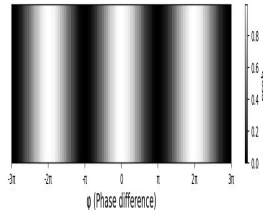# Harnessing Quantum Power: Algorithms and Simulations

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Recap

cbits $\Rightarrow$ qbits (Superposition of cbits)

$$
\begin{aligned}
|\psi\rangle &= \alpha|0\rangle + \beta|1\rangle \quad \textbf{(Superposition)} \\
\langle i|j\rangle &= \delta_{ij}, \quad |j\rangle = |0\rangle \, or \, |1\rangle \quad \textbf{(Orthonormality)} \\
\langle \psi|\psi\rangle &= |\alpha|^2 + |\beta|^2 = 1 \\
|\alpha|^2 &\Rightarrow \text{Prob. for} |0\rangle \\
|\beta|^2 &\Rightarrow \text{Prob. for} |1\rangle
\end{aligned}
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2-qubit states

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|\psi\rangle = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle
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2-qubit states

$$
\begin{aligned}
|\psi\rangle &= \alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle \\
|\alpha_{ij}|^2 &\Rightarrow \text{Prob. for} |ij\rangle
\end{aligned}
$$

Recap

Measuring a qubit collapses its state into either $|0\rangle$ or $|1\rangle$

$$\alpha|0\rangle + \beta|1\rangle \xrightarrow{\text{Measurments}} |0\rangle \, or \, |1\rangle$$

Recap

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2-Qbit states

$$\alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle \xrightarrow{\text{Measurments}} \begin{array}{ccc} |00\rangle & or & |01\rangle \\ or|10\rangle or & & |11\rangle \end{array}$$

Not deterministic!!!

Recap

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$X|j\rangle = |\bar{j}\rangle$$

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$H^{\otimes n}|x_n\rangle = \frac{1}{\sqrt{2^n}} \sum_{y=0}^{2^n-1} (-1)^{x \cdot y} |y\rangle_n$$

$$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$Y|j\rangle = i(-1^j)|\bar{j}\rangle$$

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$CX = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$Z|j\rangle = (-1^j)|j\rangle$$

$$CX|ij\rangle = |i\, i \oplus j\rangle = |i\rangle \otimes |i \oplus j\rangle$$

1. S Gate (Phase Gate) : - Adds a $\frac{\pi}{2}$ phase:

$$
\begin{aligned}
S &= \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/2} \end{bmatrix} \\
S|j\rangle &= e^{i\frac{\pi}{2}j}|j\rangle
\end{aligned}
$$

1. S Gate (Phase Gate) : - Adds a $\frac{\pi}{2}$ phase:

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Universal quantum gates

- $\{H, S, CX\}$
- CX, Any 1-Qubit Gate

$$X = HS^2H$$
$$Y = S(HS^2H)S^3$$
$$Z = S^2$$
$$CZ = (I \otimes H)CX(I \otimes H)$$

Deutsch–Jozsa algorithm

Some useful notations

$$\{0,1\}^n \ = \ \{x\} = \{x_0 x_1 x_2 ... x_{n-1}\} \quad x_i \in 0, 1$$

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Examples

$$\{0,1\}^1 = \{x_0\} = \{0,1\}$$

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$$\{0,1\}^1 = \{x_0\} = \{0, 1\}$$
$$\{0,1\}^2 = \{x_0 x_1\} = \{00, 01, 10, 11\}$$

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$$\{0,1\}^n = \{2^n \text{combinations}\}$$

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Dot product or bitwise sum

$$
\begin{aligned}
x \cdot y &= x_0 y_0 \oplus x_1 y_1 \oplus \ldots \oplus x_{n-1} y_{n-1} \\
&= (x_0 y_0 + x_1 y_1 + \ldots + x_{n-1} y_{n-1}) mod(2) \\
\text{Example:} \quad & 1 \oplus 1 \oplus 1 = 3 mod(2) = 1
\end{aligned}
$$

A Boolean function:

$$f : \{0, 1\}^n \to 0, 1$$

Constant or Balanced function

A Boolean function:

$$f : \{0,1\}^n \to 0,1$$

1. Constant function (f) : $(f(x) = 0; \forall x)$ or $(f(x) = 1; \forall x)$
2. Balanced function (f) : $(f(x) = 0)$ for exactly half of the inputs and $(f(x) = 1)$ for the other half.

