Chapter 12

Probability

12.1 Permutations and Combinations

Permutations: ${}^{n}P_{m}$
Combinations: ${}^{n}C_{m}$
Whole numbers: n, m

1251. Factorial
$$n! = 1\cdot 2\cdot 3\ldots(n-2)(n-1)n$$
$$0! = 1$$

1252. ${}^{n}P_{n} = n!$

1253. $${}^{n}P_{m} = \frac{n!}{(n-m)!}$$

1254. Binomial Coefficient
$${}^{n}C_{m} = \binom{n}{m} = \frac{n!}{m!(n-m)!}$$

1255. ${}^{n}C_{m} = {}^{n}C_{n-m}$

1256. ${}^{n}C_{m} + {}^{n}C_{m+1} = {}^{n+1}C_{m+1}$

1257. ${}^{n}C_0 + {}^{n}C_1 + {}^{n}C_2 + \ldots + {}^{n}C_n = 2^n$

1258. Pascal's Triangle

Row 0							1						
Row 1						1		1					
Row 2					1		2		1				
Row 3				1		3		3		1			
Row 4			1		4		6		4		1		
Row 5		1		5		10		10		5		1	
Row 6	1		6		15		20		15		6		1

12.2 Probability Formulas

Events: A, B
Probability: P
Random variables: X, Y, Z
Values of random variables: x, y, z
Expected value of X: μ
Any positive real number: ε
Standard deviation: σ
Variance: σ^2
Density functions: $f(x)$, $f(t)$

1259. Probability of an Event

$$P(A) = \frac{m}{n},$$

where
m is the number of possible positive outcomes,
n is the total number of possible outcomes.

1260. Range of Probability Values
$0 \le P(A) \le 1$

1261. Certain Event
$P(A) = 1$

1262. Impossible Event
$P(A) = 0$

1263. Complement
$P(\overline{A}) = 1 - P(A)$

1264. Independent Events
$P(A/B) = P(A)$,
$P(B/A) = P(B)$

1265. Addition Rule for Independent Events
$P(A \cup B) = P(A) + P(B)$

1266. Multiplication Rule for Independent Events
$P(A \cap B) = P(A) \cdot P(B)$

1267. General Addition Rule
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$,
where
$A \cup B$ is the union of events A and B,
$A \cap B$ is the intersection of events A and B.

1268. Conditional Probability
$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

1269. $P(A \cap B) = P(B) \cdot P(A/B) = P(A) \cdot P(B/A)$

1270. Law of Total Probability

$$P(A)=\sum_{i=1}^{m} P(B_i)P(A/B_i),$$

where B_i is a sequence of mutually exclusive events.

1271. Bayes' Theorem

$$P(B/A)=\frac{P(A/B)\cdot P(B)}{P(A)}$$

1272. Bayes' Formula

$$P(B_i/A)=\frac{P(B_i)\cdot P(A/B_i)}{\sum_{k=1}^{m} P(B_i)\cdot P(A/B_i)},$$

where

B_i is a set of mutually exclusive events (hypotheses),

A is the final event,

$P(B_i)$ are the prior probabilities,

$P(B_i/A)$ are the posterior probabilities.

1273. Law of Large Numbers

$$P\left(\left|\frac{S_n}{n}-\mu\right|\geq\varepsilon\right)\to 0 \text{ as } n\to\infty,$$

$$P\left(\left|\frac{S_n}{n}-\mu\right|<\varepsilon\right)\to 1 \text{ as } n\to\infty,$$

where

S_n is the sum of random variables,

n is the number of possible outcomes.

1274. Chebyshev Inequality

$$P(|X-\mu|\geq\varepsilon)\leq\frac{V(X)}{\varepsilon^2},$$

where $V(X)$ is the variance of X.

1275. Normal Density Function

$$\varphi(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}},$$

where x is a particular outcome.

1276. Standard Normal Density Function

$$\varphi(z)=\frac{1}{\sqrt{2\pi}}e^{-\frac{z^2}{2}}$$

Average value $\mu=0$, deviation $\sigma=1$.

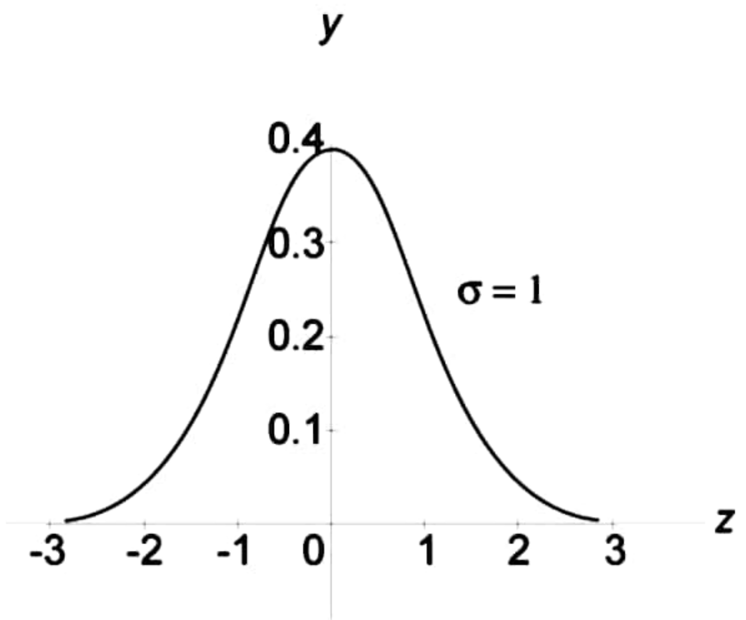

Figure 210.

