Identification of Independent attributes :-

Case study :-

Consider the ideal reactor example with 4 attributes. pressure, temperature, density, viscosity with 500 samples.

Thus 500 x 4 matrix A is formed, such that $A = [P \quad T \quad \rho \quad \eta]$

$P, T, \rho, \eta \rightarrow$ vectors of 500 samples from sensors

From domain knowledge, it can be stated that density ρ is a function of pressure & temperature.

$$\rho \sim f(P, T)$$

This implies that at least one attribute is dependent on others. This variable can be calculated as a linear combination of the other variables.

The concept of "rank" of the matrix refers to the number of linearly independent rows or columns of the matrix.

Note :- let

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 0 \\ 3 & 6 & 0 \end{bmatrix}$$

we observe that

col. 2 = 2. col (1)

col. 3 is independent.

Thus rank of matrix is 2.

R. code :-

```
> A <- matrix (c (1, 2, 3, 2, 4, 6, 1, 0, 0), 3, 3)
> library (pracma)
> Rank (A)
```

output

```
> Rank (A)
[1] 2
```

Advantages of Rank:

(i) works with reduced set of variables

(ii) dependent attributes can be calculated if they are from same data generation process.

(iii) It is independent of size of data set.

Example: Find the rank of matrix

$$A = \begin{bmatrix} 3 & 2 & 5 & 7 & 12 \\ 1 & 1 & 2 & 3 & 5 \\ 3 & 3 & 6 & 9 & 15 \end{bmatrix}$$

$R_1 \leftrightarrow R_2$
$$\begin{bmatrix} 1 & 1 & 2 & 3 & 5 \\ 3 & 2 & 5 & 7 & 12 \\ 3 & 3 & 6 & 9 & 15 \end{bmatrix}$$

$R_2 - 3R_1$
$R_3 - 3R_1$
$$\begin{bmatrix} 1 & 1 & 2 & 3 & 5 \\ 0 & -1 & -1 & 1 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\boxed{\rho(A) = 2}$$

H.W: Find rank of

(i) $A = \begin{bmatrix} 5 & 6 & 7 & 8 \\ 6 & 7 & 8 & 9 \\ 11 & 12 & 13 & 14 \\ 16 & 17 & 18 & 19 \end{bmatrix}$

(ii) $A = \begin{bmatrix} 1 & 2 & 1 & 0 \\ 3 & 2 & 1 & 2 \\ 1 & 3 & -1 & -3 \\ 2 & -1 & 2 & 5 \\ 5 & 6 & 3 & 2 \end{bmatrix}$

(iii) $A = \begin{bmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & -1 \\ 10 & 3 & 9 & 7 \\ 16 & 4 & 12 & 15 \end{bmatrix}$

Null space of Matrix A:

Null space of matrix A is the set of all the vectors that when multiplied A times, any of those vectors (x_1, x_2, x_3, x_4) which are members of null space, we get zero vector.

e.g:

$$A\bar{x} = \bar{0}$$

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{bmatrix}_{3\times 4} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}_{4\times 1} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$N(A) = \left\{ \bar{x} \in \mathbb{R}^4 \mid A \cdot \bar{x} = \bar{0} \right\}$$

i.e

$$x_1 + x_2 + x_3 + x_4 = 0$$

$$x_1 + 2x_2 + 3x_3 + 4x_4 = 0$$

$$4x_1 + 3x_2 + 2x_3 + x_4 = 0$$

solving through Row Echelon form:

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 1 & 2 & 3 & 4 & 0 \\ 4 & 3 & 2 & 1 & 0 \end{array}\right] \Rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow x_1 - x_3 - 2x_4 = 0$$

$$\Rightarrow \boxed{x_1 = x_3 + 2x_4}$$

$$\& \quad x_2 + 2x_3 + 3x_4 = 0 \quad \boxed{x_2 = -2x_3 - 3x_4}$$

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = x_3 \begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 2 \\ -3 \\ 0 \\ 1 \end{bmatrix}$$

$\therefore$ Null space is the linear combination of $\begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \end{bmatrix}$ & $\begin{bmatrix} 2 \\ -3 \\ 0 \\ 1 \end{bmatrix}$ i.e it is the span of these vectors.

$$\Rightarrow N(A) = \text{span}\left(\begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 2 \\ -3 \\ 0 \\ 1 \end{bmatrix} \right)$$

Column ~~space~~ vectors:-

let A is collection of column vectors.

