Statistics

Part 4

Decision Errors

		Decision	
		Fail to reject Ho	Reject Ho
Truth	Ho true	OK	Type 1 error
	Ha true	Type 2 error	OK

Example

- A food safety inspector is called upon to investigate a restaurant , with a few customer reports of poor sanitation practices. The inspector uses a hypothesis testing frame work to evaluate whether the regulations are met or not. The inspector's hypothesis are–
- Ho:restaurant meets food and safety regulations;

  Ha:restaurant does not meet food and safety regulations.
- If the inspector concludes that the restaurant meets the regulations and the restaurant stays open, when the restaurant is actually not safe, what type of error is made?
- Ans. Type 2

		Decision	
		Fail to reject Ho	Reject Ho
Truth	Ho true	OK	Type 1 error
	Ha true	Type 2 error	OK

Court of Law

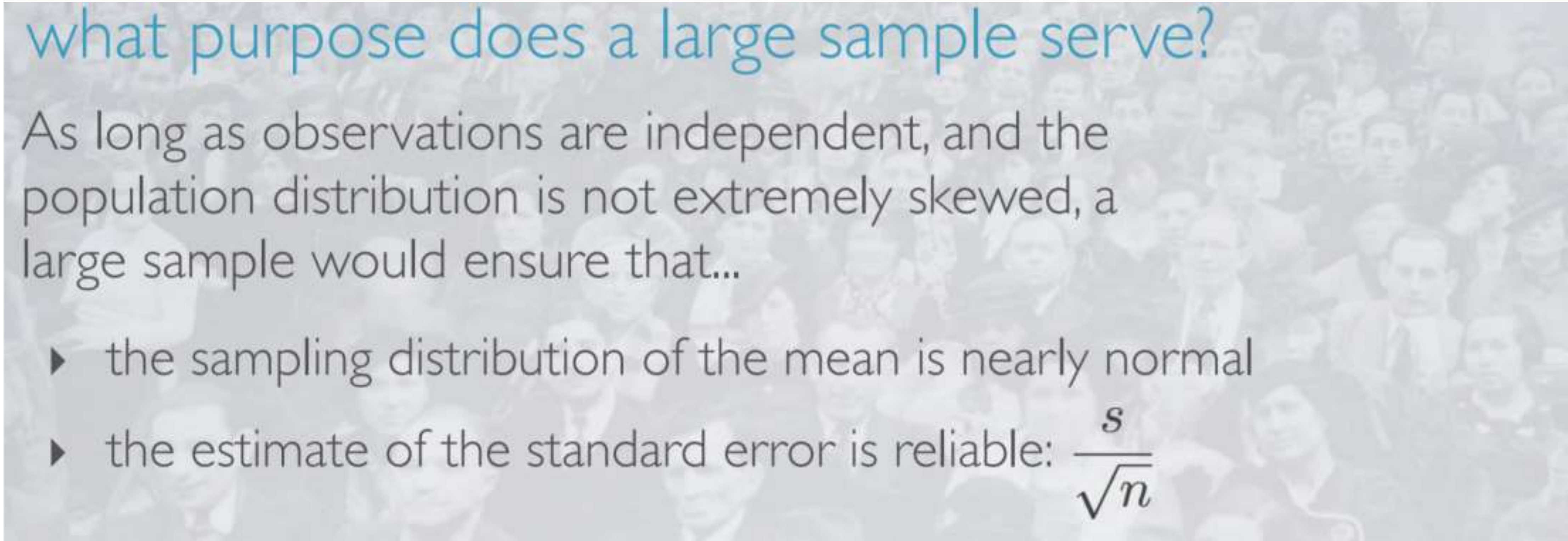
- Ho:defendant is innocent;
- Ha:defendant is guilty
- Describe the types of errors–
- Type1: defendant is innocent but announced as guilty.
- Type2:defendant is guilty but announced innocent.
- Which one would we try to minimize?
- Ans. Type 1 error

		Decision	
		Fail to reject Ho	Reject Ho
Truth	Ho true	OK	Type 1 error
	Ha true	Type 2 error	OK

