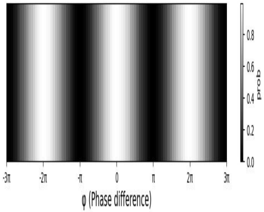
From Bits to Qubits: Foundations of Quantum Computing

Bijay K. Agrawal

Saha Institute of Nuclear Physics, Kolkata

QCFAIANGT 17 – 22 **Nov. 2025**



Questions to be addressed

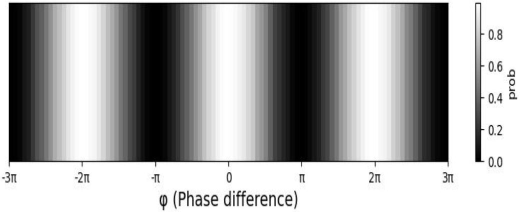
- Why Quantum Computers?
- Why Quantum computer matters?
- What are the Prerequisites?

Questions to be addressed

- Why Quantum Computers?
- Why Quantum computer matters?
- What are the Prerequisites?
- Basics of Classical Computers
- Extension to Quantum computer

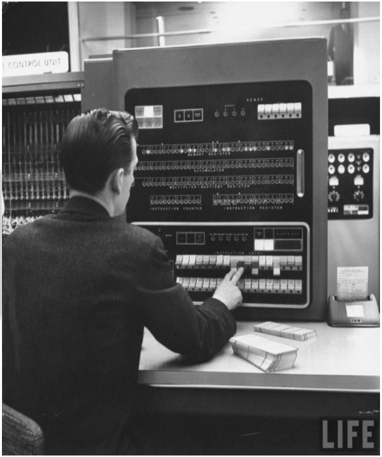
Questions to be addressed

- Why Quantum Computers?
- Why Quantum computer matters?
- What are the Prerequisites?
- Basics of Classical Computers
- Extension to Quantum computer
- Experiencing Quantum computer



First generation computers (1940-1956)

- Vacuum tubes for circuitry and magnetic drums for memory.
- They were very large in size and taking up entire room.

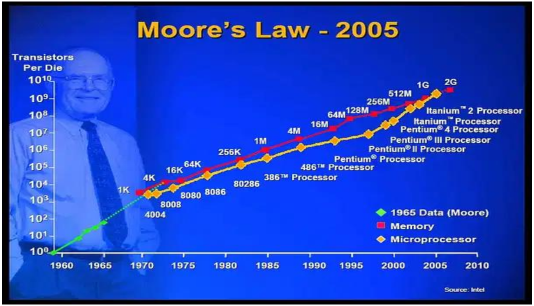


Fifth generation computers (1980- Present)

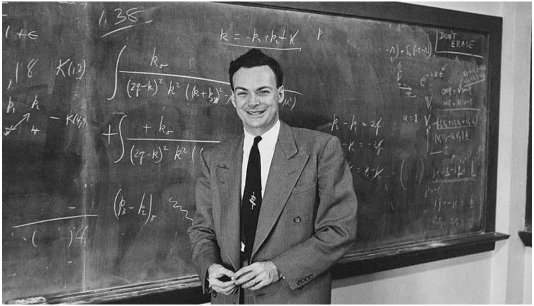
- Ultra Large-Scale Integration (ULSI):
- Millions of transistors on a single microchip
- Parallel processing Natural language understanding: Can understand human language
- Energy efficiency, Size: Small and portable, with large storage capacity
- Input/output devices Keyboard, mouse, monitor, touchscreen, speech input, printer, etc.

Moore's Law

Gordon Moore, co-founder of Intel, first described Moore's Law in 1965. Moore's Law is the observation that the number of transistors in a computer chip doubles every two years. It's a projection of a historical trend in the computing industry, and is not a law of physics.



... it seems that the laws of the physics present no barrier to reducing the size of the computers until bits are the size of the atoms, and quantum behaviour holds sway'
Richard P. Feynman



Time line of Quantum Computing

- 1980 First quantum mechanical model Paul Benioff
- 1981 Benioff & Feynmann proposed idea of quantum computing.

