ECON641 – Problem Set 1

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Contents

1 Warmup: factor intensity reversals

First, I outline the small open economy environment of the 2×2 HO model (for my own purposes).

- Two goods, 1 and 2.
- Two factors, L and K; with endogenous factor prices w and r, respectively.
- Production technology is the same in both industries, but they may differ in their relative factor intensities.
- Exogenously given goods prices, p_1 and p_2 (i.e. the demand side of the economy is pinned down).

Roughly speaking, 'no factor intensity reversals' (NFIR) means the following: for any vector of factor prices (w, r), the ordering of relative factor intensities in both industries is always the same. For example, in equilibrium the production of good 1 may be more capital intensive than production of good 2; NFIR implies that at any other vector of factor prices, the production of good 1 must always be more capital intensive compared to good 2. We can show that production technology exhibits NFIR if, given p_1 and p_2, equilibrium factor prices are uniquely pinned down.

1.1 Cobb Douglas

Cobb Douglas production clearly satisfies NFIR. To see this, suppose that $F_1(K_1, L_1) = AK_1^{\alpha} L_1^{1-\alpha}$ and $F_2(K_2, L_2) = AK_2^{\beta} L_2^{1-\beta}$. The first order conditions for the profit maximization problem for industry 1 are standard:

$$p_1 \alpha A K_1^{\alpha-1} L_1^{1-\alpha} = r, \tag{1}$$

$$p_1(1-\alpha) A K_1^{\alpha} L_1^{-\alpha} = w. \tag{2}$$

Dividing (2) by (1) gives

$$\frac{1-\alpha}{\alpha} k_1 = \frac{w}{r} \implies k_1 = \frac{\alpha}{1-\alpha} \frac{w}{r}, \text{ where } k_1 = K_1/L_1 \tag{3}$$

Now, the zero profit condition in industry 1 is

$$rK_1 + wL_1 = p_1 A K_1^{\alpha} L_1^{1-\alpha}$$

$$\implies rk_1 + w = p_1 A k_1^{\alpha} \tag{4}$$

Plugging (3) into (4) and rearranging gives

$$p_1 = C_{\alpha} r^{\alpha} w^{1-\alpha} \tag{5}$$

where $C_{\alpha} = \frac{1}{A(1-\alpha)} \left(\frac{1-\alpha}{\alpha}\right)^{\alpha}$. An analogous derivation for industry 2 gives

$$p_2 = C_{\beta} r^{\beta} w^{1-\beta} \tag{6}$$

where $C_\beta = \frac{1}{A(1-\beta)}\left(\frac{1-\beta}{\beta}\right)^\beta$. Clearly, given p_1 and p_2, there is a unique solution to (5) and (6), (w^*, r^*), (unless $\alpha = \beta$).

Another (perhaps more intuitive) way to establish NFIR would be to use equation (3) and the equivalent expression for industry 2. These expressions imply that in equilibrium:

$$\frac{k_1}{k_2} = \frac{\alpha(1-\beta)}{\beta(1-\alpha)}.$$

That is, the relative factor intensities between the two industries is independent of factor prices.

1.2 CES

CES production *does not* exhibit NFIR. To see this, suppose $F_i(K_i, L_i) = \left[K_i^{\frac{\sigma_i-1}{\sigma_i}} + L_i^{\frac{\sigma_i-1}{\sigma_i}}\right]^{\frac{\sigma_i}{\sigma_i-1}}$ for $i = 1, 2$. The FOCs for industry i are

$$p_i\left[K_i^{\frac{\sigma_i-1}{\sigma_i}} + L_i^{\frac{\sigma_i-1}{\sigma_i}}\right]^{\frac{1}{\sigma_i-1}} K_i^{-1/\sigma_i} = r \tag{7}$$

$$p_i\left[K_i^{\frac{\sigma_i-1}{\sigma_i}} + L_i^{\frac{\sigma_i-1}{\sigma_i}}\right]^{\frac{1}{\sigma_i-1}} L_i^{-1/\sigma_i} = w \tag{8}$$

