Chapter 4

Trigonometry

Angles: α, β
Real numbers (coordinates of a point): x, y
Whole number: k

4.1 Radian and Degree Measures of Angles

362. $1\ \text{rad} = \dfrac{180°}{\pi} \approx 57°17'45''$

363. $1° = \dfrac{\pi}{180}\ \text{rad} \approx 0.017453\ \text{rad}$

364. $1' = \dfrac{\pi}{180 \cdot 60}\ \text{rad} \approx 0.000291\ \text{rad}$

365. $1'' = \dfrac{\pi}{180 \cdot 3600}\ \text{rad} \approx 0.000005\ \text{rad}$

366.

Angle (degrees)	0	30	45	60	90	180	270	360
Angle (radians)	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π

4.2 Definitions and Graphs of Trigonometric Functions

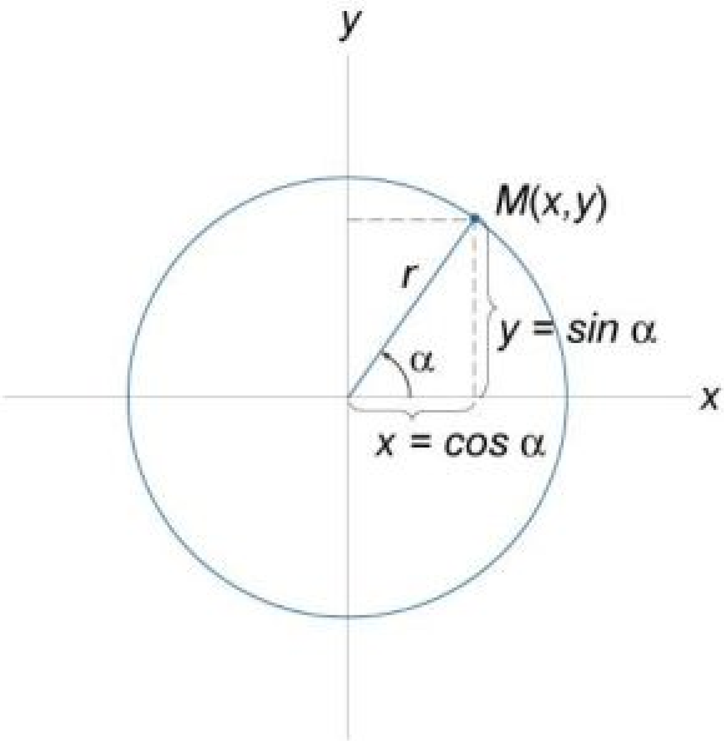

Figure 58.

367. $\sin\alpha = \frac{y}{r}$

368. $\cos\alpha = \frac{x}{r}$

369. $\tan\alpha = \frac{y}{x}$

370. $\cot\alpha = \frac{x}{y}$

371. $\sec \alpha = \frac{r}{x}$

372. $\operatorname{cosec} \alpha = \frac{r}{y}$

373. Sine Function
$y = \sin x\,,\; -1 \le \sin x \le 1.$

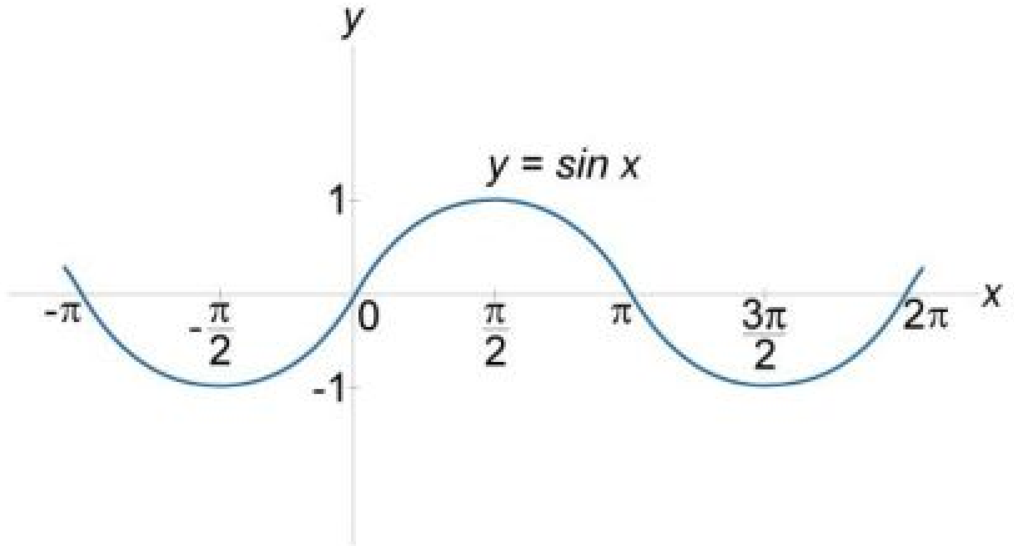

Figure 59.

374. Cosine Function
$y = \cos x\,,\; -1 \le \cos x \le 1.$

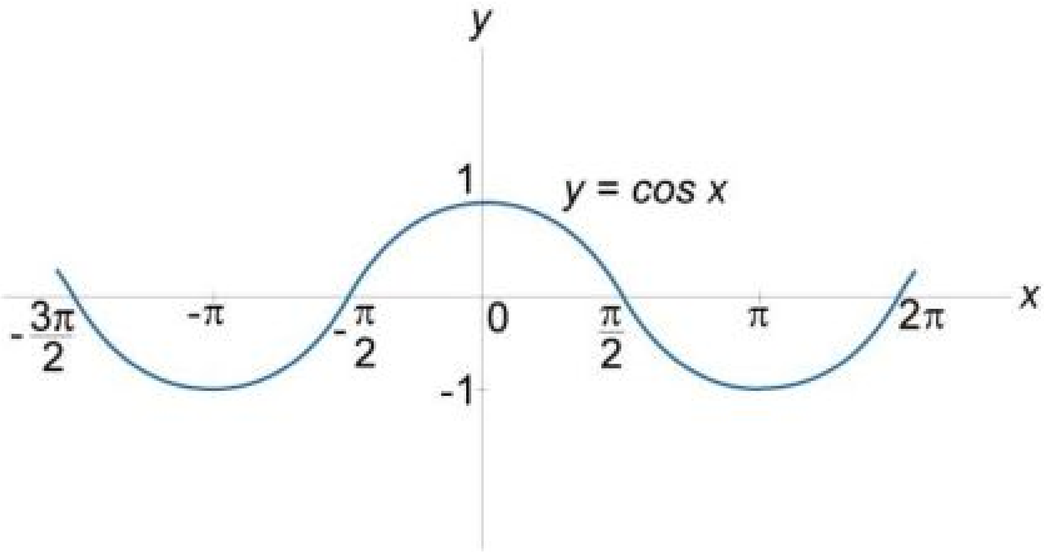

Figure 60.