Time line of Quantum Computing

- 1980 First quantum mechanical model Paul Benioff
- 1981 Benioff & Feynmann proposed idea of quantum computing.
- 1984 Quantum cryptography, Bennett and Brassard (BB84)
- 1985 Devid Deutsch , First universal quantum computer
- 1985 Asher Peres - Quantum error correction

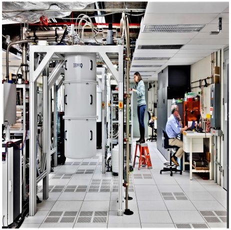
Time line of Quantum Computing

- 1980 First quantum mechanical model Paul Benioff
- 1981 Benioff & Feynmann proposed idea of quantum computing.
- 1984 Quantum cryptography, Bennett and Brassard (BB84)
- 1985 Devid Deutsch , First universal quantum computer
- 1985 Asher Peres - Quantum error correction
- 1992 Deutsch - Jozsa algorithm
- 1992 Bernstein Vazirani algorithm
- 1993 Simon algorithum
- 1994 Shor algorithm
- 1995: Expt. realization of first quantum logic gate with trapped ions
- 1996: Grover's algorithm

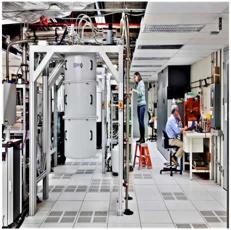
Time line of Quantum Computing

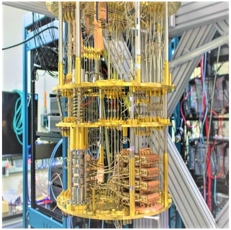
- 1980 First quantum mechanical model Paul Benioff
- 1981 Benioff & Feynmann proposed idea of quantum computing.
- 1984 Quantum cryptography, Bennett and Brassard (BB84)
- 1985 Devid Deutsch , First universal quantum computer
- 1985 Asher Peres - Quantum error correction
- 1992 Deutsch - Jozsa algorithm
- 1992 Bernstein Vazirani algorithm
- 1993 Simon algorithum
- 1994 Shor algorithm
- 1995: Expt. realization of first quantum logic gate with trapped ions
- 1996: Grover's algorithm
- 2000: Quantum computation and information, Nielsen and Chuang cited 58500

Superconducting quantum processor

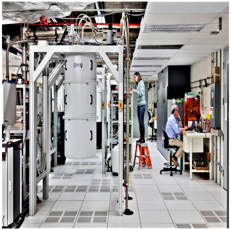


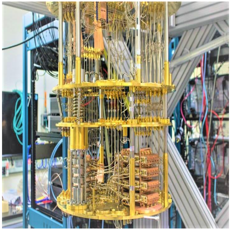
Superconducting quantum processor

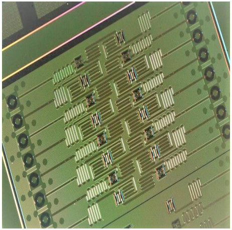




Superconducting quantum processor







Why Quantum computer matters?

- Finance
- Health
- Energy & Agriculture
- Global climate

Prerequisite for Quantum computing ?
Physicist? Computer Engineer?

Prerequisite for Quantum computing ?
Physicist? Computer Engineer?
NOTHING of that sort!!

Basis for classical computing

$$c = \begin{cases} |0\rangle & \text{(low)} \\ |1\rangle & \text{(high)} \end{cases}$$

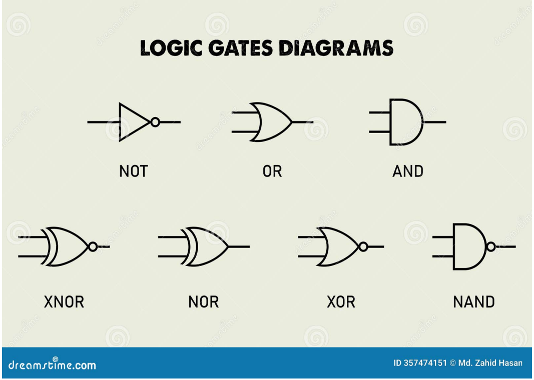
cbits: deterministic & can be read directly.

Basis for classical computing

$$c = \begin{cases} |0\rangle & \text{(low)} \\ |1\rangle & \text{(high)} \end{cases}$$

cbits: deterministic & can be read directly.

