Chapter 5

* QUADRATICS EQUATIONS *

EXERCISE 5.1

Formula and Concepts

$ax^2 + bx + c = 0$ $\}$ $(\ldots)^2$ + rational

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Nature of roots

i) $b^2 - 4ac > 0$	unequal real (rational or irrational
ii) $b^2 - 4ac < 0$	imaginary and unequal
iii) $b^2 - 4ac = 0$	real and equal

1) $x^2 - 6x + 5 = 0$

Comparing with $ax^2 + bx + c = 0$
we get

$a = 1, \; b = -6, \; c = 5$

now,

$b^2 - 4ac$

$= (-6)^2 - 4 \times 1 \times 5$

$= 16$ which is greater than 0

So,

$b^2 - 4ac > 0$ it is real, unequal, rational

b. $x^2 - 4x - 3 = 0$

here,

Comparing with $ax^2 + bx + c = 0$

we get,

$a = 1, \ b = -4, \ c = -3$

now,

$b^2 - 4ac$

$\cancel{(-6)^2} = (-4)^2 - 4 \times 1 \times -3$

$= 16 + 12$

$= 28$

$\therefore$ real, irrational and unequal

c. $x^2 - 6x + 9 = 0$

Soln,

$a = 1, \ b = -6, \ c = 9$

$b^2 - 4ac$

$= (-6)^2 - 4 \times 1 \times 9$

$= 36 - 36 = 0$

$\therefore$ real, rational and equal

d. $4x^2 - 4x + 1 = 0$

Soln

$a = 4, \ b = -4, \ c = 1$

$b^2 - 4ac$

$= (-4)^2 - 4 \times 4 \times 1 = 16 - 16 = 0$

$\therefore$ real, rational and equal

e. $2x^2 - 9x + 35 = 0$

here,

$a = 2$, $b = -9$, $c = 35$

now,

$b^2 - 4ac$

$81 - 4 \times 2 \times 35$

$= -131$

$\therefore$ imaginary and unequal

f. $4x^2 + 8x - 5 = 0$

here,

$a = 4$, $b = 8$, $c = -5$

now,

$b^2 - 4ac$

$64 - 4 \times 4 \times -5$

$= 64 + 80$

$= 144$

$\therefore$ real, rational and unequal

2.

Soln,

Comparing and we get

$a = 5$, $b = -p$, $c = 45$

then,

$b^2 - 4ac = 0$ {$\therefore$ Being equal roots}

$p^2 - 4 \times 5 \times 45 = 0$

$p^2 = 900$

$\therefore p = \pm 30$

here,

$x^2 + 2(k+2)x + 9k = 0$

or, $x^2 + 2kx + 4x + 9k = 0$

or, $x^2 + 4x + 2kx + 9k = 0$

Comparing with $ax^2 + bx + c = 0$

$a = 1, \quad b = 2(k+2), \quad c = 9k$

now,

$b^2 - 4ac = 0$ {∵ Acc. to qn}

$[2(k+2)^2]^2 - 4 \times 1 \times 9k = 0$

or, $\cancel{4}(k^2 + 4k + 4) = \cancel{36}k$

or, $k^2 + 4k + 4 = 9k$

or, $k^2 - 5k + 4 = 0$

or, $k^2 - 4k - k + 4 = 0$

or, $k(k-4) - 1(k-4) = 0$

$(k-1)(k-4) = 0$

Either,

$k = 4$ or $k = 1$

here,

Comparing and we get

$a = 1, \quad b = -(3a-1), \quad c = 2(a^2-1)$

$b^2 - 4ac = 0$

$(-(3a-1))^2 - 4 \times 1 \times 2(a^2-1) = 0$

$\Rightarrow (3a)^2 - 2.3a.1 + 1 - 8(a^2-1) = 0$

$\Rightarrow a^2 - 6a + 9 = 0$

$\Rightarrow \cancel{a^2} \; (a-3)^2 = 0$

$\therefore a = 3$

here,

Comparing with $\cancel{a} x^2 + bx + c = 0$

$a = a^2 + b^2$, $b = -2(ac+bd)$, $c = c^2 + d^2$

now,

$b^2 - 4ac = 0$

$\Rightarrow (-2(ac+bd))^2 - 4 \times (a^2+b^2) \times (c^2+d^2) = 0$

$\Rightarrow 4(a^2c^2 + 2ac.bd + b^2d^2) - 4(a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2) = 0$

