Christian Chu, Nafis Isfar

HYDROELECTRIC

X = 6, Y=3

The water starting constant $T_w$ of a hydroelectric power plant is 2.6s. At some point in the day it operates at a 35% mechanical power output. The inertia constant of the turbine-generator couple is 3.3s.

- Even-numbered groups have a transient droop controller per Hovey's method.
- They also have a separate same hydroelectric power plant, but with a governor of droop R=X% and surge tank that emulates Hovey's method.

a. Simulate the model for a time of 30s, subject to at least 3 changes in the electric power demand $P_e$ that are at least 5s apart, both in increasing and decreasing directions.
    a. Standard hydro
    b. Surge Tank

b. For a non-ideal turbine:

- Even-numbered groups repeat (a) for starting mechanical power 5% and changes that do not exceed a total of 30% or decrease below 0%.

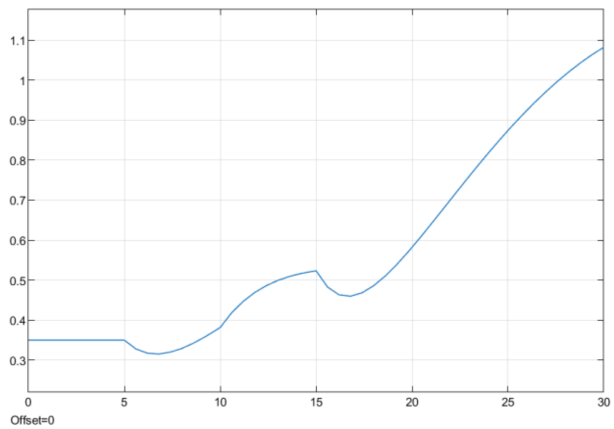
Table of Contents:

RESULTS:

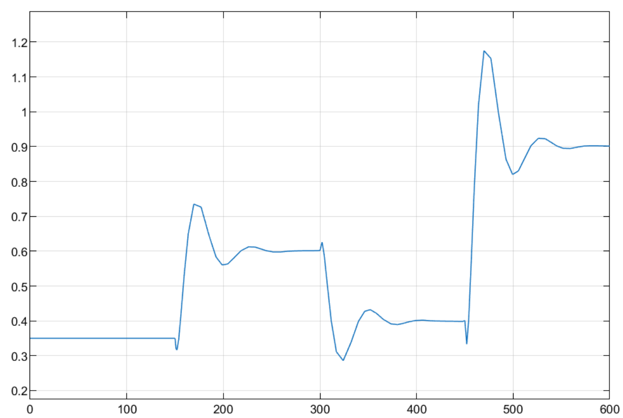
1.)

a.)

a.) (standard) We found experimentally that 30s was not enough to showcase the full response, so we increased the time to 600s.




30s




600s

b.) (with surge tank)

The transfer function for the **mechanical power of a hydraulic turbine with a surge tank** is given as:

$$\frac{\Delta p_m}{\Delta g} = \frac{1 - T_w \cdot s + s^2/\omega^2{}_n}{1 + 0.5 \cdot T_w \cdot s + s^2/\omega^2{}_n},$$

where $\omega_n = \sqrt{\frac{A \cdot g_r}{A_s \cdot L}}$

(with $g_r$: acceleration of gravity, A: cross section of penstock, $A_s$: cross section of surge tank, L: length of penstock)

We could not determine how to emulate Hovey's method with the surge tank, so $\omega_n$ was determined such that the resulting transfer function $\frac{\Delta p_m}{\Delta g}$ was critically damped...

surge tank: $\dfrac{1 - T_w \cdot s + s^2/\omega_n^2}{1 + 0.5 \cdot T_w \cdot s + s^2/\omega_n^2}$ stability

