ECON675 – Assignment 4

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1 Estimating equations

1.1 Moment conditions

The goal of this question is to show that the four given functions are valid moment conditions for the parameter $\theta_t(g)$. That is, we want to show that

$$\mathbb{E}[\psi_{\texttt{f},t}(\boldsymbol{Z}_i;\theta_t(g))] = 0,$$

for each $\texttt{f} \in \{\texttt{IPW}, \texttt{RI1}, \texttt{RI2}, \texttt{DR}\}$. Note that in the derivations below I invoke LIE a lot without specifically mentioning it.

Start with the inverse probability weighting function

$$\begin{aligned}
\mathbb{E}[\psi_{\texttt{IPW},t}(\boldsymbol{Z}_i;\theta_t(g))] &= \mathbb{E}\left[\frac{D_i(t)\cdot g(Y_i(t))}{p_t(\boldsymbol{X}_i)}\right] - \theta_t(g) \\
&= \mathbb{E}\left[\mathbb{E}\left[\frac{D_i(t)\cdot g(Y_i(t))}{p_t(\boldsymbol{X}_i)}|\boldsymbol{X}_i\right]\right] - \theta_t(g) \\
&= \mathbb{E}\left[\frac{1}{p_t(\boldsymbol{X}_i)}\mathbb{E}\left[D_i(t)|\boldsymbol{X}_i\right]\mathbb{E}\left[g(Y_i(t))|\boldsymbol{X}_i\right]\right] - \theta_t(g)
\end{aligned}$$

Now,

$$\mathbb{E}\left[D_i(t)|\boldsymbol{X}_i\right] = \Pr[D_i(t)=1|\boldsymbol{X}_i] = \Pr[T_i = t|\boldsymbol{X}_i] = p_t(\boldsymbol{X}_i).$$

Thus,

$$\begin{aligned}
\mathbb{E}[\psi_{\texttt{IPW},t}(\boldsymbol{Z}_i;\theta_t(g))] &= \mathbb{E}\left[\mathbb{E}\left[g(Y_i(t))|\boldsymbol{X}_i\right]\right] - \theta_t(g) \\
&= \mathbb{E}[g(Y_i(t))] - \theta_t(g) \\
&= 0.
\end{aligned}$$

Next, consider

$$\begin{aligned}
\mathbb{E}[\psi_{\texttt{RI1},t}(\boldsymbol{Z}_i;\theta_t(g))] &= \mathbb{E}[e_t(g;\boldsymbol{X}_i)] - \theta_t(g) \\
&= \mathbb{E}[\mathbb{E}[g(Y_i(t)|\boldsymbol{X}_i]] - \theta_t(g) \\
&= \mathbb{E}[g(Y_i(t)] - \theta_t(g) \\
&= 0.
\end{aligned}$$

And,

$$\begin{aligned}
\mathbb{E}[\psi_{\texttt{RI2},t}(\boldsymbol{Z}_i;\theta_t(g))] &= \mathbb{E}\left[\frac{D_i(t)\cdot e_t(g;\boldsymbol{X}_i)}{p_t(\boldsymbol{X}_i)}\right] - \theta_t(g) \\
&= \mathbb{E}\left[\mathbb{E}\left[\frac{D_i(t)\cdot e_t(g;\boldsymbol{X}_i)}{p_t(\boldsymbol{X}_i)}|\boldsymbol{X}_i\right]\right] - \theta_t(g) \\
&= \mathbb{E}\left[\mathbb{E}\left[e_t(g;\boldsymbol{X}_i)|\boldsymbol{X}_i\right]\right] - \theta_t(g) \\
&= \mathbb{E}[e_t(g;\boldsymbol{X}_i)] - \theta_t(g) \\
&= 0.
\end{aligned}$$

Finally, consider the doubly robust function

$$\mathbb{E}[\psi_{\texttt{DR},t}(\boldsymbol{Z}_i;\theta_t(g))] = \mathbb{E}\left[\frac{D_i(t)\cdot g(Y_i(t))}{p_t(\boldsymbol{X}_i)}\right] - \theta_t(g) - \mathbb{E}\left[\frac{e_t(g;\boldsymbol{X}_i)}{p_t(\boldsymbol{X}_i)}(D_i(t) - p_t(\boldsymbol{X}_i))\right].$$

Using the IPW result above, we know that the first two terms cancel each other out, so that

$$\begin{aligned}
\mathbb{E}[\psi_{\texttt{DR},t}(\boldsymbol{Z}_i;\theta_t(g))] &= -\mathbb{E}\left[\frac{e_t(g;\boldsymbol{X}_i)}{p_t(\boldsymbol{X}_i)}(D_i(t) - p_t(\boldsymbol{X}_i))\right] \\
&= -\mathbb{E}\left[\frac{e_t(g;\boldsymbol{X}_i)D_i(t)}{p_t(\boldsymbol{X}_i)}\right] + \mathbb{E}[e_t(g;\boldsymbol{X}_i)] \\
&= -\theta_t(g) + \theta_t(g) \\
&= 0.
\end{aligned}$$

So each of the four functions is a valid moment condition for $\theta_t(g)$.