PRACTICAL - I

LIMITS & CONTINUITY

1]

$$\lim_{x \to a} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \right]$$

$$\lim_{x \to a} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \times \frac{\sqrt{a+2x} + \sqrt{3x}}{\sqrt{a+2x} + \sqrt{3x}} \times \frac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{3a+x} + 2\sqrt{x}} \right]$$

$$\lim_{x \to a} \frac{(a+2x-3x)}{(3a+x-4x)} \frac{(\sqrt{3a+x} + 2\sqrt{x})}{(\sqrt{a+2x} + \sqrt{3x})}$$

$$\lim_{x \to a} \frac{(a-x)(\sqrt{3a+x} + 2\sqrt{x})}{(3a-3x)(\sqrt{a+2x} + \sqrt{3x})}$$

$$\frac{1}{3} \lim_{x \to a} \frac{(a-x)(\sqrt{3a+x} + 2\sqrt{x})}{(a-x)(\sqrt{a+2x} + \sqrt{3x})}$$

$$\frac{1}{3} \frac{\sqrt{3a+a} + 2\sqrt{a}}{\sqrt{a+2a} + \sqrt{3a}}$$

$$\frac{1}{3} \times \frac{\sqrt{4a} + 2\sqrt{a}}{\sqrt{3a} + \sqrt{3a}}$$

$$\frac{1}{3} \times \frac{2\sqrt{a} + 2\sqrt{a}}{\sqrt{3a} + \sqrt{3a}}$$

$$\frac{1}{3} \times \frac{\cancel{4}^{2} \cancel{\sqrt{a}}}{\cancel{2}\sqrt{3}\cancel{\sqrt{a}}}$$

$$= \frac{2}{3\sqrt{3}}$$

2. $\lim_{y \to 0} \left[\frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \right]$

$\lim_{y \to 0} \left[\frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \times \frac{\sqrt{a+y} + \sqrt{a}}{\sqrt{a+y} + \sqrt{a}} \right]$

$\lim_{y \to 0} \frac{a+y-a}{y\sqrt{a+y}\,(\sqrt{a+y}+\sqrt{a})}$

$\lim_{y \to 0} \frac{\cancel{y}}{\cancel{y}\sqrt{a+y}\,(\sqrt{a+y}+\sqrt{a})}$

~~lim~~ $= \frac{1}{\sqrt{a+0}\,(\sqrt{a+0}+\sqrt{a})}$

$= \frac{1}{\sqrt{a}\,(\sqrt{a}+\sqrt{a})}$

$= \frac{1}{\sqrt{a}\,(2\sqrt{a})} = \frac{1}{2a}$ //

3. $\lim_{x \to \pi/6} \frac{\cos x - \sqrt{3}\sin x}{\pi - 6x}$

By substituting $x - \frac{\pi}{6} = h$

$x = h + \frac{\pi}{6}$

when $h \to 0$.

$\lim_{h \to 0} \frac{\cos\left(h+\frac{\pi}{6}\right) - \sqrt{3}\sin\left(h+\pi/6\right)}{\pi - 6\left(h+\pi/6\right)}$

Using

$\cos(A+B) = \cos A \cdot \cos B + \sin A \cdot \sin B$

$\sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B$

$\lim_{h \to 0} \frac{\cos h \cdot \cos\frac{\pi}{6} - \sin h \sin\frac{\pi}{6} - \sqrt{3}\left(\sin h \cos\frac{\pi}{6} + \cos h \sin\frac{\pi}{6}\right)}{\pi - \cancel{6}\left(\frac{6h+\pi}{\cancel{6}}\right)}$

$\cos\frac{\pi}{6} = \cos 30^\circ = \sqrt{3}/2$

$\sin\pi/6 = \sin 30^\circ = 1/2$

$\lim_{h \to 0} \frac{\cos h \cdot \frac{\sqrt{3}}{2} - \sin h \frac{1}{2} - \sqrt{3}\left(\sin h \sqrt{3}/2 + \cos h \cdot \frac{1}{2}\right)}{\pi - 6h + \pi}$

$\lim_{h \to 0} \frac{\cancel{\cos\frac{\sqrt{3}}{2}h} - \sin h/2 - \sin\frac{3h}{2} - \cancel{\cos\frac{\sqrt{3}}{2}h}}{-6h}$

$\lim_{h \to 0} \frac{+\sin\frac{4h}{2}}{+6h}$

$\lim_{h \to 0} \frac{\sin 4h}{3\cancel{2}h}$

$\frac{1}{3} \lim_{h \to 0} \frac{\sin h}{h} = \frac{1}{3} \times 1 = \frac{1}{3}$ //

4] $\lim_{x\to\infty}\left[\dfrac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}}\right]$

By rationalizing Numerator & denominator both

$$\lim_{x\to\infty}\left[\frac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}} \times \frac{\sqrt{x^2+5}+\sqrt{x^2-3}}{\sqrt{x^2+5}+\sqrt{x^2-3}} \times \frac{\sqrt{x^2+3}+\sqrt{x^2+1}}{\sqrt{x^2+3}+\sqrt{x^2+1}}\right]$$

$$\lim_{x\to\infty}\left[\frac{(\cancel{x^2}+5-\cancel{x^2}+3)(\sqrt{x^2+3}+\sqrt{x^2+1})}{(\cancel{x^2}+3-\cancel{x^2}-1)(\sqrt{x^2+5}+\sqrt{x^2-3})}\right]$$

$$\lim_{x\to\infty}\frac{\overset{4}{\cancel{8}}(\sqrt{x^2+3}+\sqrt{x^2+1})}{\cancel{2}(\sqrt{x^2+5}+\sqrt{x^2-3})}$$

$$4\lim_{x\to\infty}\frac{\sqrt{x^2\left(1+\frac{3}{x^2}\right)}+\sqrt{x^2\left(1+\frac{1}{x^2}\right)}}{\sqrt{x^2\left(1+\frac{5}{x^2}\right)}+\sqrt{x^2\left(1-\frac{3}{x^2}\right)}}$$

After applying limit

we get, $= 4$ //

5i) $f(x) = \dfrac{\sin 2x}{\sqrt{1-\cos 2x}}$, for $0 < x \le \pi/2$

$= \dfrac{\cos x}{\pi - 2x}$, for $\dfrac{\pi}{2} < x < \pi$ $\Bigg\}$ at $x = \dfrac{\pi}{2}$.

$f(\pi/2) = \dfrac{\sin 2\left(\frac{\pi}{2}\right)}{\sqrt{1-\cos 2(\pi/2)}}$ $\quad \therefore f(\pi/2) = 0$

f at ~~x~~ $= \pi/2$ define

ii) $\lim_{x\to\pi/2^+} f(x) = \lim_{x\to\frac{\pi}{2}^+}\dfrac{\cos x}{\pi - 2x}$

By Substituting Method

$x - \frac{\pi}{2} = h$

$x = h + \frac{\pi}{2}$.

where $h \to 0$

$$\lim_{h\to 0}\frac{\cos(h+\pi/2)}{\pi-2(h+\pi/2)}$$

$$\lim_{h\to 0}\frac{\cos\left(h+\frac{\pi}{2}\right)}{\pi-\cancel{2}\left(\frac{2h+\pi}{\cancel{2}}\right)}$$

$$\lim_{h\to 0}\frac{\cos\left(h+\frac{\pi}{2}\right)}{-2h}$$ Using $\cos(A+B) = \cos A\cos B - \sin A\,s$

$$\lim_{h\to 0}\frac{\cos h\cdot\cos\frac{\pi}{2}\ .\sin h\cdot\sin\frac{\pi}{2}}{-2h}$$

$$\lim_{h\to 0}\frac{\cos h\cdot 0 - \sin h}{-2h}$$

$$\lim_{h\to 0}\frac{\cancel{-}\sin h}{\cancel{-}2h}$$

$$\frac{1}{2}\lim_{h\to 0}\frac{\sin h}{h}$$

$$\frac{1}{2}\times 1 = \frac{1}{2}.$$

b) $\lim_{x\to\frac{\pi}{2}^-} f(x) = \lim_{x\to\pi/2} \frac{\sin 2x}{\sqrt{1-\cos 2x}}$ Using $\sin 2x = 2\sin x \cdot \cos x$.

$\lim_{x\to\pi/2^-} \frac{2\sin x \cdot \cos x}{\sqrt{2\sin^2 x}}$

$\lim_{x\to\pi/2^-} \frac{2\cancel{\sin x}\cdot\cos x}{\sqrt{2}\,\cancel{\sin x}}$

$\lim_{x\to\pi/2^-} \frac{2\cos x}{\sqrt{2}}$

$\frac{2}{\sqrt{2}} \lim_{x\to\frac{\pi}{2}^-} \cos x$

$\therefore$ L.H.L $\neq$ R.HL

$\therefore$ f is not continous at $x = \pi/2$.

i) $f(x) = \frac{x^2-9}{x-3}$ $0 < x < 3$

$= x+3$ $3 \le x \le 6$

$= \frac{x^2-9}{x+3}$ $6 \le x \le 9$

$\left.\right\}$ at $x = 3$ & $x = 6$

at $x = 3$

i) $f(3) = \frac{x^2-9}{x-3} = 0$

f at $x = 3$ define

ii $\lim_{x\to3^+} f(x) = \lim_{x\to3^+} x+3$

$f(3) = x+3 = 3+3 = 6$

f is define at $x = 3$.

$\lim_{x\to3^+} f(x) = \lim_{x\to3^+} (x+3) = 6$

$\lim_{x\to3^-} f(x) = \lim_{x\to3^-} \frac{x^2-9}{x-3} = \frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}}$

$\therefore$ L.HL. = R.HL

f is ~~not~~ continuous at $x=3$.

for $x = 6$

$f(6) = \frac{x^2+9}{x+3} = \frac{36-9}{6+3} = \frac{\cancel{27}\,3}{\cancel{9}} = 3$

$\lim_{x\to6^+} \frac{x^2-9}{x+3}$

$\lim_{x\to6^+} \frac{(x-3)\cancel{(x+3)}}{\cancel{(x+3)}}$

$\lim_{x\to6^+} (x-3) = 6-3 = 3$

$\lim_{x\to6^-} x+3 = 3+6 = 9$

$\therefore$ LHL $\neq$ RHL

function is not continuous.

$$f(x) = \begin{cases} \dfrac{1-\cos 4x}{x^2} & x<0 \\ = k & x=0 \end{cases} \quad \text{at } x=0$$

soln:- f is continuous at $x=0$

$$\lim_{x\to 0} f(x) = f(0)$$

$$\lim_{x\to 0} \frac{1-\cos 4x}{x^2} = k$$

$$\lim_{x\to 0} \frac{2\sin^2 2x}{x^2} = k$$

$$2\lim_{x\to 0} \frac{\sin^2 2x}{x^2} = k$$

$$2\lim_{x\to 0} \left(\frac{\sin 2x}{x}\right)^2 = k$$

$$2(2)^2 = k$$

$$\therefore k = 8$$

ii) $$f(x) = \begin{cases} (\sec^2 x)^{\cot^2 x} & x=0 \\ = k & x=0 \end{cases} \quad \text{at } x=0.$$

soln:- $f(x) = (\sec^2 x)^{\cot^2 x}$

Using
$\tan^2 x - \sec^2 x = 1$
$\therefore \sec^2 x = 1+\tan^2 x$
&
$\cot^2 x = \dfrac{1}{\tan^2 x}$.

$$\lim_{x\to 0} (\sec^2 x)^{\cot^2 x}.$$

$$\lim_{x\to 0} (1+\tan^2 x)^{\frac{1}{\tan^2 x}}$$

we know that,

$$\lim_{x\to 0} (1+px)^{1/px} = e$$

$$\therefore k = e.$$

iii) $$f(x) = \begin{cases} \dfrac{\sqrt{3}-\tan x}{\pi - 3x} & x = \pi/3 \\ = k & x = \pi/3 \end{cases} \quad \text{at } x = \frac{\pi}{3}.$$

$$x - \frac{\pi}{3} = h$$

$$x = h + \frac{\pi}{3}$$

where $h \to 0$

$$f\left(\frac{\pi}{3}+h\right) = \frac{\sqrt{3}-\tan(\pi/3+h)}{\pi - 3(\pi/3+h)}$$

$$\lim_{h\to 0} \frac{\sqrt{3}-\tan(\pi/3+h)}{\pi-3(\pi/3+h)}$$

Using
$\tan(A+B) = \dfrac{\tan A + \tan B}{1-\tan A \cdot \tan B}$

$$\lim_{h\to 0} \frac{\sqrt{3} - \dfrac{\tan\frac{\pi}{3}+\tan h}{1-\tan\frac{\pi}{3}\tan h}}{\cancel{\pi} - \cancel{\pi} - 3h}$$

$$\lim_{h\to0} \frac{\dfrac{\sqrt{3}\left(1-\tan\frac{\pi}{3}\cdot\tan h\right)-\left(\tan\frac{\pi}{3}+\tan h\right)}{1-\tan\pi/3\cdot\tan h}}{-3h}$$

