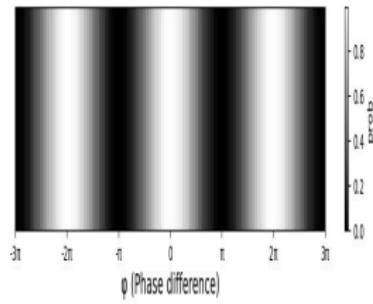


# From Bits to Qubits: Foundations of Quantum Computing

Bijay K. Agrawal

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**QCFAIANGT 17 – 22 Nov. 2025**



# Questions to be addressed

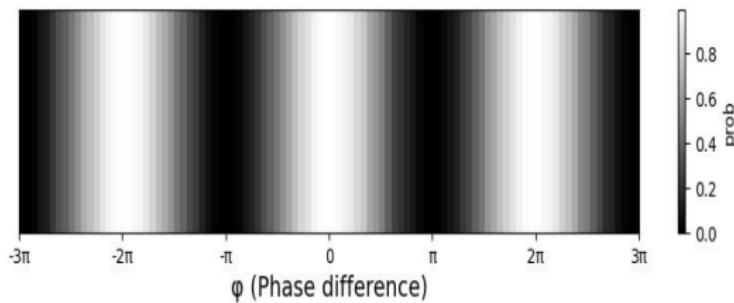
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- Why Quantum computer matters?
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- Extension to Quantum computer

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- Why Quantum Computers?
- Why Quantum computer matters?
- What are the Prerequisites?
- Basics of Classical Computers
- Extension to Quantum computer
- Experiencing Quantum computer



# First generation computers (1940-1956)

- Vacuum tubes for circuitry and magnetic drums for memory.
- They were very large in size and taking up entire room.



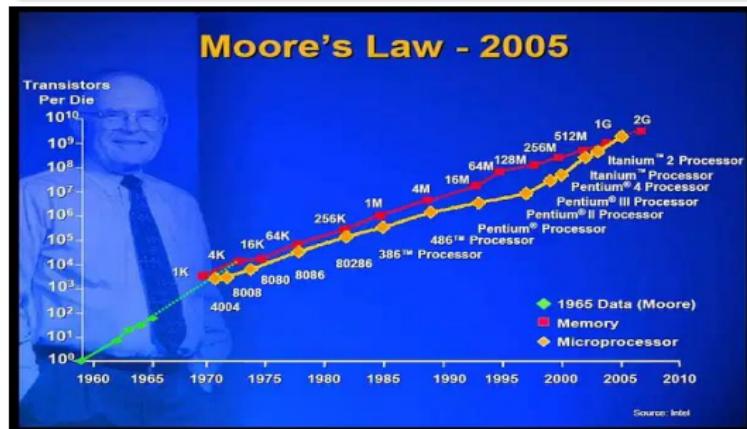
# Fifth generation computers (1980- Present)

- Ultra Large-Scale Integration (ULSI):
- Millions of transistors on a single microchip
- Parallel processing Natural language understanding: Can understand human language
- Energy efficiency, Size: Small and portable, with large storage capacity
- Input/output devices Keyboard, mouse, monitor, touchscreen, speech input, printer, etc.



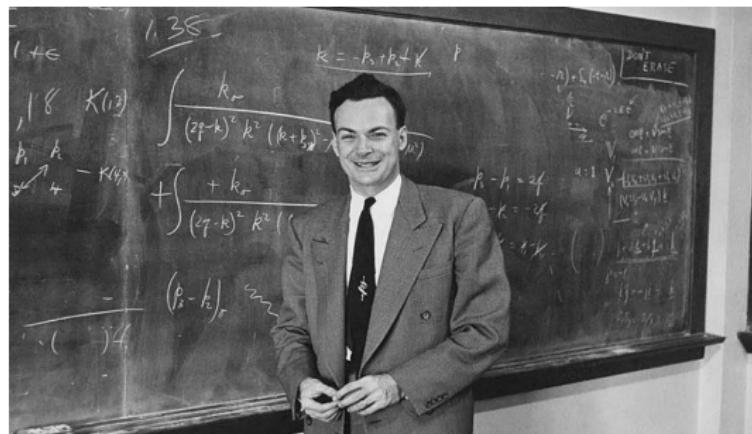
# Moore's Law

Gordon Moore, co-founder of Intel, first described Moore's Law in 1965. Moore's Law is the observation that the number of transistors in a computer chip doubles every two years. It's a projection of a historical trend in the computing industry, and is not a law of physics.