$s^2 + \frac{1}{2} T_w \cdot (\omega_n)^2 s + \omega_n^2$

| | | | |
|---|---|---|---|
| $s^2$ | $1$ | $\omega_n^2$ | ??? |
| $s^1$ | $\frac{1}{2} T_w \omega_n^2$ | $0$ | |
| $s^0$ | $\omega_n^2$ | | |

critical damping... real & repeated roots

$4 \cdot 4 = 16$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-\frac{T_w}{2}(\omega_n)^2 \pm \sqrt{\left(\frac{T_w \cdot \omega_n^2}{2}\right)^2 - 4 \cdot \omega_n^2}}{2}$$

$$\frac{1 - T_w \cdot s + s^2/\omega_n^2}{1 + 0.5 \cdot T_w \cdot s + s^2/\omega_n^2}$$

$$\cancel{-\frac{T_w}{2}(\omega_n)^2} + \sqrt{\left(\frac{T_w \cdot \omega_n^2}{2}\right)^2 - 4\omega_n^2} = \cancel{-\frac{T_w}{2}(\omega_n)^2} - \sqrt{\left(\frac{T_w \cdot \omega_n^2}{2}\right)^2 - 4\omega_n^2}$$

$$2\sqrt{\left(\frac{T_w \cdot \omega_n^2}{2}\right)^2 - 4\omega_n^2} = 0$$

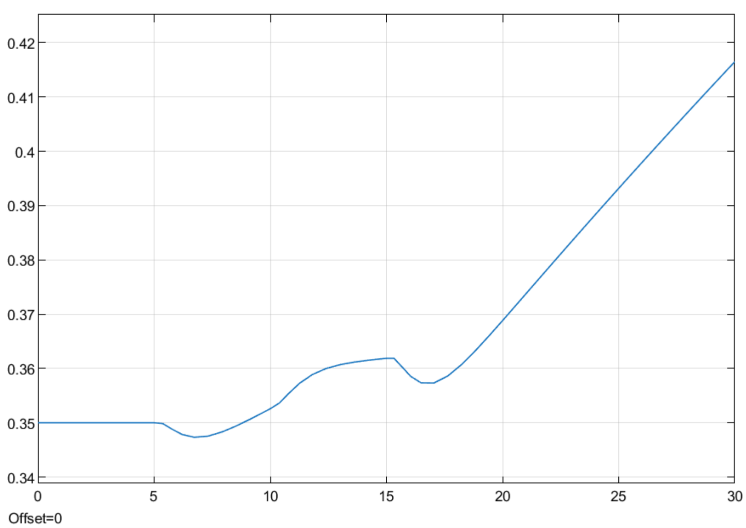
$$\frac{T_w^2 \cdot \omega_n^{\cancel{4}\,2}}{4} = 4\cancel{\omega_n^2}$$

$$\omega_n^2 = \left(\frac{16}{T_w^2}\right)$$

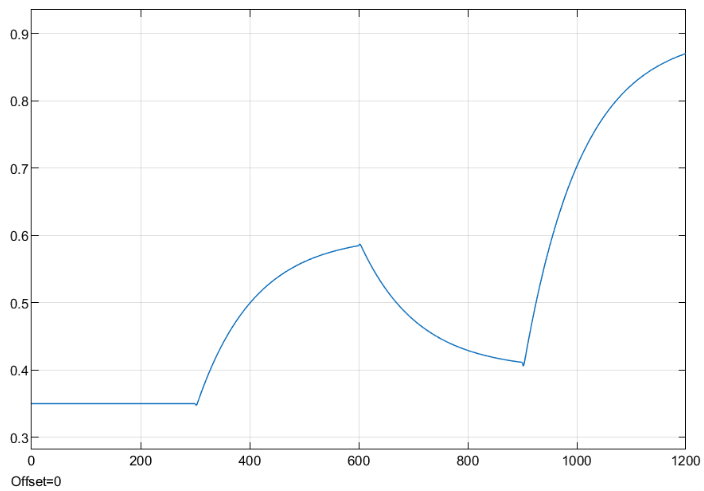
$$\omega_n = \frac{4}{T_w}$$

This gave us an $\omega_n = \frac{4}{T_\omega}$

This time, simulation time was increased to 1200 seconds... with the spacing between changes in $P_e$ being changed accordingly