Trade off

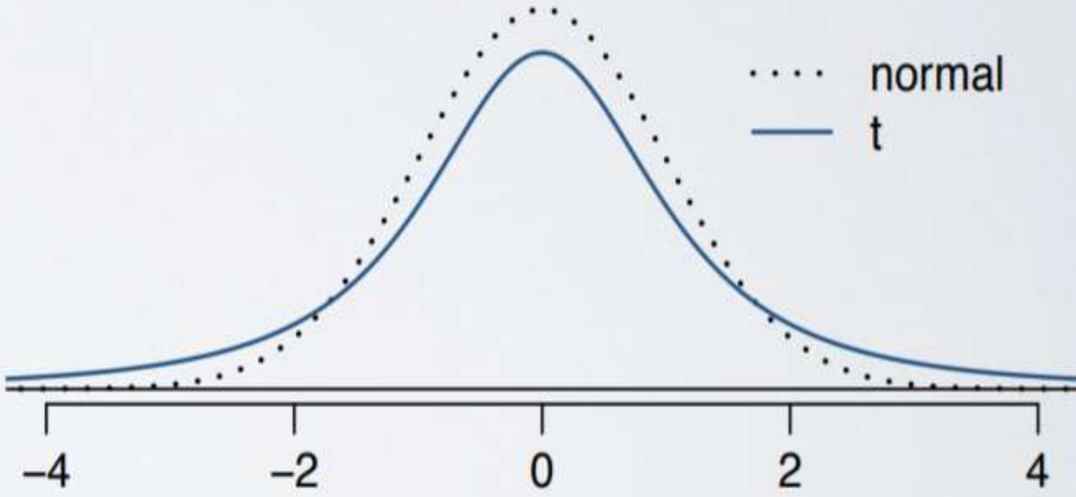
- When out of the 2, committing type1 error has more severe consequences,we would reduce significance level α
- That is why we prefer small values of α as increasing α increases Type1 error rate. The goal here is that we want to be very cautious about rejecting Ho, so we demand very strong evidence favoring Ha.
- If on the other hand, the Type2 error is more dangerous or much more costly, we would choose a higher significance level.The goal in this case is that we want to be cautious about failing to reject Ho when the null hypothesis is actually false.
- Type 1 error is rejecting Ho, when you shouldn't have and probability of doing so is α
- Type 2 error is failing to reject Ho when you should have done so, and the probability of doing so is called β
- Our goal in general in hypothesis testing is to keep both α and β low, at the same time. As we push one down, the other one shoots up, so a delicate balance needs to be stuck up.