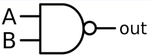
Logic gates are $\Rightarrow$ not reversible



a) NAND Gate
- Truth Table:

A	B	$\overline{A \cdot B}$
0	0	1
0	1	1
1	0	1
1	1	0

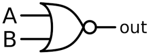
$$Y = \overline{A \cdot B}$$



NOR Gate - Truth Table:

A	B	$\overline{A + B}$
0	0	1
0	1	0
1	0	0
1	1	0

$$Y = \overline{A + B}$$



NAND gate implementation of NOT, AND & OR

1. NOT Gate:

$$\mathrm{NOT}(A) = \overline{A} = \overline{A \cdot A} = \mathrm{NAND}(A, A)$$

NAND gate implementation of NOT, AND & OR

1. NOT Gate:

$$\mathrm{NOT}(A) = \overline{A} = \overline{A \cdot A} = \mathrm{NAND}(A, A)$$

2. AND Gate

$$\begin{aligned}
\mathrm{AND}(A, B) &= (A \cdot B)\overline{\overline{A \cdot B}} \\
&= \mathrm{NOT}(\mathrm{NAND}(A, B)) \\
&= \mathrm{NAND}(\mathrm{NAND}(A, B), \mathrm{NAND}(A, B))
\end{aligned}$$

NAND gate implementation of NOT, AND & OR

1. NOT Gate:

$$\text{NOT}(A) = \overline{A} = \overline{A \cdot A} = \text{NAND}(A, A)$$

2. AND Gate

$$\begin{aligned} \text{AND}(A, B) &= (A \cdot B)\overline{\overline{A \cdot B}} \\ &= \text{NOT}(\text{NAND}(A, B)) \\ &= \text{NAND}(\text{NAND}(A, B), \text{NAND}(A, B)) \end{aligned}$$

3. OR Gate

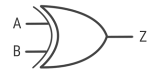
$$\begin{aligned} A + B &= \overline{(\overline{A} \cdot \overline{B})} \\ &= \text{NAND}(\overline{A}, \overline{B}) \\ &= \text{NAND}(\text{NAND}(A, A), \text{NAND}(B, B)) \end{aligned}$$

XOR Gate (Exclusive OR)

Outputs 1 if inputs are different.

Truth Table:

A	B	$A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

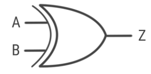


XOR Gate (Exclusive OR)

Outputs 1 if inputs are different.
Truth Table:

A	B	$A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

$$\begin{aligned} XOR &\rightarrow mod(2) \\ A \oplus B &= (A+B)mod(2) \\ A \oplus B \oplus C &= (A+B+C)mod(2) \\ 1 \oplus 1 \oplus 1 &= 3mod(2) = 1 \\ 1 \oplus 1 \oplus 1 \oplus 1 &= 4mod(2) = 0 \end{aligned}$$



XOR using NAND gate

$$A \oplus B = (A \cdot \overline{B}) + (\overline{A} \cdot B)$$

XOR using NAND gate

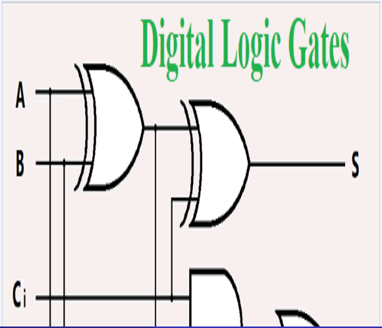
$$\begin{aligned}
A \oplus B &= (A \cdot \overline{B}) + (\overline{A} \cdot B) \\
P &= \text{NAND}(A, B) = \overline{(A \cdot B)} \\
Q &= \text{NAND}(A, P) = \text{NAND}(A, \text{NAND}(A, B)) \\
R &= \text{NAND}(B, P) = \text{NAND}(B, \text{NAND}(A, B)) \\
A \oplus B &= \text{NAND}(Q, R)
\end{aligned}$$

$$A \oplus B = \text{NAND}\big(\text{NAND}(A, \text{NAND}(A, B)),\ \text{NAND}(B, \text{NAND}(A, B))\big)$$

Full Adders - Basic arithmatic units

$$
\begin{aligned}
& A + B + Cin \\
S &= A \oplus B \oplus Cin \\
C_{out} &= (A \cdot B) + (A \oplus B) \cdot Cin \\
C_{out} &= (A \cdot B) + (B \cdot C_{in}) + (A \cdot C_{in})
\end{aligned}
$$

A	B	C_{in}	Sum (S)	Carry (C_out)
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0



Classical $\rightarrow$ Quantum

- Quantum superposition/parallelism
- Wave function collapse
- Quantum entanglement
- Interference