Combining these expressions gives

$$\begin{aligned} k_i^{-1/\sigma_i} &= \frac{r}{w} \\ \implies k_i &= \left(\frac{r}{w}\right)^{-\sigma_i}. \end{aligned}$$

Thus, in equilibrium, the relative factor intensities between the two industries is

$$\frac{k_1}{k_2} = \left(\frac{r}{w}\right)^{\sigma_2-\sigma_1},$$

which clearly depends on factor prices (unless $\sigma_1 = \sigma_2$).

1.3 Leontief

Clearly the Leontief production function exhibits NFIR. Suppose both industries have the same production function $F(K, L) = \min\{K, L\}$. Then in equilibrium, both industries must have $k_i = 1$. Then, relative factor intensities do not depend on factor prices. More generally, suppose $F_i(K_i, L_i) = \min\{\alpha_i K, \beta_i L\}$. Then in equilibrium, each industry's capital-labor ratio will be $k_i = \beta_i/\alpha_i$. Again, relative factor intensities are independent of factor prices.

2 $2 \times 2 \times 2$ HO Model

3 Technology growth in a parameterized version of DFS

3.1

I follow the derivation in EK (2005). We are given the distribution of efficiencies for producing goods j at Home and Foreign:

$$F_i(z) = \Pr[Z_i(j) \leq z] = \exp(-T_i z^{-\theta})$$

Now, we want to derive the DFS-type $A(j)$ curve, which is defined as the ratio of H's efficiency of producing j to F's corresponding efficiency.

In the EK setup the efficiencies are realizations of a random variable. Accordingly, we think of j as the *probability* that the H's relative efficiency of producing j is less than some number:

$$\begin{aligned} j &= \Pr\left[\frac{Z}{Z^*} \leq A\right] \\ &= \Pr\left[Z \leq AZ^*\right] \\ &= \int_0^\infty \exp(-T(Az_*)^{-\theta}) f(z_*) dz_*. \end{aligned}$$

Now,

$$f(z_*) = \frac{d}{dz}\exp(-T^* z^{-\theta}) = \theta T^* z^{-\theta-1}\exp(-T^* z^{-\theta})$$

Substituting into the above integral gives

$$\begin{aligned} j &= T^* \int_0^\infty \exp(-T(Az_*)^{-\theta}) \times \theta z_*^{-\theta-1}\exp(-T^* z_*^{-\theta}) dz_* \\ &= T^* \int_0^\infty \exp(-(TA^{-\theta} + T^*)z_*^{-\theta}) \times \theta z_*^{-\theta-1} dz_* \\ &= \frac{T^*}{(TA^{-\theta} + T^*)} \int_0^\infty \exp(-(TA^{-\theta} + T^*)z_*^{-\theta}) \times -\theta z_*^{-\theta-1}(TA^{-\theta} + T^*) dz_* \\ &= \frac{T^*}{(TA^{-\theta} + T^*)} \int_0^\infty \exp(-(TA^{-\theta} + T^*)z_*^{-\theta}) \times \theta z_*^{-\theta-1}(TA^{-\theta} + T^*) dz_* \\ &= \frac{T^*}{(TA^{-\theta} + T^*)}, \end{aligned}$$

since $\int_0^\infty \exp(-(TA^{-\theta} + T^*)z_*^{-\theta}) \times \theta z_*^{-\theta-1}(TA^{-\theta} + T^*) dz_* = 1$ (because it is the integral of the Frechet pdf with scale $(T^* A^{-\theta} + T)$). Thus, rearranging to get an expression for $A(j)$ gives

$$A(j) = \left[\left(\frac{1-j}{j}\right)\frac{T^*}{T}\right]^{-1/\theta}. \tag{9}$$

3.2

First note that there are no trade costs so that $d_{ni} = 1$ for all $n, i \in \{F, H\}$.

