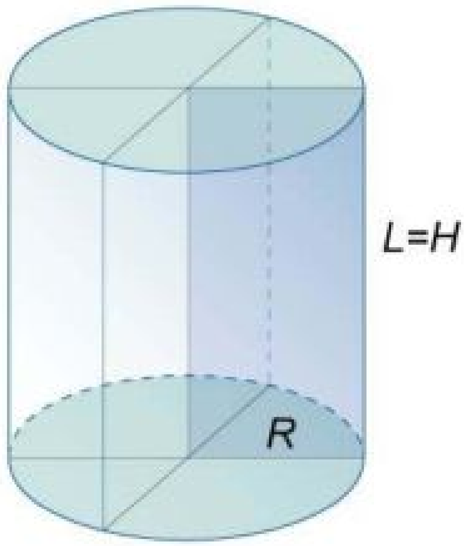


Figure 47.

321. $S_L = 2\pi RH$

322. $S = S_L + 2S_B = 2\pi R(H+R) = \pi d\left(H + \frac{d}{2}\right)$

323. $V = S_B H = \pi R^2 H$

3.31 Right Circular Cylinder with an Oblique Plane Face

Radius of base: R
The greatest height of a side: h_1
The shortest height of a side: h_2
Lateral surface area: S_L
Area of plane end faces: S_B
Total surface area: S
Volume: V

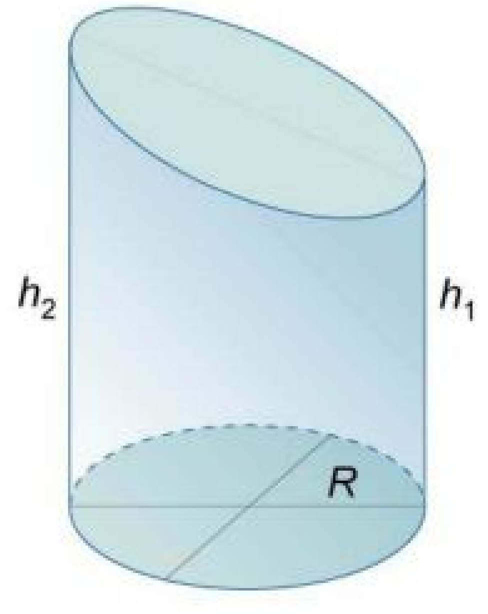


Figure 48.

324. $S_L = \pi R(h_1 + h_2)$

325. $S_B = \pi R^2 + \pi R\sqrt{R^2 + \left(\frac{h_1 - h_2}{2}\right)^2}$

326. $S = S_L + S_B = \pi R\left[h_1 + h_2 + R + \sqrt{R^2 + \left(\dfrac{h_1 - h_2}{2}\right)^2}\right]$

327. $V = \dfrac{\pi R^2}{2}\left(h_1 + h_2\right)$

3.32 Right Circular Cone

Radius of base: R
Diameter of base: d
Height: H
Slant height: m
Lateral surface area: S_L
Area of base: S_B
Total surface area: S
Volume: V

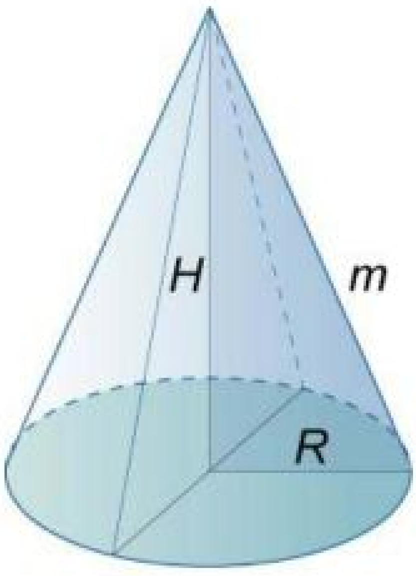


Figure 49.

328. $H = \sqrt{m^2 - R^2}$

329. $S_L = \pi R m = \dfrac{\pi m d}{2}$

330. $S_B = \pi R^2$

331. $S = S_L + S_B = \pi R(m + R) = \dfrac{1}{2}\pi d\left(m + \dfrac{d}{2}\right)$

332. $V = \dfrac{1}{3} S_B H = \dfrac{1}{3}\pi R^2 H$

3.33 Frustum of a Right Circular Cone

Radius of bases: R, r
Height: H
Slant height: m
Scale factor: k
Area of bases: S_1, S_2
Lateral surface area: S_L
Total surface area: S
Volume: V

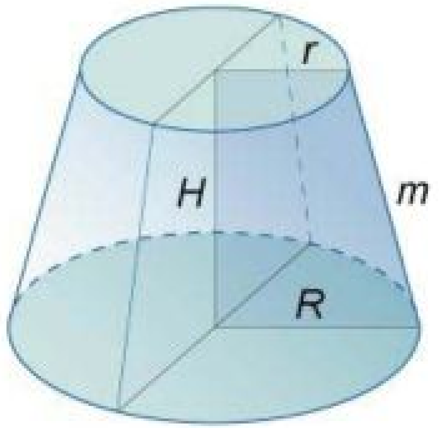


Figure 50.