375. Tangent Function

$$y = \tan x,\ x \ne (2k+1)\frac{\pi}{2},\ -\infty \le \tan x \le \infty.$$

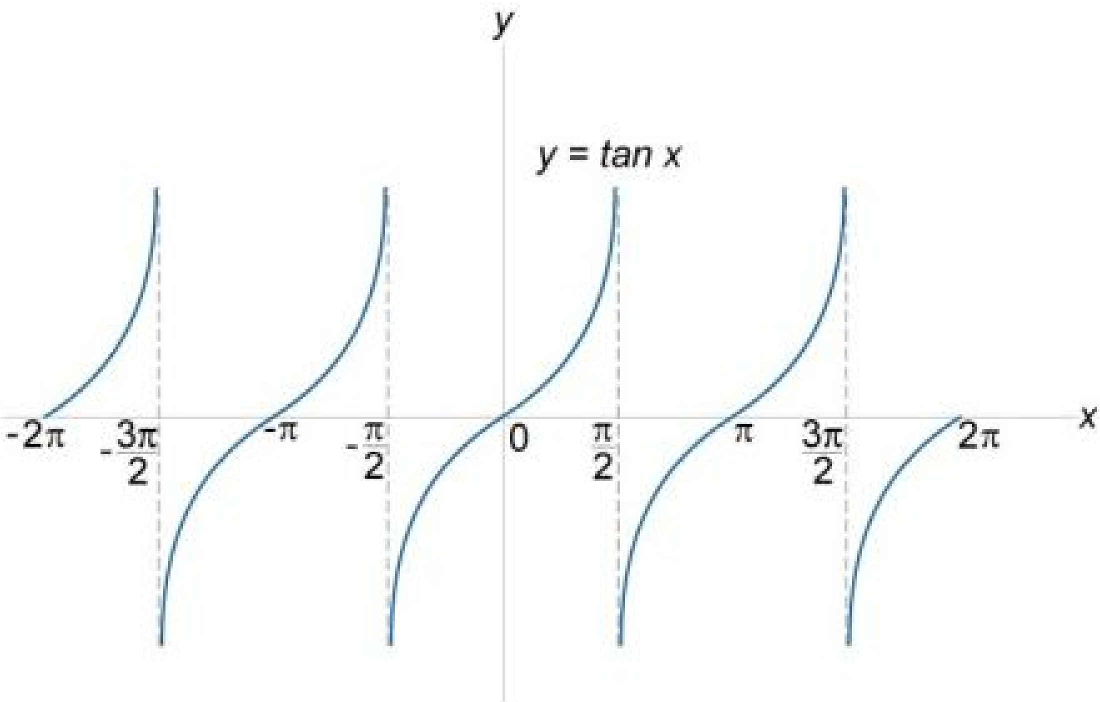

Figure 61.

376. Cotangent Function

$y = \cot x,\ x \neq k\pi,\ -\infty \leq \cot x \leq \infty.$

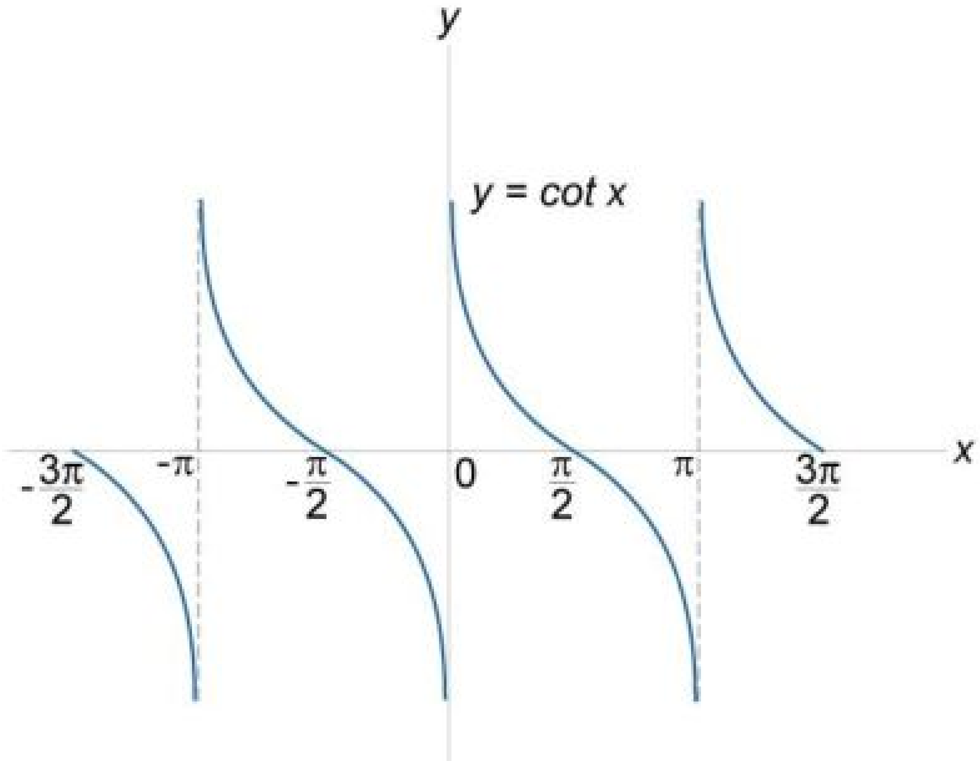

Figure 62.