... it seems that the laws of the physics present no barrier to reducing the size of the computers until bits are the size of the atoms, and quantum behaviour holds sway'

Richard P. Feynman



# Time line of Quantum Computing

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- 1994 Shor algorithm
- 1995: Expt. realization of first quantum logic gate with trapped ions
- 1996: Grover's algorithm

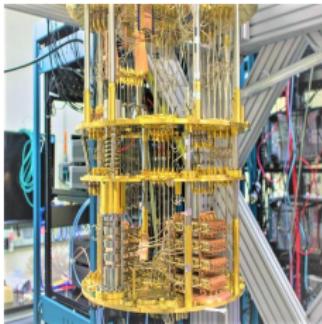
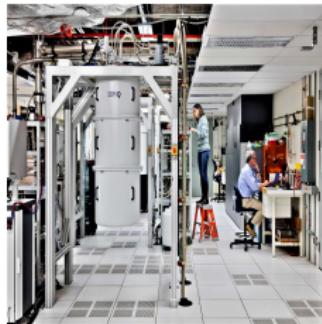
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- 2000: Quantum computation and information, Nielsen and Chuang cited 58500

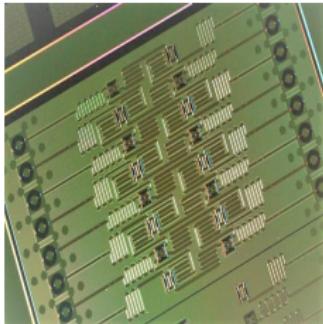
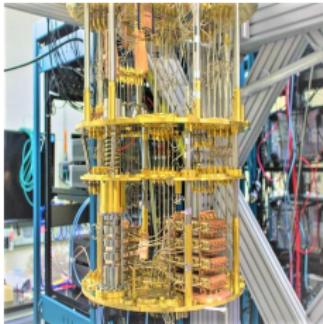
# Superconducting quantum processor



# Superconducting quantum processor



# Superconducting quantum processor



## Why Quantum computer matters?

- Finance
- Health
- Energy & Agriculture
- Global climate

Prerequisite for Quantum computing ?  
Physicist? Computer Engineer?

Prerequisite for Quantum computing ?  
Physicist? Computer Engineer?  
**NOTHING of that sort!!**

# Basis for classical computing

$$c = \begin{cases} |0\rangle & (\text{low}) \\ |1\rangle & (\text{high}) \end{cases}$$

**cbits: deterministic  
& can be read  
directly.**

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**cbits: deterministic  
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Logic gates are  $\Rightarrow$  not reversible

## LOGIC GATES DIAGRAMS



NOT



OR



AND



XNOR



NOR



XOR



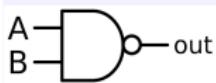
NAND

# Universal logic Gates (classical)

a) NAND Gate  
- Truth Table:

A	B	$\overline{A \cdot B}$
0	0	1
0	1	1
1	0	1
1	1	0

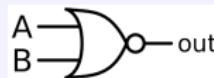
$$Y = \overline{A \cdot B}$$



NOR Gate - Truth Table:

A	B	$\overline{A + B}$
0	0	1
0	1	0
1	0	0
1	1	0

$$Y = \overline{A + B}$$



# NAND gate implementation of NOT, AND & OR

## 1. NOT Gate:

$$\text{NOT}(A) = \overline{A} = \overline{\overline{A} \cdot \overline{A}} = \text{NAND}(A, A)$$