333. $H = \sqrt{m^2 - (R - r)^2}$

334. $\frac{R}{r} = k$

335. $\frac{S_2}{S_1} = \frac{R^2}{r^2} = k^2$

336. $S_L = \pi m(R + r)$

337. $S = S_1 + S_2 + S_L = \pi\left[R^2 + r^2 + m(R + r)\right]$

338. $V = \frac{h}{3}\left(S_1 + \sqrt{S_1S_2} + S_2\right)$

339. $V = \frac{hS_1}{3}\left[1 + \frac{R}{r} + \left(\frac{R}{r}\right)^2\right] = \frac{hS_1}{3}\left[1 + k + k^2\right]$

3.34 Sphere

Radius: R
Diameter: d
Surface area: S
Volume: V

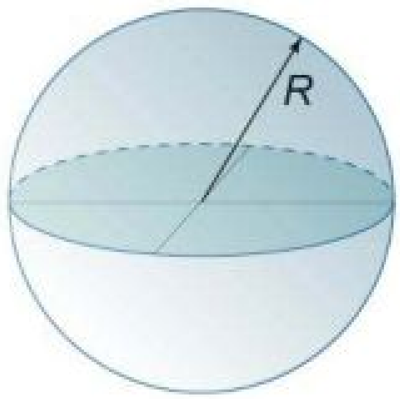


Figure 51.

340. $S = 4\pi R^2$

341. $V = \frac{4}{3}\pi R^3 H = \frac{1}{6}\pi d^3 = \frac{1}{3}SR$

3.35 Spherical Cap

Radius of sphere: R
Radius of base: r
Height: h
Area of plane face: S_B
Area of spherical cap: S_C
Total surface area: S
Volume: V

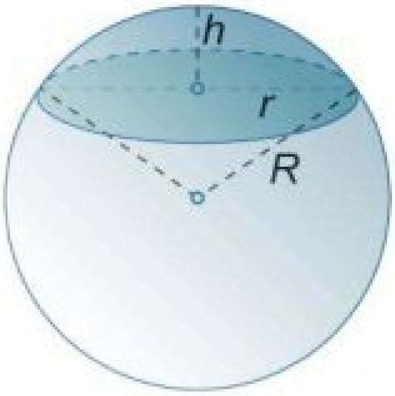


Figure 52.

342. $R = \dfrac{r^2 + h^2}{2h}$

343. $S_B = \pi r^2$

344. $S_C = \pi\left(h^2 + r^2\right)$

345. $S = S_B + S_C = \pi\left(h^2 + 2r^2\right) = \pi\left(2Rh + r^2\right)$

346. $V = \dfrac{\pi}{6} h^2 (3R - h) = \dfrac{\pi}{6} h\left(3r^2 + h^2\right)$

3.36 Spherical Sector

Radius of sphere: R
Radius of base of spherical cap: r
Height: h
Total surface area: S
Volume: V

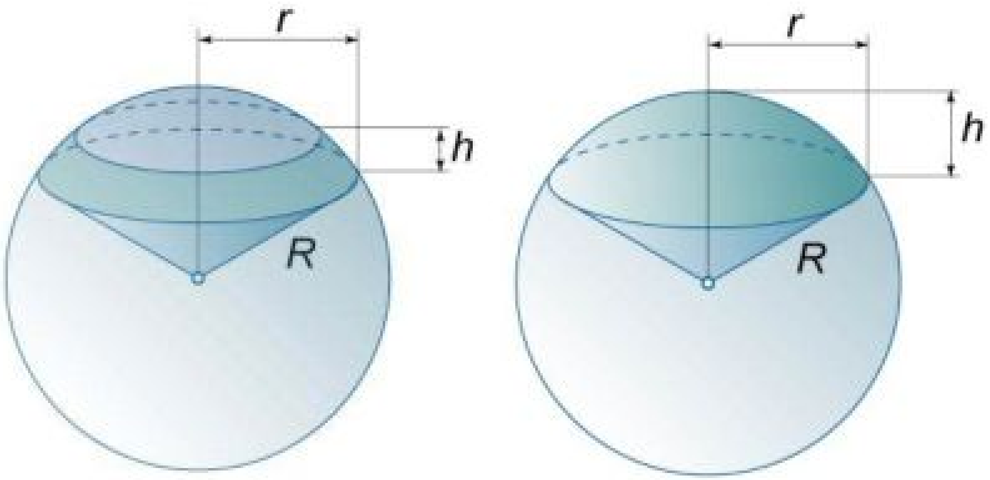


Figure 53.

347. $S = \pi R(2h + r)$

348. $V = \frac{2}{3}\pi R^2 h$

Note: The given formulas are correct both for "open" and "closed" spherical sector.