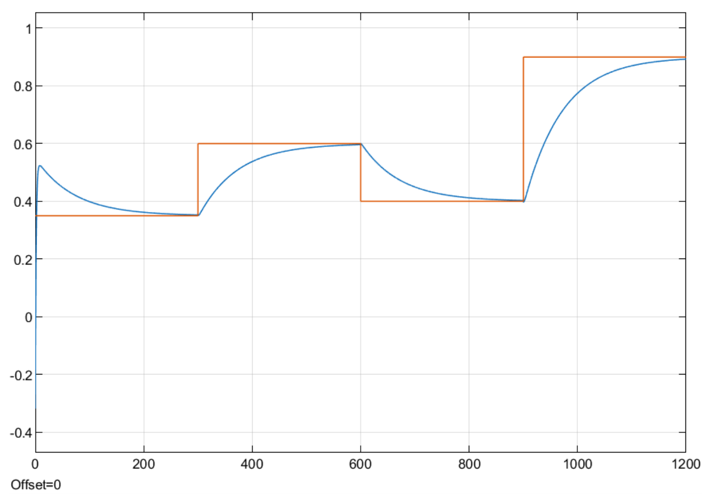



30s simulation

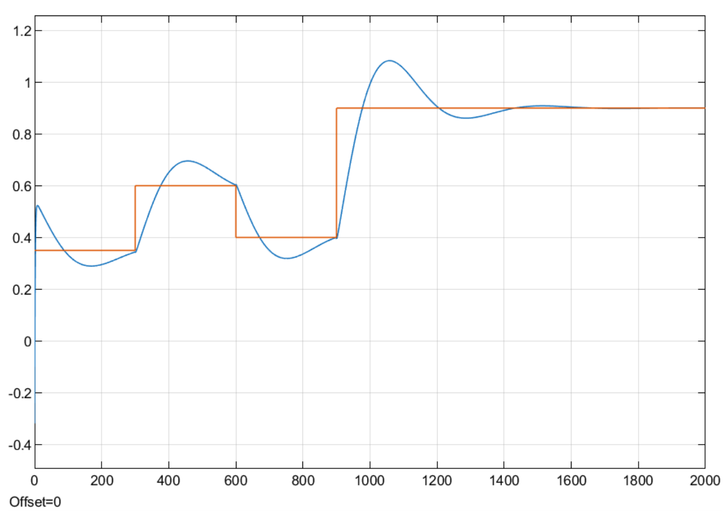



1200s simulation

1b) (non-ideal case)




1200s simulation

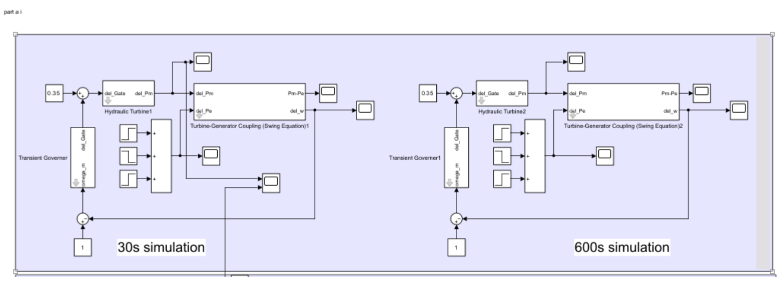



2000s simulation

w/ PI controller for $\omega$

METHODOLOGY:

For 1a.a)




Standard Hydro

The standard hydro implementation was used. However, with the standard values for $T_\omega$ and H, the simulation took too long to converge, so they were reduced to their current values:

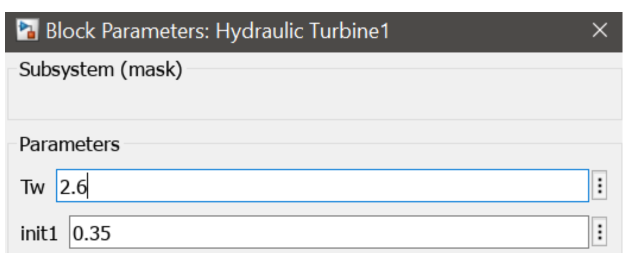
Tw = 2.6s H = 3.3s

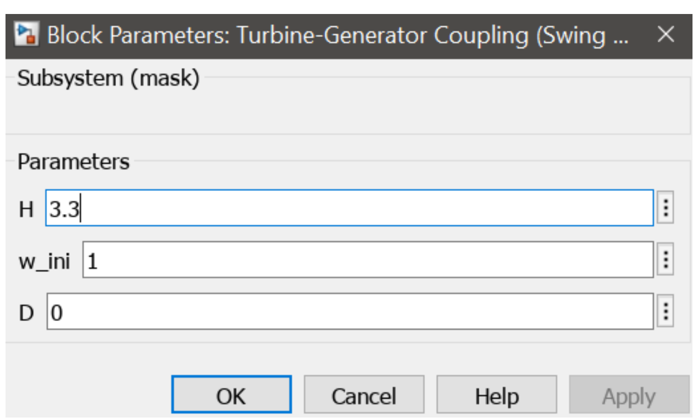
As per Hovey (1962)

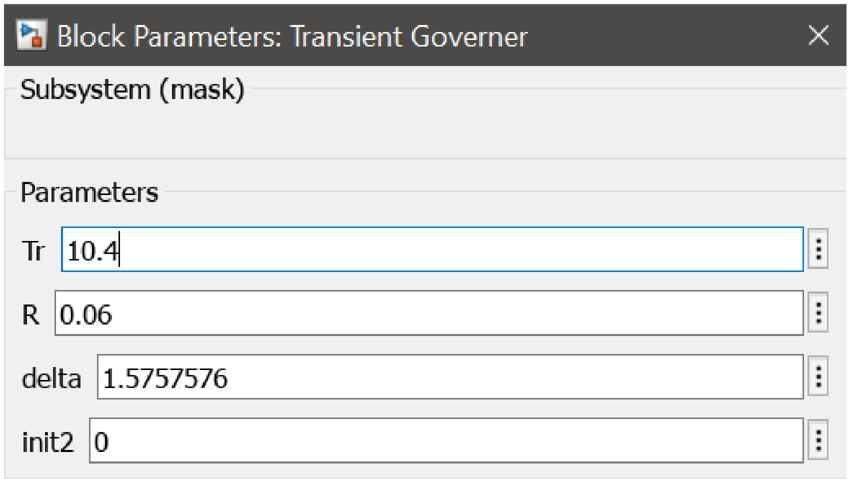
$$\delta = \frac{2T_W}{H}, \qquad T_R = 4T_W$$

$$\delta = \frac{2 \cdot 2.6}{3.3}, \qquad T_R = 10.4$$

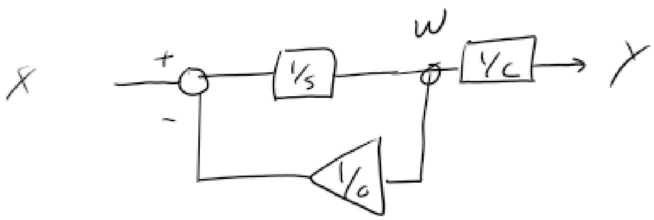
$$\delta = 1.575757575, \qquad T_R = 10.4$$






The blocks inside the subsystems were implemented as such:




$$W = \frac{1}{s}\left(X - \frac{1}{C}W\right)$$

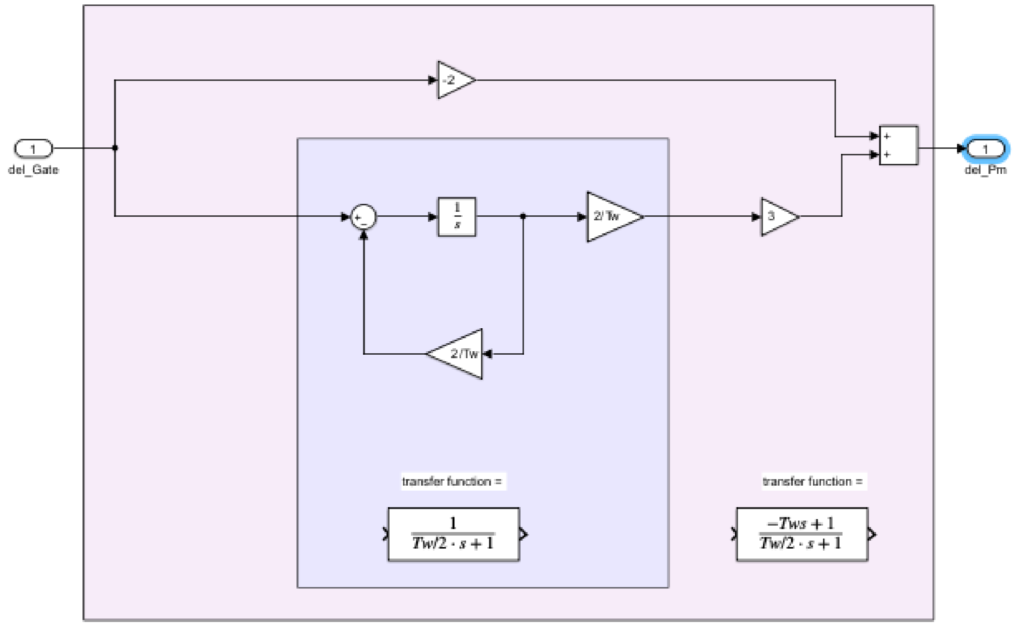
$$W\left(1 + \frac{1}{Cs}\right) = \frac{1}{s}X$$

$$\frac{W}{X} = \frac{\frac{1}{s}}{1 + \frac{1}{Cs}} = \frac{1}{s + \frac{1}{C}} = \frac{C}{Cs + 1}$$

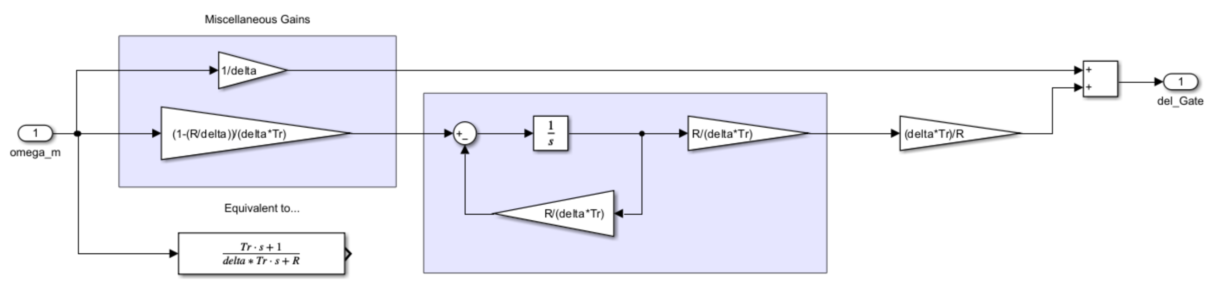
$$\boxed{\frac{Y}{X} = \frac{1}{1 + Cs}}$$

$$\frac{Y}{X} = \frac{C}{1 + sC}$$

## Hydraulic Turbine1

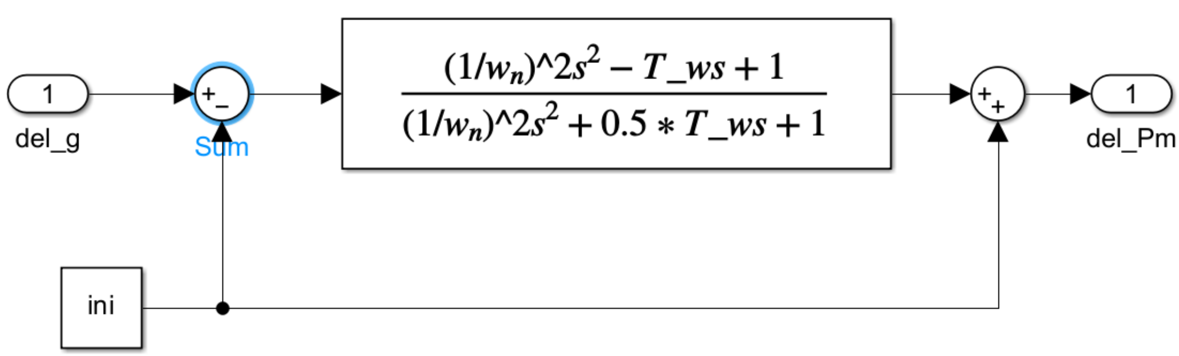



### Transient Governer




For 1a.b)