Basis for Quantum computing

cbits $\Rightarrow$ qbits (Superposition of cbits)

$$
\begin{aligned}
|\psi\rangle &= \alpha|0\rangle + \beta|1\rangle \qquad \textbf{(Superposition)} \\
\langle\psi| &= \alpha^*\langle 0| + \beta^*\langle 1| \\
|0\rangle &= \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\
\langle 0| &= \begin{bmatrix} 1 & 0 \end{bmatrix} \qquad \langle 1| = \begin{bmatrix} 0 & 1 \end{bmatrix} \textbf{(Matrix Rep.)} \\
\langle 0|0\rangle &= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 1
\end{aligned}
$$

Basis for Quantum computing

cbits $\Rightarrow$ qbits (Superposition of cbits)

$$
\begin{aligned}
|\psi\rangle &= \alpha|0\rangle + \beta|1\rangle \qquad \textbf{(Superposition)} \\
\langle\psi| &= \alpha^*\langle 0| + \beta^*\langle 1| \\
|0\rangle &= \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\
\langle 0| &= [1 \qquad 0] \qquad \langle 1| = [0 \qquad 1] \textbf{(Matrix Rep.)} \\
\langle 0|0\rangle &= [1 \qquad 0] \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 1 \\
\langle i|j\rangle &= \delta_{ij}, \qquad |j\rangle = |0\rangle or |1\rangle \qquad \textbf{(Orthonormality)}
\end{aligned}
$$

Basis for Quantum computing

cbits $\Rightarrow$ qbits (Superposition of cbits)

$$
\begin{aligned}
|\psi\rangle &= \alpha|0\rangle + \beta|1\rangle \qquad \textbf{(Superposition)} \\
\langle\psi| &= \alpha^*\langle 0| + \beta^*\langle 1| \\
|0\rangle &= \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\
\langle 0| &= [1 \quad 0] \qquad \langle 1| = [0 \quad 1] \textbf{(Matrix Rep.)} \\
\langle 0|0\rangle &= [1 \quad 0] \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 1 \\
\langle i|j\rangle &= \delta_{ij}, \qquad |j\rangle = |0\rangle or |1\rangle \qquad \textbf{(Orthonormality)} \\
\langle\psi|\psi\rangle &= \alpha^*\alpha\,\langle 0|0\rangle + \alpha^*\beta\,\langle 0|1\rangle + \beta^*\alpha\,\langle 1|0\rangle + \beta^*\beta\,\langle 1|1\rangle \\
&= \alpha^*\alpha + \beta^*\beta = \mid \alpha \mid^2 + \mid \beta \mid^2 = 1 \\
\mid \alpha \mid^2 &\Rightarrow \text{Prob. for}|0\rangle \qquad , \mid \beta \mid^2 \Rightarrow \text{Prob. for}|1\rangle
\end{aligned}
$$

Basis for Quantum computing

cbits $\Rightarrow$ qbits (Superposition of cbits)

$$
\begin{aligned}
|\psi\rangle &= \alpha|0\rangle + \beta|1\rangle \qquad \textbf{(Superposition)} \\
\langle\psi| &= \alpha^*\langle 0| + \beta^*\langle 1| \\
|0\rangle &= \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\
\langle 0| &= [1 \qquad 0] \qquad \langle 1| = [0 \qquad 1] \textbf{(Matrix Rep.)} \\
\langle 0|0\rangle &= [1 \qquad 0] \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 1 \\
\langle i|j\rangle &= \delta_{ij}, \qquad |j\rangle = |0\rangle or |1\rangle \qquad \textbf{(Orthonormality)} \\
\langle\psi|\psi\rangle &= \alpha^*\alpha\,\langle 0|0\rangle + \alpha^*\beta\,\langle 0|1\rangle + \beta^*\alpha\,\langle 1|0\rangle + \beta^*\beta\,\langle 1|1\rangle \\
&= \alpha^*\alpha + \beta^*\beta = \mid \alpha \mid^2 + \mid \beta \mid^2 = 1 \\
\mid \alpha \mid^2 &\Rightarrow \text{Prob. for}|0\rangle \qquad , \mid \beta \mid^2 \Rightarrow \text{Prob. for}|1\rangle
\end{aligned}
$$