$\Rightarrow 4\{(\cancel{a^2c^2} + 2ac.bd + \cancel{b^2d^2} - \cancel{a^2c^2} - a^2d^2 - b^2c^2 - \cancel{b^2d^2})\} = 0$

$\Rightarrow 2ac.bd - a^2d^2 - b^2c^2 = 0$

$\Rightarrow -(ad-bc)^2 = 0 \quad \{\therefore (a-b)^2\}$

$ad - bc = 0$

$\therefore \frac{a}{b} = \frac{c}{d}$ proved.

here.

Comparing and we get

$a = a^2 - bc$, $b = 2(b^2 - 4ac)$, $c = c^2 - ab$

now,

$b^2-4ac=0$

$\Rightarrow \cancel{2}[2(b^2-ac)]^2-4(a^2-bc)(c^2-ab)=0$

$\Rightarrow 4(b^4-2.b^2.ac+a^2c^2)-4(a^2c^2-a^3b-bc^2$

$ab^2c)=0$

$\Rightarrow b^4-2ab^2c+\cancel{a^2c^2}-\cancel{a^2c^2}+a^3b+bc^3-$

$ab^3c=0$

$\Rightarrow b^4-3ab^2+a^3b+bc^3=0$

$\Rightarrow b(b^3-3ab+a^3+c^3)=0$

if Either

$\therefore b=0$

or,

$b^3-3ab+a^3+c^3=0$

proved.

7.

here.

a, b & c are rational

$a+b+c=0 \Rightarrow a+c=-b,\ a+b=-c,\ a+b \Rightarrow -c$

Comparing and we get.

$a=b+c-a,\quad b=c+a-b,\quad c=a+b-c$

now,

$b^2-4ac=(c+a-b)^2-4(b+c-a)(a+b-c)$

$=(-b-b)^2-4(-a-a)(-c-c)$

$=4b^2-16ac$

$=4b^2-16a(-a-b)$

$=4b^2+16a^2+16ab$

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$= (2b)^2 + 16ab + (4a)^2$

$= (2b+4a)^2$ which is > 0, and perfect square

$\therefore$ Root of given eqn are rational

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Soln,

$(x-a)(x-b) = k^2$

$x^2 - ax + ab - bx - k^2 = 0$

$x^2 - x(a+b) + (ab-k^2) = 0$

$\therefore$ $a = 1$, $b = a+b$, $c = ab-k^2$

$b^2 - 4ac = (a+b)^2 - 4 \times 1 \times (ab-k^2)$

$= (a+b)^2 + 4k^2$

$= (a+b)^2 + (2k)^2$

here,

$(a+b)^2$ and $(2k)^2$ is always positive.

So $b^2 - 4ac$ is always > 0

$\therefore$ all ~~o~~ the value of k is real

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9. Soln

Comparing we get,

$a = 1, \quad b = -4ab, \quad c = (a^2 + 2b^2)^2$

now,

$b^2 - 4ac = (-4ab)^2 - 4 \times 1 \times (a^2 + 2b^2)^2$

$= \cancel{16a^2b^2} - 4a^4 - \cancel{16a^2b^2} - 16b^4$

$= -4a^4 - 16b^4$

$\therefore = -4(a^4 + 4b^4)$ which is

less than 0

So The roots of eqn are imaginary

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Soln

For 1st equation.

$a = q, \quad b = 2p, \quad c = 2q$

$b^2 - 4ac = (2p)^2 - 4 \times q \times 2q$

$= 4p^2 - 8q^2$

real, unequal $\therefore = 4(p^2 - 2q^2) > 0$

For 2nd eqn

$a = p+q, \quad b = 2q, \quad c = p - q$

$b^2 - 4ac = 4q^2 - 4(p+q)(p-q)$

$= 4q^2 - 4(p^2 - \cancel{pq} + \cancel{pq} - q^2)$

$= \cancel{4q^2} - 4p^2 + \cancel{4q^2}$

$\therefore = -4p^2 < 0$

imaginary