377. Secant Function

$y = \sec x,\ x \neq (2k+1)\dfrac{\pi}{2}.$

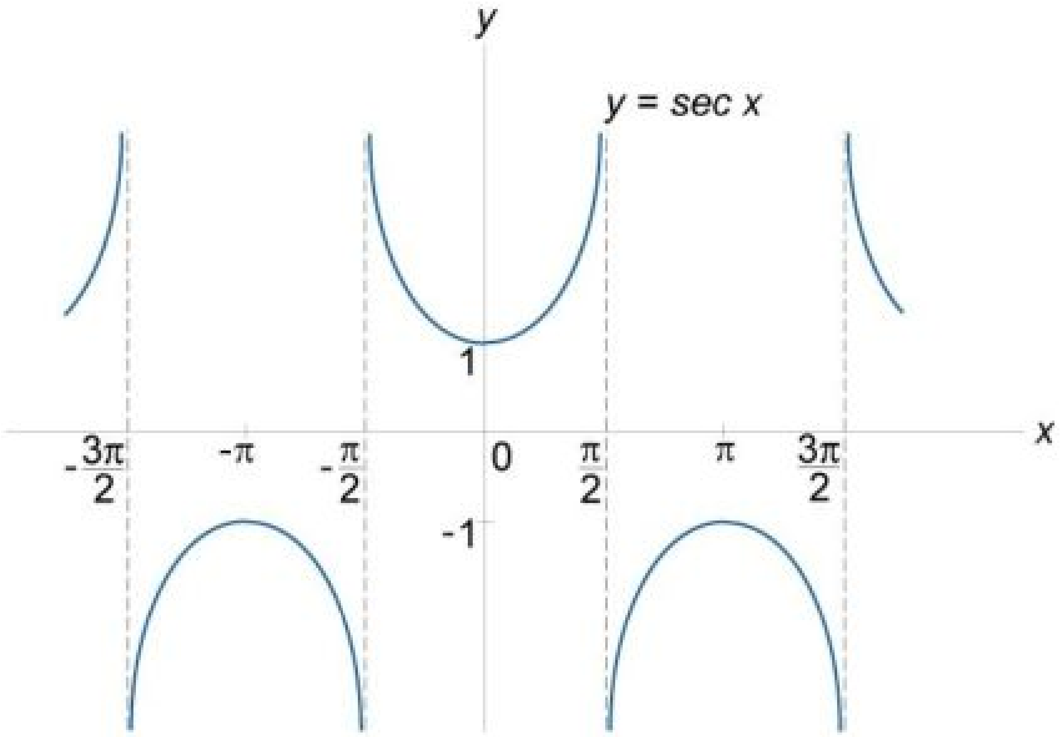

Figure 63.

378. Cosecant Function
$y = \operatorname{cosec} x, \; x \neq k\pi.$

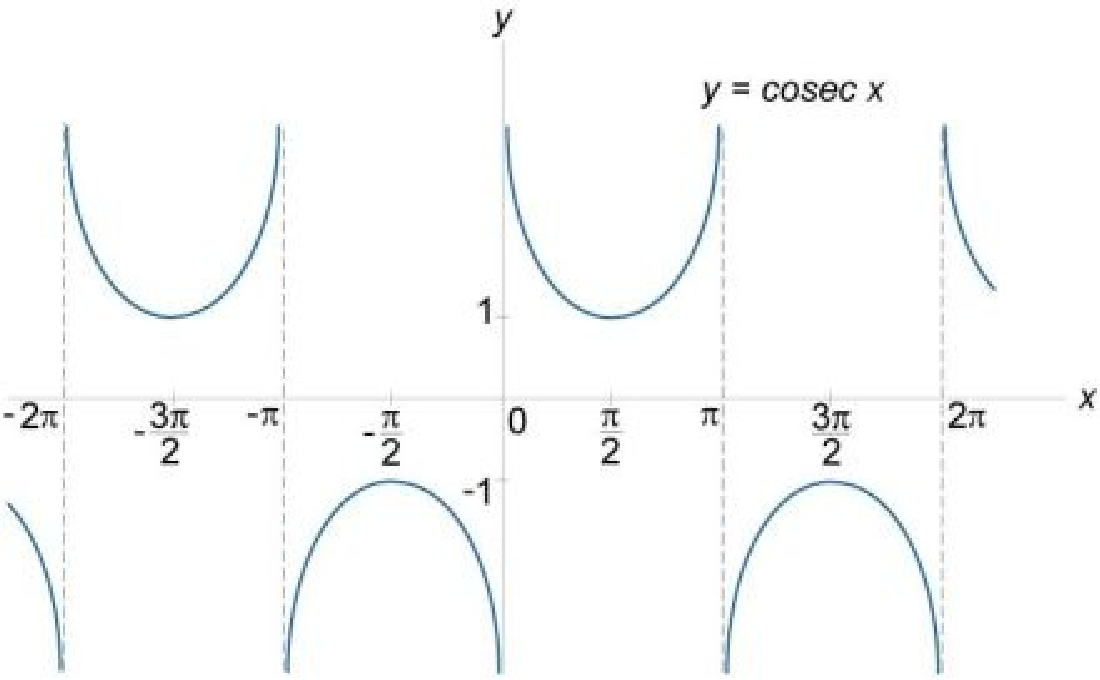

Figure 64.

4.3. Signs of Trigonometric Functions

379.

Quadrant	Sin α	Cos α	Tan α	Cot α	Sec α	Cosec α
I	+	+	+	+	+	+
II	+					+
III			+	+		
IV		+			+	

380.

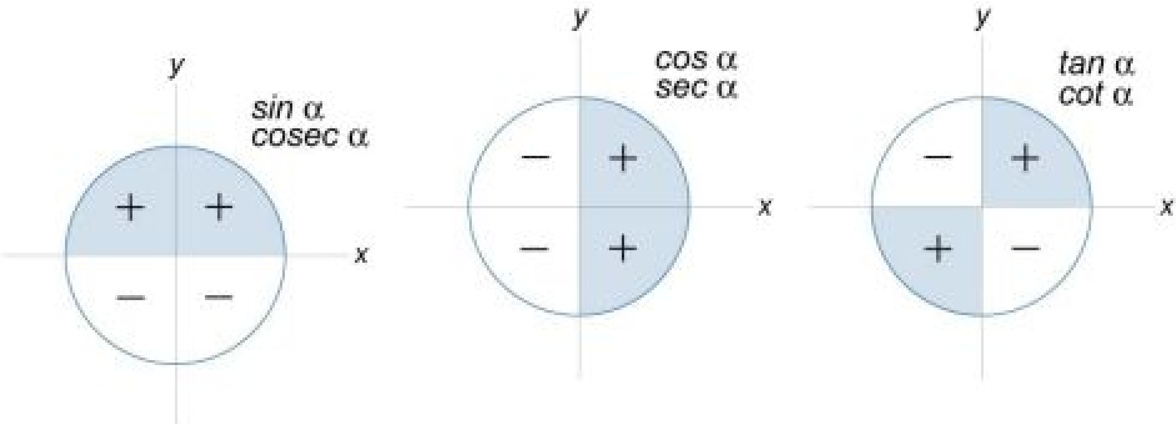

Figure 65.