Using $\tan\pi/3 = \tan 60 = \sqrt{3}$

$$\lim_{h\to0} \frac{\dfrac{(\sqrt{3}-\sqrt{3}\times\sqrt{3}\cdot\tan h)-(\sqrt{3}+\tan h)}{1-\tan\pi/3\cdot\tan h}}{-3h}$$

$$\lim_{h\to0} \frac{\dfrac{(\sqrt{3}\cdot3\cdot\tan h)-(\sqrt{3}+\tan h)}{1-\sqrt{3}\cdot\tan h}}{-3h}$$

$$\lim_{h\to0} \frac{\dfrac{(\cancel{\sqrt{3}}-3\tan h-\cancel{\sqrt{3}}-\tan h)}{1-\sqrt{3}\cdot\tan h}}{-3h}$$

$$\lim_{h\to0} \frac{\cancel{-}4\tan h}{\cancel{-}3h(1-\sqrt{3}\tan h)}$$

$$\lim_{h\to0} \frac{4\tan h}{3h(1-\sqrt{3}\tan h)}$$

$$\frac{4}{3}\lim_{h\to0}\frac{\tan h}{h}\ \lim_{h\to0}\frac{1}{(1-\sqrt{3}\tan h)} \qquad \frac{\tan h}{h}=1$$

$$\frac{4}{3}\cdot\frac{1}{(1-\sqrt{3}\,0)} = \frac{4}{3}$$

7 i) $f(x) = \dfrac{1-\cos3x}{x\tan x}$ $\quad x=0$
$= 9 \quad x=0$ $\Big\}$ at $x=0$

$$f(x) = \frac{1-\cos3x}{x\tan x}$$

$$\lim_{x\to0} = \frac{2\sin^2 3/2\,x}{x\tan x}$$

$$\lim_{x\to0} = \frac{\dfrac{2\sin^2\frac{3x}{2}}{x^2}\times x^2}{x-\dfrac{\tan x}{x^2}\times x^2}$$

$$= 2\lim_{x\to0}\frac{\left(\frac{3}{2}\right)^2}{1} = 2\times\frac{9}{4_2} = \frac{9}{2}$$

$$\lim_{x\to0} f(x) = \frac{9}{2} \qquad 9 = f(0)$$

$\therefore$ f is not continuous at $x=0$.

Redefine function.

$$f(x) = \begin{cases} \dfrac{1-\cos3x}{x\tan x} & x\neq0, \\ \dfrac{9}{2} & x=0 \end{cases}$$

Now $\lim_{x\to0} f(x) = f(0)$

f has removable discontinuity at $x=0$.

7) ii $f(x) = \begin{cases} \dfrac{(e^{3x}-1)\sin x^\circ}{x^2} & x \neq 0 \\ = \pi/6 & x = 0 \end{cases}$ at $x = 0$

$\lim_{x\to 0} \dfrac{(e^{3x}-1)\sin\left(\frac{\pi x}{180}\right)}{x^2}$

$\lim_{x\to 0} \dfrac{e^{3x}-1}{x} \lim_{x\to 0} \dfrac{\sin\left(\frac{\pi x}{180}\right)}{x}$

$\lim_{x\to 0} \dfrac{3 \cdot e^{3x}-1}{3x} \lim_{x\to 0} \dfrac{\sin\left(\frac{\pi x}{180}\right)}{x}$.

$3 \lim \dfrac{e^{3x}-1}{3x} \lim_{x\to 0} \dfrac{\sin\left(\frac{\pi x}{180}\right)}{x}$

$3 \log e \dfrac{\pi}{180} = \dfrac{\pi}{60} = f(0)$

f is continous at $x=0$.

8) $f(x) = e^{x^2} - \cos x \quad x = 0$
is continuous at $x = 0$

$\therefore$ Given,
f is continuous at $x = 0$.
$\lim_{x\to 0} f(x) = f(0)$

$= \lim_{x\to 0} \dfrac{e^{x^2} - \cos x}{x^2} = f(0)$

$= \lim_{x\to 0} \dfrac{e^{x^2} - \cos x - 1 + 1}{x^2}$

$= \lim_{x\to 0} \dfrac{(e^{x^2}-1) + (1-\cos x)}{x^2}$

$\lim_{x\to 0} \dfrac{e^{x^2}-1}{x^2} + \lim_{x\to 0} \dfrac{1-\cos x}{x^2}$

$\log e + \lim_{x\to 0} \dfrac{2\sin^2 x/2}{x^2}$

$\log e + 2 \lim_{x\to 0} \left(\dfrac{\sin x/2}{x}\right)^2$

Multiply with 2 on Numerator & Denominator.

$= 1 + \cancel{2} \times \dfrac{1}{\cancel{4}\,2} = \dfrac{3}{2} = f(0)$ //

No.
1)
2)
3)

9. $f(x) = \dfrac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x}$ $\quad x = \dfrac{\pi}{2}$.

$f(x)$ is continuous at $x = \pi/2$.

$$\lim_{x \to \pi/2} \frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x} \times \frac{\sqrt{2} + \sqrt{1+\sin x}}{\sqrt{2} + \sqrt{1+\sin x}}$$

$$\lim_{x \to \pi/2} \frac{2 - 1 + \sin x}{\cos^2 x (\sqrt{2} + \sqrt{1+\sin x})}$$

$$= \lim_{x \to \pi/2} \frac{1 + \sin x}{1 - \sin^2 x (\sqrt{2} + \sqrt{1+\sin x})}$$

$$= \lim_{x \to \pi/2} \frac{\cancel{1+\sin x}}{(1-\sin x)\cancel{(1+\sin x)}(\sqrt{2} + \sqrt{1+\sin x})}$$

$$= \lim_{x \to \pi/2} \frac{1}{(1-\sin x)(\sqrt{2} + \sqrt{1+\sin x})}$$

$$= \cancel{\frac{\sqrt{2}}{2\sqrt{2}}} \qquad \frac{1}{2(\sqrt{2} + \sqrt{2})}$$

$$\frac{1}{2(2\sqrt{2})} = \frac{1}{4\sqrt{2}}$$

$$\therefore f\left(\frac{\pi}{2}\right) = \frac{1}{4\sqrt{2}}$$

5/12/19

PRACTICAL - 2

TOPIC - DERIVATIVES

Q.1] Show that the following function defined from $\mathbb{R}$ to $\mathbb{R}$ are differentiable.

i] $\cot x$

$f(x) = \cot x$

$$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{\cot x - \cot a}{x - a}$$

$$= \lim_{x \to a} \frac{\frac{1}{\tan x} - \frac{1}{\tan a}}{x - a}$$

$$= \lim_{x \to a} \frac{\tan a - \tan x}{(x-a) \tan x \tan a}$$

put $x - a = h$

$x = a + h$

as $x \to a$, $h \to 0$

$$Df(h) = \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{(a+h-a) \tan(a+h) \tan a}$$

$$= \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{h \times \tan(a+h) + a \tan a}$$

formula: $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A . \tan B}$

$\tan A - \tan B = \tan(A-B)(1 + \tan A . \tan B)$

$$= \lim_{h \to 0} \frac{\tan(a - a - h) \cdot (1 + \tan a \tan(a+h))}{h \times \tan(a+h) \cdot \tan a.}$$

$$= \lim_{h\to 0} \frac{-\tan h}{h} \times \frac{1+\tan a \tan(a+h)}{\tan(a+h)\tan a}$$

$$= -1 \times \frac{1+\tan^2 a}{\tan^2 a}$$

$$= -\frac{\sec^2 a}{\tan^2 a}$$

$$= \frac{1}{\cos^2 a} \times \frac{\cos^2 a}{\sin^2 a}$$

$$= -\operatorname{cosec}^2 a$$

$\therefore Df(a) = -\cos^2 a$.

$\therefore f$ is differentiable at $a \in R$.

ii) $\operatorname{cosec} x$

$f(x) = \operatorname{cosec} x$

$$Df(a) = \lim_{x\to a} \frac{f(x) - f(a)}{x-a}$$

$$= \lim_{x\to a} \frac{\operatorname{cosec} x - \operatorname{cosec} a}{x-a}$$

$$= \lim_{x\to a} \frac{\frac{1}{\sin x} - \frac{1}{\sin a}}{x-a}$$

$$= \lim_{x\to a} \frac{\sin a - \sin x}{(x-a)\sin a \sin x}$$

put $x - a = h$

$x = a+h$

as $x \to a$, $h \to 0$

$$Df(a) = \lim_{h\to 0} \frac{\sin a - \sin(a+h)}{(a+h-a)\sin a \sin(a+h)}$$

Formula: $\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

$$\lim_{h\to 0} \frac{2\cos\left(\frac{a+a+h}{2}\right) \cdot \sin\left(\frac{a-a-h}{2}\right)}{h \times \sin a \cdot \sin(a+h)}$$

$$= \lim_{h\to 0} -\frac{\sin\frac{h}{2}}{h/2} \times \frac{1}{2} \times \frac{2\cos\left(\frac{2a+h}{2}\right)}{\sin a \sin(a+h)}$$

$$= -\frac{1}{2} \times \frac{2\cos\left(\frac{2a+0}{2}\right)}{\sin(a+0)}$$

$$= -\frac{\cos a}{\sin^2 a} = -\cot a \operatorname{cosec} a.$$

iii) $\sec x$

$f(x) = \sec x$

$$Df(a) = \lim_{x\to a} \frac{f(x) - f(a)}{x-a}$$

$$= \lim_{x\to a} \frac{\sec x - \sec a}{x-a}$$

$$= \lim_{x\to a} \frac{\frac{1}{\cos x} - \frac{1}{\cos a}}{x-a}$$

$$= \lim_{x\to a} \frac{\cos a - \cos x}{(x-a)\cos a \cos x}$$

Put $x - a = h$

$x = a+h$

as $x \to a$, $h \to 0$

$$Df(a) = \lim_{h\to 0} \frac{\cos a - \cos(a+h)}{h \times \cos a \cos(a+h)}$$

Formula: $-2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

$$= \lim_{h\to 0} \frac{-2\sin\left(\frac{a+a+h}{2}\right)\sin\left(\frac{a-a-h}{2}\right)}{h \times \cos a \cos(a+h)}$$

$$= \lim_{h\to 0} \frac{-2\sin\left(\frac{2a+h}{2}\right)\sin -\frac{h}{2}}{\cos a \cos(a+h) \times -\frac{h}{2}} \times -\frac{1}{4}$$

$$= -\frac{1}{2} \times \frac{-2\sin\left(\frac{2a+0}{2}\right)}{\cos a \cos(a+0)}$$

$$= -\frac{1}{\cancel{2}} \times -\cancel{2} \frac{\sin a}{\cos a \times \cos a}$$

$$= \tan a \sec a$$

Q.2) If $f(x) = 4x+1, \quad x \leq 2$

$= x^2+5 \quad x > 0$ at $x = 2$ then

find function is differentiable or not.

Soln: LHD :-

$$Df(2^-) = \lim_{x\to 2^-} \frac{f(x)-f(2)}{x-2}$$

$$= \lim_{x\to 2^-} \frac{4x+1-(4\times 2+1)}{x-2}$$

$$= \lim_{x\to 2^-} \frac{4x+1-9}{x-2}$$

$$= \lim_{x\to 2^-} \frac{4x-8}{x-2}$$

$$= \lim_{x\to 2^-} \frac{4\cancel{(x-2)}}{\cancel{(x-2)}} \quad \cancel{=4}$$

$$= 4$$

$Df(2^-) = 4$

RHD:

$$Df(2^+) = \lim_{x\to 2^+} \frac{x^2+5-9}{x-2}$$

$$= \lim_{x\to 2^+} \frac{x^2-4}{x-2}$$

$$= \lim_{x\to 2^+} \frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$$

$= 2+2 = 4$

$\therefore$ RHD = LHD

f is differentiable at $x = 2$

Q3) If $f(x) = 4x+7, \quad x < 3$

$= x^2+3x+1, \quad x \geq 3$ at $x = 3$ then

find it is differentiable or not?

Solution :- RHD:

$$Df(3^+) = \lim_{x\to 3^+} \frac{f(x)-f(3)}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x^2+3x+1-(3^2+3\times 3\cancel{x}+1)}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x^2+3x+1-19}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x^2+3x\cancel{+1}-18}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x^2+6x-3x-18}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x(x+6)-3(x+6)}{x-3}$$

$$= \lim_{x\to 3^+} \frac{(x+6)\cancel{(x-3)}}{\cancel{(x-3)}}$$

$$= 3+6 = 9$$

$Df(3^+) = 9$

LHD $= Df(3^-)$

$$= \lim_{x\to 3^-} \frac{f(x)-f(3)}{x-3}$$

$$= \lim_{x\to 3^-} \frac{4x+7-19}{x-3}$$

$$= \lim_{x \to 3^-} \frac{4x-12}{x-3}$$

$$= \lim_{x \to 3^-} \frac{4\cancel{(x-3)}}{\cancel{(x-3)}}$$

$Df(3^-) = 4$

RHD $\neq$ LHD

$\therefore f$ is not differentiable at $x = 3$

Q 4) If $f(x) = 8x - 5$, $x \leq 2$

$= 3x^2 - 4x + 7$, $x > 2$, at $x = 2$, Then

find f is differentiable or not.

$\Rightarrow$ $f(2) = 8 \times 2 - 5 = 16 - 5 = 11$.