Examples

2-bit function - Constant or Balanced ?

x_0	x_1	$f(x)$	$f(x)$
0	0	0	1
0	1	0	1
1	0	0	1
1	1	0	1

x_0	x_1	$f(x)$	$f(x)$
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	1

Examples

2-bit function - Constant or Balanced ?

x_0	x_1	$f(x)$	$f(x)$
0	0	0	1
0	1	0	1
1	0	0	1
1	1	0	1

x_0	x_1	$f(x)$	$f(x)$
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	1

3-bit functuion ?

x_0	x_1	x_2	$f(x)$	$f(x)$
0	0	0	0	0
0	0	1	0	1
0	1	0	0	0
0	1	1	0	0
1	0	0	1	0
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

Determine whether (f) is constant or balanced.

Classical Computer- $(2^{n-1} + 1)$ evaluations in worst case.
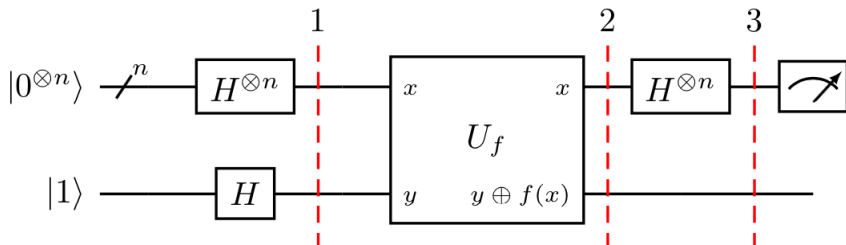Clock speed = 1 GHz = 10 evaluations per second, Number of evaluations
= (2^{79})

$$
\begin{aligned}
\text{Time (years)} &= \frac{2^{79}}{10^9} \text{ eval/sec} \div (3.156 \times 10^7 \text{sec/year}) \\
&= 1.9168 \times 10^7 \text{years.}
\end{aligned}
$$

Estimated time for n=80

Speed of processor eval/s	Time
1 GHz = 10^9	1.7×10^7 Years
1 TFLOP $\approx 10^{12}$	1.7×10^4 Years
1 PFLOP = 10^{15}	17years
1 EFLOP = 10^{18}	6.2 days

Deutsch–Jozsa algorithm.

Quantum computer: only one execution!!

$$|\psi_0\rangle = |0\rangle^{\otimes n} \otimes |1\rangle \qquad |\psi_1\rangle = H^{\otimes n}|0\rangle^{\otimes n} \otimes H|1\rangle$$

$$|\psi_2\rangle = U_f|\psi_1\rangle \qquad |\psi_3\rangle = H^{\otimes n}|\psi_2\rangle$$

Deutsch–Jozsa algorithm.

1. Prepare two quantum registers. $|x\rangle = 0^{\otimes n}$ and $|y\rangle = 1$

$$|\psi_0\rangle = |x\rangle \otimes |y\rangle = |0\rangle^{\otimes n} \otimes |1\rangle$$

2. Apply a Hadamard gate to each qubit:

$$|\psi_1\rangle = H^{\otimes n}|0\rangle^{\otimes n} \otimes H|1\rangle$$

3. Apply the quantum oracle:

$$
\begin{aligned}
U_f|x\rangle \otimes |y\rangle &= |x\rangle \otimes |y \oplus f(x)\rangle \\
|\psi_2\rangle &= U_f|\psi_1\rangle a
\end{aligned}
$$

4. Apply Hadamard gate to each qubit in the first register:

$$|\psi_3\rangle = H^{\otimes n}|\psi_2\rangle$$

5. Measure the first register.

$$
\begin{aligned}
|\psi_0\rangle &= |0\rangle^{\otimes n} \otimes |1\rangle \\
|\psi_1\rangle &= H^{\otimes n}|0\rangle^{\otimes n} \otimes H|1\rangle \\
|\psi_1\rangle &= \frac{1}{\sqrt{2^n}} \sum_{x=0}^{2^n-1} |x_n\rangle \otimes \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right) \\
|\psi_1\rangle &= \frac{1}{\sqrt{2^{n+1}}} \sum_{x=0}^{2^n-1} |x_n\rangle \otimes \left(|0\rangle - |1\rangle\right) \\
|\psi_2\rangle &= U_f|\psi_1\rangle \\
&= U_f\left(\frac{1}{\sqrt{2^{n+1}}} \sum_{x=0}^{2^n-1} |x_n\rangle \otimes \left(|0\rangle - |1\rangle\right)\right) \\
|\psi_2\rangle &= \frac{1}{\sqrt{2^{n+1}}} \sum_{x=0}^{2^n-1} |x_n\rangle(|0 \oplus f(x)\rangle - |1 \oplus f(x)\rangle) \\
&= \frac{1}{\sqrt{2^{n+1}}} \sum_{x=0}^{2^n-1} (-1)^{f(x)}|x_n\rangle(|0\rangle - |1\rangle)
\end{aligned}
$$