Now, we know that within a country, goods will be purchased from the lowest cost source. Since Home has a comparative advantage at lower values of j, we know that Home will produce the range of goods $[0, \bar{j}]$ where

$$\frac{w}{z(\bar{j})} = \frac{w^*}{z^*(\bar{j})}.$$

The LHS of the above expression is the unit cost of producing $\bar{j}$ at home, and the RHS is the cost of buying the good from Foreign. Rearranging the above expression gives

$$\frac{z(\bar{j})}{z^*(\bar{j})} = \frac{w}{w^*}$$
$$\implies A(\bar{j}) = \omega, \tag{10}$$

where $\omega = w/w^*$. Similarly, Foreign will produce a range of goods $[\underline{j}, 1]$ domestically, such that

$$\frac{w^*}{z^*(\underline{j})} = \frac{w}{z(\underline{j})}$$
$$\implies A(\underline{j}) = \omega. \tag{11}$$

Thus, there is a unique cuttoff good.

Next, we need to invoke market clearing. Here, we note that preferences are Cobb Douglas, with equal weights across each good. Thus, each country spends a constant share of its income on each good. We know that Home produces $[0, \bar{j}]$ goods domestically, and exports $[0, \underline{j}]$ goods to Foreign. Thus, market clearing at Home requires

$$wL = \bar{j}wL + \underline{j}w^*L^* \tag{12}$$
$$\tag{13}$$

Substituting (10) and (11) into (12) and rearranging gives

$$L = LA^{-1}(\omega) + \frac{1}{\omega}L^*A^{-1}(\omega)$$
$$\implies \frac{1}{A^{-1}(\omega)} = 1 + \frac{1}{\omega}\frac{L^*}{L}$$

And, from the above derivation we know: $A^{-1}(\omega) = \frac{T^*}{(T\omega^{-\theta} + T^*)}$. Substituting this into the above expression gives

$$\frac{T\omega^{-\theta} + T^*}{T^*} = 1 + \frac{1}{\omega}\frac{L^*}{L}$$
$$\implies \frac{T}{T^*}\omega^{-\theta} = \frac{1}{\omega}\frac{L^*}{L}$$
$$\implies \omega = \left[\frac{T^*}{T}\frac{L^*}{L}\right]^{1/(1-\theta)}.$$

And the equilibrium cutoff good is

$$\bar{j} = A^{-1}(\omega)$$

3.3

With no trade costs goods prices are identical in H and F. Thus, ω measures H's welfare relative to F's. From the above expression for equilibrium ω, if $\theta > 1$, then an increase in T^* reduces relative welfare in H. The comparative static is

$$\frac{d\omega}{dT^*} = \left[\frac{1}{T}\frac{L^*}{L}\right]^{1/(1-\theta)} \frac{1}{1-\theta}(T^*)^{\theta/(1-\theta)}.$$

So, if $\theta > 1$, then the effect of an increase in T^* is decreasing in L^*/L, which is observable in the data.

4 Key implications of EK's Ricardian model

4.1

The unit cost of sending good j from i to n is given by

$$C_{ni}(j) = \frac{c_i}{Z_i(j)} d_{ni} \tag{14}$$

4.2

We want to compute the probability that i will sell good j to n. Since the derivation below is the same for all goods, I suppress the index j.