R.H.D :

$$Df(2^+) = \lim_{x \to 2^+} \frac{f(x) - f(2)}{x-2}$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 4x + 7 - 11}{x-2}$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 4x - 4}{x-2}$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 6x + 2x - 4}{x-2}$$

$$= \lim_{x \to 2^+} \frac{3x(x-2) + 2(x-2)}{x-2}$$

$$= \lim_{x \to 2^+} \frac{(3x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$$

$= 3 \times 2 + 2 = 8$

$Df(2^+) = 8$

LHD:

$$Df(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x-2}$$

$$= \lim_{x \to 2^-} \frac{8x - 5 - 11}{x-2}$$

$$= \lim_{x \to 2^-} \frac{8x - 16}{x-2}$$

$$= \lim_{x \to 2^-} \frac{8\cancel{(x-2)}}{\cancel{(x-2)}}$$

$= 8$

$Df(2^-) = 8$

LHD = RHD

$\therefore f$ is differentiable at $x = 8$.

A+ 19/12/17

PRACTICAL . 3

Topic : Application of Derivatives

Q.1] Find the intervals in which function is increasing or decreasing.

a] $f(x) = x^3 - 5x - 11$

$\Rightarrow$: f is increasing if and only if $f'(x) > 0$

$\because f(x) = x^3 - 5x - 11$

$f'(x) = 3x^2 - 5$

$3x^2 - 5 > 0$

$x = \pm\sqrt{\frac{5}{3}}$

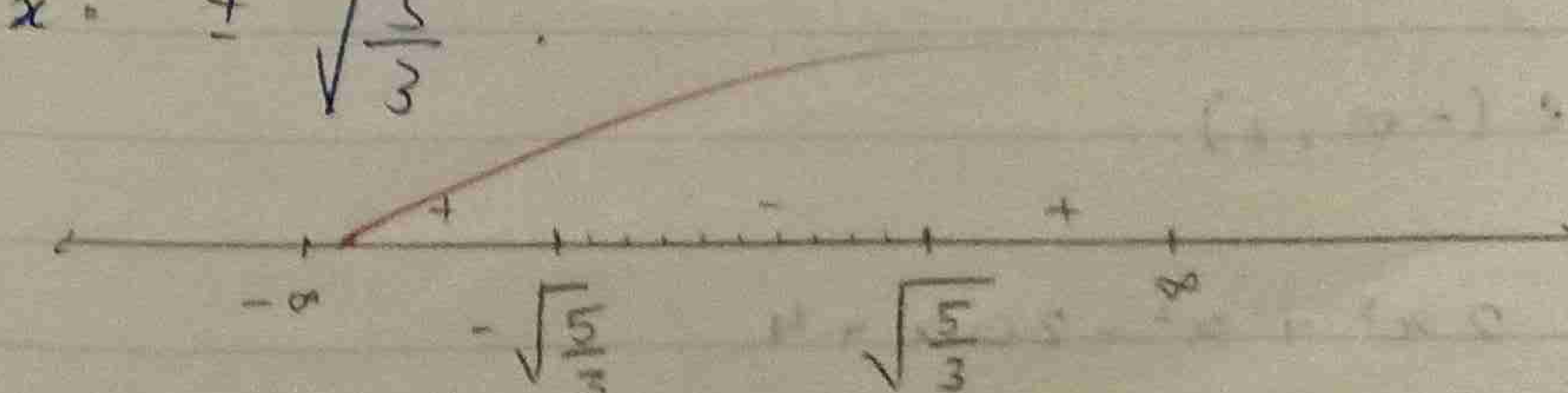
$\therefore x \in \left(-\infty, -\sqrt{\frac{5}{3}}\right) \cup \left(\sqrt{\frac{5}{3}}, \infty\right)$

Now f is decreasing if and only if $f'(x) < 0$

$f'(x) < 0$

$\therefore 3x^2 - 5 < 0$

$\therefore x = \pm\sqrt{\frac{5}{3}}$



$\therefore x \in \left(-\sqrt{\frac{5}{3}}, \sqrt{\frac{5}{3}}\right)$

b) $f(x) = x^2 - 4x$

Solution : f is increasing if and only if $f'(x) > 0$

$\because f(x) = x^2 - 4x$

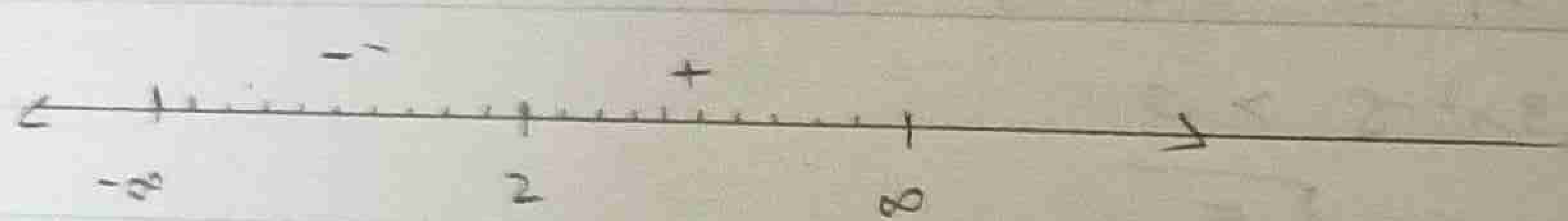
$\therefore f'(x) = 2x - 4$

$\therefore 2x - 4 > 0$

$\therefore 2(x-2) > 0$

$\therefore x - 2 > 0$

$\therefore x = 2$



$\therefore x \in (2, \infty)$

Now f is decreasing if and only if $f'(x) < 0$

$\therefore 2x - 4 > 0$

$\therefore 2(x-2) < 0$

$\therefore x - 2 < 0$

$\therefore x = 2$

$\therefore x \in (-\infty, 2)$

$f(x) = 2x^3 + x^2 - 20x + 4$

Solution: f is increasing if and only if $f'(x) > 0$

$\because f(x) = 2x^3 + x^2 - 20x + 4$

$\therefore f'(x) = 6x^2 + 2x - 20$

$\therefore 6x^2 + 2x - 20 > 0$

$\therefore 6x^2 + 12x - 10x - 20 > 0$

$\therefore 6x(x+2) - 10(x+2) > 0$

$\therefore (x+2)(6x - 10) > 0$

$\therefore x = -2, \frac{5}{3}$

$\therefore x \in (-\infty, -2) \cup (5/3, \infty)$

Now f is decreasing if and only if $f'(x) < 0$

$\therefore 6x^2 + 2x - 20 < 0$

$\therefore (x+2)(6x-10) < 0$

$\therefore x = -2, 5/3$

$\therefore x \in \left(-2, \frac{5}{3}\right)$

d] $f(x) = x^3 - 27x + 5$

Solution: f is increasing if and only if $f'(x) > 0$

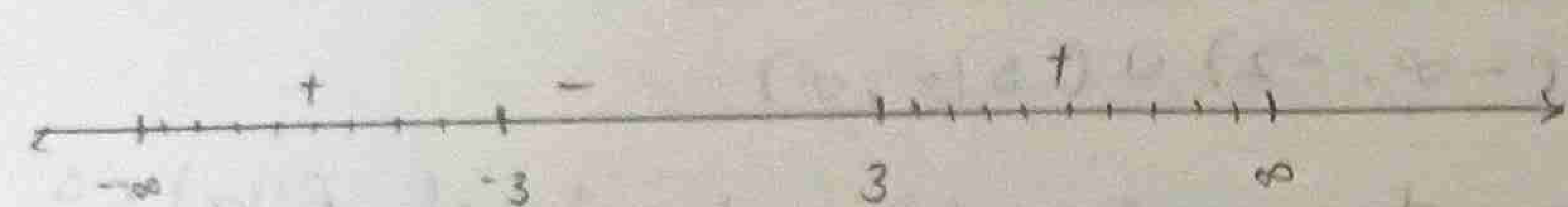
$\therefore f(x) = x^3 + 2x + 5$

$\therefore f'(x) = 3x^2 - 27$

$\therefore 3x^2 - 27 > 0$

$\therefore 3(x^2 - 9) > 0$

$\therefore x = 3, -3$



$\therefore x \in (-\infty, -3) \cup (3, \infty)$

Now f is decreasing if and only if $f'(x) < 0$

$\therefore 3x^2 - 27 < 0$

$\therefore 3(x^2 - 9) < 0$

$\therefore x^2 - 9 < 0$

$\therefore x = 3, -3$

$\therefore x \in (-3, 3)$

e) $f(x) = 69 - 24x - 9x^2 + 2x^3$

Solution: f is increasing if and only if $f'(x) > 0$

$\therefore f(x) = 69 - 24x - 9x^2 + 2x^3$

$\therefore f'(x) = -24 - 18x + 6x^2$

$\therefore -24 - 18x + 6x^2 > 0$

$\therefore 6(-4 - 3x + x^2) > 0$

$\therefore x^2 - 3x - 4 > 0$

$\therefore \cancel{x}(\cancel{x} - x^2 - 4x + x - 4) > 0$

$\therefore x(x - 4) \; 1 \; (x - 4) > 0$

$\therefore (x - 4)(x + 1) > 0$

$\therefore x = 4, x - 1$

$x \in (-\infty, -1) \cup (4, \infty)$

Now f is decreasing if and only if $f'(x) < 0$

$\therefore -24 - 18x + 6x^2 < 0$

$\therefore 6(-4 - 3x + x^2) < 0$

$\therefore (x - 4)(x + 1) < 0$

$\therefore x = 4, -1$

$x \in (-1, 4)$.

Q.2) find the intervals in which functions is concave upward concave downward.

a] $y = 3x^2 - 2x^5$

Solution :- $\therefore f(x) = 3x^2 - 2x^3$

$\therefore f'(x) = 6x - 6x^2$

$\therefore f''(x) = 6 - 12x$

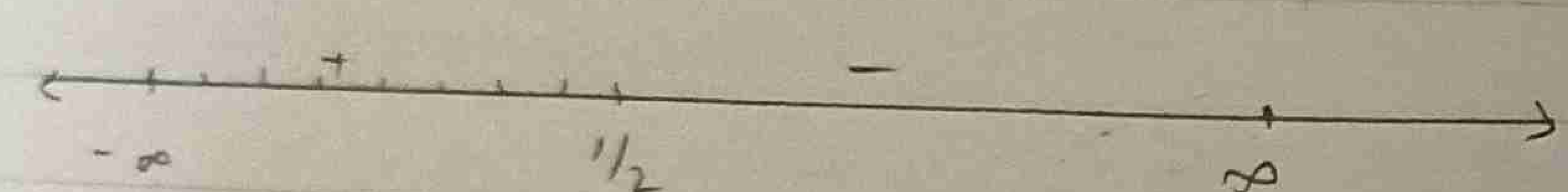
$\because$ f is concave upward if and only if $f''(x) > 0$

$\therefore 6 - 12x > 0$

$\therefore 6(1 - 2x) > 0$

$\therefore 1 - 2x > 0$

$\therefore -(2x - 1) > 0$



$x \in (-\infty, \frac{1}{2})$

$\because$ f is concave downward if and only if $f''(x) < 0$

$\therefore 6(1 - 2x) < 0$

$\therefore -(2x - 1) < 0$

$x \in (\frac{1}{2}, \infty)$

b] $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

Solution :- $\therefore y = f(x)$

$\therefore f(x) = x^4 - 6x^3 + 12x^2 + 5x + 7$

$\therefore f'(x) = 4x^3 - 18x^2 + 24x + 5$

$\therefore f''(x) = 12x^2 - 36x + 24$

$\therefore$ f is concave upward if and only if $f''(x) > 0$

$\therefore 12x^2 - 36x + 24 > 0$

$\therefore 12(x^2 - 3x + 2) > 0$

$\therefore x^2 - 3x + 2 > 0$

$\therefore x = 2, 1$

$x \in (-\infty, 1) \cup (2, \infty)$

$\therefore$ f is concave downward if and only if $f''(x) < 0$

$\therefore 12x^2 - 36x + 24 < 0$

$\therefore 12(x^2 - 3x + 2) < 0$

$\therefore x^2 - 3x + 2 < 0$

$\therefore (x - 2)(x - 1) < 0$

$\therefore x = 2, 1$

$x \in (1, 2)$

c] $y = x^3 - 27x + 5$

Solution:-

$\therefore y = f(x)$

$\therefore f(x) = x^3 - 27x + 5$

$\therefore f'(x) = 3x^2 - 27$

$\therefore f''(x) = 6x$

$\therefore f$ is concave upward if and only if

$\therefore f''(x) > 0$

$\therefore 6x > 0$

$\therefore x > 0$

$\therefore x = 0$

$\therefore x \in (0, \infty)$

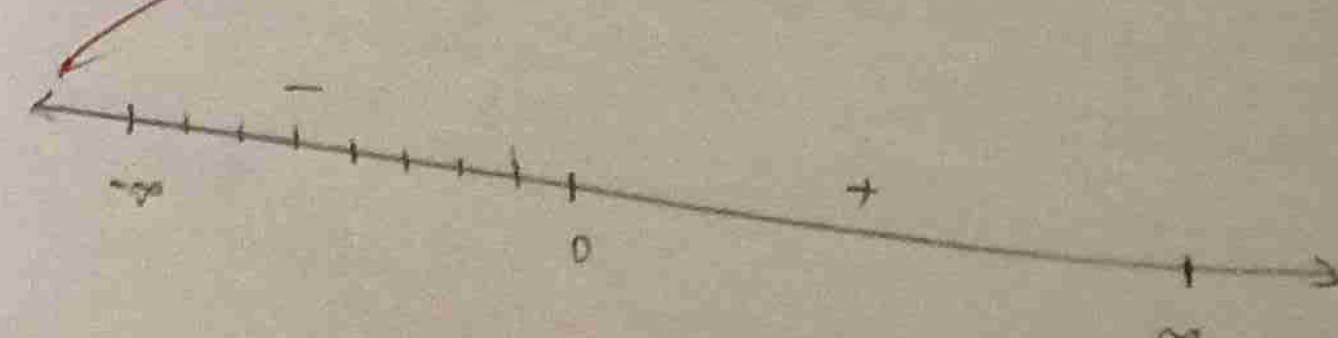
$\therefore f$ is concave downward if and only if

