ECON675 – Assignment 3

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1 Non-linear least squares

1.1 Identifiability

This is a standard M-estimation problem. The parameter vector $\boldsymbol{\beta}_0$ is assumed to solve the population problem

$$\boldsymbol{\beta}_0 = \arg\min_{\beta \in \mathbb{R}^d} \mathbb{E}[(y_i - \mu(\boldsymbol{x}_i'\boldsymbol{\beta}))^2].$$

For $\boldsymbol{\beta}_0$ to be identified, it must be the *unique* solution to the above population problem (i.e. the unique minimizer). In math, this means for all $\epsilon > 0$ and for some $\delta > 0$:

$$\sup_{||\beta - \beta_0|| > \epsilon} M(\boldsymbol{\beta}) \geq M(\boldsymbol{\beta}_0) + \delta$$

where $M(\boldsymbol{\beta}) = \mathbb{E}[(y_i - \mu(\boldsymbol{x}_i'\boldsymbol{\beta}))^2]$. Of course $\boldsymbol{\beta}_0$ can be written in closed form if $\mu(\cdot)$ is linear. In this case, we know that

$$\boldsymbol{\beta}_0 = \mathbb{E}[\boldsymbol{x}_i\boldsymbol{x}_i']^{-1}\mathbb{E}[\boldsymbol{x}_i y_i].$$

1.2 Asymptotic normality