### Hydraulic Turbine with Surge Tank



For 1b)

I) $\Delta V = \alpha_{11} \Delta h + \alpha_{12} \Delta g$

II) $\Delta p_m = \alpha_{21} \Delta h + \alpha_{22} \Delta g$

III) $\Delta p_m = \Delta h + \Delta V$

IV) $-\Delta h = T_w \frac{dV}{dt} \xrightarrow{\frac{d}{dt} \to s} -\Delta h = T_w.s.V$

Remove $\Delta h$...

$\downarrow \quad \Delta h = -T_w.s.V$

Ii) $\Delta V = \alpha_{11}(-T_w.s.V) + \alpha_{12} \Delta g$

IIi) $\Delta p_m = \alpha_{21}(-T_w.s.V) + \alpha_{22} \Delta g$

IIIi) $\Delta p_m = (-T_w.s.V) - \Delta V$

$$\Delta V\left(1 + \alpha_{11}.T_w.s\right) = \alpha_{12} \Delta g$$

$$\Delta V = \frac{\alpha_{12} \Delta g}{1 + \alpha_{11}.T_w.s}$$

$\downarrow$ remove $\Delta V$

IIi) $\Delta p_m = \alpha_{21}\left(-T_w.s.\left(\frac{\alpha_{12}\Delta g}{1+\alpha_{11}.T_w.s}\right)\right) + \alpha_{22} \Delta g$

$$\frac{\Delta p_m}{\Delta g} = \alpha_{22} - \alpha_{21}\left(\frac{T_w.s.\alpha_{12}}{1+\alpha_{11} T_w.s}\right)$$

$$= \frac{\alpha_{22} + \alpha_{22}\alpha_{11} T_w.s - \alpha_{21}\alpha_{12} T_w.s}{1 + \alpha_{11} T_w.s}$$

$$\frac{\Delta p_m}{\Delta g} = \frac{\alpha_{22} + (\alpha_{22}\alpha_{11} - \alpha_{21}\alpha_{12}) T_w.s}{1 + \alpha_{11}.T_w.s}$$

ideal case: $\alpha_{11} = \frac{1}{2}$ $\alpha_{12} = 1$

$\alpha_{21} = \frac{3}{2}$ $\alpha_{22} = 1$

$$\frac{1 + \left(\left(\frac{1}{2}\right) - \frac{3}{2}\right) T_w.s)}{1 + \frac{1}{2} T_w.s} = \frac{1 - T_w.s}{1 + \frac{T_w}{2} s}$$

$$\frac{\alpha_{22}+(\alpha_{22}\alpha_{11}-\alpha_{21}\alpha_{12})T_w \cdot S}{1+\alpha_{11}T_w \cdot S} = X + \frac{Y}{1+\alpha_{11}T_w \cdot S}$$

$$X+\alpha_{11}XT_wS + Y = \alpha_{22}+(\alpha_{22}\alpha_{11}-\alpha_{21}\alpha_{12})T_w \cdot S$$

$$Y+X=\alpha_{22}$$

$$\alpha_{11}XT_w\cancel{S} = (\alpha_{22}\alpha_{11}-\alpha_{21}\alpha_{12})T_w \cancel{S}$$

$$X = \frac{\alpha_{22}\alpha_{11}-\alpha_{21}\alpha_{12}}{\alpha_{11}}$$

$\alpha_{12}=1.1$
$\alpha_{22}=1.5$

$$\frac{\frac{3}{2}\cdot\alpha_{11}-1.1\,\alpha_{21}}{\alpha_{11}} = \boxed{\frac{3}{2}-1.1\cdot\frac{\alpha_{21}}{\alpha_{11}}}$$