$f''(x) < 0$

$\therefore 6x < 0$

$\therefore x < 0$

$\therefore x = 0$



$x \in (-\infty, 0)$

d] $y = 69 - 24x - 9x^2 + 2x^3$

Solution: $\therefore y = f(x)$

$\therefore f(x) = 69 - 24x - 9x^2 + 2x^3$

$\therefore f'(x) = -24 - 18x + 6x^2$

$\therefore f''(x) = -18 + 12x$

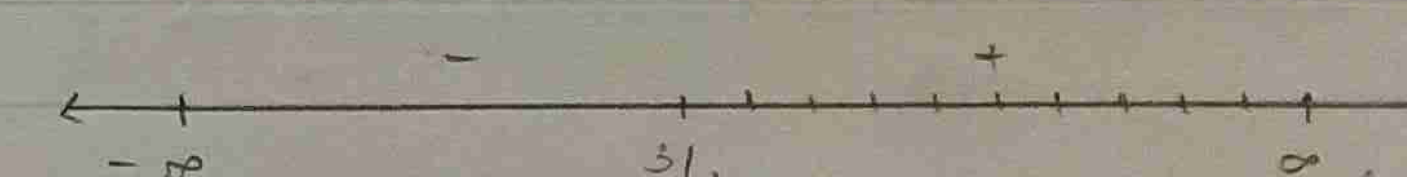
$\therefore f$ is concave upward if and only if $f''(x) > 0$

$\therefore -18 + 12x > 0$

$\therefore 6(2x - 3) > 0$

$\therefore 2x - 3 > 0$

$\therefore x = 3/2$.



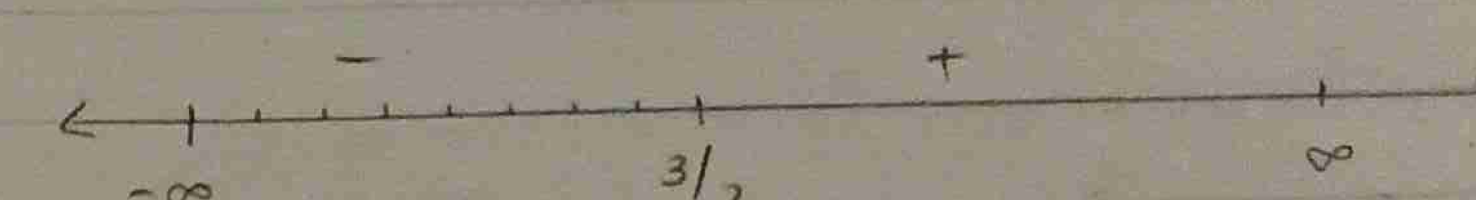
$x \in (\frac{3}{2}, \infty)$

$\therefore f$ is concave downwards if and only if $f''(x) < 0$

$\therefore -18 + 12x < 0$

$\therefore 6(2x - 3) < 0$

$\therefore x = 3/2$



$\therefore x \in (-\infty, \frac{3}{2})$

e) $y = 2x^3 + x^2 - 20x + 4$

Solution:- $\therefore y = f(x)$

$\therefore f(x) = 2x^3 + x^2 - 20x + 4$

$\therefore f'(x) = 6x^2 + 2x - 20$

$\therefore f''(x) = 12x + 2$

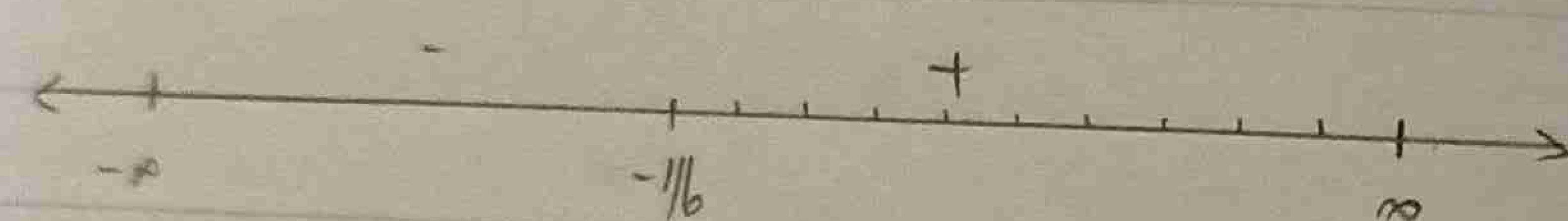
$\therefore f$ is concave upwards if and only if $f''(x) > 0$

$\therefore 12x + 2 > 0$

$\therefore 2(6x+1) > 0$

$\therefore 6x + 1 > 0$

$\therefore x = -1/6$



$x \in \left(-\frac{1}{6}, \infty\right)$

$\therefore f$ is concave downward if and only if $f''(x) < 0$

$\therefore 12x + 2 < 0$

$\therefore 2(6x+1) < 0$

$\therefore 6x + 1 < 0$

$\therefore x = -1/6$

$x \in \left(-\infty, -\frac{1}{6}\right)$

AIM - Application of derivatives & newton method.

Q.1] Find maximum & minimum value of following

i] $f(x) = x^2 + \frac{16}{x^2}$

ii] $f(x) = 3 - 5x^3 + 3x^5$

iii] $f(x) = x^3 - 3x^2 + 1 \quad [-1/2, 4]$

iv] $f(x) = 2x^3 - 3x^2 - 12x + 1 \quad [-2, 3]$

Find the root of the following equation by newton's method (Take 4 iteration only correct upto + decimal)

i] $f(x) = x^3 - 3x^2 - 55x + 95$ (take $x_0 = 0$)

ii] $f(x) = x^3 - 4x - 9$ in $[2, 5]$

iii] $f(x) = x^3 - 18x^2 - 10x + 17$ in $[1, 2]$.

Solution :

$f(x) = x^2 + \frac{16}{x^2}$

$f'(x) = 2x - 32/x^3 = 0$

$\cancel{2x} =$ Now consider $f'(x) = 0$

$2x - 32/x^3 = 0$

$2x = 32/x^3$

$x^4 = 16$

$x = \pm 2$

$f''(x) = 2 + 96/x^4$

$f''(x) = 2 + 96/24$

$= 2 + 96/16$

$= 2 + 6$

$= 8 > 0$

$\therefore$ f has maximum value at $x = 2$

$f(2) = 2^2 + 16/22$

$= 4 + 16/4$

$= 4 + 4$

$= 8$

$\therefore f''(-2) = 2 + 96/(-2)^4$

$= 2 + 96/16$

$= 2 + 6$

$= 8 > 0$

$\therefore$ f has minimum value at $x = 2$

function reaches minimum value at $x = 2$, & $x = -2$.

ii) $f(x) = 35x^2 + 3x^5$

$f'(x) = 15x^2 + 15x^4$

Consider $f'(x) = 0$

$15x^2 + 15x^4 = 0$

$15x^4 = 15x^2$

$x^2 = 1$

$x = \pm 1$

$\therefore f''(x) = 30x + 60x^3$

$f(1) = -30 + 60$

$= 30 > 0$

$\therefore$ f has maxi

$f(1) = 3 - 5(1)^3 + 3(1)^5$

$= 6 - 5 = 1$

$f''(x) = 3 - 5(-1)^3 + 3(-1)^5$

$= 3 + 5 = 3 - 5$

$\therefore$ f has maximum value ~~e~~ at $x = 1$ & has minimum value at $x = -1$.

1. $f(x) = x^3 - 3x^2 + 1$

$f'(x) = 3x^2 \cdot 6(x)$

Consider, $f'(x) = 0$

$3x^2 - 6x = 0$

$3x(x - 2) = 0$

$x = 0$ or $x = 2$

$f''(x) = 6x - 6$

$f'(0) = 6(0) - 6$

$= 6 < 0$ $\therefore$ f has maximum value.

$f(0) = (0)^3 - 3(0)^2 + 1 = 1$

$f''(2) = 6(2) - 6$

$= 12 - 6$

$= 6 > 0$

$\therefore$ f has maximum value at $x = 0$

$f(0) = (0)^3 - 3(0)^2 + 1 = 1$

$f''(2) = 6(2) - 6$

$= 12 - 6$

$= 6 > 0$

f has minimum value at $x = 2$.

$f(2) = (2)^3 - 3(2)^2 + 1$
$= 8 - 3(4) + 1$
$= 9 - 12$
$= -3$

$\therefore$ f has maximum value 1 at $x = 0$ & f has minimum value -3 at $x = 2$.

iv) $f(x) = 2x^3 - 3x^2 - 12x + 1$
$f'(x) = 6x^2 - 6x - 12$
consider, $f'(x) = 0$
$6x^2 - 6x - 12 = 0$
$6(x^2 - x - 2) = 0$
$x^2 + x - 2x - 2 = 0$
$x(x+1) - 2(x+1) = 0$
$x = 2$ or $x = -1$
$f''(x) = 12x - 6$
$f''(2) = 12(2) - 6$
$= 24 - 6$
$= 18 > 0$

$\therefore$ f has minimum value of $x = \pm 2$.
$f(2) = 2(2)^3 - 3(2)^2 - 12(2) + 1$
$= 2(8) - 3(4) - 24 + 1$
$= 16 - 18 - 24 + 1$
$= -19$
$f''(-1) = 12(-1) - 6$
$= -12 - 6$
$= -18 < 0$
f has maximum value at $x = -1$
$\therefore f(-1) = 2(-1)^3 - 3(-1)^2 - 12(1) + 1$

$\therefore$ f has maximum value at $x = -1$ &
f has minimum value -19 at $x = 2$.

2.2) $f(x) = x^3 - 3x^2 - 55x + 95$, $x = 0$ given
$f'(x) = 3x^2 - 6x - 55$
By Newton's method,
$x_{n+1} = x_n - f(x_n) / f'(x_n)$
$x_1 = x_0 - f(x_0) / f'(x_0)$
$x_1 = 0 + 9.5/55$
$x_1 = 0.1727$
$f(x_1) = (0.1727)^3 - 3(0.1727)^2 - 55(0.1727) + 9.5$
$= 0.0051 - 0.895 - 9.4985 + 9.5$
$= -0.0829$
$f'(x_1) = 3(0.1727)^2 - 6(0.1727) - 55$
$= 0.0895 - 1.0362 - 55$
$= -55.9467$
$\therefore x_2 = x_1 - f(x_1) / f'(x_1)$
$= 0.1727 - 0.0829 / 55.9467$
$= 0.1712$.
$\therefore f(x_2) = (0.1712)^3 - 3(0.1712)^2 - 55(0.1712) + 9.5$
$= 0.0050 - 0.0879 - 9.416 + 9.5$
$= 0.0011$.
$f'(x_2) = 3(0.1712)^3 - 3(0.1712) - 55$
$= 0.0879 - 1.0272 - 55$
$= -55.9393$.
$x_3 = x_2 - f(x_2) / f'(x_2)$
$= 0.1712 + 0.0011 / 55.9393$
$= 0.1712$.
The root of the equation is 0.1712.

ii) $f(x) = x^3 - 4x - 9$

$f'(x) = 3x^2 - 4$

$f(2) = 2^3 - 4(2) - 9$

$= \cancel{8} - \cancel{8} - 9$

$= -9$

$f(3) = 3^3 - 4(3) - 9$

$= 27 - 12 - 9$

$= 6$

Let $x_0 = 3$ be the initial approximation by newton's method.