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \qquad A = [\bar{v}_1 \ \bar{v}_2 \cdots \bar{v}_n]$$

$$\bar{v}_1 \quad \bar{v}_2 \quad \cdots \quad \bar{v}_n$$

For Null space $\Rightarrow$

ie $A = [\bar{v}_1 \ \bar{v}_2 \cdots \bar{v}_n] \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Now if $v_1 \ v_2 \cdots v_n$ are linearly independent it means that there exist only one solution ie $x_1 \ x_2 \cdots x_n = 0$

i.e $N(A)$ contains only zero vector.

$$N(A) = \{0\}.$$

Thus it can said that,

If column vectors of matrix A are linearly independent it means that null space is zero vector.

<u>OR</u>

If null space is a zero vector it indicates that column vectors are linearly independent.

Column space :-

It is defined as all possible linear combinations of column vectors $(\bar{v}_1, \bar{v}_2 \cdots \bar{v}_n)$.

i.e $C(A) = \text{span}(\bar{v}_1, \bar{v}_2 \cdots \bar{v}_n)$.

Example :

For the given matrix $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 1 & 4 & 3 \\ 3 & 4 & 1 & 2 \end{bmatrix}$ find column space & null space.

Solun :- we know column space i.e

$$C(A) = \text{span}\left(\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} \right)$$

But we are not sure that whether all these vectors are basis i.e whether they are linearly independent? Thus we need to find whether these vectors are linearly independent. we know that, if the null space is zero vector, then these vectors will be linearly independent.

Hence let us check null space of A.

N(A) can be found by Row reduced Echelon form.

$R_2 \Rightarrow R_2 - 2R_1$

$R_3 \Rightarrow R_3 - 3R_1$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 1 & 4 & 3 \\ 3 & 4 & 1 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & 2 & 1 \\ 0 & 1 & -2 & 1 \end{bmatrix}$$

$R_3 \Rightarrow R_2 + R_3 \Rightarrow$

& $R_1 = R_1 - R_2$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 3 & 2 \\ 0 & -1 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

pivot elements x_1 & x_2

& x_3 & x_4 are free variables.

$\Rightarrow$

$$x_1 = -3x_3 - 2x_4$$

&

$$x_2 = 2x_3 + x_4$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = x_3 \begin{bmatrix} -3 \\ 2 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -2 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

Thus N(A) is span $\left(\begin{bmatrix} -3 \\ 2 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \\ 0 \\ 1 \end{bmatrix} \right)$

Thus it doesnot contain only zero vector.

It indicates that column vectors are not linearly independent.

Now, finding redundant vectors.

$$x_1 = -3x_3 - 2x_4 \quad \$ \quad x_2 = 2x_3 + x_4$$

let

$$x_3 = 0 \quad \& \quad x_4 = -1$$

=)

$$x_1 = 2 \qquad x_2 = -1$$

=)

$$x_1 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix} = 2 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} - 1 \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2-1 \\ 4-1 \\ 6-4 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$$

Thus vector $\begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$ can be constructed with $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \$ \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}$

similarly

let

$$x_3 = -1 \qquad x_4 = 0$$

$$x_1 = 3 \qquad x_2 = -2$$

=)

$$3 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} - 2 \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 3-2 \\ 6-2 \\ 9-8 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix}$$

Thus vector $\begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix}$ can be constructed with $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \& \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}$

Therefore $\begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} \& \begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix}$ becomes redundant.