4.4 Trigonometric Functions of Common Angles

381.

$\alpha°$	α rad	$\sin\alpha$	$\cos\alpha$	$\tan\alpha$	$\cot\alpha$	$\sec\alpha$	$\operatorname{cosec}\alpha$
0	0	0	1	0	∞	1	∞
30	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$	$\frac{2}{\sqrt{3}}$	2
45	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1	$\sqrt{2}$	$\sqrt{2}$
60	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{1}{\sqrt{3}}$	2	$\frac{2}{\sqrt{3}}$
90	$\frac{\pi}{2}$	1	0	∞	0	∞	1
120	$\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$	$-\frac{1}{\sqrt{3}}$	-2	$\frac{2}{\sqrt{3}}$
180	π	0	-1	0	∞	-1	∞
270	$\frac{3\pi}{2}$	-1	0	∞	0	∞	-1
360	2π	0	1	0	∞	1	∞

382.

$\alpha°$	α rad	$\sin\alpha$	$\cos\alpha$	$\tan\alpha$	$\cot\alpha$
15	$\frac{\pi}{12}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$2-\sqrt{3}$	$2+\sqrt{3}$
18	$\frac{\pi}{10}$	$\frac{\sqrt{5}-1}{4}$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\sqrt{\frac{5-2\sqrt{5}}{5}}$	$\sqrt{5+2\sqrt{5}}$
36	$\frac{\pi}{5}$	$\frac{\sqrt{10-2\sqrt{5}}}{4}$	$\frac{\sqrt{5}+1}{4}$	$\frac{\sqrt{10-2\sqrt{5}}}{\sqrt{5}+1}$	$\frac{\sqrt{5}+1}{\sqrt{10-2\sqrt{5}}}$
54	$\frac{3\pi}{10}$	$\frac{\sqrt{5}+1}{4}$	$\frac{\sqrt{10-2\sqrt{5}}}{4}$	$\frac{\sqrt{5}+1}{\sqrt{10-2\sqrt{5}}}$	$\frac{\sqrt{10-2\sqrt{5}}}{\sqrt{5}+1}$
72	$\frac{2\pi}{5}$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\frac{\sqrt{5}-1}{4}$	$\sqrt{5+2\sqrt{5}}$	$\sqrt{\frac{5-2\sqrt{5}}{5}}$
75	$\frac{5\pi}{12}$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$2+\sqrt{3}$	$2-\sqrt{3}$

4.5 Most Important Formulas

383. $\sin^2\alpha+\cos^2\alpha=1$

384. $\sec^2\alpha-\tan^2\alpha=1$

385. $\csc^2\alpha-\cot^2\alpha=1$

386. $\tan\alpha=\dfrac{\sin\alpha}{\cos\alpha}$

387. $\cot\alpha = \dfrac{\cos\alpha}{\sin\alpha}$

388. $\tan\alpha \cdot \cot\alpha = 1$

389. $\sec\alpha = \dfrac{1}{\cos\alpha}$

390. $\operatorname{cosec}\alpha = \dfrac{1}{\sin\alpha}$

4.6 Reduction Formulas

391.

β	$\sin\beta$	$\cos\beta$	$\tan\beta$	$\cot\beta$
$-\alpha$	$-\sin\alpha$	$+\cos\alpha$	$-\tan\alpha$	$-\cot\alpha$
$90°-\alpha$	$+\cos\alpha$	$+\sin\alpha$	$+\cot\alpha$	$+\tan\alpha$
$90°+\alpha$	$+\cos\alpha$	$-\sin\alpha$	$-\cot\alpha$	$-\tan\alpha$
$180°-\alpha$	$+\sin\alpha$	$-\cos\alpha$	$-\tan\alpha$	$-\cot\alpha$
$180°+\alpha$	$-\sin\alpha$	$-\cos\alpha$	$+\tan\alpha$	$+\cot\alpha$
$270°-\alpha$	$-\cos\alpha$	$-\sin\alpha$	$+\cot\alpha$	$+\tan\alpha$
$270°+\alpha$	$-\cos\alpha$	$+\sin\alpha$	$-\cot\alpha$	$-\tan\alpha$
$360°-\alpha$	$-\sin\alpha$	$+\cos\alpha$	$-\tan\alpha$	$-\cot\alpha$
$360°+\alpha$	$+\sin\alpha$	$+\cos\alpha$	$+\tan\alpha$	$+\cot\alpha$

4.7 Periodicity of Trigonometric Functions

392. $\sin(\alpha \pm 2\pi n) = \sin\alpha$, period 2π or $360°$.

393. $\cos(\alpha \pm 2\pi n) = \cos\alpha$, period 2π or $360°$.

394. $\tan(\alpha \pm \pi n) = \tan\alpha$, period π or $180°$.

395. $\cot(\alpha \pm \pi n) = \cot\alpha$, period π or $180°$.

4.8 Relations between Trigonometric Functions

396. $$\sin\alpha = \pm\sqrt{1-\cos^2\alpha} = \pm\sqrt{\frac{1}{2}(1-\cos 2\alpha)} = 2\cos^2\left(\frac{\alpha}{2}-\frac{\pi}{4}\right)-1$$
$$= \frac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$$

397. $$\cos\alpha = \pm\sqrt{1-\sin^2\alpha} = \pm\sqrt{\frac{1}{2}(1+\cos 2\alpha)} = 2\cos^2\frac{\alpha}{2}-1$$
$$= \frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$$

398. $$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \pm\sqrt{\sec^2\alpha - 1} = \frac{\sin 2\alpha}{1+\cos 2\alpha} = \frac{1-\cos 2\alpha}{\sin 2\alpha}$$

$$= \pm\sqrt{\frac{1-\cos 2\alpha}{1+\cos 2\alpha}} = \frac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$$

