1.

a) $x^3 - 8x + 15 = 0$ & $2x^3 - x - 15 = 0$

Soln.

$$\begin{array}{cccc} 1 & -8 & 15 & 1 \\ 2 & -1 & -15 & 2 \end{array}$$

1st exp. $\left((1 \times -1) - (2 \times -8)\right)\left((15 \times 8) - (-15 \times -1)\right)$

$= \cancel{17 \times 45}$

$= 2025$

2nd exp $= \left(15 \times 2 - (-15 \times 1)\right)^2$

$= (45)^2 = 2025$

b. $3x^2 - 8x + 4 = 0$ & $4x^2 - 7x - 2 = 0$

Soln,

$$\begin{array}{cccc} 3 & -8 & 4 & 3 \\ 4 & -7 & -2 & 4 \end{array}$$

1st exp $= (-21 - (32)) - (16 - (-28))$

$= 11 \times 44$

$= 484$

2nd exp $= (16 - (-6))^2$

$= (22)^2$

$= 484$

2.

a) $4x^2 + px - 12 = 0$ & $4x^2 + 3p - 4 = 0$

here,

4	p	-12	4
4	3p	-4	4

1st exp = 2nd exp

$(12p - 4p)(-4p - (-36p)) = (-48 - (-16))^2$

$(8p)(32p) = (-32)^2$

$\cancel{32} \times 8p^2 = \cancel{32} \times 32$

$p^2 = 4$

$p = \sqrt{4}$

$p = \pm 2$

b)

2	p	-1	2
3	-2	-5	3

now,

$(2 \times (-2) - 3p)(-5p - 2) = (-1 \times 3 - (-5 \times 2))^2$

$(-4 - 3p)(-5p - 2) = (-3 + 10)^2$

$20p + 8 + 15p^2 + 6p = 49$

$15p^2 + 26p - 41 = 0$

$p^2 + \frac{26p}{15} - \frac{41}{15} = 0$

$15p^2 - \cancel{\frac{41p}{15}} + 26p - 41 = 0$

$\therefore p = 1$ or $p = -41/15$

3.

Soln,

$$\frac{x^2}{\begin{vmatrix} p & q \\ p' & q' \end{vmatrix}} = \frac{-x}{\begin{vmatrix} 1 & q \\ 1 & q_1 \end{vmatrix}} = \frac{-1}{\begin{vmatrix} 1 & p \\ 1 & p_1 \end{vmatrix}}$$

$$x = \frac{-(pq' - p'q)}{q' - q}$$

$$x = \frac{qp' - q'p}{q' - q} = \frac{pq' - p'q}{q - q'}$$

(∴ taking -'ve common)

$$-x = \frac{q' - q}{p' - p}$$

$$x = \frac{q - q'}{p' - p}$$

∴ either $\frac{pq' - p'q}{q - q'}$ or $\frac{q - q'}{p' - p}$

4.

Soln,

$$x^2 + px + q = 0 \quad -(\text{i})$$

$$x^2 + qx + p = 0 \quad -(\text{ii})$$

now,

$$\overbrace{x^2 = -x}^{1st} \; \overbrace{= 1}^{2nd}$$

$$\begin{array}{cccccc} p & q & 1 & q & 1 & p \\ q & p & 1 & p & 1 & q \end{array}$$

1st
$$x = \frac{-(p^2 - q^2)}{(p-q)}$$

$$x = \frac{-(p+q)\cancel{(p-q)}}{\cancel{(p-q)}}$$

$$x = -(p+q) \quad -(\text{iii})$$

2nd.
$$-x = \frac{p-q}{q-p}$$

$$x = 1$$

from (iii)

$$-p - q = 1$$

$$\therefore p + q + 1 = 0$$

Soln.

$$ax^2 + bx + c = 0$$
$$bx^2 + cx + a = 0$$

x^2	$= -x$	$= 1$
$b \quad c$	$a \quad c$	$a \quad b$
$c \quad a$	$b \quad a$	$b \quad c$

1st exp $= \dfrac{-(ab - c^2)}{(a^2 - bc)}$

$$= \frac{c^2 - ab}{a^2 - bc} \quad \text{--- (i)}$$

2nd exp, $-x = \dfrac{a^2 - bc}{ac - b^2}$ (ii)

From (i) & (ii)

$$\frac{c^2 - ab}{a^2 - bc} = \frac{-(a^2 - bc)}{ac - b^2}$$

$$\frac{c^2 - ab}{a^2 - bc} = \frac{bc - a^2}{ac - b^2}$$

$$(c^2 - ab)(ac - b^2) = (bc - a^2)(a^2 - bc)$$

or, $ac^3 - \cancel{c^2b^2} - a^2bc + ab^3 = bca^2 - \cancel{b^2c^2} - a^4 + a^2bc$

or, $ac^3 + a^4 + ab^3 - 3a^2bc = 0$

$c^3 + a^3 + b^3 - 3abc = 0$

$(a+b+c)(a^2+b^2+c^2-ab-bc-ca) = 0$

$a+b+c = 0$ {$\because a^3+b^3+c^3$}

$\therefore a+b+c = 0$

proved

here,

$x^2 + bx + ca = 0$ — (1)

$x^2 + cx + ab = 0$ — (2)

$$\frac{x^2}{\begin{vmatrix} b & ca \\ c & ab \end{vmatrix}} = \frac{-x}{\begin{vmatrix} 1 & ca \\ 1 & ab \end{vmatrix}} = \frac{1}{\begin{vmatrix} 1 & b \\ 1 & c \end{vmatrix}}$$

1st, $x = -\frac{(ab^2 - c^2a)}{ab - ca}$

$= \frac{c^2 - b^2}{b - c}$

$= -(c+b)$ — (i)

2nd. $x = -\frac{(ab - ac)}{c - b}$

$= a$ — (ii)

now, $a+b+c = 0$ {from (i) & (ii)}

in (1)

$\alpha + \beta = -b$

$\alpha \cdot \beta = ca$

in (2)

$\alpha + \boxed{\beta'} = -c$

$\alpha \cdot \boxed{\beta'} = ab$

now,

$\beta = -b - \alpha$

and,

$\beta' = -c - \alpha$

$x^2 - (\beta + \beta')x + \beta \cdot \beta' = 0$

$x^2 - [-b - a + (-c - a)]x + [-(b+a) \cdot -(c+a)] = 0$

$x^2 + (b + a + c + a)x + (bc + ba + ac + a^2) = 0$

$x^2 + ax + (bc + a(a+b+c)) = 0 \quad \{\because a+b+c$

$x^2 + ax + bc = 0$

hence proved #