Hence

$$C(A) = \text{span}\left(\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix} \right).$$

Orthogonal vectors -

Two vectors are orthogonal to each other, when their dot product i
Dot product of two 'n' dimensional vectors $\bar{A}$ and $\bar{B}$ is

$$\bar{A}\cdot\bar{B} = \sum_{i=1}^{n} a_i b_i$$

Thus, the vectors A and B are orthogonal to each other if and only i

$$\bar{A}\cdot\bar{B} = \sum_{i=1}^{n} a_i b_i = A^T B = 0.$$

Consider the vectors V_1 and V_2 in 3D space. Identify if they are orthogonal to each other.

$$V_1 = \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix} \qquad V_2 = \begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix}$$

$$\therefore \bar{V_1}\cdot\bar{V_2} = V_1^T V_2 = \begin{bmatrix} 1 & -2 & 4 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix} = 0$$

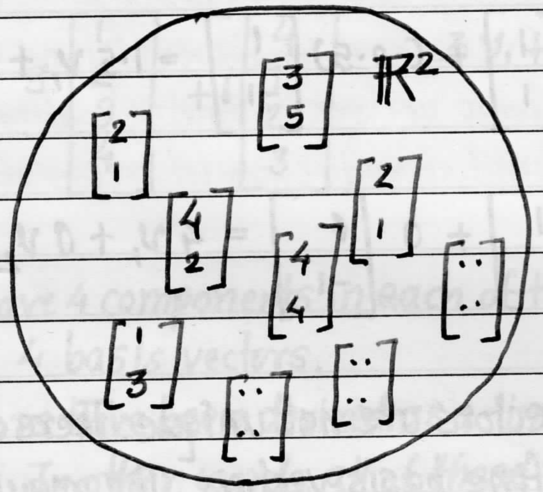
Hence, the vectors are orthogonal.

Orthonormal vectors - These are orthogonal vectors with unit magnitude.

$$V_1 = \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix} \Big/ \sqrt{(1)^2 + (-2)^2 + (4)^2}$$

All orthonormal vectors are orthogonal.

Basis vectors - Consider a 2-dimensional region of space R^2.
I can get infinite no. of points in the 2-dimensional space. All these points can be represented as vectors.



Consider two vectors $v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $v_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

If we take any vector from $\mathbb{R}^2$, say $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$, I can write $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ as some linear combination of v_1 and v_2.

$$\begin{bmatrix} 2 \\ 1 \end{bmatrix} = 2 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 2v_1 + 1v_2$$

$$\begin{bmatrix} 1 \\ 3 \end{bmatrix} = 1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 3 \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 1v_1 + 3v_2 \quad \text{\& so on}$$

In a way v_1 and v_2 characterize the space or they form a basis for this space, and any vector in $\mathbb{R}^2$ can be written as linear combination of these two vectors.

Thus,
Basis vectors are set of vectors that are independent and span the space. Span means that any vector from the space can be represented as a linear combination of basis vectors. They are independent and hence form the basis of $\mathbb{R}^2$ space.

Now consider two vectors $v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $v_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

These two are linearly independent of each other.

$$\begin{bmatrix} 2 \\ 1 \end{bmatrix} = 1.5 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + (-0.5) \begin{bmatrix} 1 \\ -1 \end{bmatrix} = 1.5\, v_1 + (-0.5)\, v_2$$

$$\begin{bmatrix} 4 \\ 4 \end{bmatrix} = 4 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + 0 \begin{bmatrix} 1 \\ -1 \end{bmatrix} = 4\, v_1 + 0\, v_2$$

Thus, the basis vectors are not unique. There are many ways with which one can define basis vectors. The requirements are that they should be independent and should span the whole space. Also in each of the variety of basis vectors, the no. of vectors (2 i.e. v_1 & v_2) in this case need to be the same.

Now, consider some $\mathbb{R}^4$

$\mathbb{R}^4$

$$\begin{bmatrix} 6 \\ 5 \\ 8 \\ 11 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \quad \begin{bmatrix} 9 \\ 4 \\ 7 \\ 10 \end{bmatrix} \cdots \cdots \quad \begin{bmatrix} 7 \\ 7 \\ 11 \\ 15 \end{bmatrix}$$

Consider two vectors

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \text{ and } v_2 = \begin{bmatrix} 4 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$

Hence,

$$\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = 1 \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} + 0 \begin{bmatrix} 4 \\ 1 \\ 2 \\ 8 \end{bmatrix} = 1\, v_1 + 0\, v_2$$

$$\begin{bmatrix}7\\7\\11\\15\end{bmatrix} = 3\begin{bmatrix}1\\2\\3\\4\end{bmatrix} + 1\begin{bmatrix}4\\1\\2\\3\end{bmatrix} = 3v_1 + 1v_2$$

Thus, though I have 4 components in each of these vectors, $\begin{bmatrix}7\\7\\11\\15\end{bmatrix}$ they do not need 4 basis vectors.
All of these vectors have been derived as a linear combination of just 2 basis vectors. In other words, all of these vectors would occupy a 2-dimensional subspace in $\mathbb{R}^4$.