399. $$\cot\alpha = \frac{\cos\alpha}{\sin\alpha} = \pm\sqrt{\csc^2\alpha - 1} = \frac{1+\cos 2\alpha}{\sin 2\alpha} = \frac{\sin 2\alpha}{1-\cos 2\alpha}$$

$$= \pm\sqrt{\frac{1+\cos 2\alpha}{1-\cos 2\alpha}} = \frac{1-\tan^2\frac{\alpha}{2}}{2\tan\frac{\alpha}{2}}$$

400. $$\sec\alpha = \frac{1}{\cos\alpha} = \pm\sqrt{1+\tan^2\alpha} = \frac{1+\tan^2\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}}$$

401. $$\csc\alpha = \frac{1}{\sin\alpha} = \pm\sqrt{1+\cot^2\alpha} = \frac{1+\tan^2\frac{\alpha}{2}}{2\tan\frac{\alpha}{2}}$$

4.9 Addition and Subtraction Formulas

402. $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$

403. $\sin(\alpha - y) = \sin\alpha\cos\beta - \sin\beta\cos\alpha$

404. $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

405. $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

406. $\tan(\alpha+\beta)=\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

407. $\tan(\alpha-\beta)=\dfrac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$

408. $\cot(\alpha+\beta)=\dfrac{1-\tan\alpha\tan\beta}{\tan\alpha+\tan\beta}$

409. $\cot(\alpha-\beta)=\dfrac{1+\tan\alpha\tan\beta}{\tan\alpha-\tan\beta}$

4.10 Double Angle Formulas

410. $\sin 2\alpha = 2\sin\alpha\cdot\cos\alpha$

411. $\cos 2\alpha = \cos^2\alpha-\sin^2\alpha = 1-2\sin^2\alpha = 2\cos^2\alpha-1$

412. $\tan 2\alpha = \dfrac{2\tan\alpha}{1-\tan^2\alpha} = \dfrac{2}{\cot\alpha-\tan\alpha}$

413. $\cot 2\alpha = \dfrac{\cot^2\alpha-1}{2\cot\alpha} = \dfrac{\cot\alpha-\tan\alpha}{2}$

4.11 Multiple Angle Formulas

414. $\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha = 3\cos^2\alpha \cdot \sin\alpha - \sin^3\alpha$

415. $\sin 4\alpha = 4\sin\alpha \cdot \cos\alpha - 8\sin^3\alpha \cdot \cos\alpha$

416. $\sin 5\alpha = 5\sin\alpha - 20\sin^3\alpha + 16\sin^5\alpha$

417. $\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha = \cos^3\alpha - 3\cos\alpha \cdot \sin^2\alpha$

418. $\cos 4\alpha = 8\cos^4\alpha - 8\cos^2\alpha + 1$

419. $\cos 5\alpha = 16\cos^5\alpha - 20\cos^3\alpha + 5\cos\alpha$

420. $\tan 3\alpha = \dfrac{3\tan\alpha - \tan^3\alpha}{1 - 3\tan^2\alpha}$

421. $\tan 4\alpha = \dfrac{4\tan\alpha - 4\tan^3\alpha}{1 - 6\tan^2\alpha + \tan^4\alpha}$

422. $\tan 5\alpha = \dfrac{\tan^5\alpha - 10\tan^3\alpha + 5\tan\alpha}{1 - 10\tan^2\alpha + 5\tan^4\alpha}$

423. $\cot 3\alpha = \dfrac{\cot^3\alpha - 3\cot\alpha}{3\cot^2\alpha - 1}$

424. $\cot 4\alpha = \dfrac{1 - 6\tan^2\alpha + \tan^4\alpha}{4\tan\alpha - 4\tan^3\alpha}$

425. $\cot 5\alpha = \dfrac{1 - 10\tan^2\alpha + 5\tan^4\alpha}{\tan^5\alpha - 10\tan^3\alpha + 5\tan\alpha}$

4.12 Half Angle Formulas

426. $\sin\dfrac{\alpha}{2} = \pm\sqrt{\dfrac{1-\cos\alpha}{2}}$

427. $\cos\dfrac{\alpha}{2} = \pm\sqrt{\dfrac{1+\cos\alpha}{2}}$

428. $\tan\dfrac{\alpha}{2} = \pm\sqrt{\dfrac{1-\cos\alpha}{1+\cos\alpha}} = \dfrac{\sin\alpha}{1+\cos\alpha} = \dfrac{1-\cos\alpha}{\sin\alpha} = \csc\alpha - \cot\alpha$

429. $\cot\dfrac{\alpha}{2} = \pm\sqrt{\dfrac{1+\cos\alpha}{1-\cos\alpha}} = \dfrac{\sin\alpha}{1-\cos\alpha} = \dfrac{1+\cos\alpha}{\sin\alpha} = \csc\alpha + \cot\alpha$

4.13 Half Angle Tangent Identities

430. $\sin\alpha = \dfrac{2\tan\dfrac{\alpha}{2}}{1+\tan^2\dfrac{\alpha}{2}}$

431. $$\cos\alpha = \frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$$

432. $$\tan\alpha = \frac{2\tan\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}}$$

433. $$\cot\alpha = \frac{1-\tan^2\frac{\alpha}{2}}{2\tan\frac{\alpha}{2}}$$

4.14 Transforming of Trigonometric Expressions to Product

434. $$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

435. $$\sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

436. $$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

437. $$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

438. $\tan\alpha+\tan\beta=\dfrac{\sin(\alpha+\beta)}{\cos\alpha\cdot\cos\beta}$

439. $\tan\alpha-\tan\beta=\dfrac{\sin(\alpha-\beta)}{\cos\alpha\cdot\cos\beta}$

440. $\cot\alpha+\cot\beta=\dfrac{\sin(\beta+\alpha)}{\sin\alpha\cdot\sin\beta}$

441. $\cot\alpha-\cot\beta=\dfrac{\sin(\beta-\alpha)}{\sin\alpha\cdot\sin\beta}$

442. $\cos\alpha+\sin\alpha=\sqrt{2}\cos\left(\dfrac{\pi}{4}-\alpha\right)=\sqrt{2}\sin\left(\dfrac{\pi}{4}+\alpha\right)$

443. $\cos\alpha-\sin\alpha=\sqrt{2}\sin\left(\dfrac{\pi}{4}-\alpha\right)=\sqrt{2}\cos\left(\dfrac{\pi}{4}+\alpha\right)$