1277. Standard Z Value

$$Z=\frac{X-\mu}{\sigma}$$

1278. Cumulative Normal Distribution Function

$$F(x)=\frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{x}e^{-\frac{(t-\mu)^2}{2\sigma^2}}dt,$$

where
x is a particular outcome,
t is a variable of integration.

1279. $P(\alpha < X < \beta) = F\left(\frac{\alpha - \mu}{\sigma}\right) - F\left(\frac{\beta - \mu}{\sigma}\right)$,

where
X is normally distributed random variable,
F is cumulative normal distribution function,
$P(\alpha < X < \beta)$ is interval probability.

1280. $P(|X - \mu| < \varepsilon) = 2F\left(\frac{\varepsilon}{\sigma}\right)$,

where
X is normally distributed random variable,
F is cumulative normal distribution function.

1281. Cumulative Distribution Function

$F(x) = P(X < x) = \int_{-\infty}^{x} f(t)dt$,

where t is a variable of integration.

1282. Bernoulli Trials Process

$\mu = np \quad , \sigma^2 = npq$,
where
n is a sequence of experiments,
p is the probability of success of each experiments,
q is the probability of failure, $q = 1 - p$.

1283. Binomial Distribution Function

$b(n,p,q) = \binom{n}{k} p^k q^{n-k}$,

$\mu = np$, $\sigma^2 = npq$,
$f(x) = (q + pe^x)^n$,
where
n is the number of trials of selections,
p is the probability of success,
q is the probability of failure, $q = 1 - p$.

1284. Geometric Distribution
$P(T = j) = q^{j-1}p$,
$\mu = \frac{1}{p}$, $\sigma^2 = \frac{q}{p^2}$,
where
T is the first successful event is the series,
j is the event number,
p is the probability that any one event is successful,
q is the probability of failure, $q = 1 - p$.

1285. Poisson Distribution
$P(X = k) \approx \frac{\lambda^k}{k!}e^{-\lambda}$, $\lambda = np$,
$\mu = \lambda$, $\sigma^2 = \lambda$,
where
λ is the rate of occurrence,
k is the number of positive outcomes.

1286. Density Function
$$P(a \le X \le b) = \int_a^b f(x)dx$$

1287. Continuous Uniform Density
$f = \frac{1}{b-a}$, $\mu = \frac{a+b}{2}$,

where f is the density function.

1288. Exponential Density Function
$f(t)=\lambda e^{-\lambda t}$, $\mu=\lambda$, $\sigma^2=\lambda^2$
where t is time, λ is the failure rate.

1289. Exponential Distribution Function
$F(t)=1-e^{-\lambda t}$,
where t is time, λ is the failure rate.

1290. Expected Value of Discrete Random Variables
$$\mu=E(X)=\sum_{i=1}^{n} x_i p_i,$$
where x_i is a particular outcome, p_i is its probability.

1291. Expected Value of Continuous Random Variables
$$\mu=E(X)=\int_{-\infty}^{\infty} x f(x)dx$$

1292. Properties of Expectations
$E(X+Y)=E(X)+E(Y)$,
$E(X-Y)=E(X)-E(Y)$,
$E(cX)=cE(X)$,
$E(XY)=E(X)\cdot E(Y)$,
where c is a constant.

1293. $E(X^2)=V(X)+\mu^2$,
where
$\mu=E(X)$ is the expected value,
$V(X)$ is the variance.

1294. Markov Inequality

$$P(X > k) \le \frac{E(X)}{k},$$

where k is some constant.

1295. Variance of Discrete Random Variables

$$\sigma^2 = V(X) = E\left[(X-\mu)^2\right] = \sum_{i=1}^{n} (x_i - \mu)^2 p_i,$$

where

x_i is a particular outcome,

p_i is its probability.

1296. Variance of Continuous Random Variables

$$\sigma^2 = V(X) = E\left[(X-\mu)^2\right] = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$$

1297. Properties of Variance

$$V(X+Y) = V(X) + V(Y),$$

$$V(X-Y) = V(X) + V(Y),$$

$$V(X+c) = V(X),$$

$$V(cX) = c^2 V(X),$$

where c is a constant.

1298. Standard Deviation

$$D(X) = \sqrt{V(X)} = \sqrt{E\left[(X-\mu)^2\right]}$$

1299. Covariance

$$\operatorname{cov}(X,Y) = E\left[(X-\mu(X))(Y-\mu(Y))\right] = E(XY) - \mu(X)\mu(Y),$$

where

X is random variable,

$V(X)$ is the variance of X,

μ is the expected value of X or Y.

1300. Correlation

$$\rho(X,Y)=\frac{\operatorname{cov}(X,Y)}{\sqrt{V(X)V(Y)}},$$

where

$V(X)$ is the variance of X,

$V(Y)$ is the variance of Y.