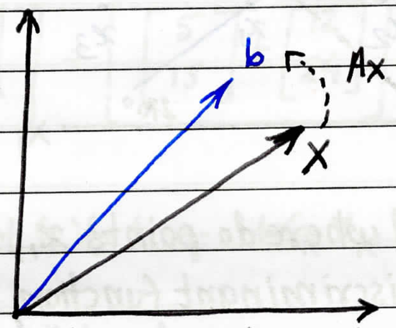
To find basis vectors of the given set of vectors

$$\begin{bmatrix} 6 & 1 & 9 & -3 & 3 & 14 & 11 & 7 & 2 & 7 \\ 5 & 2 & 4 & 1 & -1 & 7 & 8 & 0 & -3 & 7 \\ 8 & 3 & 7 & 1 & -1 & 12 & 13 & 1 & -4 & 11 \\ 11 & 4 & 10 & 1 & -1 & 17 & 18 & 2 & -5 & 15 \end{bmatrix}$$

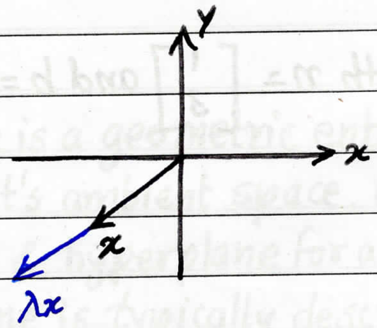
There is some data generation process, which is generating vectors such as above. I have got these 10 samples. If you give me these vectors in $\mathbb{R}^4$, how many basis vectors do I need to represent them.
If you identify the rank of this matrix, it would provide information about no. of linearly independent columns.
The rank of the matrix would be 2, when calculated. So there are only 2 basis vectors.
Any 2 independent columns can be picked from the above matrix as basis vectors.

Suppose I have 200 such samples and I want to store these 200 samples, i.e. I would be storing 800 numbers. The rank of the matrix with 200 vectors would still be 2. Hence any 2 independent columns would be picked as basis vectors and 8 numbers would have been stored. For the remaining 198 samples, to represent each of those I would use only 2 nos.

Eigenvalues and Eigenvectors –
We have previously seen linear equations of the form $Ax = b$.
The geometrical interpretation of this will be, when vector x is operated on by A, we obtain a new vector b, with a different orientation.



Are there directions for a matrix A, such that when the matrix operates on these directions, they maintain their orientation, save for multiplication by a positive or negative scalar?
i.e. $Ax = \lambda x$.
The constant λ (positive) represents the amount of stretch or shrinkage, the attributes x go through in the same direction.



The solutions (x) are known as Eigenvectors and their corresponding λ are called Eigenvalues.

We can find the Eigenvalues as follows–

$$Ax = \lambda x$$
$$\therefore \; Ax - \lambda I x = 0$$
$$\therefore \; (A - \lambda I)x = 0.$$

Thus, the Eigenvalues of the equation can be identified using
$$|A - \lambda I| = 0.$$

If this equation is solvable, then x is in the null space of $(A-\lambda I)$ matrix, and we also know that the rank nullity theorem says (rank) + (nullity) = n.

Consider the given matrix $A = \begin{bmatrix} 8 & 7 \\ 2 & 3 \end{bmatrix}$

$$\begin{bmatrix} 8 & 7 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \lambda\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \lambda x_1 \\ \lambda x_2 \end{bmatrix}$$

$$|A-\lambda I| = \left|\begin{bmatrix} 8 & 7 \\ 2 & 3 \end{bmatrix} - \lambda\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right| = \left|\begin{bmatrix} 8-\lambda & 7 \\ 2 & 3-\lambda \end{bmatrix}\right| = 0$$

$$\therefore (8-\lambda)(3-\lambda) - 14 = 0$$

$$\lambda^2 - 11\lambda + 10 = 0$$

$\therefore \lambda = (10, 1)$ → The outcome is real. It could be a conjugate also.