444. $\tan\alpha+\cot\beta=\dfrac{\cos(\alpha-\beta)}{\cos\alpha\cdot\sin\beta}$

445. $\tan\alpha-\cot\beta=-\dfrac{\cos(\alpha+\beta)}{\cos\alpha\cdot\sin\beta}$

446. $1+\cos\alpha=2\cos^2\dfrac{\alpha}{2}$

447. $1-\cos\alpha=2\sin^2\dfrac{\alpha}{2}$

448. $1+\sin\alpha = 2\cos^2\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)$

449. $1-\sin\alpha = 2\sin^2\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)$

4.15 Transforming of Trigonometric Expressions to Sum

450. $\sin\alpha\cdot\sin\beta = \frac{\cos(\alpha-\beta)-\cos(\alpha+\beta)}{2}$

451. $\cos\alpha\cdot\cos\beta = \frac{\cos(\alpha-\beta)+\cos(\alpha+\beta)}{2}$

452. $\sin\alpha\cdot\cos\beta = \frac{\sin(\alpha-\beta)+\sin(\alpha+\beta)}{2}$

453. $\tan\alpha\cdot\tan\beta = \frac{\tan\alpha+\tan\beta}{\cot\alpha+\cot\beta}$

454. $\cot\alpha\cdot\cot\beta = \frac{\cot\alpha+\cot\beta}{\tan\alpha+\tan\beta}$

455. $\tan\alpha\cdot\cot\beta = \frac{\tan\alpha+\cot\beta}{\cot\alpha+\tan\beta}$

4.16 Powers of Trigonometric Functions

456. $\sin^2\alpha = \dfrac{1-\cos 2\alpha}{2}$

457. $\sin^3\alpha = \dfrac{3\sin\alpha - \sin 3\alpha}{4}$

458. $\sin^4\alpha = \dfrac{\cos 4\alpha - 4\cos 2\alpha + 3}{8}$

459. $\sin^5\alpha = \dfrac{10\sin\alpha - 5\sin 3\alpha + \sin 5\alpha}{16}$

460. $\sin^6\alpha = \dfrac{10 - 15\cos 2\alpha + 6\cos 4\alpha - \cos 6\alpha}{32}$

461. $\cos^2\alpha = \dfrac{1+\cos 2\alpha}{2}$

462. $\cos^3\alpha = \dfrac{3\cos\alpha + \cos 3\alpha}{4}$

463. $\cos^4\alpha = \dfrac{\cos 4\alpha + 4\cos 2\alpha + 3}{8}$

464. $\cos^5\alpha = \dfrac{10\cos\alpha + 5\sin 3\alpha + \cos 5\alpha}{16}$

465. $\cos^6\alpha = \dfrac{10 + 15\cos 2\alpha + 6\cos 4\alpha + \cos 6\alpha}{32}$

4.17 Graphs of Inverse Trigonometric Functions

466. Inverse Sine Function

$$y = \arcsin x\,,\ -1 \le x \le 1\,,\ -\frac{\pi}{2} \le \arcsin x \le \frac{\pi}{2}\,.$$

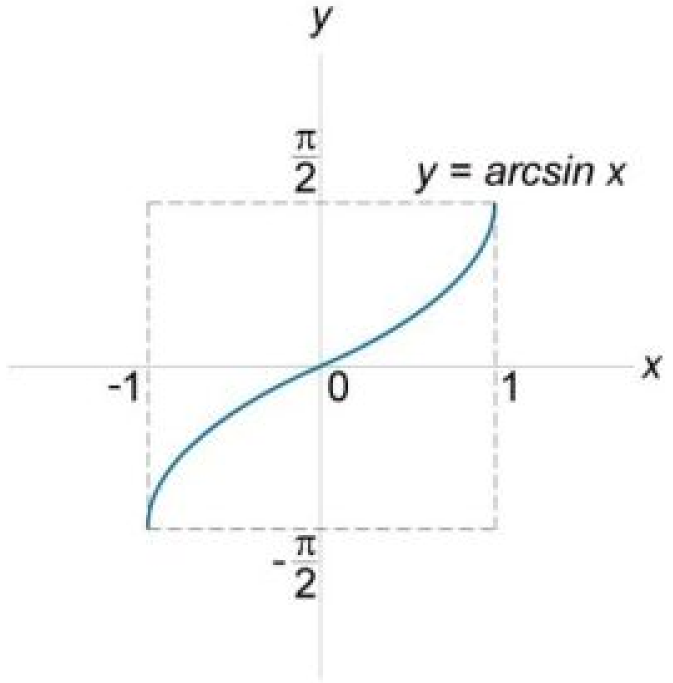

Figure 66.

467. Inverse Cosine Function

$$y = \arccos x\,,\ -1 \le x \le 1\,,\ 0 \le \arccos x \le \pi\,.$$

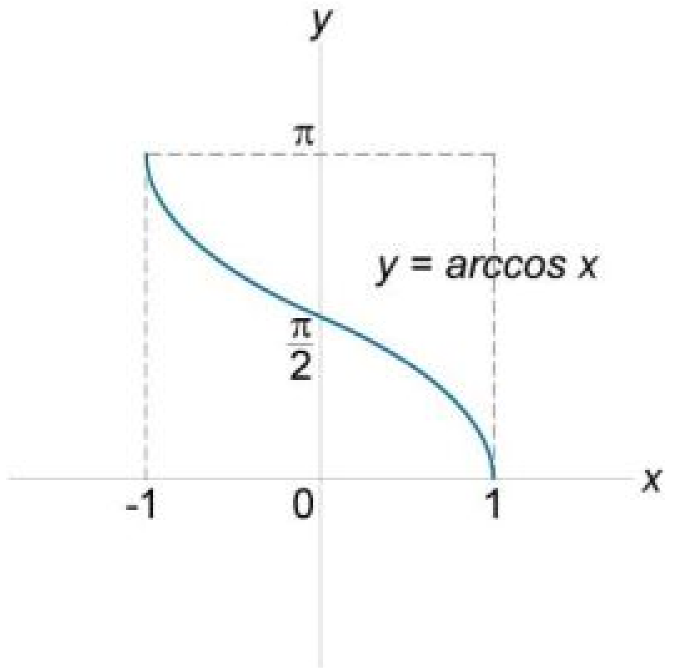

Figure 67.