$$
\begin{aligned}
|\psi_3\rangle &= H^{\otimes n}|\psi_2\rangle \\
&= \frac{1}{\sqrt{2^{n+1}}} \sum_{x=0}^{2^n-1} (-1)^{f(x)} H^{\otimes n}|x_n\rangle (|0\rangle - |1\rangle) \\
|\psi_3\rangle &= \frac{1}{2^n} \sum_{x=0}^{2^n-1} (-1)^{f(x)} \left[\sum_{y=0}^{2^n-1} (-1)^{x \cdot y}|y_n\rangle \right] \otimes \frac{(|0\rangle - |1\rangle)}{\sqrt{2}} \\
&= \frac{1}{2^n} \sum_{y=0}^{2^n-1} \left[\sum_{x=0}^{2^n-1} (-1)^{f(x)} (-1)^{x \cdot y} \right] |y_n\rangle \otimes \frac{(|0\rangle - |1\rangle)}{\sqrt{2}}
\end{aligned}
$$

Amplitude for $|y\rangle$ is

$$\alpha_y = \frac{1}{2^n} \sum_{x=0}^{2^n-1} (-1)^{f(x)} (-1)^{x \cdot y}$$

for $|y\rangle = |0\rangle^{\otimes n}$ is

$$\alpha_0 = \frac{1}{2^n} \sum_{x=0}^{2^n-1} (-1)^{f(x)}$$
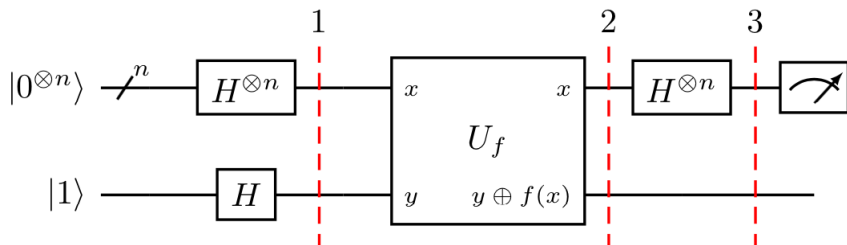
If $f(x) = 0$ (constant function)

$$\alpha_0 = \frac{1}{2^n} \sum_{x=0} 1 = 1$$

If $f(x)$ is balanced, i.e., $f(x) = 0$ and 1 equal number of times,

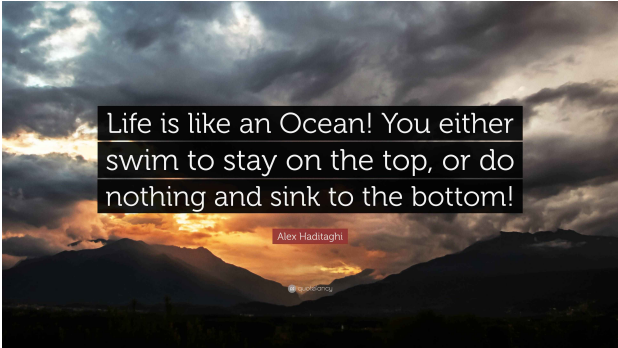$$\alpha_0 = \frac{1}{2^n} \sum_{x=0}^{2^n-1} (-1)^{f(x)} = 0$$

Therefore, measuring the amplitude of $|y\rangle = |0\rangle^{\otimes n}$ will inform us about the nature of $f(x)$.

$$\begin{aligned}
|\psi\rangle &= \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle) \\
\langle\psi|\mathcal{O}|\psi\rangle &= \frac{1}{2}(\langle 000| + \langle 111|)\mathcal{O}(|000\rangle + |111\rangle) \\
&= (\langle 000|\mathcal{O}\langle 000|) + (\langle 000|\mathcal{O}\langle 111|) \\
&\quad + (\langle 111|\mathcal{O}\langle 000|) + (\langle 111|\mathcal{O}\langle 111|)
\end{aligned}$$

$$\begin{aligned}
ZZZ|000\rangle &= |000\rangle \\
ZZZ|111\rangle &= -|111\rangle \\
\langle 000|ZZZ|000\rangle &= 1 \\
\langle 000|ZZZ|111\rangle &= 0 \\
\langle 111|ZZZ|000\rangle &= 0 \\
\langle 111|ZZZ|111\rangle &= -1 \\
\langle\psi|ZZZ|\psi\rangle &= 0
\end{aligned}$$