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2. AND Gate

$$\begin{aligned}\text{AND}(A, B) &= (A \cdot B) \overline{\overline{A} \cdot \overline{B}} \\ &= \text{NOT}(\text{NAND}(A, B)) \\ &= \text{NAND}(\text{NAND}(A, B), \text{NAND}(A, B))\end{aligned}$$

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3. OR Gate

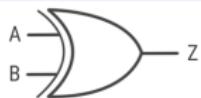
$$\begin{aligned}A + B &= \overline{(\overline{A} \cdot \overline{B})} \\ &= \text{NAND}(\overline{A}, \overline{B}) \\ &= \text{NAND}(\text{NAND}(A, A), \text{NAND}(B, B))\end{aligned}$$

## XOR Gate (Exclusive OR)

Outputs 1 if inputs  
are different.

Truth Table:

A	B	$A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0



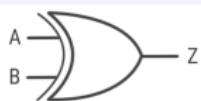
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$$\begin{aligned}XOR &\rightarrow mod(2) \\ A \oplus B &= (A + B)mod(2) \\ A \oplus B \oplus C &= (A + B + C)mod(2) \\ 1 \oplus 1 \oplus 1 &= 3mod(2) = 1 \\ 1 \oplus 1 \oplus 1 \oplus 1 &= 4mod(2) = 0\end{aligned}$$



## XOR using NAND gate

$$A \oplus B = (A \cdot \overline{B}) + (\overline{A} \cdot B)$$

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$$A \oplus B = (A \cdot \overline{B}) + (\overline{A} \cdot B)$$

$$P = \text{NAND}(A, B) = \overline{(A \cdot B)}$$

$$Q = \text{NAND}(A, P) = \text{NAND}(A, \text{NAND}(A, B))$$

$$R = \text{NAND}(B, P) = \text{NAND}(B, \text{NAND}(A, B))$$

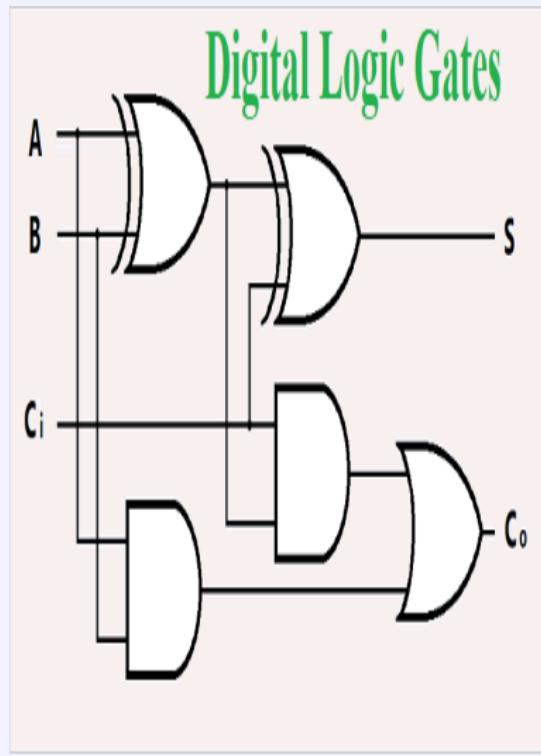
$$A \oplus B = \text{NAND}(Q, R)$$

$$A \oplus B = \text{NAND}(\text{NAND}(A, \text{NAND}(A, B)), \text{NAND}(B, \text{NAND}(A, B)))$$

# Full Adders - Basic arithmetic units

$$\begin{aligned} S &= A + B + Cin \\ S &= A \oplus B \oplus Cin \\ C_{out} &= (A \cdot B) + (A \oplus B) \cdot Cin \\ C_{out} &= (A \cdot B) + (B \cdot Cin) + (Cin \cdot A) \end{aligned}$$

A	B	C <sub>in</sub>	Sum (S)	Carry (C <sub>out</sub> )
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1



# Classical → Quantum

- Quantum superposition/parallelism
- Wave function collapse
- Quantum entanglement
- Interference

# Basis for Quantum computing

cbits  $\Rightarrow$  qbits (Superposition of cbits)