t-distribution



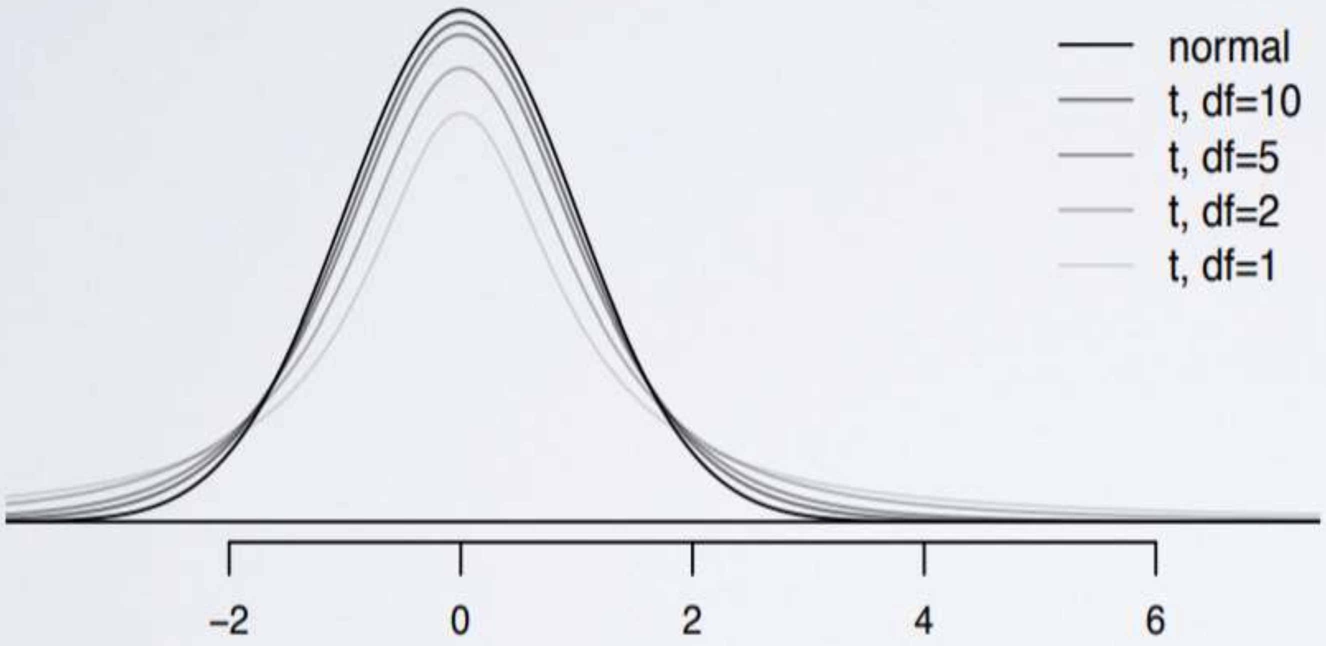
t distribution

- when σ unknown (almost always), use the t distribution to address the uncertainty of the standard error estimate
- bell shaped but thicker tails than the normal
  - observations more likely to fall beyond 2 SDs from the mean
  - extra thick tails helpful for mitigating the effect of a less reliable estimate for the standard error of the sampling distribution



t distribution

- always centered at 0 (like the standard normal)
- has one parameter: degrees of freedom (df) - determines thickness of tails
  - remember, the normal distribution has two parameters: mean and SD



What happens to the shape of the t-distribution as degrees of freedom increases?

approaches the normal dist

t statistic

- for inference on a mean where
  - σ unknown, which is almost always
- calculated the same way

$$T = \frac{obs - null}{SE}$$

- p-value (same definition)
  - one or two tail area, based on H_A
  - using R, applet, or table

Suppose you have a two sided hypothesis test, and your test statistic is 2. Under which of these scenarios would you be able to reject the null hypothesis at the 5% sig. level?

Find the following probabilities.

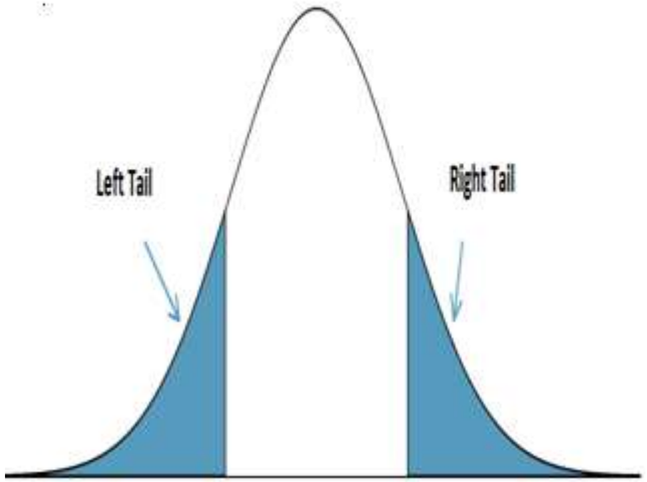
a. $P(|Z| > 2)$ 0.0455 → reject

```
> pnorm(2, lower.tail = FALSE) * 2
[1] 0.0455
```

b. $P(|t_{df=50}| > 2)$ 0.0509 → fail to reject?

```
> pt(2, df = 50, lower.tail = FALSE) * 2
[1] 0.0509
```

c. $P(|t_{df=10}| > 2)$ 0.0734 → fail to reject



Conclusion:

- Thus when we decrease the degree of freedom, we most likely fail to reject the null hypothesis(as its p-value increases).

Case Study: Playing computer game during lunch affects fullness, memory for lunch and later snack intake.

Researchers evaluated the relation between being distracted and recall of food consumed and snacking with the idea if you are distracted while eating you may not remember what you eat. They also consdier that failure of recalling the consumption of food increase snacking later on.

Playing a computer game during lunch affects fullness, memory for lunch, and later snack intake

distraction and recall of food consumed and snacking

sample: 44 patients: 22 men and 22 women

study design:

- randomized into two groups:

(1) play solitaire while eating - "win as many games as possible"