3.37 Spherical Segment

Radius of sphere: R
Radius of bases: r_1, r_2
Height: h
Area of spherical surface: S_S
Area of plane end faces: S_1, S_2
Total surface area: S
Volume: V

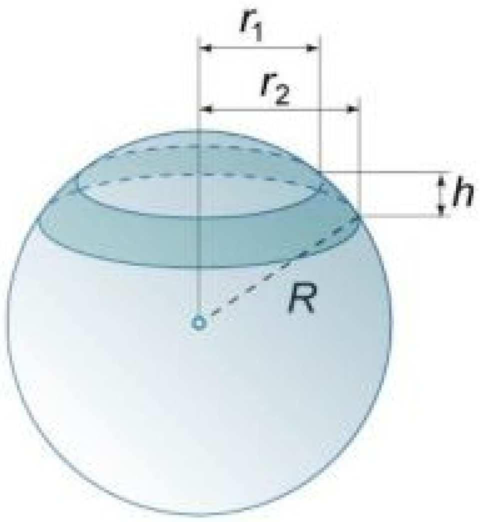


Figure 54.

349. $S_S = 2\pi Rh$

350. $S = S_S + S_1 + S_2 = \pi\left(2Rh + r_1^2 + r_2^2\right)$

351. $V = \dfrac{1}{6}\pi h\left(3r_1^2 + 3r_2^2 + h^2\right)$

3.38 Spherical Wedge

Radius: R
Dihedral angle in degrees: x
Dihedral angle in radians: α
Area of spherical lune: S_L
Total surface area: S
Volume: V

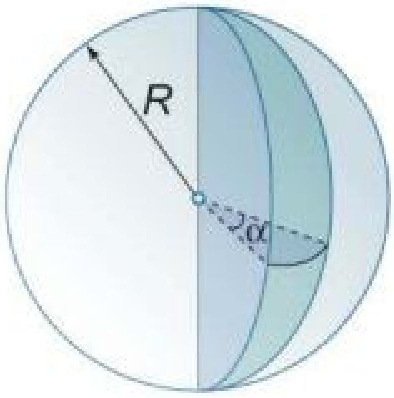


Figure 55.

352. $S_L = \dfrac{\pi R^2}{90}\alpha = 2R^2x$

353. $S = \pi R^2 + \dfrac{\pi R^2}{90}\alpha = \pi R^2 + 2R^2x$

354. $V = \dfrac{\pi R^3}{270}\alpha = \dfrac{2}{3}R^3x$

3.39 Ellipsoid

Semi-axes: a, b, c
Volume: V

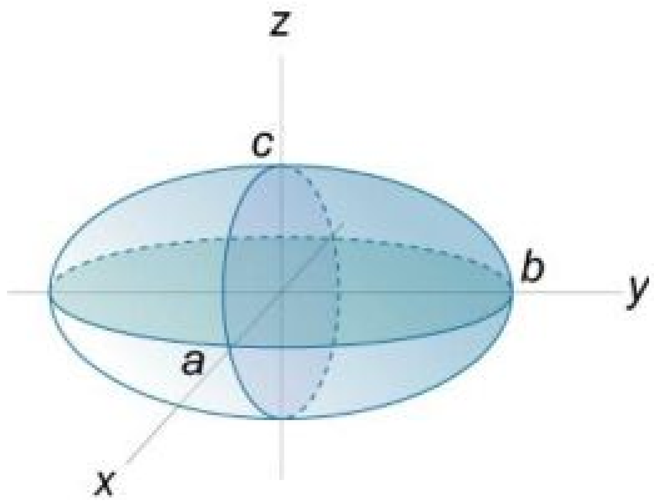


Figure 56.

355. $V = \frac{4}{3}\pi abc$

Prolate Spheroid

Semi-axes: a, b, b ($a > b$)
Surface area: S
Volume: V

356. $S = 2\pi b\left(b + \frac{a \arcsin e}{e}\right),$

where $e = \frac{\sqrt{a^2 - b^2}}{a}.$

357. $V = \frac{4}{3}\pi b^2 a$

Oblate Spheroid

Semi-axes: a, b, b ($a < b$)
Surface area: S
Volume: V

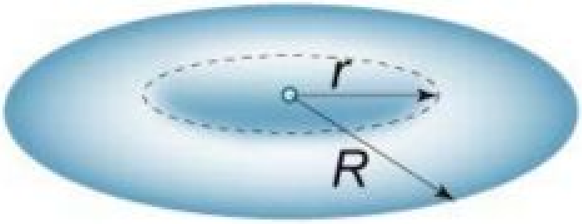
358. $$S = 2\pi b\left(b + \frac{a \operatorname{arcsinh}\left(\frac{be}{a}\right)}{be/a}\right),$$

where $e = \dfrac{\sqrt{b^2 - a^2}}{b}$.

359. $$V = \frac{4}{3}\pi b^2 a$$

3.40 Circular Torus

Major radius: R
Minor radius: r
Surface area: S
Volume: V



Picture 57.

360. $S = 4\pi^2 Rr$

361. $V = 2\pi^2 Rr^2$