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \quad (\text{Superposition})$$

$$\langle\psi| = \alpha^*\langle 0| + \beta^*\langle 1|$$

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\langle 0| = [1 \quad 0] \quad \langle 1| = [0 \quad 1] \quad (\text{Matrix Rep.})$$

$$\langle 0|0\rangle = [1 \quad 0] \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 1$$

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$$\langle i|j\rangle = \delta_{ij}, \quad |j\rangle = |0\rangle \text{ or } |1\rangle \quad (\text{Orthonormality})$$

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$$\begin{aligned}\langle\psi|\psi\rangle &= \alpha^*\alpha \langle 0|0\rangle + \alpha^*\beta \langle 0|1\rangle + \beta^*\alpha \langle 1|0\rangle + \beta^*\beta \langle 1|1\rangle \\ &= \alpha^*\alpha + \beta^*\beta = |\alpha|^2 + |\beta|^2 = 1\end{aligned}$$

$$|\alpha|^2 \Rightarrow \text{Prob. for } |0\rangle, \quad |\beta|^2 \Rightarrow \text{Prob. for } |1\rangle$$

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## 2-qubit states

$$|\psi\rangle = \alpha_{00}|q_0 0 q_1 0\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle$$

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$$|00\rangle = |0\rangle \otimes |0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$|01\rangle = |0\rangle \otimes |1\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$|10\rangle = |1\rangle \otimes |0\rangle = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \quad |11\rangle = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

# Measurements

Measuring a qubit collapses its state

collapses into either  $|0\rangle$  or  $|1\rangle$

with probabilities  $|\alpha|^2$  and  $|\beta|^2$  respectively.

$$\alpha|0\rangle + \beta|1\rangle \xrightarrow{\text{Measurements}} |0\rangle \text{ or } |1\rangle$$

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## 2-Qbit states

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$$\alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle \xrightarrow{\text{Measurements}} |00\rangle \text{ or } |01\rangle \\ \text{or } |10\rangle \text{ or } |11\rangle$$

**Not deterministic!!!**

# Entanglement

Qubits can become entangled, meaning the state of one qubit is directly related to the state of another.

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}} \left( |_{q_0 q_1}^{00}\rangle + |_{q_0 q_1}^{11}\rangle \right)$$

$$|\Phi^-\rangle = \frac{1}{\sqrt{2}} (|00\rangle - |11\rangle)$$

$$|\Psi^+\rangle = \frac{1}{\sqrt{2}} (|01\rangle + |10\rangle)$$

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}} (|01\rangle - |10\rangle)$$

# Quantum gates

Quantum gates- "the building blocks of quantum computation", analogous to classical logic gates. They operate on qubits and manipulate their states by applying unitary transformations.

X (NOT Gate) :

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$X|0\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = |1\rangle$$

$$X|1\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle$$

$$X|j\rangle = |\bar{j}\rangle$$

# Quantum gates

Y Gate :

$$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$Y|j\rangle = i(-1^j)|\bar{j}\rangle$$

# Quantum gates

Y Gate :

$$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

Z Gate :

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$Y|j\rangle = i(-1^j)|\bar{j}\rangle$$

$$Z|j\rangle = (-1^j)|j\rangle$$

$$X, Y, Z = X_1, X_2, X_3$$

$$[X_i, X_j] = 2i\epsilon_{ijk}X_k$$

$$\{X_i, X_j\} = 2\delta_{ij}\mathbb{1}$$

# Quantum gates

Y Gate :