$x_{n+1} = x_n - f(x_n) / f'(x_n)$

$x_1 = x_0\ f(x_0) / f'(x_0)$

$= 3 - 6/23$

$= 2.7392$

$f(x_1) = (2.7392)^3 - 4(2.7392) - 9$

$= 8528 - 10.9568 - 9$

$= 0.596$

$f'(x_1) = 3(2.7392)^2 - 4$

$= 22.5096 - 4$

$= 18.8096$

$x_2 = x_1 - f(x_1) / f'(x_1)$

$= 2.7392 - 6.846 / 18.5096$

$= 2.7071$

$f(x_2) = (2.7071)^3 - 4(2.7071)$

$= 13.886 - 10.8284$

$= 0.0102$

$f'(x_2) = 3(2.7071)^2 - 4$

$= 21.9881 - 4$

$= 17.9851$

$2.7071 - \frac{0.0102}{17.9851}$

$= 2.7071 - 0.0056 = 2.7015$

$f(x_3) = (2.7015)^3 - 4(2.7015) - 9$

$= (9.7)68 - 10.806 - 9 = -0.0901$

$f'(3) = 3(2.7015)^2 - 4 = 2.18943 - 4 = 17.8943$

$x_4 = 2.7015 + 0.0901 / 17.8943 = 2.7015 + 0.0030$

$= 2.7065$

$f(x) = x^3 - 18x^2 - 10x + 17 \quad [1, 2]$

$f'(x) = 3x^2 - 36x - 10$

$f(1) = (1)^3 - 1.8(1)^2 - 10(1) + 17$

$= 1.8 - 10 + 17$

$= 6.2$

$f(2) = (2)^3 - 1.8(2)^2 - 10(2) + 17$

$= 8 - 7.2 - 20 + 17 = -2.2$

Let $x_0 = 2$ be initial approximation by newton method.

$x_{n+1} = x_n - f(x_n) / f'(x_n)$

$x_1 = x_0 - f(x_0) / f'(x_0)$

$= 2 - 2.2/5.2$

$= 2 - 0.4230 = 1.577$

$f(x_1) = (1.577)^3 - 1.8(1.577)^2 - 10(1.577) + 17$

$= 3.9219 - 44769 - 15.77 + 17$

$= 0.6755$

No.

$f'(x_1) = 3(1.577)^2 - 3.6(1.577) - 10$

$= 7.4608 - 5.6772 - 10$

$= -8.2164$

$x_2 = x_1 - f(x_1) / f'(x_1)$

$= 1.577 + 0.6755 / 8.2164$

$= 1.577 + 0.0822$

$= 1.6592$

$f(x_2) = (1.6592)^3 - 1.8(1.6592)^2 - 10(1.6592) + 17$

$= 4.5677 - 4.9553 - 16.592 + 17$

$= 0.0204$

$f'(x_2) = 3(1.6592)^2 - 3.6(1.6592) - 10$

$= 8.2588 - 5.97312 - 10$

$= -7.7143$

$x_3 = x_2 - f(x_2) / f'(x_2)$

$= 1.6592 / 0.0204 / 7.7143$

$= 1.6592 + 0.0026$

$= 1.6618$

$f(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 10(1.6618)$

$= 4.5892 - 4.9708 - 16.618 + 17$

$= 0.0004$

$f'(x_3) = 3(1.6618)^2 - 3.6(1.6618) - 10$

$= 8.2847 - 5.9824 - 10$

$= 7.697$

$x_4 = x_3 - f(x_3) / f'(x_3)$

$= 1.8618 + \frac{0.0004}{7.6975}$

$= 1.6618.$

AK
02/01/2020

PRACTICAL - 5

AIM - Integration

2) Solve the following integration :-

i) $\int \frac{dx}{\sqrt{x^2 + 2x - 3}}$

ii) $\int (4e^{3x+1}) dx$

iii) $\int (2x^2 - 3\sin x + 5\sqrt{x}) dx$

iv) $\int \frac{x^3 + 3x + 4}{\sqrt{x}} dx$

v) $\int t^7 \sin(2t^4) dx$

vi) $\int \sqrt{x} (x^2 - 1) dx$

vii) $\int \frac{-1}{x^3} \sin\left(\frac{1}{x^2}\right) dx$

viii) $\int \frac{\cos x}{3\sqrt{\sin^2 x}} dx$

ix) $\int e^{\cos^2 x} \sin 2x \, dx$

x) $\int \left(\frac{x^2 - 2x}{x^3 - 3x^2 + 1} \right) dx$

1) $\int \frac{1}{x^2+2x-3}\,dx$

$= \int \frac{1}{\sqrt{x^2+2x-3}}\,dx$

$= \int \frac{1}{\sqrt{x^2+2x+1-4}}\,dx$

$= a^2+2ab+b^2 = (a+b)^2$

$= \int \frac{1}{\sqrt{(x+1)^2-4}}\,dx$

Substitute put $x+1=t$

$dx = 1/t \times dt$ where $t=1$, $t=x+1$

$\int \frac{1}{\sqrt{t^2-4}}\,dt$

using

$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln(|x+\sqrt{x^2-a^2}|)$

$= \ln(|t+\sqrt{t^2-4}|)$

$= \ln(|x+1\ \sqrt{(x+1)^2-4}|)$

$= \ln(|x+1+\sqrt{x^2+2x-3}|)$

$= \ln(|x+1+\sqrt{x^2+2x-3}|)+c$

) $\int(4e^{3x}+1)\,dx$

$= \int 4e^{3x}\,dx + \int 1\,dx$

$= 4\int e^{3x}\,dx + \int 1\,dx$ # $\int e^{ax}\,dx = 1/a\, e^{ax}$

$= \frac{4e^{3x}}{3} + x$

$= \frac{4e^{3x}}{3} + x + c$

3) $\int 2x^2 - 3\sin(x) + 5\sqrt{x}\,dx$

$= \int 2x^2 - 3\sin(x) + 5x^{1/2}\,dx$ # $\sqrt[n]{a^m} = a^{m/n}$.

$= \int 2x^2\,dx - \int 3\sin(x)\,dx + \int 5x^{1/2}\,dx$.

$= \frac{2x^3}{3} + 3\cos x + \frac{\cos\sqrt{x}}{3} + c$

$= \frac{2x^3+\cos\sqrt{x}}{3} + 3\cos x + c$

4) $\int \frac{x^3+3x+4}{\sqrt{x}}\,dx$

$= \int \frac{x^3+3x+4}{x^{1/2}}\,dx$

split the denominator

$= \int \frac{x^3}{x^{1/2}} + \frac{3x}{x^{1/2}} + 4/x^{1/2}\,dx$

$= \int x^{5/2} + 3x^{1/2} + 4/x^{1/2}\,dx$

$= \int x^{5/2}\,dx + \int 3x^{1/2}\,dx + \int 4/x^{1/2}\,dx$.

$= \frac{x^{5/2+1}}{5/2+1}$

$= \frac{2x^3\sqrt{x}}{7} + 2x\sqrt{x} + 8\sqrt{x} + c$

5) $\int t^7 \times \sin(2t^4)\, dt$

Put $u = 2t^4$

$du = 2 \times 4t^3$

$= \int t^7 \times \sin(2t^4) \times \frac{1}{2\times 4t^3}\, dx$

$= \int t^4 \sin(2t^4) \times 1/_{2\times4}\, dx$

$= \int t^4 \sin(2t^4) \times \frac{1}{8}\, dx = \frac{t^4 \times \sin(2t^4)\, dx}{8}$

Substitute t^4 with $u/2$

$= \int \frac{u/2 \times \sin(u)}{8}\, dx$

$= \int \frac{u \times \sin(u)}{2} / 8\, dx$

$= \int \frac{u \times \sin(u)}{16}\, dx$

$= 1/16 \int u \times \sin(u)\, dx$

$\int u\,dv = uv - \int v\,du$

where $u = u$

$dv = \sin(u) \times du$

$du = 1\,du \quad v = -\cos(u)$

$= 1/16\, (u \times (-\cos(u)) - \int -\cos(u)\, dx$

$= 1/16 \times (u \times (-\cos u)) + \int \cos(u)\, du$

$\int \cos x\, dx = \sin(x)$

$= 1/16 \times (u \times (-\cos(u)) + \sin(u))$

Return the substitution $u = 2t^4$

$= 1/16 \times (2t^4 \times (-\cos(2t^4)) + \sin(2t^4))$

$= -\frac{t^4 \times \cos(2t^4)}{8} + \frac{\sin(2t^4)}{16} + c$

6) $\int \sqrt{x}\,(x^2 - 1)\, dx$

$= \int \sqrt{x}\, dx^2 - \sqrt{x}\, dx$

$= \int x^{1/2} \times x^2 - x^{1/2}\, dx$

$= \int x^{5/2} - x^{1/2}\, dx$

$= \int x^{5/2}\, dx - \int x^{1/2}\, dx$

$= I_1 = \frac{x^{5/2+1}}{5/2+1} = \frac{x^{7/2}}{7/2} = \frac{2x^{7/2}}{7} = \frac{2\sqrt{x^7}}{7} = \frac{2x^3\sqrt{x}}{7}$

$= I_2 = \frac{x^{1/2+1}}{1/2+1} = \frac{x^{3/2}}{3/2} = \frac{2x^{3/2}}{3/2} = \frac{2\sqrt{x^3}}{3}$

$= \frac{2x^3\sqrt{x}}{7} + \frac{2\sqrt{x^3}}{3} + c$

iii) $\int \frac{\cos x}{\sqrt[3]{\sin(x)^2}}\, dx$

$= \int \frac{\cos x}{\sin x^{2/3}}\, dx$

Put $t = \sin(x)$

$t = \cos x$

$= \int \frac{\cos(x)}{\sin(x)^{5/2}} \times \frac{1}{\cos(x)}\, dt$

$= \frac{1}{\sin x^{5/2}}\, dt = \frac{1}{t^{2/3}}\, dt.$

$$I = \int \frac{1}{t^{2/3}}\, dt = \frac{-1}{(2/3 - 1)\, t^{2/3 - 1}} = \frac{-1}{-1/3\, t^{2/3-1}}$$

$$= \frac{1}{1/3\, t^{-1/3}} = \frac{t^{1/3}}{1/3} = 3t^{1/3}$$

$$= 3^{2}\sqrt{t}$$

Return substitution $t = \sin(x)$

$$= 3\sqrt[3]{\sin(x)} + c$$

x) $\displaystyle\int \frac{x^2 - 2x}{x^3 - 3x^2 + 1}\, dx$

put $x^3 - 3x^2 + 1 = dt$

$$I = \int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} \times \frac{1}{3x^2 - 3 \times 2x}\, dt$$

$$= \int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} \times \frac{1}{3x^2 - 6x}\, dt$$

$$= \int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} \times \frac{1}{3(x^2 - 2x)}\, dt$$

$$= \int \frac{1}{x^3 - 3x^2 + 1} \times \frac{1}{3}\, dt$$

$$= \int \frac{1}{3(x^3 - 3x^2 + 1)}\, dt = \int \frac{1}{3t}\, dt$$

$$= \frac{1}{3}\int \frac{1}{t}\, dt \qquad \int \frac{1}{x}\, dx = \ln(x)$$

$$= \frac{1}{3} \times \ln|t| + c$$

$$= \frac{1}{3} \times \ln(|x^3 - 3x^2 + 1|) + c.$$

PRACTICAL - 6

Application of Integration & Numerical Integration

1) $y = \sqrt{4-x^2}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{4-x^3}}$$

$$= \frac{-x}{\sqrt{4-x^2}}$$

$$I = \int_{-2}^{2} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_{-2}^{2} \sqrt{1+\frac{x^2}{4-x^2}}\, dx$$

$$= \int_{-2}^{2} \sqrt{\frac{4-x^2+x^2}{4-x^2}}$$

$$= \int_{-2}^{2} \sqrt{\frac{4}{4-x^2}}\, dx$$

$$= 2\int_{-2}^{2} \frac{1}{\sqrt{2-x^2}}\, dx$$

$$= 2\left[\sin^{-1}(x/2)\right]^{2}$$

$$= 2\left[\sin^{-1}(1) - \sin^{-1}(-1)\right]$$

$$= 2\left[\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right] = 2\pi$$

3] $y = x^{3/2} \qquad x \in [0,4]$

$\frac{dy}{dx} = \frac{3}{2} x^{1/2}$

$l = \int_0^{4} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$

$= \int_0^4 \sqrt{1+\frac{9}{4}x}\; dx$

$= \frac{1}{2}\int_0^4 \sqrt{4+9x}\; dx$

$= \frac{1}{2}\left[\frac{[4+9x]^{3/2}}{3/2} \times \frac{1}{9}\right]_0^4$

$= \frac{1}{27}\left[(4+9x)^{3/2}\right]$

$= \frac{1}{27}\left[(4+0)^{7/2} - (4+31)^{7/5}\right]$

$= \frac{1}{27}\,(4^{3/2} - 8)$ unib

4] $x = 3\sin t \qquad y = 3\cos t$

$\frac{dx}{dt} = 3\cos t \qquad \frac{dy}{dt} = -3\sin t$

$I = \int_0^{2\pi} \sqrt{\left(\frac{dy}{dx}\right)^2 + \left(\frac{dx}{dt}\right)^2}\, dt$