468. Inverse Tangent Function

$$y = \arctan x, \ -\infty \le x \le \infty, \ -\frac{\pi}{2} < \arctan x < \frac{\pi}{2}.$$

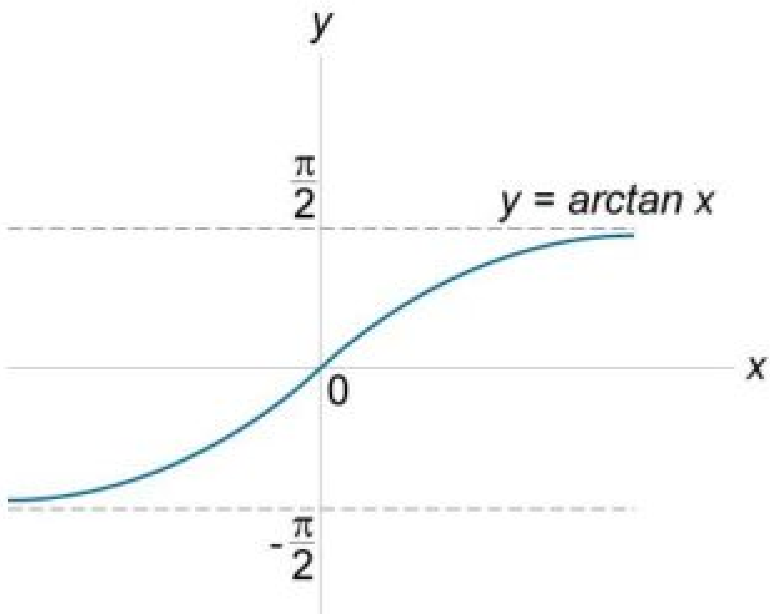

Figure 68.

469. Inverse Cotangent Function

$y = \operatorname{arccot} x$, $-\infty \le x \le \infty$, $0 < \operatorname{arccot} x < \pi$.

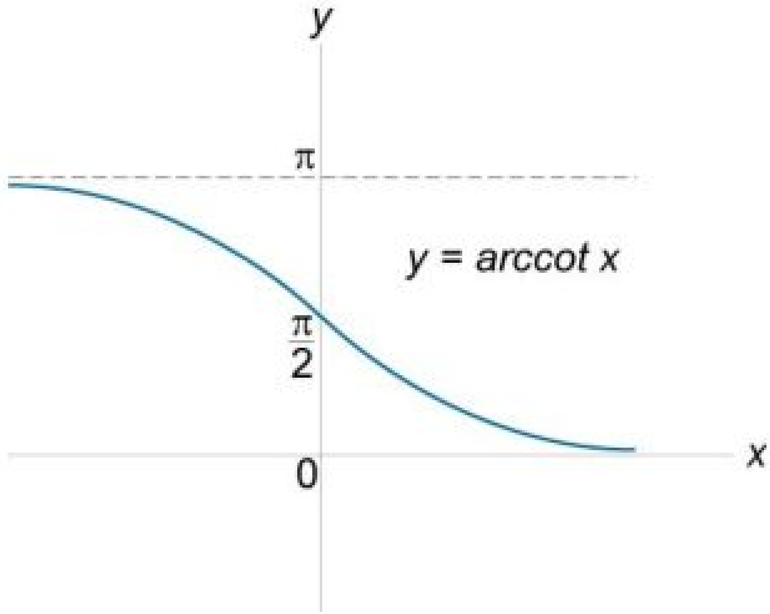

Figure 69.

470. Inverse Secant Function

$y = \operatorname{arcsec} x$, $x \in (-\infty, -1] \cup [1, \infty)$, $\operatorname{arcsec} x \in \left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$.

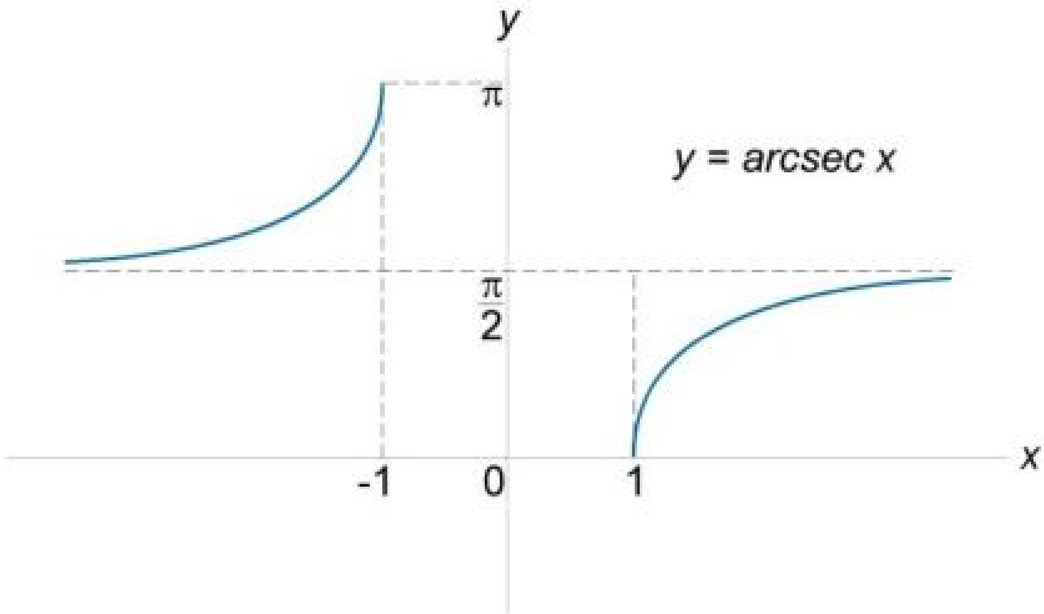

Figure 70.