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$$X, Y, Z = X_1, X_2, X_3$$

$$[X_i, X_j] = 2i\epsilon_{ijk}X_k$$

$$\{X_i, X_j\} = 2\delta_{ij}\mathbb{1}$$

$$X_i \equiv \sigma_i$$

$$|0\rangle, |1\rangle \equiv |\uparrow\rangle|\downarrow\rangle$$

# Quantum gates

Hadamard Gate (H) - Creates superposition:

$$\begin{aligned} H &= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \\ &= \frac{1}{\sqrt{2}} \left[ \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right] \\ &= \frac{1}{\sqrt{2}}(Z + X) \end{aligned}$$

$$\begin{aligned} H|0\rangle &= \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \\ H|1\rangle &= \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \end{aligned}$$

$$H|j\rangle = \frac{1}{\sqrt{2}}(|0\rangle + (-1)^j|1\rangle)$$

# Working with H gate

$$H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$\begin{aligned} H^{\otimes 2}|00\rangle &= \frac{1}{\sqrt{2^2}}(|0\rangle + |1\rangle) \otimes (|0\rangle + |1\rangle) \\ &= \frac{1}{\sqrt{2^2}}(|00\rangle + |01\rangle + |10\rangle + |11\rangle) \\ &= \frac{1}{\sqrt{2^2}}(|0\rangle_2 + |1\rangle_2 + |2\rangle_2 + |3\rangle_2) \end{aligned}$$

$$= \frac{1}{\sqrt{2^2}} \sum_{y=0}^{2^2-1} |y\rangle_2$$

$$H^{\otimes n}|0_n\rangle = \frac{1}{\sqrt{2^n}} \sum_{y=0}^{2^n-1} |y\rangle_n$$

# Working with H gate

$$H^{\otimes n}|0_n\rangle = \frac{1}{\sqrt{2^n}} \sum_{y=0}^{2^n-1} |y\rangle_n$$

$$H^{\otimes n}|x_n\rangle = \frac{1}{\sqrt{2^n}} \sum_{y=0}^{2^n-1} (-1^{x \cdot y}) |y\rangle_n$$

$$\begin{aligned} x \cdot y &= x_0 y_0 \oplus x_1 y_1 \oplus \dots \oplus x_{n-1} y_{n-1} \\ &= (x_0 y_0 + x_1 y_1 + \dots + x_{n-1} y_{n-1}) \text{mod}(2) \end{aligned}$$

# Working with H gate

$$H^{\otimes 3} |0_3\rangle = \text{????}$$

# Working with H gate

$$H^{\otimes 3}|0_3\rangle = \text{?????}$$

$$\begin{aligned} H^{\otimes 3}|0_3\rangle &= H^{\otimes 3}|000\rangle = \frac{1}{\sqrt{2^3}} \sum_{y=0}^{2^3-1} |y\rangle_3 \\ &= \frac{1}{\sqrt{2^3}} (|0\rangle_3 + |1\rangle_3 + |2\rangle_3 + \dots |7\rangle_3) \\ &= \frac{1}{\sqrt{2^3}} (|000\rangle + |001\rangle + |010\rangle + \dots |111\rangle) \end{aligned}$$

# Working with H gate

$$H^{\otimes n}|x_n\rangle = \frac{1}{\sqrt{2^n}} \sum_{y=0}^{2^n-1} -1^{x \cdot y} |y\rangle_n$$
$$H^{\otimes 2}|3_2\rangle = ????$$

# Working with H gate

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$$H^{\otimes 2}|3_2\rangle = \text{????}$$

$$\begin{aligned} H^{\otimes 2}|3_2\rangle &= \frac{1}{\sqrt{2^2}} \sum_{y=0}^{2^2-1} -1^{3 \cdot y} |y\rangle_2 \\ &= \frac{1}{2} (-1^{3 \cdot 0}|0\rangle_2 + (-1^{3 \cdot 1}|1\rangle_2) + (-1^{3 \cdot 2}|2\rangle_2) + (-1^{3 \cdot 3}|3\rangle_2)) \\ &= \frac{1}{2} (|00\rangle - |01\rangle - |10\rangle + |11\rangle) \end{aligned}$$

$$x \cdot y = x_0 y_0 \oplus x_1 y_1 = y_0 + y_1$$

$$\text{if } y = 0, y_0 = 0, y_1 = 0$$

$$\text{if } y = 1, y_0 = 0, y_1 = 1$$

# Phase Shift Gates

1. S Gate (Phase Gate) : - Adds a  $\frac{\pi}{2}$  phase:

$$S = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/2} \end{bmatrix}$$

$$S|j\rangle = e^{i\frac{\pi}{2}j}|j\rangle$$

$$S^\dagger|j\rangle = e^{-i\frac{\pi}{2}j}e^{i\frac{\pi}{2}j}|j\rangle = |j\rangle$$

