ECON641 – Problem Set 1

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1 Warmup: factor intensity reversals

First, I outline the small open economy environment of the 2×2 HO model (for my own purposes).

- Two goods, 1 and 2.
- Two factors, L and K; with endogenous factor prices w and r, respectively.
- Production technology is the same in both industries, but they may differ in their relative factor intensities.
- Exogenously given goods prices, p_1 and p_2 (i.e. the demand side of the economy is pinned down).

Roughly speaking, 'no factor intensity reversals' (NFIR) means the following: for any vector of factor prices (w, r), the ordering of relative factor intensities in both industries is always the same. For example, in equilibrium the production of good 1 may be more capital intensive than production of good 2; NFIR implies that at any other vector of factor prices, the production of good 1 must always be more capital intensive compared to good 2. We can show that production technology exhibits NFIR if, given p_1 and p_2, equilibrium factor prices are uniquely pinned down.

1.1 Cobb Douglas

Cobb Douglas production clearly satisfies NFIR. To see this, suppose that $F_1(K_1, L_1) = AK_1^{\alpha}L_1^{1-\alpha}$ and $F_2(K_2, L_2) = AK_2^{\beta}L_2^{1-\beta}$. The first order conditions for the profit maximization problem for industry 1 are standard:

$$p_1 \alpha A K_1^{\alpha-1} L_1^{1-\alpha} = r, \tag{1}$$
$$p_1(1-\alpha) A K_1^{\alpha} L_1^{-\alpha} = w. \tag{2}$$

Dividing (2) by (1) gives

$$\frac{1-\alpha}{\alpha} k_1 = \frac{w}{r} \implies k_1 = \frac{\alpha}{1-\alpha}\frac{w}{r}, \text{ where } k_1 = K_1/L_1 \tag{3}$$

Now, the zero profit condition in industry 1 is

$$rK_1 + wL_1 = p_1 A K_1^{\alpha} L_1^{1-\alpha}$$
$$\implies rk_1 + w = p_1 A k_1^{\alpha} \tag{4}$$

Plugging (3) into (4) and rearranging gives

$$p_1 = C_{\alpha} r^{\alpha} w^{1-\alpha} \tag{5}$$

where $C_{\alpha} = \frac{1}{A(1-\alpha)}\left(\frac{1-\alpha}{\alpha}\right)^{\alpha}$. An analogous derivation for industry 2 gives

$$p_2 = C_{\beta} r^{\beta} w^{1-\beta} \tag{6}$$

where $C_\beta = \frac{1}{A(1-\beta)}\left(\frac{1-\beta}{\beta}\right)^\beta$. Clearly, given p_1 and p_2, there is a unique solution to (5) and (6), (w^*, r^*), (unless $\alpha = \beta$).

Another (perhaps more intuitive) way to establish NFIR would be to use equation (3) and the equivalent expression for industry 2. These expressions imply that in equilibrium:

$$\frac{k_1}{k_2} = \frac{\alpha(1-\beta)}{\beta(1-\alpha)}.$$

That is, the relative factor intensities between the two industries is independent of factor prices.

1.2 CES

CES production *does not* exhibit NFIR. To see this, suppose $F_i(K_i, L_i) = \left[K_i^{\frac{\sigma_i-1}{\sigma_i}} + L_i^{\frac{\sigma_i-1}{\sigma_i}}\right]^{\frac{\sigma_i}{\sigma_i-1}}$ for $i = 1, 2$. The FOCs for industry i are

$$p_i\left[K_i^{\frac{\sigma_i-1}{\sigma_i}} + L_i^{\frac{\sigma_i-1}{\sigma_i}}\right]^{\frac{1}{\sigma_i-1}} K_i^{-1/\sigma_i} = r \tag{7}$$

$$p_i\left[K_i^{\frac{\sigma_i-1}{\sigma_i}} + L_i^{\frac{\sigma_i-1}{\sigma_i}}\right]^{\frac{1}{\sigma_i-1}} L_i^{-1/\sigma_i} = w \tag{8}$$

Combining these expressions gives

$$k_i^{-1/\sigma_i} = \frac{r}{w}$$
$$\Longrightarrow k_i = \left(\frac{r}{w}\right)^{-\sigma_i}.$$

Thus, in equilibrium, the relative factor intensities between the two industries is

$$\frac{k_1}{k_2} = \left(\frac{r}{w}\right)^{\sigma_2-\sigma_1},$$

which clearly depends on factor prices (unless $\sigma_1 = \sigma_2$).

1.3 Leontief

Clearly the Leontief production function exhibits NFIR. Suppose both industries have the same production function $F(K, L) = \min\{K, L\}$. Then in equilibrium, both industries must have $k_i = 1$. Then, relative factor intensities do not depend on factor prices. More generally, suppose $F_i(K_i, L_i) = \min\{\alpha_i K, \beta_i L\}$. Then in equilibrium, each industry's capital-labor ratio will be $k_i = \beta_i/\alpha_i$. Again, relative factor intensities are independent of factor prices.

2 $2 \times 2 \times 2$ HO Model