2-qubit states

$$|\psi\rangle = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle$$

2-qubit states

$$
\begin{aligned}
|\psi\rangle &= \alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle \\
|ij\rangle &= |i\rangle \otimes |j\rangle = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \otimes \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \\
&= \begin{bmatrix} a_1 b_1 \\ a_1 b_2 \\ a_2 b_1 \\ a_2 b_2 \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
|00\rangle &= |0\rangle \otimes |0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \\
|01\rangle &= |0\rangle \otimes |1\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \\
|10\rangle &= |1\rangle \otimes |0\rangle = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \qquad |11\rangle = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}
\end{aligned}
$$

Measurments

Measuring a qubit collapses its state

collapses into either $|0\rangle$ or $|1\rangle$
with probabilities $\mid \alpha \mid^2$ and $\mid \beta \mid^2$respectively.

$$\alpha|0\rangle + \beta|1\rangle \xrightarrow{\text{Measurments}} |0\rangle or |1\rangle$$

Measurments

Measuring a qubit collapses its state

collapses into either $|0\rangle$ or $|1\rangle$
with probabilities $|\ \alpha\ |^2$ and $|\ \beta\ |^2$respectively.

$$\alpha|0\rangle + \beta|1\rangle \xRightarrow{\text{Measurments}} |0\rangle or |1\rangle$$

2-Qbit states

$$\alpha|0\rangle + \beta|1\rangle \xRightarrow{\text{Measurments}} |0\rangle or |1\rangle$$

$$\alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle \xRightarrow{\text{Measurments}} \begin{array}{l} |00\rangle \quad or \quad |01\rangle \\ or|10\rangle or \quad |11\rangle \end{array}$$

Not deterministic!!!

Entanglement

Qubits can become entangled, meaning the state of one qubit is directly related to the state of another.

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}\left(|\underset{q_0 q_1}{00}\rangle + |\underset{q_0 q_1}{11}\rangle\right)$$

$$|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$

$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

Quantum gates

Quantum gates- "the building blocks of quantum computation", analogous to classical logic gates. They operate on qubits and manipulate their states by applying unitary transformations.

X (NOT Gate) :

$$
\begin{aligned}
X &= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \\
X|0\rangle &= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = |1\rangle \\
X|1\rangle &= \begin{bmatrix} 0 & 1 \\ 1 & \\ 0 & \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = |0\rangle
\end{aligned}
$$

$$\boxed{X|j\rangle = |\bar{j}\rangle}$$

Quantum gates

Y Gate :

$$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$\boxed{Y|j\rangle = i(-1^{j})|\bar{j}\rangle}$$

Quantum gates

Y Gate :

$$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$\boxed{Y|j\rangle = i(-1^{j})|\bar{j}\rangle}$$

Z Gate :

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\boxed{Z|j\rangle = (-1^{j})|j\rangle}$$

$$\begin{aligned} X, Y, Z &= X_1, X_2, X_3 \\ [X_i, X_j] &= 2i\epsilon_{ijk}X_k \\ \{X_i, X_j\} &= 2\delta_{ij}\mathbb{1} \end{aligned}$$

Quantum gates

Y Gate :

$$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$\boxed{Y|j\rangle = i(-1^j)|\bar{j}\rangle}$$

Z Gate :

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\boxed{Z|j\rangle = (-1^j)|j\rangle}$$

$$\begin{aligned} X, Y, Z &= X_1, X_2, X_3 \\ [X_i, X_j] &= 2i\epsilon_{ijk}X_k \\ \{X_i, X_j\} &= 2\delta_{ij}\mathbb{1} \\ X_i &\equiv \sigma_i \\ |0\rangle, |1\rangle &\equiv |\uparrow\rangle|\downarrow\rangle \end{aligned}$$

Quantum gates

Hadamard Gate (H) - Creates superposition:

$$\begin{aligned} H &= \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \\ &= \frac{1}{\sqrt{2}}\left[\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\right] \\ &= \frac{1}{\sqrt{2}}(Z + X) \end{aligned}$$

$$\begin{aligned} H|0\rangle &= \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \\ H|1\rangle &= \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \end{aligned}$$

$$\boxed{H|j\rangle = \frac{1}{\sqrt{2}}(|0\rangle + (-1)^j|1\rangle)}$$

Working with H gate

$$
\begin{aligned}
H|0\rangle &= \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \\
H^{\otimes 2}|00\rangle &= \frac{1}{\sqrt{2^2}}(|0\rangle + |1\rangle) \otimes (|0\rangle + |1\rangle) \\
&= \frac{1}{\sqrt{2^2}}(|00\rangle + |01\rangle + |10\rangle + |11\rangle) \\
&= \frac{1}{\sqrt{2^2}}(|0\rangle_2 + |1\rangle_2 + |2\rangle_2 + |3\rangle_2) \\
&= \frac{1}{\sqrt{2^2}}\sum_{y=0}^{2^2-1}|y\rangle_2 \\
H^{\otimes n}|0_n\rangle &= \frac{1}{\sqrt{2^n}}\sum_{y=0}^{2^n-1}|y\rangle_n
\end{aligned}
$$