$$S^2|0\rangle = |0\rangle$$

$$S^2|1\rangle = e^{i\frac{\pi}{2}}e^{i\frac{\pi}{2}}|1\rangle = |-1\rangle$$

## Two qubits Gate: CNOT or CX (Controlled-NOT)

$$|00\rangle = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad |01\rangle = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad |10\rangle = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \quad |11\rangle = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

## Two qubits Gate: CNOT or CX (Controlled-NOT)

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$$\text{CX} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

## Two qubits Gate: CNOT or CX (Controlled-NOT)

$$|00\rangle = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad |01\rangle = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad |10\rangle = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \quad |11\rangle = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$CX = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$CX|00\rangle = |00\rangle \quad CX|01\rangle = |01\rangle$$

$$CX|10\rangle = |11\rangle \quad CX|11\rangle = |10\rangle$$

$$\begin{aligned} CX|0j\rangle &= |0j\rangle \\ CX|1j\rangle &= |1\bar{j}\rangle \end{aligned}$$

$$CX|ij\rangle = |i i \oplus j\rangle = |i\rangle \otimes |i \oplus j\rangle$$

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$|i\rangle \rightarrow \text{Control}$   
 $|j\rangle \rightarrow \text{Target}$

## Two qubits CZ Gate

Controlled-Z (CZ) Gate : Adds a phase flip to the target qubit if the control qubit is  $|1\rangle$ :

$$CZ = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$CZ|ij\rangle = (-1^{i,j})|ij\rangle$$

# Three qubit Toffoli Gate (CCNOT or CCX Gate)

$$CCX|ijk\rangle = |i\ j\ i \cdot j \oplus k\rangle = |ij\rangle \otimes |i \cdot j \oplus k\rangle$$

$|ij\rangle \rightarrow \text{Control}$

$|k\rangle \rightarrow \text{Target}$

# Universal quantum gates

- $\{H, S, CX\}$
- CX, Any 1-Qubit Gate

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- $\{H, S, CX\}$
- CX, Any 1-Qubit Gate

$$X = HS^2H$$

$$Y = S(HS^2H)S^3$$

$$Z = S^2$$

$$CZ = (I \otimes H)CX(I \otimes H)$$

# Basic quantum circuits

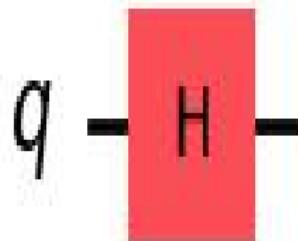


Figure:  $|\psi\rangle = H|q\rangle$

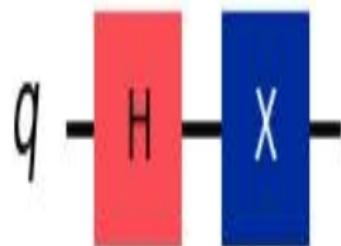


Figure:  $|\psi\rangle = XH|q\rangle$

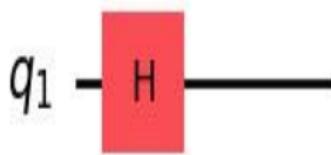
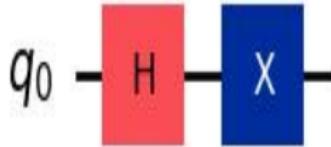


Figure:  $|\psi\rangle = XH|q_0\rangle \otimes H|q_1\rangle$

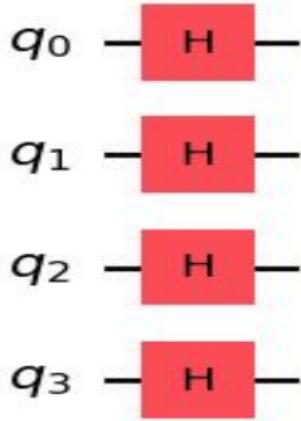


Figure:  $|\psi\rangle = H|q_0\rangle \otimes H|q_1\rangle \otimes H|q_2\rangle \otimes H|q_3\rangle$