$X$

$$\frac{3}{2}-1.1\cdot\frac{\alpha_{21}}{\alpha_{11}}+Y=\alpha_{22}$$

$$Y=\alpha_{22}+1.1\left(\frac{\alpha_{21}}{\alpha_{11}}\right)-\frac{3}{2}$$

$$X+Y=\alpha_{22}$$

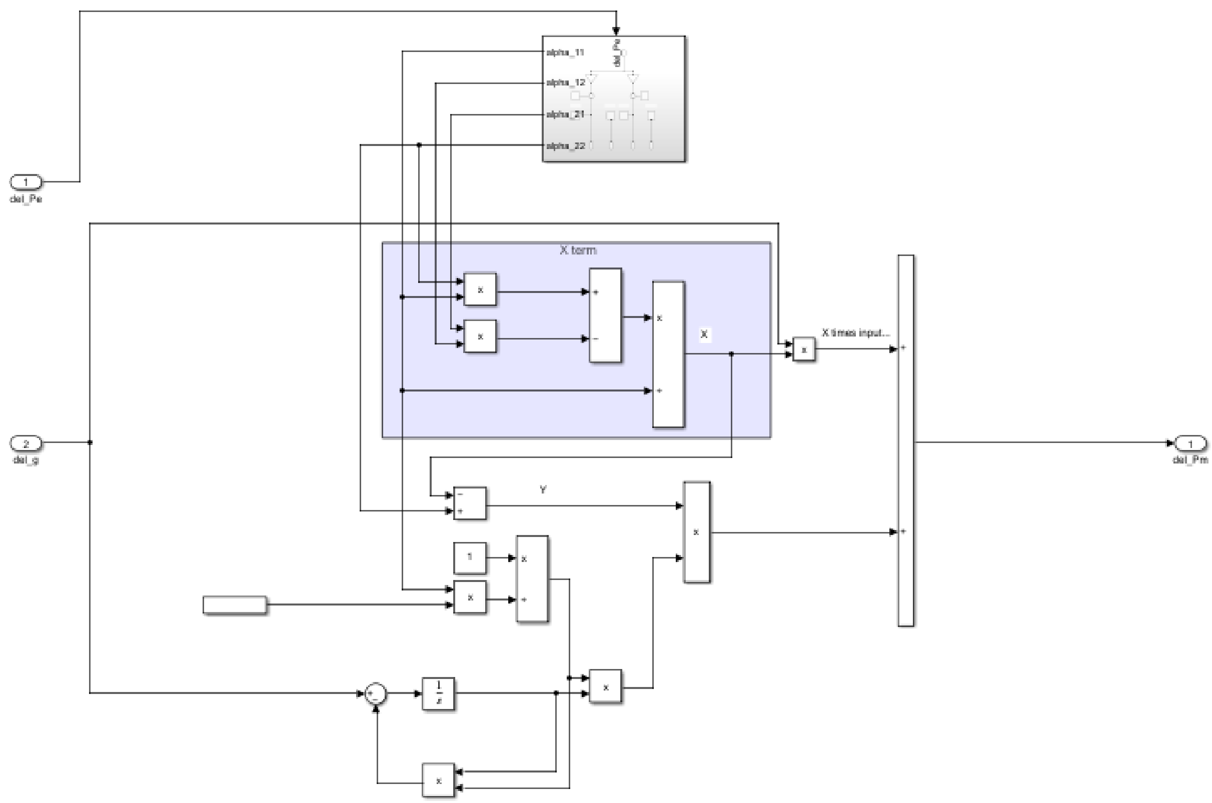
$$\frac{1}{1+\alpha_{11}T_w \cdot S}$$

$$Y=\alpha_{22}-X$$

$$C=\alpha_{11}T_w$$

it is better to open the Simulink file for this one, it is very complex

Non-Ideal Hydraulic Turbine ▸




MISTAKES MADE:

1. COULD NOT FIND $\omega_n$ TO EMULATE HOVEY'S METHOD FOR SURGE TANK
2. THE NONIDEAL CASE DOES NOT EXHIBIT A WATER HAMMER... THERE IS A MISTAKE SOMEWHERE IN THE CALCULATION
3. COULD NOT INITIALIZE PROPERLY THE NONIDEAL CASE... AS A RESULT, INITIALIZES AT 0...
4. (and others, very likely...)