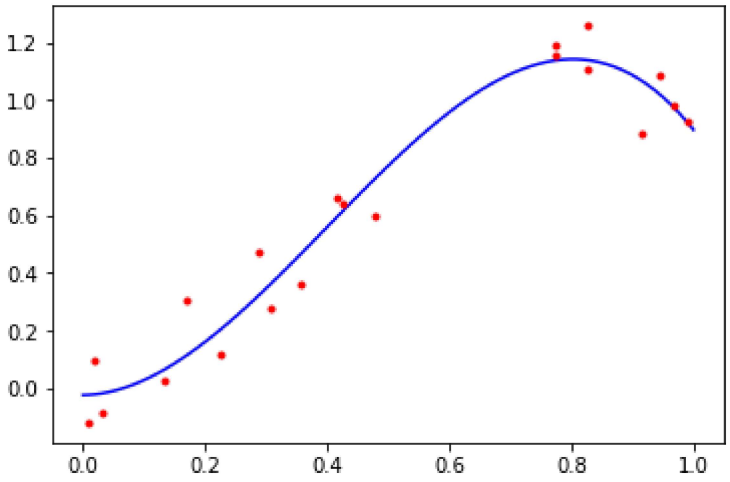
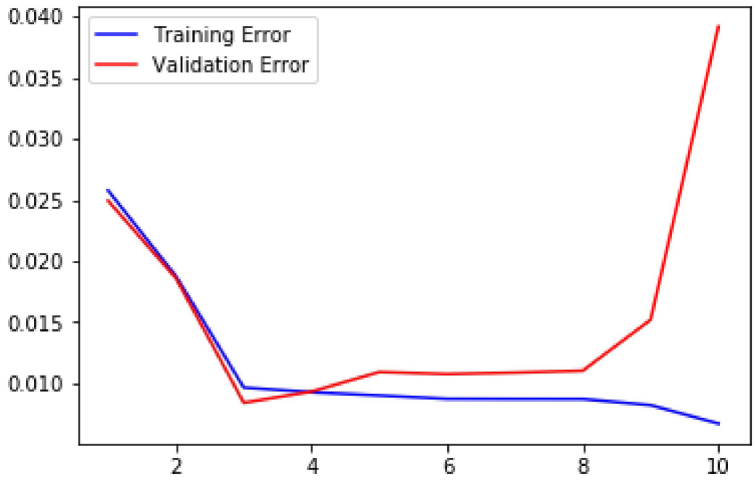
In [1]:
```
import numpy as np
import matplotlib.pyplot as plt
import pandas

# Function that creates the X matrix as defined for fitting our model
def create_X(x,deg):
    X = np.ones((len(x),deg+1))
    for i in range(1,deg+1):
        X[:,i] = x**i
    return X

# Function for predicting the response
def predict_y(x,beta):
    return np.dot(create_X(x,len(beta)-1),beta)

# Function for fitting the model
def fit_beta(df,deg):
    return np.linalg.lstsq(create_X(df.x,deg),df.y,rcond=None)[0]

# Function for computing the MSE
def mse(y,yPred):
    return np.mean((y-yPred)**2)

# Loading training, validation and test data
dfTrain = pandas.read_csv('Data_Train.csv')
dfVal = pandas.read_csv('Data_Val.csv')
dfTest = pandas.read_csv('Data_Test.csv')

############ TRAINING A MODEL

# Fitting model
deg = 3
#X = create_X(dfTrain.x,deg)
beta = fit_beta(dfTrain,deg)

# Computing training error
yPredTrain = predict_y(dfTrain.x,beta)
err = mse(dfTrain.y,yPredTrain)
print('Training Error = {:2.3}'.format(err))

# Computing test error
yPredTest = predict_y(dfTest.x,beta)
err = mse(dfTest.y,yPredTest)
print('Test Error = {:2.3}'.format(err))
```

```
Training Error = 0.00967
Test Error = 0.0116
```

In [2]:
```
############ PLOTTING FITTED MODEL
x = np.linspace(0,1,100)
y = predict_y(x,beta)

plt.plot(x,y,'b-',dfTrain.x,dfTrain.y,'r.')
plt.show()
```



In [3]:
```
############ HYPER-PARAMETER TUNING

# Initializing range of degree values to be tested and errors
degRange = list(range(1,11))
errTrain = np.zeros(len(degRange))
errVal = np.zeros(len(degRange))

# Computing error as a function of degree
# ...
for i in range(len(degRange)):

    # Computing error for training dataset
    betaTrain = fit_beta(dfTrain,degRange[i])
    yPredTrain = predict_y(dfTrain.x,betaTrain)
    errTrain[i] = mse(dfTrain.y,yPredTrain)

    # Computing error for validation dataset
    yPredVal = predict_y(dfVal.x,betaTrain)
    errVal[i] = mse(dfVal.y,yPredVal)

# Plotting training and validation errors
plt.plot(degRange,errTrain,'b-',degRange,errVal,'r-')
plt.legend(('Training Error','Validation Error'))
plt.show()
```



In [4]:
```
############ TRAINING SELECTED MODEL

# Concatenating data training and validation data frames
df = pandas.concat([dfTrain, dfVal])

# Fit model using the optimal degree found in the previous cell
degOpt = degRange[np.argmin(errVal)]
print('Optimal degree : ', degOpt)
betaFinal = fit_beta(df, degOpt)

# Compute and print training and test errors
# ...
yPredTrain = predict_y(df.x,betaFinal)
err = mse(df.y,yPredTrain)
print('Training Error = {:2.3}'.format(err))

yPredTest = predict_y(dfTest.x,betaFinal)
err = mse(dfTest.y,yPredTest)
print('Test Error = {:2.3}'.format(err))
```

```
Optimal degree :  3
Training Error = 0.0087
Test Error = 0.0108
```

In [5]:
```
############ PLOTTING FITTED MODEL
# Plot the fitted model as in the second cell
# ...
x = np.linspace(0,1,100)
y = predict_y(x,betaFinal)

plt.plot(x,y,'b-',df.x,df.y,'r.')
plt.show()
```