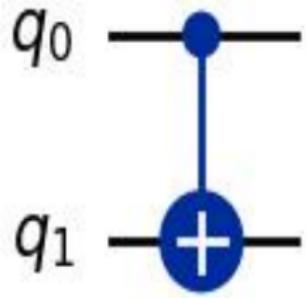


Figure:  $|\psi\rangle = CX^{01}|q_0q_1\rangle$

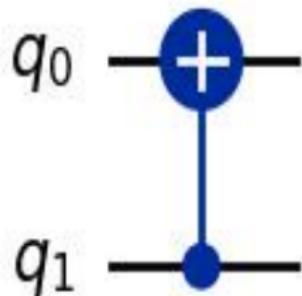


Figure:  $|\psi\rangle = CX^{10}|q_0q_1\rangle$

# QC - Entanglement

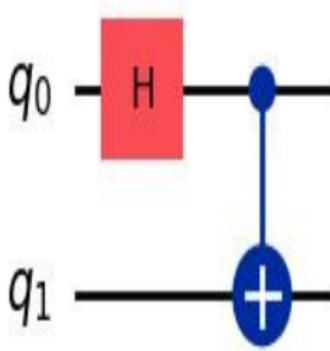
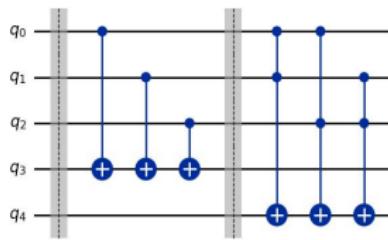


Figure:  $|\psi\rangle = CX^{01}H^0|q_0q_1\rangle$

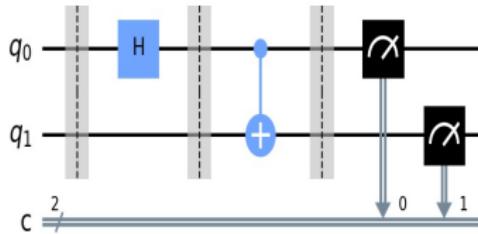
# Full Adder

$$\begin{aligned} S &= A \oplus B \oplus Cin \\ C_{out} &= (A \cdot B) + (B \cdot Cin) + (Cin \cdot A) \end{aligned}$$

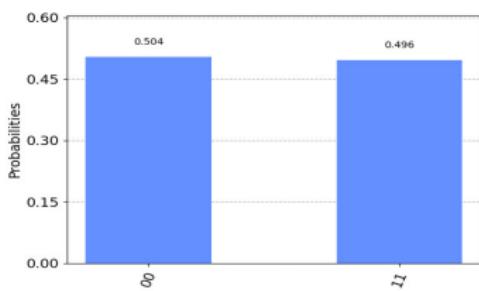
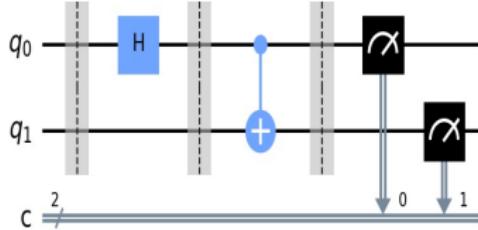


$$\begin{aligned} |\psi\rangle &= |q_3 q_4\rangle \\ |q_3\rangle &= |0 \oplus q_0 \oplus q_1 \oplus q_2\rangle \\ |q_4\rangle &= |0 \oplus q_0 \cdot q_1 \oplus q_0 \cdot q_2 \oplus q_1 \cdot q_2\rangle \\ \text{For } q_0 = q_1 = q_2 = 1 \\ |q_3\rangle &= |0 \oplus 1 \oplus 1 \oplus 1\rangle = |1\rangle \\ |q_4\rangle &= |0 \oplus 1 \cdot 1 \oplus 1 \cdot 1 \oplus 1 \cdot 1\rangle = 1 \\ |\psi\rangle &= |q_3 q_4\rangle = |11\rangle \end{aligned}$$