$= \int_0^{2\pi} \sqrt{(3\cos t)^3 + (-3\sin t)^2}\, dt$

$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}\; dt$

$= \int_0^{2\pi} 3\, dt$

$= 3\int_0^{2\pi} dt$

$= 3[x]_0^{2\pi}$

$= 3[2\pi - 0]$

$I = 6\pi$ units

5] $x = \frac{1}{6}y^5 + \frac{1}{2y}$

$\frac{dx}{dy} = \frac{y^3}{2} - \frac{1}{2y^2}$

$\frac{dx}{dy} = \frac{y^4-1}{2y^2}$

$I = \int_0^2 \sqrt{1+\left(\frac{dx}{dy}\right)^2}\; dy$

$= \int_0^2 \sqrt{1+\left(\frac{y^4-1}{4y^4}\right)^2}\; dy$

$= \int_0^2 \sqrt{\frac{(y^4-1)+4\times y^4\times 1}{(2y^2)^2}}\; dy$

(iii)

$$= \int_0^2 \frac{y^4+1}{2y^2}\, dy$$

$$= \frac{1}{2}\int_1^2 y^2\,dy + \frac{1}{2}\int_1^2 y^{-2}\,dy$$

$$= \frac{1}{2}\left[\frac{y^3}{3} - \frac{y^{-1}}{1}\right]_1^2$$

$$= \frac{1}{2}\left[\frac{8}{3} - \frac{1}{2} - \frac{1}{3} + 1\right]$$

$$= \frac{1}{2}\left[\frac{7}{3} + \frac{1}{2}\right]$$

$$= \frac{1}{2}\left[\frac{17}{6}\right]$$

$$= \frac{17}{12} \text{ unit}$$

Q.2) i) $\int_0^2 e^x\,dx$ with $n=4$

$$h = \frac{b-a}{n} = \frac{2-0}{4} = 0.5$$

x	0	0.5	1	1.5	2
y	1	1.284	2.7183	9.4877	54.5982
	y_0	y_1	y_2	y_3	y_4

$$\int_0^2 e^{x^2}\,dx = \frac{2}{3}\left[(y_0+y_4) + 4(y_1+y_3) + 2(y_2)\right]$$

$$= \frac{0.5}{3}\left[(1+54.5982) + 5(1.284+9.487) + 2.23885\right]$$

$$= \frac{0.5}{3}\left[55.5982 + 43.0866 + 5.456\right]$$

$$\int_0^2 e^{x^2}\,dx = 17.3535.$$

ii) $\int_0^4 x^2\,dx$ $n=4$

$$h = \frac{4-0}{4} = 1$$

x	0	1	2	3	4
y	0	1	4	9	16
	y_0	y_1	y_2	y_3	y_4

$$\int_0^4 x^2\,dx = \frac{h}{3}\left[(y_0 - y_4) + 4(y_1+y_3) + 2y_2\right]$$

$$= \frac{1}{3}\left[0 + 16 + 4(1+9) + 2\times 4\right]$$

$$= \frac{1}{3}\left[16 + 4(10) + 8\right]$$

$$= \frac{64}{3}$$

$$\int_0^4 x^2\,dx = 21.3333.$$

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iii) $\int_0^{\pi/3} \sqrt{\sin x}\, dx \quad n=6$

$h = \frac{\frac{\pi}{3} - 0}{6} = \frac{\pi}{18}$

x	0	$\frac{\pi}{18}$	$\frac{2\pi}{18}$	$\frac{3\pi}{18}$	$\frac{4\pi}{18}$	$\frac{5\pi}{18}$	$\frac{6\pi}{18}$
y	0	0.4167	0.4885	0.7071	0.8017	0.8752	0.9306
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

$\int_0^{\pi/3} \sqrt{\sin x}\, dx = \frac{h}{3}\left[y_0 + y_6 + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)\right]$

$= \frac{\pi/18}{3}\left[0.4167 + 0.9306 + 4(0.4167 + 0.7071 + 0.9752) + 2(0.5\ldots\right.$ 0.8017

$= \frac{\pi}{54}\left[1.3473 + 4(1.999) + 2(1.3865)\right]$

$= \frac{\pi}{54}\left[1.3473 + 1.996 + 2.773\right]$

$= \frac{\pi}{54} \times 12.1163$

$\int_0^{\pi/3} \sqrt{\sin x}\, dx = 0.7049$

09/01/2020

PRACTICAL - 7

Topic : Differential Equation

1] $x\frac{dy}{dx} + y = e^x$

$\frac{dy}{dx} + \frac{1}{x} y = \frac{e^x}{x}$

$P(x) = \frac{1}{x} \qquad Q(x) = \frac{e^x}{x}$

$I.F = e^{\int P(x)\, dx}$

$= e^{\int 1/x\, dx}$

$= e^{\ln x}$

$I.F = x$

$y(I.F) = \int Q(x)(I.F)\, dx + c$

$= \int \frac{e^x}{x} \cdot x \cdot dx + c$

$= \int e^x\, dx + c$

$xy = e^x + c.$

Q.2] $e^x \frac{dy}{dx} + 2e^x y = 1$

$\frac{dy}{dx} + 2\frac{e^x}{e^x} y = \frac{1}{e^x} \qquad (\div \text{ by } e^x)$

$\frac{dy}{dx} + 2y = \frac{1}{e^x}$

$\frac{dy}{dx} + 2y = e^{-x}$

$p(x) = 2$ $\quad Q(x) = e^{-x}$

$\int p(x)\,dx$

$I.F = e\int 2\,dx$

$= e^{2x}$

$y(I.F) = \int Q(x)(I.F)\,dx + c$

$y.e^{2x} \int e^{-x} + 2x\,dx + c$

$= \int e^{x}\,dx + c$

$y.e^{2x} = e^{x} + c$

3] $x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$\therefore \frac{dy}{dx} + \frac{2y}{x} = \frac{\cos x}{x^2}$

$p(x) = 2(x)$ $\quad Q(x) = \frac{\cos x}{x^2}$

$I.F = e\int p(x)\,dx$

$= e\int 2/x\,dx$

$= e^{2/x}\,x$

$= \ln x^2$

$y(I.F) = \int Q(x)(I.F)\,dx + c$

$= \int \frac{\cos x}{x^2} \cdot x^2\,dx + c$

$= \int \cos x + c$

$x^2 y = \sin x + c.$

4) $x\frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$

$\frac{dy}{dx} + \frac{3y}{x} = \frac{\sin x}{x^3}$ ($\div$ by x on both sides)

$p(x) = 3/x$ $\quad Q(x) = \sin x / x^3$

$= e\int p(x)\,dx$

$= e\int 3/x\,dx$

$= e^{3/x}\,dx$

$= e^{\ln x^3}$

$I.F = x^3$

$y(I.F) = \int Q(x)(I.F)\,dx + c$

$= \int \frac{\sin x}{x^3} \cdot x^3\,dx + c$

$= \int \sin x\,dx + c$

$x^3 y = -\cos x + c$

$e^{2x}\frac{dy}{dx} + 2e^{2x}y = 2x$

$\frac{dy}{dx} + 2y = \frac{2x}{e^{2x}}$

$p(x) = 2$ $\quad Q(x) = 2x/e^{2x} = 2xe^{-2x}$

$I.F = e\int p(x)\,dx$

$= e\int 2\,dx$

$= e^{2x}$

$y(I.F) = \int Q(x)(I.F)\,dx + c$

$= \int 2x e^{-2x} e^{2x}\,dx + c$

$= \int 2x\,dx + c$

$ye^{2x} = x^2 + c$

vi) $\sec^2 x \cdot \tan y\,dx + \sec^2 y \tan x\,dy = 0$

$\sec^2 x \cdot \tan y\,dx = -\sec^2 y \cdot \tan x\,dy$

$\frac{\sec^2 x}{\tan x} = \frac{-\sec^2 y\,dy}{\tan y}$

$\int \frac{\sec^2 x\,dx}{\tan x} = -\int \frac{\sec^2 y\,dy}{\tan y}$

$\log|\tan x| = -\log|\tan y| + c$

$\log|\tan x \cdot \tan y| = c$

$\tan x \cdot \tan y = e^c$

vii) $\frac{dy}{dx} = \sin^2(x-y+1)$

Put $x-y+1 = v$

Differentiating on both sides

$x-y+1 = v$

$\cancel{1-\frac{dy}{dx} = \frac{dv}{dx}}$.

$1-\frac{dy}{dx} = \frac{dv}{dx}$

$1-\frac{dv}{dx} = \sin^2 v$

$\frac{dv}{dx} = 1-\sin^2 v$

$\frac{dv}{dx} = \cos^2 v$

$\frac{dv}{\cos^2 v} = dx$

$\int \sec^2 v\,dv = \int dx$

$\tan v = x + c$

$\tan(x+y-1) = x + c$.

viii) $\frac{dy}{dx} = \frac{2x+3y-1}{6x+9y+6}$

Put $2x+3y = v$

$2 + 3\frac{dy}{dx} = \frac{dv}{dx}$

$\frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx} - 2\right)$

$\frac{1}{3}\left(\frac{dv}{dx} - 2\right) = \frac{1}{3}\left(\frac{(v-1)}{(v+2)}\right)$

$\frac{dv}{dx} = \frac{v-1}{v+2} + 2$

$\frac{dv}{dx} = \frac{v-1+2v+4}{v+2}$

$= \frac{3v+3}{v+2}$

$= \frac{3(v+1)}{v+2}$

$$\int \left(\frac{v+2}{v+1}\right) dx = 3dx$$

$$= \int \frac{v+1}{v}\, dx + \int \frac{1}{v+1}\, dx = 3x$$

$$v + \log|x| = 3x + c$$

$$2x + 3y + \log|2x + 3y + 1| = 3x + c$$

$$3y = x - \log|2x + 3y - 1| + c$$

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PRACTICAL-8

TOPIC - Euler's Rule

1) $\frac{dy}{dx} = y + e^x - 2$ $y(0) = 2$ $h = 0.5$ find $y(2) = ?$

$\Rightarrow$ $f(x) = y + e^x - 2$ $x_0 = 0$ $y(0) = 2$ $h = 0.5$

n	x_n	y_n	$f(x_n)(x_n)$	y_{n+1}
0	0	2	1	2.5
1	0.5	2.5	2.1487	3.5743
2	1	3.5743	4.2925	5.7205
3	1.5	5.7205	8.2021	9.8215
4	2	9.8215		

$\therefore y(2) = 9.8215$

2) $\frac{dy}{dx} = 1 + y^2$, $y(0) = 1$ $h = 0.2$ find $y(1) = ?$

$y_0 = 0$, $y_0 = 0$ $h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	0	1	0.2
1	0.2	0.2	1.04	0.408
2	0.4	0.408	1.1664	0.6412
3	0.6	0.6412	1.411	0.9234
4	0.8	0.9234	1.8526	1.2939
5	1	1.2939		

$y(1) = 1.2939$

Q3 $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$ $y(0) = 1$ $h = 0.2$ find $y(1) = ?$

$x_0 = 0$ $y(0) = 1$ $h = 0.2$.

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	1	0	1
1	0.2	1	0.4472	1.0894
2	0.4	1.0894	0.6059	1.2105
3	0.6	1.2105	0.7040	1.3513
4	0.8	1.3513	0.7696	1.5051
5	1	1.5051		

$\therefore y(1) = 1.5051$

Q.4) $\frac{dy}{dx} = 3x^2+1$ $y(1) = 2$ find $y(2)$ $h = 0.5$

$h = 0.25$

$y_0 = 2$ $x_0 = 1$ $h = 0.5$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	4
1	1.5	4	7.75	7.875
2	2	7.875		

$y(2) = 7.875$

Q4] $y_0 = 2$ $x_0 = 1$ $h = 0.25$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	3
1	1.25	3	5.6875	4.4218
2	1.5	4.4218	59.6569	19.3360
3	1.75	19.3360	1122.6426	299.9960
4	2	299.9960		

$y(2) = 299.9960$

Q5] $\frac{dy}{dx} = \sqrt{xy} + 2$ $y(1) = 1$ $h = 0.2$

$x_0 = 1$, $y_0 = 1$, $h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	1	3	3.6
1	1.2	3.6		

$y(1.2) = 3.6$

23/01/2020

PRACTICAL - 9

Limits & Partial Order Derivatives

① Evaluate the following limits :

i) $\lim\limits_{(x,y)\to(-4,-1)} \dfrac{x^3-3y+y^2-1}{xy+5}$

Applying limits

$\dfrac{(-4)^3-3(-1)+(-1)^2-1}{(-4)(-1)+5}$

$\dfrac{-64+3+1-1}{4+5} = \dfrac{-61}{9}$

ii) $\lim\limits_{(x,y)\to(2,0)} \dfrac{(y+1)(x^2+y^2-4x)}{x+3y}$

Applying limits

$\dfrac{(0+1)((2)^2+(0)^2-4(2))}{2+3(0)}$

$\dfrac{1(4-8)}{2} = \dfrac{-4}{2} = -2.$

iii) $\lim\limits_{(x,y,z)\to(1,1,1)} \dfrac{x^2-y^2z^2}{x^3-x^2yz}$

$\lim\limits_{(x,y,z)\to(1,1,1)} \dfrac{(x)^2-(yz)^2}{x^2(x-yz)}$

$\lim\limits_{(x,y,z)\to(1,1,1)} \dfrac{(x+yz)(x-yz)}{x^2(x-yz)}$ $[\because (a)^2-b^2 = (a+b)(a-b)]$

$\lim\limits_{(x,y,z)\to(1,1,1)} \dfrac{x+yz}{x}$

$\dfrac{1+(1)(1)}{1^2} = 2.$

②) Find f_x, f_y for each of the following

i) $f(x,y) = xy\,e^{x^2+y^2}$

$f(x) = \dfrac{\partial f}{\partial x}$

$= \dfrac{\partial(xy\,e^{x^2+y^2})}{\partial x}$

$= y\,\dfrac{\partial(x.\,e^{x^2+y^2})}{\partial x}$

$= y\left[x.\dfrac{d}{dy}(e^{x^2+y^2}) + e^{x^2+y^2}.\dfrac{d}{dx}(x)\right]$

$\left[\because \dfrac{d}{dx}(uv) = u.v' + v.u'\right]$

$= y[x.\,e^{x^2+y^2}.2x + e^{x^2+y^2}(1)]$

$= y.\,e^{x^2+y^2}[2x+1]$

Now, $f(y) = \dfrac{\partial f}{\partial y}$

$= \dfrac{\partial(xy\,e^{x+y^2})}{\partial y}$

$= x.\dfrac{\partial}{\partial y}(y.\,e^{x^2+y^2})$

$= x\left[y.\dfrac{d}{dy}(e^{x^2+y^2}) + e^{x^2+y^2}.\dfrac{d}{dy}(y)\right]$

$= x.[2y^2.\,e^{x+y^2} + e^{x^2+y^2}]$

$= x.\,e^{x^2+y^2}[2y^2+1].$

ii) $f(x,y) = e^x \cos y$

$\therefore f(x) = e^x \cos y$

$f(y) = e^x \frac{\partial}{\partial y}(\cos y)$

$= e^x(-\sin y)$

$= -e^x \sin y$.

iii) $f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

$f(x) = \frac{\partial f}{\partial x} = \frac{\partial(x^3y^2 - 3x^2y + y^3 + 1)}{\partial x}$

$= 3x^2y^2 - 3(2x)y$

$= 3x^2y^2 - 6xy$.