Working with H gate

$$H^{\otimes n}|0_n\rangle = \frac{1}{\sqrt{2^n}} \sum_{y=0}^{2^n-1} |y\rangle_n$$

$$\boxed{H^{\otimes n}|x_n\rangle = \frac{1}{\sqrt{2^n}} \sum_{y=0}^{2^n-1} (-1^{x\cdot y})|y\rangle_n}$$

$$\begin{aligned} x \cdot y &= x_0 y_0 \oplus x_1 y_1 \oplus \ldots \oplus x_{n-1} y_{n-1} \\ &= (x_0 y_0 + x_1 y_1 + \ldots + x_{n-1} y_{n-1}) mod(2) \end{aligned} \tag{1}$$

Working with H gate

$$H^{\otimes 3}|0_3\rangle \quad = \quad ?????$$

Working with H gate

$$
\begin{aligned}
H^{\otimes 3}|0_3\rangle &= \text{?????} \\
H^{\otimes 3}|0_3\rangle &= H^{\otimes 3}|000\rangle = \frac{1}{\sqrt{2^3}} \sum_{y=0}^{2^3-1} |y\rangle_3 \\
&= \frac{1}{\sqrt{2^3}} (|0\rangle_3 + |1\rangle_3 + |2\rangle_3 + \ldots|7\rangle_3) \\
&= \frac{1}{\sqrt{2^3}} (|000\rangle + |001\rangle + |010\rangle + \ldots|111\rangle)
\end{aligned}
$$

Working with H gate

$$
\begin{aligned}
H^{\otimes n}|x_n\rangle &= \frac{1}{\sqrt{2^n}} \sum_{y=0}^{2^n-1} -1^{x \cdot y} |y\rangle_n \\
H^{\otimes 2}|3_2\rangle &= ????
\end{aligned}
$$

Working with H gate

$$
\begin{aligned}
H^{\otimes n}|x_n\rangle &= \frac{1}{\sqrt{2^n}}\sum_{y=0}^{2^n-1} -1^{x\cdot y}|y\rangle_n \\
H^{\otimes 2}|3_2\rangle &= ????
\end{aligned}
$$

$$
\begin{aligned}
H^{\otimes 2}|3_2\rangle &= \frac{1}{\sqrt{2^2}}\sum_{y=0}^{2^2-1} -1^{3\cdot y}|y\rangle_2 \\
&= \frac{1}{2}(-1^{3\cdot 0}|0\rangle_2 + (-1^{3\cdot 1}|1\rangle_2) + (-1^{3\cdot 2}|2\rangle_2) + (-1^{3\cdot 3}|3\rangle_2)) \\
&= \frac{1}{2}(|00\rangle - |01\rangle - |10\rangle + |11\rangle) \\
x\cdot y &= x_0 y_0 \oplus x_1 y_1 = y_0 + y_1 \\
if \quad & y = 0, y_0 = 0, y_1 = 0 \\
if \quad & y = 1, y_0 = 0, y_1 = 1
\end{aligned}
$$

Phase Shift Gates

1. S Gate (Phase Gate) : - Adds a $\frac{\pi}{2}$ phase:

$$\begin{aligned} S &= \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/2} \end{bmatrix} \\ S|j\rangle &= e^{i\frac{\pi}{2}j}|j\rangle \\ S^\dagger|j\rangle &= e^{-i\frac{\pi}{2}j}e^{i\frac{\pi}{2}j}|j\rangle = |j\rangle \\ S^2|0\rangle &= |0\rangle \\ S^2|1\rangle &= e^{i\frac{\pi}{2}}e^{i\frac{\pi}{2}}|1\rangle = |-1\rangle \end{aligned}$$