(2) eat lunch without distractions

- both groups provided same amount of lunch
- offered biscuits to snack on after lunch

biscuit intake	$\bar{x}$	s	n
solitaire	52.1 g	45.1 g	22
no distraction	27.1 g	26.4 g	22

estimating the mean

point estimate ± margin of error

$$\bar{x} \pm t^\star_{df} SE_{\bar{x}}$$

$$\bar{x} \pm t^\star_{df} \frac{s}{\sqrt{n}}$$

$$\bar{x} \pm t^\star_{n-1} \frac{s}{\sqrt{n}}$$

Degrees of freedom for t statistic for inference on one sample mean $df = n - 1$

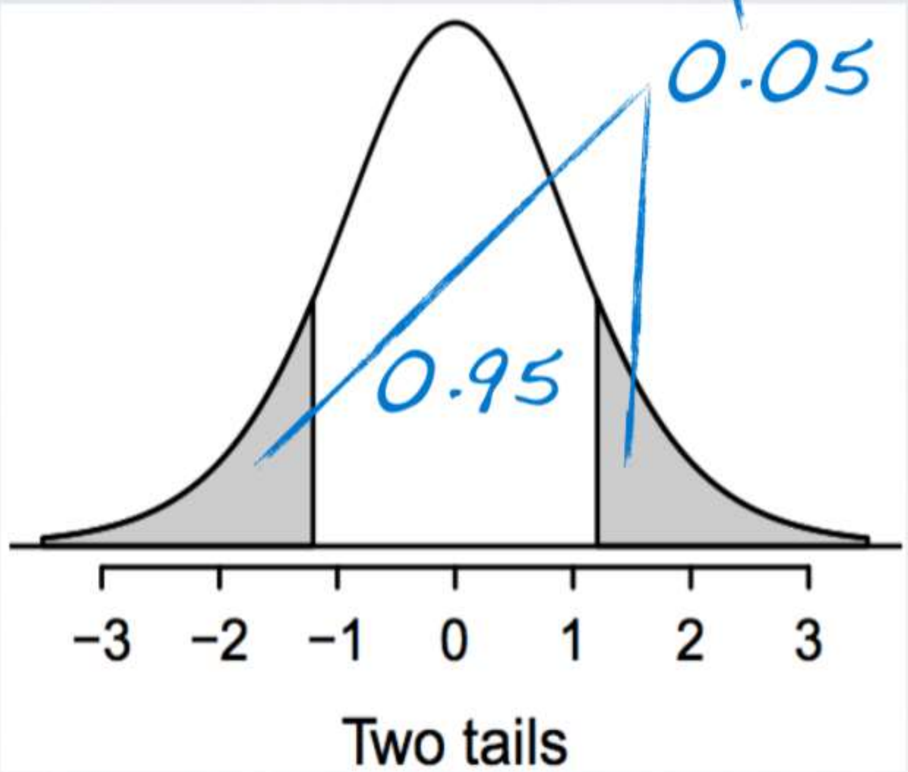
finding the critical t score

using the table

1. determine df

$df = 22 - 1 = 21$

2. find corresponding tail area for desired confidence level



one tail		0.100	0.050	0.025	0.010	0.005
two tails		0.200	0.100	0.050	0.020	0.010
df	1	3.08	6.31	12.71	31.82	63.66
	2	1.89	2.92	4.30	6.96	9.92
	3	1.64	2.35	3.18	4.54	5.84
	4	1.53	2.13	2.78	3.75	4.60
	5	1.48	2.02	2.57	3.36	4.03
	6	1.44	1.94	2.45	3.14	3.71
	7	1.41	1.89	2.36	3.00	3.50
	8	1.40	1.86	2.31	2.90	3.36
	9	1.38	1.83	2.26	2.82	3.25
	10	1.37	1.81	2.23	2.76	3.17
	11	1.36	1.80	2.20	2.72	3.11
	12	1.36	1.78	2.18	2.68	3.05
	13	1.35	1.77	2.16	2.65	3.01
	14	1.35	1.76	2.14	2.62	2.98
	15	1.34	1.75	2.13	2.60	2.95
	16	1.34	1.75	2.12	2.58	2.92
	17	1.33	1.74	2.11	2.57	2.90
	18	1.33	1.73	2.10	2.55	2.88
	19	1.33	1.73	2.09	2.54	2.86
	20	1.33	1.72	2.09	2.53	2.85
	21	1.32	1.72	2.08	2.52	2.83
	22	1.32	1.72	2.07	2.51	2.82
	23	1.32	1.71	2.07	2.50	2.81
	24	1.32	1.71	2.06	2.49	2.80
	25	1.32	1.71	2.06	2.49	2.79
	26	1.31	1.71	2.06	2.48	2.78
	27	1.31	1.70	2.05	2.47	2.77

Estimate the average after-lunch snack consumption (in grams) of people who eat lunch **distracted** using a 95% confidence interval.

$$\bar{x} = 52.1\ g$$
$$s = 45.1\ g$$
$$n = 22$$
$$t^\star_{21} = 2.08$$

$$\bar{x} \pm t^* SE = 52.1 \pm 2.08 \times \frac{45.1}{\sqrt{22}}$$

$$= 52.1 \pm 2.08 \times 9.62$$

$$= 52.1 \pm 20 = (32.1, 72.1)$$

We are 95% confident that distracted eaters consume between 32.1 to 72.1 grams of snacks post-meal.