$f(y) = \frac{\partial f}{\partial y} = \frac{\partial(x^3y^2 - 3x^2y + y^3 + 1)}{\partial y}$

$= x^3(2y) - 3(1)x^2 + 3y^2$

$= 2x^3y - 3x^2 + 3y^2$

Q.3) Using definition find values of f_x, f_y at $(0,0)$ for $f(x,y) = \frac{2x}{1+y^2}$

$f_x(a,b) = \lim_{h\to 0} \frac{f(a+h,b) - f(a,b)}{h}$

where $(a,b) = (0,0)$

$f_x(0,0) = \lim_{h\to 0} \frac{f(h,0) - f(0,0)}{h}$

$= \lim_{h\to 0} \frac{2h - 0}{h} = 2$.

Similarly,

$f_y(0,0) = \lim_{h\to 0} \frac{f(h,0) - f(0,0)}{h}$

$= \lim_{h\to 0} \frac{0-0}{h} = 0$

$\therefore \underline{f_x = 2}, \underline{f_y = 0}$.

Q.4) Find all second order partial derivatives of f. Also verify whether $f_{xy} = f_{yx}$.

i) $f(x,y) = \frac{y^2 - xy}{x^2}$

$\therefore f(x) = \frac{\partial f}{\partial x} = \frac{\partial(y^2 - xy)}{x^2}$

$= \frac{x^2 \cdot \frac{d}{dx}(y^2 - xy) - (y^2 - xy)\frac{d}{dx}(x^2)}{(x^2)^2}$

$= \frac{x^2(-y) - (y^2 - xy)(2x)}{x^4}$

$= \frac{-x^2y - 2xy^2 + 2x^2y}{x^4} = \frac{x(xy - 2y^2)}{x^4}$

$f(x) = \frac{xy - 2y^2}{x^3}$

$f(y) = \frac{\partial f}{\partial y} = \frac{\partial(y^2 - xy)}{x^2}$

$= \frac{\partial\left(\frac{y^2}{x^2} - \frac{xy}{x^2}\right)}{\partial y}$

$= \partial\left(\frac{y^2}{x^2} - \frac{y}{x}\right) / \partial y$

$= \frac{1}{x^2} 2y - \frac{1}{x}$

$f(y) = \frac{2y - x}{x^2}$

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$$f(x,x) = \frac{\partial\left(\frac{xy-2y^2}{x^3}\right)}{\partial x}$$

$$= \frac{x^3 \frac{d}{dx}(xy-2y^2) - (xy-2y^2)\frac{d}{dx}(x^3)}{(x^3)^2}$$

$$= \frac{x^3(y) - (xy-2y)^2(3x^2)}{6}$$

$$= \frac{x^3y - 3x^3y + 6x^2y^2}{6}$$

$$= \frac{6x^2y^2 - 2x^3y}{x^6} = \frac{x^2(6y^2 - 2xy)}{x^6}$$

$$= \frac{6y^2 - 2xy}{x^4}$$

$$f(yy) = \frac{\partial\left(\frac{2y-x}{x^2}\right)}{\partial y}$$

$$= \frac{\frac{1}{x^2}\partial(2y-x)}{\partial y} = \frac{1}{x}(2) = \frac{2}{x^2}$$

$$f(xy) = \frac{\partial\left(\frac{xy-2y^2}{x^3}\right)}{\partial y} = \frac{\partial\left(\frac{xy}{x^3} - \frac{2y^2}{x^3}\right)}{\partial y}$$

$$= \frac{\partial\left(\frac{y}{x^2} - \frac{2y^2}{x^3}\right)}{\partial y}$$

$$= \frac{1}{x^2} - \frac{1}{x^3}2(2y)$$

$$= \frac{1}{x^2} - \frac{4y}{x^3} = \frac{x^3 - 4yx^2}{x^6}$$

$$= \frac{x^2(x-4y)}{x^6}$$

$$= \frac{x-4y}{x^4}$$

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$$f(yx) = \frac{\partial\left(\frac{2y-x}{x^2}\right)}{\partial x}$$

$$= \partial\left(\frac{2y}{x^2} - \frac{x}{x^2}\right) = \frac{\partial\left(\frac{2y}{x^2} - \frac{1}{x}\right)}{\partial x}$$

$$= 2y\left(\frac{-2}{x^3}\right) - \left(\frac{-1}{x^2}\right)$$

$$= \frac{-4y}{x^3} + \frac{1}{x^2}$$

$$= \frac{-4yx^2 + x^3}{x^6}$$

$$= \frac{x^2(x-4y)}{x^6}$$

$$= \frac{x-4y}{x^4}$$

$$\therefore f(xy) = f(yx) = \frac{x-4y}{x^4}$$

Hence Verified.

ii) $f(x,y) = x^3 + 3x^2y^2 - \log(x^2+1)$

$$\therefore f(x) = \frac{\partial f}{\partial x} = \frac{\partial(x^3 + 3x^2y^2 - \log(x^2+1))}{\partial x}$$

$$= 3x^2 + 3(2x)y^2 - \frac{1}{x^2+1}(2x)$$

$$f(x) = 3x^2 + 6xy^2 - \frac{2x}{x^2+1}$$

$$f(y) = \frac{\partial f}{\partial y} = \frac{\partial(x^3 + 3x^2y^2 - \log(x^2+1)}{\partial y}$$

$= 0 + 3(2y)(x^2) + 0$

$f(y) = 6x^2y$.

$f(xx) = \frac{\partial fx}{\partial x} = \frac{\partial}{\partial x}\left(3x^2 + 6xy^2 - \frac{2x}{x^2+1}\right)$

$= 6x + 6y^2(1) - 2\left[\frac{x^2 + (1)(1) + x(2x)}{(x^2+1)^2}\right]$

$= 6x + 6y^2 - 2\left(\frac{x^2+1-2x^2}{(x^2+1)^2}\right)$

$= 6x + 6y^2 - 2\left(\frac{-x^2+1}{(x^2+1)^2}\right)$

$= f(yy) = \frac{\partial}{\partial y} fy = \frac{\partial(6x^2y)}{\partial y}$

$= 6x^2(1) = 6x^2$.

$f(xy) = \frac{\partial\left(3x^2 + 6xy^2 - \frac{2x}{x^2+1}\right)}{\partial y}$

$= 0 + 6x(2y)$

$= 12xy$

$f(yx) = \frac{\partial}{\partial x} fy$

$= \frac{\partial(6x^2y)}{\partial x}$

$= 12xy$

$\therefore f(xy) = f(yx) = 12xy$

Hence verified.

iii) $f(x,y) = \sin(xy) + e^{x+y}$

$f(x) = \frac{\partial f}{\partial x} = \frac{\partial(\sin(xy) + e^{x+y})}{\partial x}$

$= \cos(xy)(y) + e^{x+y}(1)$

$= y\cos xy + e^{x+y}$

$f(y) = \frac{\partial f}{\partial y} = \frac{\partial(\sin(xy) + e^{x+y})}{\partial y}$

$= \cos(xy)(x) + e^{x+y}(1)$

$= x\cos xy + e^{x+y}$

$f(xx) = \frac{\partial fx}{\partial x} = \frac{\partial(y\cos xy + e^{x+y})}{\partial x}$

$= y\cos xy(y) + e^{x+y}(1)$

$= y^2\cos xy + e^{x+y}$

$f(yy) = \frac{\partial fy}{\partial y} = \frac{\partial(x\cos xy + e^{x+y})}{\partial y}$

$= x\cos xy(x) + e^{x+y}(1)$

$= x^2\cos xy + e^{x+y}$

$f(xy) = \frac{\partial fx}{\partial y} = \frac{\partial(y\cos xy + e^{x+y})}{\partial y}$

$= y[-\sin(xy)(x) + \cos xy(1)] + e^{x+y}(1)$

$= -xy\sin(xy) + \cos xy + e^{x+y}$.

$f(yx) = \frac{\partial fy}{\partial x} = \frac{\partial(x\cos xy + e^{x+y})}{\partial x}$

$= \cos xy(1) + x(-\sin(xy))(y) + e^{x+y}$

$= -xy\sin(xy) + \cos(xy) + e^{x+y}$.

$= -xy\sin(xy) + \cos(xy) + e^{x+y}$.

$\therefore f(xy) = f(yx) = -xy\sin(xy) + \cos(xy) + e^{x+y}$.

Q.5) Find the linearization of $f(x,y)$ at given point.

i) $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

$f(1,1) = \sqrt{(1)^2+(1)^2}$

$= \sqrt{2}$.

$f(x) = \frac{1}{2\sqrt{x^2+y^2}} \cdot 2x = \frac{x}{\sqrt{x^2+y^2}}$

$f(y) = \frac{1}{2\sqrt{x^2+y^2}} \cdot 2y = \frac{y}{\sqrt{x^2+y^2}}$

$f_x(1,1) = \frac{1}{\sqrt{(1)^2+(1)^2}} = \frac{1}{\sqrt{2}}$

$f_y(1,1) = \frac{1}{\sqrt{(1)^2+(1)^2}} = \frac{1}{\sqrt{2}}$

$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1) + \frac{1}{\sqrt{2}}(y-1)$

$= \frac{2+x-1+(y-1)}{\sqrt{2}}$

$= \frac{2+x+y-2}{\sqrt{2}}$

$= \frac{x+y}{\sqrt{2}}$

ii) $f(x,y) = 1-x+y\sin x$ at $(\pi/2, 0)$

$f(\pi/2, 0) = 1 - \frac{\pi}{2} + 0\left(\sin\left(\frac{\pi}{2}\right)\right)$

$= 1 - \frac{\pi}{2}$

$f(x) = -1 + y\cos x$

$f(y) = 1$

$f_x\left(\frac{\pi}{2}, 0\right) = -1 + 0\cos\left(\frac{\pi}{2}\right)$

$= -1$

$f_y\left(\frac{\pi}{2}, 0\right) = 1$

$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= 1 - \frac{\pi}{2} + (-1)\left(x - \frac{\pi}{2}\right) + 1(y-0)$

$= 1 - \frac{\pi}{2} - x + \frac{\pi}{2} + y$

$= y - x + 1$

iii) $f(x,y) = \log x + \log y$ at $(1,1)$

$f(1,1) = \log(1) + \log(1)$

$= 0 + 0 = 0$

$f(x) = \frac{1}{x}$ $\qquad f_y = \frac{1}{y}$

$f(1,1) = 1$ $\qquad f_y(1,1) = 1$

$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a-b)(y-b)$

$= 0 + 1(x-1) + 1(y-1)$

$= x-1+y-1$

$= x+y-2$.

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Practical - 10

Q.1] Find the directional derivative of the following functions at given points & in the direction of given vector:-

i] $f(x,y) = x+2y-3$ $\quad a=(1,-1), u = 3i-j$

Here $u \cdot 3i-j$ is not a unit vector.

$|\bar{u}| = \sqrt{(3)^2+(-1)^2} = \sqrt{9+1} = \sqrt{10}$.

Unit vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{10}}(3,-1)$

$= \left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, -\frac{1}{\sqrt{10}}\right)$

$f(a)$ ~~that~~ $= f(1,-1) = (1) + 2(-1) - 3 = 1-2-3 = -4$.