R code

```
A = matrix(c(8,7,2,3),2,2,byrow=TRUE)
ev= eigen(A)
values = ev$values
```

Console Output

```
> values
[1] 10 1
```

Computation of Eigenvectors -

$$\begin{bmatrix} 8 & 7 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \lambda\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

with $\lambda = 1$,

$$\begin{bmatrix} 8 & 7 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\begin{bmatrix} 8x_1 + 7x_2 \\ 2x_1 + 3x_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

i.e. $8x_1 + 7x_2 - x_1 = 0$ i.e. $7x_1 + 7x_2 = 0$ i.e. $x_1 + x_2 = 0$

i.e. $2x_1 + 3x_2 - x_2 = 0$ i.e. $2x_1 + 2x_2 = 0$ i.e. $x_1 + x_2 = 0$

i.e. $x_1 = -x_2$

The Eigenvector would be chosen with magnitude = 1.

$$\therefore \quad X = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix}$$

With $\lambda = 10$

$$\begin{bmatrix} 8 & 7 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 10x_1 \\ 10x_2 \end{bmatrix}$$

$$\begin{bmatrix} 8x_1 + 7x_2 \\ 2x_1 + 3x_2 \end{bmatrix} = \begin{bmatrix} 10x_1 \\ 10x_2 \end{bmatrix}$$

i.e. $8x_1 + 7x_2 - 10x_1 = 0$ i.e. $-2x_1 + 7x_2 = 0$ i.e. $2x_1 = 7x_2$

$2x_1 + 3x_2 - 10x_2 = 0$ i.e. $2x_1 - 7x_2 = 0$ i.e. $2x_1 = 7x_2$

$$\therefore \quad X = \begin{bmatrix} \frac{7}{\sqrt{53}} \\ \frac{2}{\sqrt{53}} \end{bmatrix}$$

R Code

```
A = matrix(c(8,7,2,3), 2, 2, byrow = TRUE)
ev = eigen(A)
vectors <- ev$vectors
```

```
> vectors
          [,1]       [,2]
[1,] 0.9615239 -0.7071068
[2,] 0.2747211  0.7071068
```

Ex:- Find eigen values $ eigen vectors for matrix

$$A = \begin{bmatrix} 8 & 0 & 3 \\ 2 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 8-\lambda & 0 & 3 \\ 2 & 2-\lambda & 1 \\ 2 & 0 & 3-\lambda \end{bmatrix}$$

C.E

$$(8-\lambda)\left[6-5\lambda+\lambda^2\right]+3\left[-4+2\lambda\right]=0$$

$$48-40\lambda+8\lambda^2-6\lambda+5\lambda^2-\lambda^3-12+6\lambda$$

$$-\lambda^3+13\lambda^2-40\lambda+36=0$$

$$\lambda^3-13\lambda^2+40\lambda-36=0$$

$$\lambda = 2, 2, 9$$

finding eigen vector. let $\lambda = 9$

$$\begin{bmatrix} -1 & 0 & 3 \\ 2 & -7 & 1 \\ 2 & 0 & -6 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

1 & 3 row are dependent. taking independ. equations.

$$-x_1+3x_3=0 \qquad \boxed{x_1 = 3x_3}$$

$$2x_1-7x_2+x_3=0$$

$\Rightarrow$

$$2(3x_3)-7x_2+x_3=0$$

$$6x_3-7x_2+x_3=0$$

$$7x_3-7x_2=0 \Rightarrow \boxed{x_2 = x_3}$$

$\Rightarrow$ let $x_2 = x_3 = 1 \Rightarrow x_1 = 3$

$\therefore$ Eigen vector is $\begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$

II'ly we get for $\lambda = 2$

Eigen vector as $\begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

<u>H.W</u> :

Find Eigen value & Eigen vectors of

$$A = \begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix}$$