Harnessing Quantum Power: Algorithms and Simulations

Bijay K. Agrawal

Saha Institute of Nuclear Physics, Kolkata

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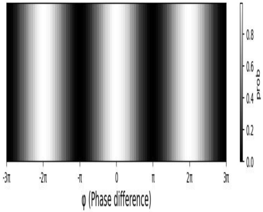

Recap

cbits $\Rightarrow$ qbits (Superposition of cbits)

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\begin{aligned}
|\psi\rangle &= \alpha|0\rangle + \beta|1\rangle \qquad \textbf{(Superposition)} \\
\langle i|j\rangle &= \delta_{ij}, \qquad |j\rangle = |0\rangle or|1\rangle \qquad \textbf{(Orthonormality)} \\
\langle\psi|\psi\rangle &= |\,\alpha\,|^2 + |\,\beta\,|^2 = 1 \\
|\,\alpha\,|^2 &\Rightarrow \text{Prob. for}|0\rangle \\
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2-qubit states

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Measuring a qubit collapses its state into either $|0\rangle$ or $|1\rangle$

$$\alpha|0\rangle + \beta|1\rangle \xrightarrow{\text{Measurments}} |0\rangle or |1\rangle$$

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$$\alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle \xrightarrow{\text{Measurments}} |00\rangle \quad or \quad |01\rangle \quad or|10\rangle or \quad |11\rangle$$

Not deterministic!!!

Recap

$$
\begin{aligned}
X &= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \\
X|j\rangle &= |\bar{j}\rangle
\end{aligned}
$$

$$
\begin{aligned}
Y &= \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \\
Y|j\rangle &= i(-1^{j})|\bar{j}\rangle \\
Z &= \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \\
Z|j\rangle &= (-1^{j})|j\rangle
\end{aligned}
$$

$$
\begin{aligned}
H &= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \\
H^{\otimes n}|x_n\rangle &= \frac{1}{\sqrt{2^n}} \sum_{y=0}^{2^n-1} (-1^{x \cdot y})|y\rangle_n
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{CX} &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \\
CX|ij\rangle &= |i\, i \oplus j\rangle = |i\rangle \otimes |i \oplus j\rangle
\end{aligned}
$$

Deutsch–Jozsa algorithm

Some useful notations

$$\{0,1\}^n \quad = \quad \{x\} = \{x_0 x_1 x_2 ... x_{n-1}\} \quad x_i \in 0,1$$

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$$\{0,1\}^3 = \{x_0 x_1 x_2\} = \{000,001,010,...,111\}$$

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$$\{0,1\}^n = \{2^n \text{combinations}\}$$

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\end{aligned}
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Dot product or bitwise sum

$$
\begin{aligned}
x \cdot y &= x_0 y_0 \oplus x_1 y_1 \oplus \ldots \oplus x_{n-1} y_{n-1} \\
&= (x_0 y_0 + x_1 y_1 + \ldots + x_{n-1} y_{n-1}) mod(2) \\
\text{Example:} \quad & 1 \oplus 1 \oplus 1 = 3 mod(2) = 1
\end{aligned}
$$

Constant or Balanced function

A Boolean function:

$$f : \{0, 1\}^n \to 0, 1$$

Constant or Balanced function

A Boolean function:

$$f : \{0, 1\}^n \rightarrow 0, 1$$

1. Constant function (f) : $(f(x) = 0; \forall x)$ or $(f(x) = 1; \forall x)$
2. Balanced function (f) : $(f(x) = 0)$ for exactly half of the inputs and $(f(x) = 1)$ for the other half.

Examples

2-bit function - Constant or Balanced ?

x_0	x_1	$f(x)$	$f(x)$
0	0	0	1
0	1	0	1
1	0	0	1
1	1	0	1

x_0	x_1	$f(x)$	$f(x)$
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	1

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x_0	x_1	$f(x)$	$f(x)$
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x_0	x_1	$f(x)$	$f(x)$
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	1

3-bit functuion ?

x_0	x_1	x_2	$f(x)$	$f(x)$
0	0	0	0	0
0	0	1	0	1
0	1	0	0	0
0	1	1	0	0
1	0	0	1	0
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

Determine whether (f) is constant** or balanced.