# Measurement - Entanglement

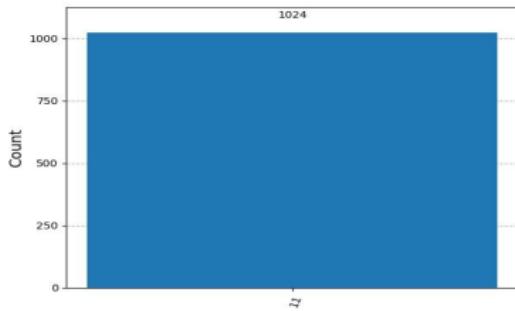
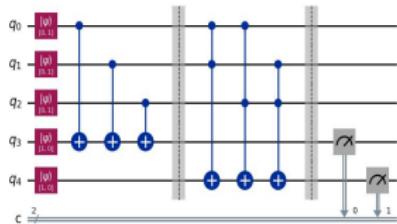


# Measurement - Entanglement

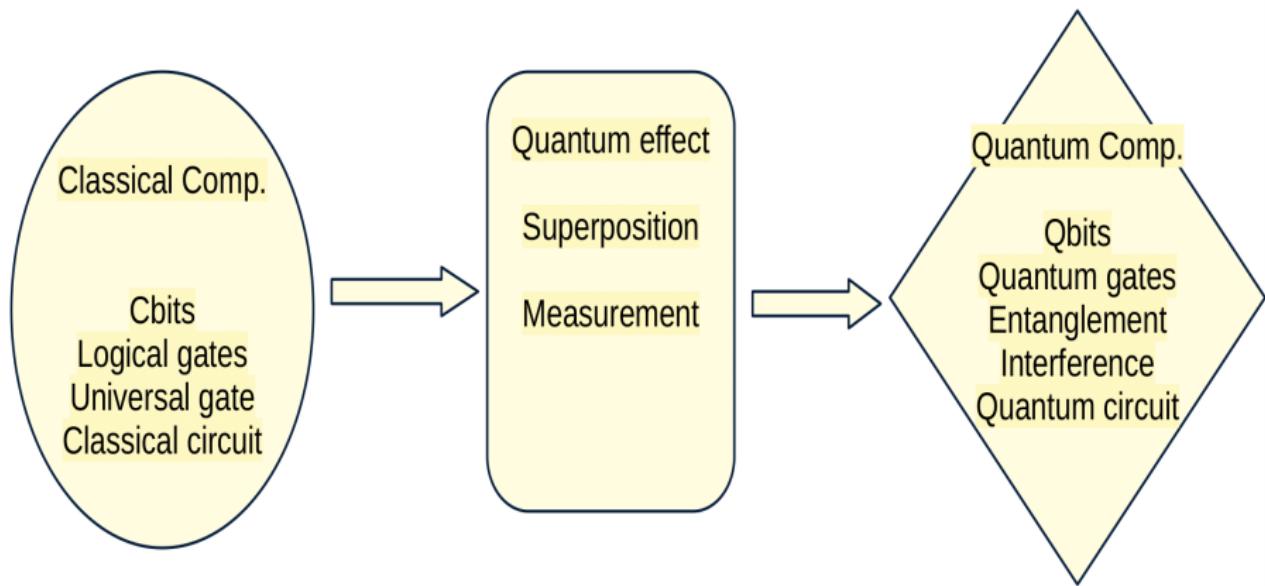


$$\begin{aligned} |\psi_0\rangle &= |0\rangle \times |0\rangle = |00\rangle \\ |\psi_1\rangle &= H|0\rangle \times |0\rangle \\ &= \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \times |0\rangle \\ &= \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle) \\ |\psi_2\rangle &= \frac{1}{\sqrt{2}}(CX|00\rangle + CX|10\rangle) \\ &= \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \end{aligned}$$

# Quantum circuits - Measurements



# Conclusions





THANK YOU!