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$= f\left(1+\frac{3}{\sqrt{10}}\right)\left(-1-\frac{h}{\sqrt{3}}\right)$

$f(a+hu) = \left(1+\frac{3}{\sqrt{10}}\right) + 2\left(-1-\frac{h}{\sqrt{10}}\right) - 3$

$= 1+\frac{3}{\sqrt{10}} - 2 - \frac{2h}{\sqrt{10}} - 3$

$f(a+hu) = \frac{-4+h}{\sqrt{10}}$

$D_u f(a) = \lim_{h\to 0} \frac{f(a+hu) - f(a)}{h}$

$= \lim_{h\to 0} \frac{-4 + h/\sqrt{10} + 4}{h}$

$D_u f(a) = \frac{1}{\sqrt{10}}$.

ii] $f(x) = y^2 - 4x + 1$ $\quad a=(3,4)$. $u = i+5j$

Here $u = i+5j$ is not a vector unit.

$|\bar{u}| = \sqrt{(1)^2+(5)^2} = \sqrt{26}$

Unit vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{26}}(1,5)$

$= \left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$

$f(a) = f(3,4) = (4)^2 - 4(3) + 1 = 5$

$f(a+hu) = f(3,4) + h\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$

$= f\left(3+\frac{h}{\sqrt{26}}, 4+\frac{5h}{\sqrt{26}}\right)$

$f(a+hu) = \left(4+\frac{5h}{\sqrt{26}}\right)^2 - 4\left(3+\frac{h}{\sqrt{26}}\right)+1$

$= 16 + \frac{25h^2}{\sqrt{26}} + \frac{40h}{\sqrt{26}} - 12 - \frac{4h}{\sqrt{26}} + 1$

$= \frac{25h^2}{\sqrt{26}} + \frac{40h}{\sqrt{26}} - \frac{4h}{\sqrt{26}} + 5$

$= \frac{25h^2}{26} + \frac{40h-4h}{\sqrt{26}} + 5$

$= \frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5$

$D_u f(a) = \lim_{h\to 0} \frac{\frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + \cancel{5} - \cancel{5}}{h}$

$\frac{h\left(\frac{25h}{26} + \frac{36}{\sqrt{26}}\right)}{h}$

$D_u f(a) = \frac{25h}{26} + \frac{36}{\sqrt{26}}$

iii) $2x+3y$ $a=(1,2)$, $u(3i+4j)$

Here $u=3i+4j$ is not a unit vector.

$|\bar{u}| = \sqrt{(3)^2+(4)^2} = \sqrt{25} = 5$.

Unit vector along u is $\frac{u}{|u|} = \frac{1}{5}(3,4)$

$= \left(\frac{3}{5}, \frac{4}{5}\right)$

$f(a) = f(1,2) = 2(1)+3(2) = 8$

$f(a+hu) = f(1,2) + h\left(\frac{3}{5}, \frac{4}{5}\right)$

$= f\left(1+\frac{3h}{5}, 2+\frac{4}{5}\right)$

$f(a+hu) = 2\left(1+\frac{3h}{5}\right) + 3\left(2+\frac{4}{5}\right)$

$= 2 + \frac{6h}{5} + 6 + \frac{12h}{5}$

$= \frac{18h}{5} + 8$

$D_u f(a) = \lim_{h\to 0} \frac{\frac{18h}{5} + 8 - 8}{h}$

$= \frac{18}{5}$

Q2] Find gradient vector for the following function at given point

i) $f(x,y) = x^y + y^x$, $a=(1,1)$

$f_x = yx^{y-1} + y^x \log y$

$f_y = x^y \log x + xy^{x-1}$

$f(x,y) = (f_x, f_y)$

$= (yx^{y-1} + y^x \log y, x^y \log x + xy^{x-1})$

$f(1,1) = (1+0, 0+1)$

$= (1,1)$

ii) $f(x,y) = (\tan^{-1}x) \cdot y^2$ $a=(1,-1)$

$f_x = \frac{1}{1+x^2} \cdot y^2$

$f_y = 2y \tan^{-1}x$

$f(x,y) = (f_x, f_y)$

$= \left(\frac{y^2}{1+x^2}, 2y\tan^{-1}x\right)$

$f(1,-1) = \left(\frac{1}{2}, \tan^{-1}(1)(-2)\right)$

$= \left(\frac{1}{2}, \frac{\pi}{4}(-2)\right)$

$= \left(\frac{1}{2}, -\frac{\pi}{2}\right)$

iii) $f(x,y,z) = xyz - e^{x+y+z}$, $a(1,-1,0)$

$f_x = yz - e^{x+y+z}$

$f_y = xz - e^{x+y+z}$

$f_z = xy - e^{x+y+z}$

$\nabla f(x,y,z) = f_x, f_y, f_z$

$= yz - e^{x+y+z}, xz - e^{x+y+z}, xy - e^{x+y+z}$

$f(1,-1,0) = (1-1)(0) - e^{1+(-1)+0}, (1)(0) - e^{1-1+0}, (1)(-1) - e^{1-1+0}$

$= (0-e^0, 0-e^0, -1-e^0)$

$= (-1, -1, -2)$

Q3) Find the equation of tangent & normal to each of the following using curves at given points.

i) $x^2\cos y + e^{xy} = 2$ at $(1,0)$.

$f_x = \cos y \; 2x + e^{xy} y$

$f_y = x^2(-\sin y) + e^{xy} x$

$(x_0, y_0) = (1, 0) \quad \therefore x_0 = 1, y_0 = 0$

eqn of tangent.

$f_x(x - x_0) + f_y(y - y_0) = 0$

$f_x(x_0, y_0) = \cos 0 \; 2(1) + e^0 \, 0$

$= 1(2) + 0$

$= 2$

$f_y(x_0, y_0) = (1)^2 - (\sin 0) + e^0 \cdot 1$

$= 0 + 1 \cdot 1$

$= 1$

$2(x-1) + 1(y-0) = 0$

$2x - 2 + y = 0$

$2x + y - 2 = 0$ → It is the required equation of tangent.

eqn of normal.

$= ax + by + c = 0$

$= bx + ay + d = 0$

$: 1(x) + 2(y) + d = 0$

$= 1 + 2y + d = 0$

$= 1 + 2(0) + d = 0$ at $(1, 0)$

$= d + 1 = 0$

$= d = -1$

ii) $x^2 + y^2 - 2x + 3y + 2 = 0$ at $(2, -2)$

$f(x) = 2x + 0 - 2 + 0 + 0$

$= 2x - 2$

$f_y = 0 + 2y - 0 + 3 + 0$

$= 2y + 3$

$(x_0, y_0) = (2, -2) \quad \therefore x_0 = 2, y_0 = -2$

$f_x(x_0, y_0) = 2(2) - 2 = 2$

$f_y(x_0, y_0) = 2(-2) + 3 = -1$

eqn of tangent

$f_x(x - x_0) + f_y(y - y_0) = 0$

$2(x-2) + (-1(y+2)) = 0$

$2x - 2 - y - 2 = 0$

$2x - y - 4 = 0$ → Required equation of tangent.

eqn of Normal

$= ax + by + c = 0$

$bx + ay + d = 0$

$= 1(x) + 2(y) + d = 0$

$-x + 2y + d = 0$ at $(2, -2)$

$-2 + 2(-2) + d = 0$

$-2 - 4 + d = 0$

$-6 + d = 0$

$\therefore d = 6$

Q.4) Find the eqn of tangent & normal line to each of the following surface

i) $x^2 - 2yz + 3y + xz = 7$ at $(2,1,0)$

$f(x) = 2x - 0 + 0 + z$

$f_x = 2x + z$

$f_y = 0 + 2z + 3 + 0$

$= 2z + 3$

$f_z = 0 - 2y + 0 + x$

$-2y + x$

$(x_0, y_0, z_0) = (2,1,0)$ ∴ $x_0 = 2, y_0 = 1, z_0 = 0$

$f_x(x_0, y_0, z_0) = 2(2) + 0 = 4$

$f_y(x_0, y_0, z_0) = 2(0) + 3 = 3$

$f_z(x_0, y_0, z_0) = -2(1) + 2 = 0$

equation of tangent

$f_x(x_0 - x_0) + f_y(y_0 - y_0) + f_z(z_0 - z_0) = 0$

$= 4(x-2) + 3(y-1) + 0(z-0) = 0$

$= 4x - 8 + 3y - 3 = 0$

$4x + 3y - 11 = 0$ → Required equation of Tangent.

Eqn of normal. at $(4,3,-11)$

$\frac{x - x_0}{f_x} = \frac{y - y_0}{f_y} = \frac{z - z_0}{f_z}$

$= \frac{x-2}{4} = \frac{y-1}{3}, \frac{z+11}{0} =$

ii) $3xy^2 - x - y + z = -4$ at $(1,-1,2)$

$3xyz - x - y + z + 4 = 0$ at $(1,-1,2)$

$f_x = 3yz - 1 - 0 + 0 + 0$

$= 3yz - 1$

$f_y = 3xz - 0 - 1 + 0 + 0$

$= 3xz - 1$

$f_z = 3xy - 0 - 0 + 1 + 0$

$= 3xy + 1$

$(x_0, y_0, z_0) = (1, -1, 2)$ ∴ $x_0 = 1, y_0 = -1, z_0 = 2$

$f_x(x_0, y_0, z_0) = 3(-1)(2) - 1 = -7$

$f_y(x_0, y_0, z_0) = 3(+1)(2) - 1 = 5$

$f_z(x_0, y_0, z_0) = 3(1)(-1) + 1 = -2$

eqn of tangent

$-7(x-1) + 5(y+1) - 2(z-2) = 0$

$-7x + 7 + 5y + 5 - 2z + 4 = 0$

$-7x + 5y - 2z + 16 = 0$ → Required eqn of tangent.

Eqn of normal at $(-7,5,-2)$.

$= \frac{x - x_0}{f_x} = \frac{y - y_0}{f_y} = \frac{z - z_0}{f_z}$

$= \frac{x-1}{-7} = \frac{y+1}{5} = \frac{z-2}{-2}$.

Q.5) Find the local maxima & minima for the following function.

i) $f(x,y) = 3x^2 + y^2 - 3xy + 6x - 4y$

$f_x = 6x + 0 - 3y + 6 - 0$
$= 6x - 3y + 6$

$f_y = 0 + 2y - 3x + 0 - 4$
$= 2y - 3x - 4$

$f_x = 0$

$6x - 3y + 6 = 0$
$3(2x - y + 2) = 0$
$2x - y + 2 = 0$
$2x - y = -2$ —— ①

$f_y = 0$

$2y - 3x - 4 = 0$
$2y - 3x = 4$ —— ②

Multiply eqⁿ 1 with 2.

$4x - 2y = -4$
$2y - 3x = 4$
$x = 0$

Substitute value of x in eq ①

$2(0) - y = -2$
$-y = -2$

$\therefore y = 2$

$\therefore$ Critical points are $(0,2)$.

$r = f_{xx} = 6$
$t = f_{yy} = 2$
$s = f_{xy} = -3$

Here $r > 0$

$= rt - s^2$
$= 6(2) - (-3)^2$
$= 12 - 9$
$= 3 > 0$

$\therefore f$ has minimum at $(0,2)$

$3x^2 + y^2 - 3xy + 6x - 4y$ at $(0,2)$

$3(0)^2 + (2)^2 - 3(0)(2) + 6(0) - 4(2)$

$0 + 4 - 0 + 0 - 8$
$= -4$

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ii) $f(x,y) = 2x^4 + 3x^2y - y^2$

$f_x = 8x^3 + 6xy$

$f_y = 3x^2 - 2y$

$f_x = 0$

$\therefore 8x^3 + 6xy = 0$
$2x(4x^2 + 3y) = 0$
$4x^2 + 3y = 0$ —— ①

$f_y = 0$

$3x^2 - 2y = 0$ —— ②

Multiply eq ① with ③
② with ④

$12x^2 + 9y = 0$
$- \;12x^2 + 8y = 0$

$\therefore 17y = 0$
$y = 0$

Substitute value of y in eq^n ①

$4x^2+3(0)=0$

$4x^2=0$

$x=0$

Critical point is $(0,0)$

$r=f_{xx}=24x^2+6x$

$t=f_{yy}=0-2=-2$

$s=f_{xy}=6x-0=6x=6(0)=0$

r at $(0,0)$

$24(0)+6(0)=0$

$\therefore r=0$

$rt-s^2=(0)(-2)-(5)^2$

$=0-0=0$

$r=0$ & $rt-s^2=0$

(nothing to say)

$f(x,y)$ at $(0,0)$

$2(0)^4+3(0)^2(0)-(0)$

$=0+0-0$

$=\underline{\underline{0}}$

iii $f(x,y)=x^2-y^2+2x+8y-70$

$f(x)=2x+2$

$f_y=-2y+8$

$f_x=0$

$\therefore 2x+2=0$

$x=\frac{-2}{2}$ $\therefore x=-1$

$f_y=0$

$-2y+8=0$

$y=\frac{-8}{-2}$

$\therefore y=4$

$\therefore$ critical point is $(-1,4)$.

$r=f_{xx}=2$

$t=f_{yy}=-2$

$s=f_{xy}=0$

$r>0$

$rt-s^2=2(-2)-(0)^2$

$=-4-0$

$=-4<0$

$f(x,y)$ at $(-1,4)$

$(-1)^2-(4)^2+2(-1)+8(4)-70$

$1+16-2+32-70$

$=17+30-70$

$=37-70$

$=\underline{\underline{33}}$.

06/02/2020