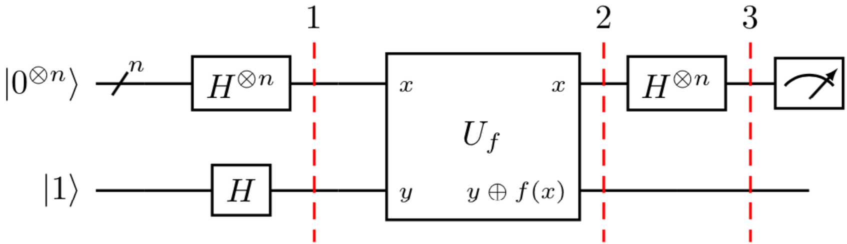
Classical Computer- $(2^{n-1}+1)$ evaluations in worst case.
Clock speed = 1 GHz = 10 evaluations per second, Number of evaluations = (2^{79})

$$\begin{aligned}\text{Time (years)} &= \frac{2^{79}}{10^9}\ \text{eval/sec} \div (3.156\times 10^7\text{sec/year})\\ &= 1.9168\times 10^7\text{years.}\end{aligned}$$

Estimated time for n=80

Speed of processor eval/s	Time
1 GHz = 10^9	1.7×10^7Years
1 TFLOP $\approx 10^{12}$	1.7×10^4Years
1 PFLOP = 10^{15}	17years
1 EFLOP = 10^{18}	6.2 days

Quantum computer: only one execution!!



$$|\psi_0\rangle = |0\rangle^{\otimes n} \otimes |1\rangle \qquad |\psi_1\rangle = H^{\otimes n}|0\rangle^{\otimes n} \otimes H|1\rangle$$

$$|\psi_2\rangle = U_f|\psi_1\rangle \qquad |\psi_3\rangle = H^{\otimes n}|\psi_2\rangle$$

Deutsch–Jozsa algorithm.

1. Prepare two quantum registers. $|x\rangle = 0^{\otimes n}$ and $|y\rangle = 1$

$$|\psi_0\rangle = |x\rangle \otimes |y\rangle = |0\rangle^{\otimes n} \otimes |1\rangle$$

2. Apply a Hadamard gate to each qubit:

$$|\psi_1\rangle = H^{\otimes n}|0\rangle^{\otimes n} \otimes H|1\rangle$$

3. Apply the quantum oracle:

$$\begin{aligned} U_f|x\rangle \otimes |y\rangle &= |x\rangle \otimes |y \oplus f(x)\rangle \\ |\psi_2\rangle &= U_f|\psi_1\rangle a \end{aligned}$$

4. Apply Hadamard gate to each qubit in the first register:

$$|\psi_3\rangle = H^{\otimes n}|\psi_2\rangle$$

5. Measure the first register.

$$
\begin{aligned}
|\psi_0\rangle &= |0\rangle^{\otimes n} \otimes |1\rangle \\
|\psi_1\rangle &= H^{\otimes n}|0\rangle^{\otimes n} \otimes H|1\rangle \\
|\psi_1\rangle &= \frac{1}{\sqrt{2^n}} \sum_{x=0}^{2^n-1} |x_n\rangle \otimes \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle) \\
|\psi_1\rangle &= \frac{1}{\sqrt{2^{n+1}}} \sum_{x=0}^{2^n-1} |x_n\rangle \otimes (|0\rangle - |1\rangle) \\
|\psi_2\rangle &= U_f|\psi_1\rangle \\
&= U_f \left(\frac{1}{\sqrt{2^{n+1}}} \sum_{x=0}^{2^n-1} |x_n\rangle \otimes (|0\rangle - |1\rangle) \right) \\
|\psi_2\rangle &= \frac{1}{\sqrt{2^{n+1}}} \sum_{x=0}^{2^n-1} |x_n\rangle(|0 \oplus f(x)\rangle - |1 \oplus f(x)\rangle) \\
&= \frac{1}{\sqrt{2^{n+1}}} \sum_{x=0}^{2^n-1} (-1)^{f(x)}|x_n\rangle(|0\rangle - |1\rangle)
\end{aligned}
$$

