For example, the conventional tabu search can be repro-duced by setting α → ∞, kr = 1 and the size of memory s as the length of the tabu list. This means that the strength of infinity/ On the other hand, it is possible for this neural network model to decrease the tabu effect exponentially with 0 < kr < 1 and a positive finite α. We will call this version the exponential tabu search in the following.

2.1.3 Chaotic neural network based on paths

The structure of the above neural network is similar to the chaotic neural network (Aihara, 1990; Aihara et al., 1990). The chaotic neural network also has the refractory effect (corresponding to the tabu effect of the above tabu search neural network) which decreases exponentially with the parameter kr. This refractory effect with exponential decay is almost the same as the tabu effect of the exponent-tial tabu search realized by the tabu search neural network described above. However, there is a significant difference on the output function. The output of each neuron xij(t) of the above tabu search neural network is determined to be 0 or 1 by detecting the maximum internal state of Eq. (3) among all neurons. This discrete output is used for deter-mining whether the corresponding move is memorized in the tabu list, and it is an essential aspect of the tabu effect in the conventional tabu search. On the other hand, the chaotic neural network originally adopts an analog sigmoi-dal function (Aihara, 1990; Aihara et al., 1990) rather than the Heaviside output function of the Caisaniello neuron model (Caianielo, 1961) and the Nagumo-Sato neuron model (Nagumo & Sato,1972). The non linear analog function produces deterministic but complicated chaotic dynamics in the chaotic neuron model. If a discrete output function, such as the Heaviside step function, which only takes 0 or 1 outputs, is used for the chaotic neuron, chaotic behavior cannot be observed (Aihara, 1990; Aihara et al., 1990). Then, it is expected that the chaotic dynamics may also be realized on the tabu search neural network (Eqs. (1) and (2)) by replacing the discrete output function to the analog sigmoidal function. This chaotic method is different from the conventional tabu search, but the effect of the tabu is still preserved, even if we use the analog output function, For the case of analog output function, the firing of neuron is defined by the condition that xij(t + 1) > 1/2, whrere xij(t + 1) = f｛εij(t + 1) +ηi,j( t + 1) + εa(i)(j)(t + 1) + εij(t + 1)}, f(y) = 1/(1 + e -y/e), and the detection of the maxi-connections with the internal state ηij(t) (Eq. (5)).

Then, the novel method, which includes both of the tabu effect and chaotic dynamics, is realized by the following equations with an asynchronous updating:

If xij(t + 1) > 1/2, the (i, j)th neuron fires and the path between cities i and j is connected with the 2-opt exchange as shown in Fig. 2.

It should be noted that this neural network is updated asynchronously. In this neural network, several neurons have possibilities to satisfy the condition for firing in a single iteration. Then, events on firing(a 2-opt updating) should be done for all those firings. However, it is impos-sible to apply more than one 2-opt exchanges simul-taneously. If we did that, it would be risky for keeping solutions feasible. Then, the asynchronous update is intro-deuced in this paper, because no simultaneous exchanges are required.

For numerical calculation, Eq. (6) can be reduced to the following forms: if t < s, where R = 0(1 - kr). Here, we assume that xij(u) = 0 for u < 0, which means that there is generally no tabu effect for initial conditions.

In the original chaotic neural network (Aihara, 1990; Aihara et al., 1990), s -1 = t in Eq. (6), which means that the chaotic neural network keeps whole previous memory effects from t = 0, which exponentially decrease with kr. The conventional tabu search, whose strength is always infinite, should not continue forever for permitting previous moves which leads to the better solutions not found so far. The performance of the conventional tabu search depends on tabu list memory size s. On the other hand, longer memory can be preserved in the proposed chaotic search. Moreover, this search includes not only tabu search but also chaotic fluctuation which is considered effective for combinatorial optimization (Chen & Aihara, 1995; Hasegawa et al., 1995; Hasegawa et al.,1997; Ishii & Satoh, 1997; Nozawa, 1992; Yamada & Aihara, 1997).

Although the proposed chaotic searching method may realize efficient search, this still requires large computer memory because it consists of neuron s of the order of n² shown in Fig. 2. We call this the two-dimensional method.

2.2. Solving an n-city TSP with n neurons (one-dimensional method)

2.2.1. Tabu search memorizing cites

Another tabu list which is based on only one -dimensional elements, cities, is introduced for the base of our chaotic search. This tabu list consists of cities which have appeared in a 2-opt updating. Since this list cannot memorize the paths which have been connected, it may not be so enough as the tabu list with paths in Section 2.1., for memorizing previously searched states. The advantage of using the one-dimensional list is that the chaotic search approach based on this list requires a much smaller size of memories paths, each path should be denoted by two cities, and the size of the tabu search neural network becomes the order of n². On the other hand, the tabu search memorizing cities can be realized by a neural network with only n neurons, because cities can be labeled by only n elements.

Here, we introduce such a tabu search as follows: when the 2-opt exchange, which links the city I with j, is done as shown in Fig. 3, only the city I is memorized in the tabu list. In each 2-opt updating, if both cities I and j are not tabu and the gain is the largest, the corresponding move is chosen.

2.2.2. Tabu search neural network memorizing cities

For implementing the above tabu search on a neural network, n neurons are prepared for an n-city TSP. The ith neuron corresponds to the city i. In the case of updating the ith neuron, the city j should be selected at first because it is a candidate for being connected with the city I(Fig.3). Then, this city j also should be not tabu. Namely, both tabu effects of the ith and the jth neurons should be small. Furthermore, the corresponding 2-opt exchange, which links the cities i and j, should offer larger gain for being selected as a real update.

Then, we construct the neural network as follows: the input to the ith neuron, εi(t), includes both the tabu effect of the jth neuron and the gain effect which is offered by the 2-opt exchange linking the cities i and j. Here, this input, εi(t), is only from the best one for connecting with the city i, namely, the smaller tabu effect εj(t) and the larger gain Δij(t). In order to select such a best j, the city corresponding to the maximum of εi(t) + Δij(t) is chosen. Then, this maximum value is applied to the ith neuron as the internal state εi(t). In this neural network, if the ith neuron fired, the 2-opt exchange is executed to connect the cities I and j as shown in Fig. 3. The internal state corresponding to the two-dimensional method (Selection 2.1.2).

Then, the tabu search neural network based on cities can be realized by the following equations: If {εi(t + 1) + εi(t + 1)} is the largest in all neurons, the corresponding move is actually executed. Namely, the city I is connected to the city which was selected as j in Eq. (10), using the 2-opt exchange.

By this neural network, if the ith neuron has fired in previous s iterations, the firing of this neuron is avoided by the internal state εi(t). Even if the tabu effect of the jth neuron selected in Eq. (10) is very small, it does not fire when the ith neuron has a strong tabu effect.

The tabu search in Section 2.2.1 can be realized by setting α → ∞, kr = 1 and the size of memory s as the length of the tabu list.