471. Inverse Cosecant Function

$$y = \operatorname{arccsc} x,\ \ x \in (-\infty, -1] \cup [1, \infty),\ \ \operatorname{arccsc} x \in \left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right].$$

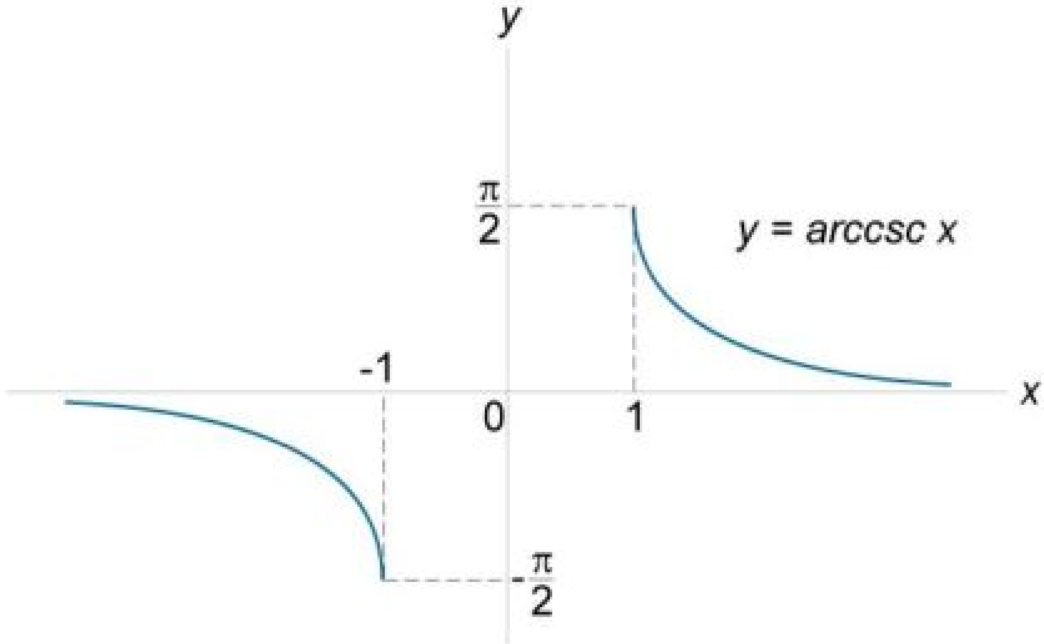

Figure 71.

4.18 Principal Values of Inverse Trigonometric Functions

472.

x	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
arcsin x	0°	30°	45°	60°	90°
arccos x	90°	60°	45°	30°	0°
x	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	
arcsin x	−30°	−45°	−60°	−90°	
arccos x	120°	135°	150°	180°	

473.

x	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$-\frac{\sqrt{3}}{3}$	-1	$-\sqrt{3}$
$\arctan x$	$0°$	$30°$	$45°$	$60°$	$-30°$	$-45°$	$-60°$
$\operatorname{arccot} x$	$90°$	$60°$	$45°$	$30°$	$120°$	$135°$	$150°$

4.19 Relations between Inverse Trigonometric Functions

474. $\arcsin(-x) = -\arcsin x$

475. $\arcsin x = \frac{\pi}{2} - \arccos x$

476. $\arcsin x = \arccos\sqrt{1-x^2}$, $0 \le x \le 1$.

477. $\arcsin x = -\arccos\sqrt{1-x^2}$, $-1 \le x \le 0$.

478. $\arcsin x = \arctan\frac{x}{\sqrt{1-x^2}}$, $x^2 < 1$.

479. $\arcsin x = \operatorname{arccot}\frac{\sqrt{1-x^2}}{x}$, $0 < x \le 1$.

480. $\arcsin x = \operatorname{arccot}\frac{\sqrt{1-x^2}}{x} - \pi$, $-1 \le x < 0$.

481. $\arccos(-x) = \pi - \arccos x$

482. $\arccos x = \frac{\pi}{2} - \arcsin x$

483. $\arccos x = \arcsin \sqrt{1-x^2}$, $0 \le x \le 1$.

484. $\arccos x = \pi - \arcsin \sqrt{1-x^2}$, $-1 \le x \le 0$.

485. $\arccos x = \arctan \frac{\sqrt{1-x^2}}{x}$, $0 < x \le 1$.

486. $\arccos x = \pi + \arctan \frac{\sqrt{1-x^2}}{x}$, $-1 \le x < 0$.

487. $\arccos x = \operatorname{arccot} \frac{x}{\sqrt{1-x^2}}$, $-1 \le x \le 1$.

488. $\arctan(-x) = -\arctan x$

489. $\arctan x = \frac{\pi}{2} - \operatorname{arccot} x$

490. $\arctan x = \arcsin \frac{x}{\sqrt{1+x^2}}$

491. $\arctan x = \arccos \frac{1}{\sqrt{1+x^2}}$, $x \ge 0$.

492. $\arctan x = -\arccos \frac{1}{\sqrt{1+x^2}}$, $x \le 0$.

493. $\arctan x = \frac{\pi}{2} - \arctan\frac{1}{x},\ x > 0.$

494. $\arctan x = -\frac{\pi}{2} - \arctan\frac{1}{x},\ x < 0.$

495. $\arctan x = \operatorname{arccot}\frac{1}{x},\ x > 0.$

496. $\arctan x = \operatorname{arccot}\frac{1}{x} - \pi,\ x < 0.$

497. $\operatorname{arccot}(-x) = \pi - \operatorname{arccot} x$

498. $\operatorname{arccot} x = \frac{\pi}{2} - \arctan x$

499. $\operatorname{arccot} x = \arcsin\frac{1}{\sqrt{1+x^2}},\ x > 0.$

500. $\operatorname{arccot} x = \pi - \arcsin\frac{1}{\sqrt{1+x^2}},\ x < 0.$

501. $\operatorname{arccot} x = \arccos\frac{x}{\sqrt{1+x^2}}$

502. $\operatorname{arccot} x = \arctan\frac{1}{x},\ x > 0.$

503. $\operatorname{arccot} x = \pi + \arctan\frac{1}{x},\ x < 0.$

4.20 Trigonometric Equations

Whole number: n

504. $\sin x = a\,,\ \ x = (-1)^n \arcsin a + \pi n$

505. $\cos x = a\,,\ \ x = \pm \arccos a + 2\pi n$

506. $\tan x = a\,,\ \ x = \arctan a + \pi n$

507. $\cot x = a\,,\ \ x = \operatorname{arccot} a + \pi n$

4.21 Relations to Hyperbolic Functions

Imaginary unit: i

508. $\sin(ix) = i \sinh x$

509. $\tan(ix) = i \tanh x$

510. $\cot(ix) = -i \coth x$

511. $\sec(ix) = \operatorname{sech} x$

512. $\csc(ix) = -i \operatorname{csch} x$