Two qubits Gate: CNOT or CX (Controlled-NOT)

$$|00\rangle = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad |01\rangle = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad |10\rangle = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \quad |11\rangle = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Two qubits Gate: CNOT or CX (Controlled-NOT)

$$|00\rangle = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad |01\rangle = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad |10\rangle = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \quad |11\rangle = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\mathrm{CX} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Two qubits Gate: CNOT or CX (Controlled-NOT)

$$|00\rangle = \begin{bmatrix}1\\0\\0\\0\end{bmatrix}, \qquad |01\rangle = \begin{bmatrix}0\\1\\0\\0\end{bmatrix} \qquad |10\rangle = \begin{bmatrix}0\\0\\1\\0\end{bmatrix} \qquad |11\rangle = \begin{bmatrix}0\\0\\0\\1\end{bmatrix}$$

$$\mathrm{CX} = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\end{bmatrix}$$

$$CX|00\rangle = |00\rangle \qquad CX|01\rangle = |01\rangle$$

$$CX|10\rangle = |11\rangle \qquad CX|11\rangle = |10\rangle$$

$$
\begin{aligned}
CX|0j\rangle &= |0j\rangle \\
CX|1j\rangle &= |1\bar{j}\rangle
\end{aligned}
$$

$$
\boxed{CX|ij\rangle = |i\, i \oplus j\rangle = |i\rangle \otimes |i \oplus j\rangle}
$$

$$
\begin{aligned}
CX|0j\rangle &= |0j\rangle \\
CX|1j\rangle &= |1\bar{j}\rangle
\end{aligned}
$$

$$
\boxed{CX|ij\rangle = |i\, i \oplus j\rangle = |i\rangle \otimes |i \oplus j\rangle}
$$

$$
\begin{aligned}
|i\rangle &\rightarrow \text{Control} \\
|j\rangle &\rightarrow \text{Target}
\end{aligned}
$$

Two qubits CZ Gate

Controlled-Z (CZ) Gate :Adds a phase flip to the target qubit if the control qubit is $|1\rangle$:

$$CZ = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$CZ|ij\rangle \quad = \quad (-1^{i \cdot j})|ij\rangle$$

Three qubit Toffoli Gate (CCNOT or CCX Gate)

$$CCX|ijk\rangle \quad = \quad |i \quad j \qquad i \cdot j \oplus k\rangle = |ij\rangle \otimes |i \cdot j \oplus k\rangle$$

$$|ij\rangle \rightarrow \text{Control}$$
$$|k\rangle \rightarrow \text{Target}$$

Universal quantum gates

- $\{H, S, CX\}$
- CX, Any 1-Qubit Gate

Universal quantum gates

- $\{H, S, CX\}$
- CX, Any 1-Qubit Gate

$$\begin{aligned} X &= HS^2H \\ Y &= S(HS^2H)S^3 \\ Z &= S^2 \\ CZ &= (I \otimes H)CX(I \otimes H) \end{aligned}$$

Basic quantum circuits

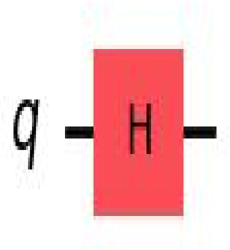


Figure: $|\psi\rangle = H|q\rangle$

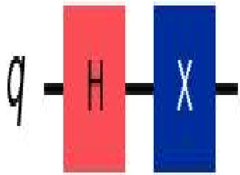


Figure: $|\psi\rangle = XH|q\rangle$

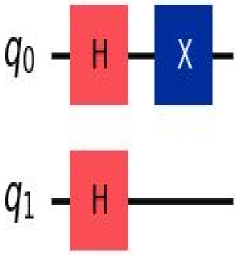


Figure: $|\psi\rangle = XH|q_0\rangle \otimes H|q_1\rangle$

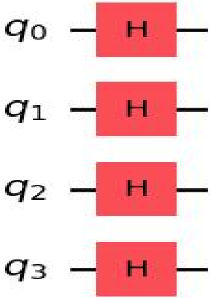


Figure: $|\psi\rangle = H|q_0\rangle \otimes H|q_1\rangle \otimes H|q_2\rangle \otimes H|q_3\rangle$