$$
\begin{aligned}
|\psi_3\rangle &= H^{\otimes n}|\psi_2\rangle \\
&= \frac{1}{\sqrt{2^{n+1}}}\sum_{x=0}^{2^n-1}(-1)^{f(x)}H^{\otimes n}|x_n\rangle(|0\rangle - |1\rangle) \\
|\psi_3\rangle &= \frac{1}{2^n}\sum_{x=0}^{2^n-1}(-1)^{f(x)}\left[\sum_{y=0}^{2^n-1}(-1)^{x\cdot y}|y_n\rangle\right]\otimes\frac{(|0\rangle - |1\rangle)}{\sqrt{2}} \\
&= \frac{1}{2^n}\sum_{y=0}^{2^n-1}\left[\sum_{x=0}^{2^n-1}(-1)^{f(x)}(-1)^{x\cdot y}\right]|y_n\rangle\otimes\frac{(|0\rangle - |1\rangle)}{\sqrt{2}}
\end{aligned}
$$

Amplitude for $|y\rangle$ is

$$\alpha_y = \frac{1}{2^n}\sum_{x=0}^{2^n-1}(-1)^{f(x)}(-1)^{x\cdot y}$$

for $|y\rangle = |0\rangle^{\otimes n}$ is

$$\alpha_0 = \frac{1}{2^n}\sum_{x=0}^{2^n-1}(-1)^{f(x)}$$

If $f(x) = 0$ (constant function)

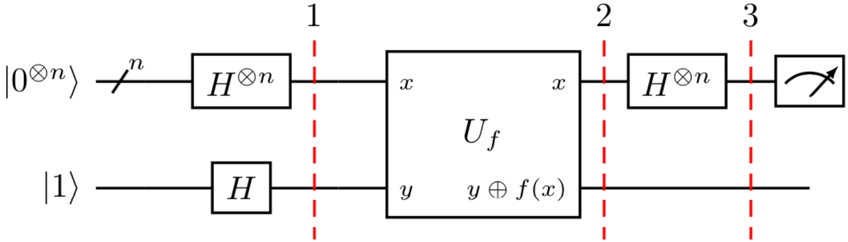
$$\alpha_0 = \frac{1}{2^n}\sum_{x=0} 1 = 1$$

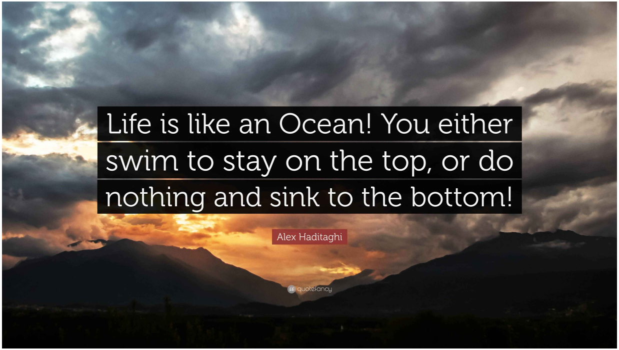
If $f(x)$ is balanced, i.e., $f(x) = 0$ and 1 equal number of times,

$$\alpha_0 = \frac{1}{2^n}\sum_{x=0}^{2^n-1}(-1)^{f(x)} = 0$$

Therefore, measuring the amplitude of $|y\rangle = |0\rangle^{\otimes n}$ will inform us about the nature of $f(x)$.

Conclusions




Life is like an Ocean! You either swim to stay on the top, or do nothing and sink to the bottom!
Alex Haditaghi
quotefancy


THANK YOU!