Inference for comparing 2 means

estimating the difference between independent means

point estimate ± margin of error

$$(\bar{x}_1 - \bar{x}_2) \pm t^\star_{df} SE_{(\bar{x}_1 - \bar{x}_2)}$$

Standard error of difference between two independent means:	$SE_{(\bar{x}_1 - \bar{x}_2)} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$
DF for t statistic for inference on difference of two means	$df = min(n_1 - 1, n_2 - 1)$

Estimate the difference between the average post-meal snack consumption between those who eat with and without distractions.

biscuit intake	$\bar{x}$	s	n
solitaire	52.1 g	45.1 g	22
no distraction	27.1 g	26.4 g	22

$$(\bar{X}_{wd} - \bar{X}_{wod}) \pm t^{*}_{df}\, SE = (52.1 - 27.1) \pm 2.08 \times \sqrt{\frac{45.1^2}{22} + \frac{26.4^2}{22}}$$

$$= 25 \pm 2.08 \times 11.14$$

$$= 25 \pm 23.17$$

$$= (1.83,\ 48.17)$$

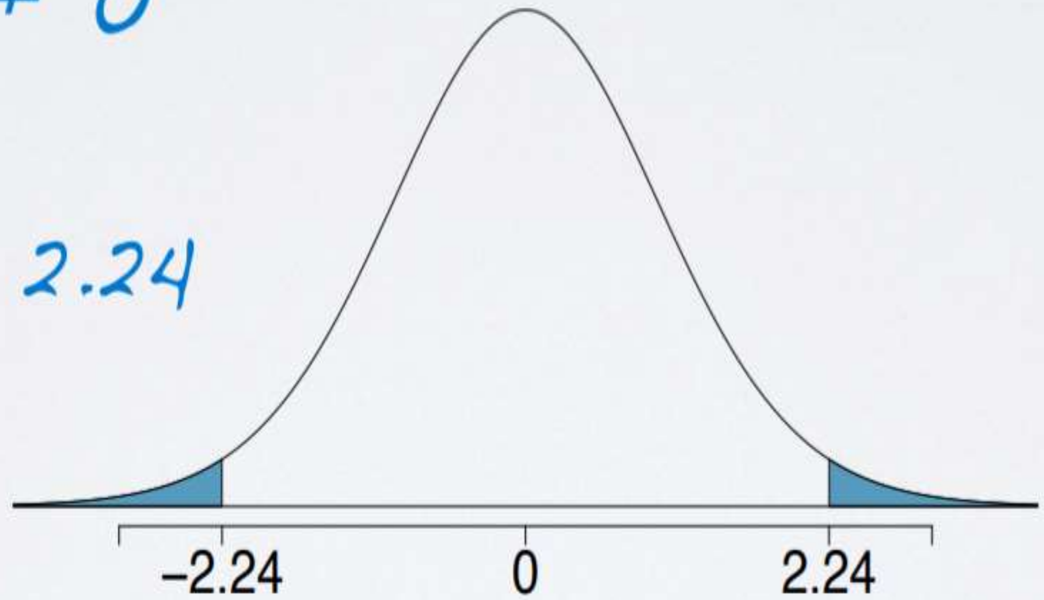
Do these data provide convincing evidence of a difference between the average post-meal snack consumption between those who eat with and without distractions?

biscuit intake	$\bar{x}$	s	n
solitaire	52.1 g	45.1 g	22
no distraction	27.1 g	26.4 g	22

$H_0: \mu_{wd} - \mu_{wod} = 0$

$H_A: \mu_{wd} - \mu_{wod} \neq 0$

$$T_{21} = \frac{25 - 0}{11.14} = 2.24$$



recap

biscuit intake	$\bar{x}$	s	n
solitaire	52.1 g	45.1 g	22
no distraction	27.1 g	26.4 g	22

95% confidence interval: (1.83g, 48.17g)

$$H_0 : \mu_{wd} - \mu_{wod} = 0$$
$$H_A : \mu_{wd} - \mu_{wod} \neq 0$$

p-value $\approx$ 0.04

Reject H_0

agree