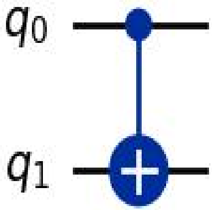


Figure: $|\psi\rangle = CX^{01}|q_0 q_1\rangle$

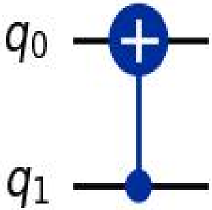


Figure: $|\psi\rangle = CX^{10}|q_0 q_1\rangle$

QC - Entanglement

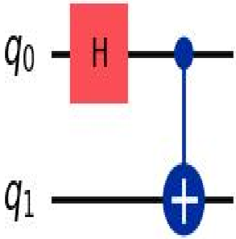
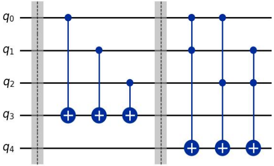


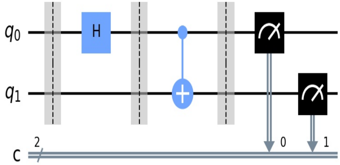
Figure: $|\psi\rangle = CX^{01} H^{0} |q_0 q_1\rangle$

Full Adder

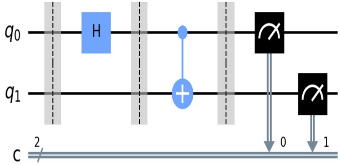


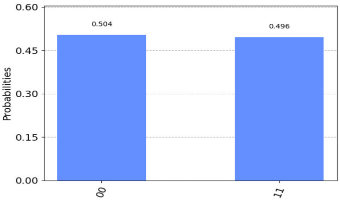
$$
\begin{aligned}
|\psi\rangle &= |q_3 q_4\rangle \\
|q_3\rangle &= |0 \oplus q_0 \oplus q_1 \oplus q_2\rangle \\
|q_4\rangle &= |0 \oplus q_0 \cdot q_1 \oplus q_0 \cdot q_2 \oplus q_1 \cdot q_2\rangle \\
\text{For } q_0 &= q_1 = q_2 = 1 \\
|q_3\rangle &= |0 \oplus 1 \oplus 1 \oplus 1\rangle = |1\rangle \\
|q_4\rangle &= |0 \oplus 1 \cdot 1 \oplus 1 \cdot 1 \oplus 1 \cdot 1\rangle = 1 \\
|\psi\rangle &= |q_3 q_4\rangle = |11\rangle
\end{aligned}
$$

Measurement - Entanglement



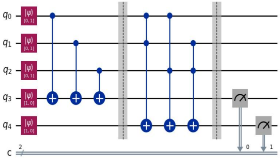
Measurement - Entanglement

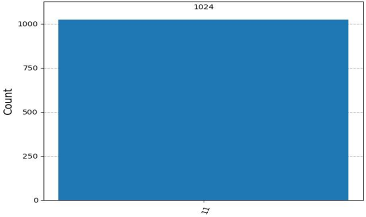




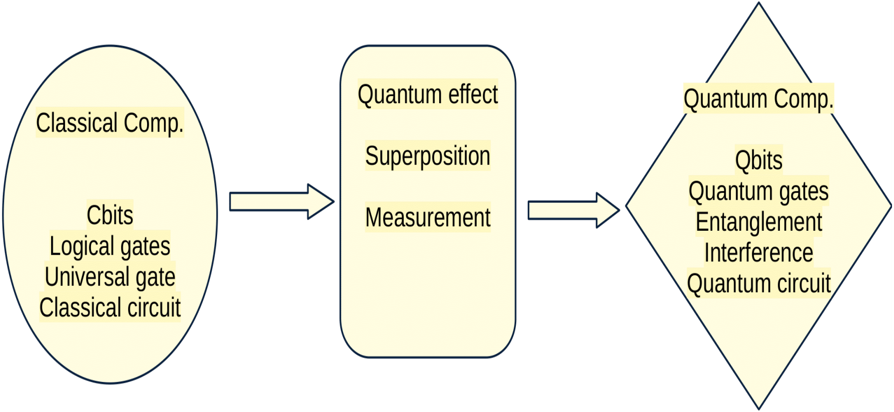
$$
\begin{aligned}
|\psi_0\rangle &= |0\rangle \times |0\rangle = |00\rangle \\
|\psi_1\rangle &= H|0\rangle \times |0\rangle \\
&= \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \times |0\rangle \\
&= \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle) \\
|\psi_2\rangle &= \frac{1}{\sqrt{2}}(CX|00\rangle + CX|10\rangle) \\
&= \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)
\end{aligned}
$$

Quantum circuits - Measurements





Conclusions




THANK YOU!