(a) From (14) note that

$$Z_i = \frac{c_i d_{ni}}{C_{ni}}$$

Now,

$$\Pr[Z_i \leq p] = \Pr[c_i d_{ni}/C_{ni} \leq p]$$

Thus,

$$\begin{aligned}
\Pr[Z_i \leq c_i d_{ni}/p] &= \Pr[c_i d_{ni}/C_{ni} \leq c_i d_{ni}/p] \\
&= \Pr[p \leq C_{ni}] \\
&= \Pr[C_{ni} \geq p] \\
&= 1 - \Pr[C_{ni} \leq p] \\
\therefore \Pr[C_{ni} \leq p] &= 1 - \Pr[Z_i \leq c_i d_{ni}/p] \\
&= 1 - F_i(c_i d_{ni}/p) \\
&= 1 - \exp(-T_i(c_i d_{ni})^{-\theta} p^{\theta}),
\end{aligned}$$

which is the probability distribution of C_{ni}. Denote this distribution as G_{ni} so that

$$G_{ni}(p) = 1 - \exp(-T_i(c_i d_{ni})^{-\theta} p^{\theta}) \tag{15}$$

(b) Next we want to compute the probability that i is the cheapest supplier for n. Denote this probability as π_{ni}. We have

$$\begin{aligned}
\pi_{ni} \equiv \Pr[C_{ni} &= \min\{C_{ns}; s \neq i\}] \\
&= \Pr[C_{ns} \geq C_{ni} \text{ for all } s \neq i] \\
&= \int_0^{\infty} \prod_{s \neq i} [1 - G_{ns}(C_{ni})] dG_{ni}(C_{ni}) \\
&= \int_0^{\infty} \prod_{s \neq i} [1 - G_{ns}(p)] dG_{ni}(p), \qquad (16)
\end{aligned}$$

where I just re-denote the integration dummy as p to ease notation.

(c) Now,

$$dG_{ni}(p) = \frac{d}{dp}G_{ni}(p) = \exp(-T_i(c_i d_{ni})^{-\theta}p^{\theta})T_i(c_i d_{ni})^{-\theta}\theta p^{\theta-1} \tag{17}$$

Substituting (15) and (17) into (16) gives

$$\begin{aligned}
\pi_{ni} &= \int_0^{\infty}\left[\prod_{s\neq i}\exp(-T_s(c_s d_{ns})^{-\theta}p^{\theta})\right]\exp(-T_i(c_i d_{ni})^{-\theta}p^{\theta})T_i(c_i d_{ni})^{-\theta}\theta p^{\theta-1}dp \\
&= \int_0^{\infty}\exp(-\sum_{s\neq i}T_s(c_s d_{ns})^{-\theta}p^{\theta})\exp(-T_i(c_i d_{ni})^{-\theta}p^{\theta})T_i(c_i d_{ni})^{-\theta}\theta p^{\theta-1}dp \\
&= \int_0^{\infty}\exp(-\sum_{i=1}^{N}T_i(c_{ni} d_{ni})^{-\theta}p^{\theta})T_i(c_i d_{ni})^{-\theta}\theta p^{\theta-1}dp \\
&= \int_0^{\infty}\exp(-\Phi_n p^{\theta})T_i(c_i d_{ni})\theta p^{\theta-1}dp \\
&= T_i(c_i d_{ni})^{-\theta}\int_0^{\infty}\exp(-\Phi_n p^{\theta})\theta p^{\theta-1}dp \\
&= \frac{T_i(c_i d_{ni})^{-\theta}}{\Phi_n}\int_0^{\infty}\exp(-\Phi_n p^{\theta})\Phi_n\theta p^{\theta-1}dp \\
&= \frac{T_i(c_i d_{ni})^{-\theta}}{\Phi_n},
\end{aligned}$$

since the integral in the second last line evaluates to 1, because it is a pdf of a probability distribution of the form (15).

4.3

Next we want to compute the probability distribution of goods prices actually bought in market n. Call this price P_n and recall that n buys the lowest cost good

$$P_n = \min_{i=1,..,N}\{C_{ni}\}$$

Thus

$$\begin{aligned}
\Pr[P_n \leq p] &= \Pr[\min_{i=1,..,N}\{C_{ni}\} \leq p] \\
&= 1 - \Pr[C_{ni} > p \text{ for all } i] \\
&= 1 - \prod_{i=1}^{N}[1 - G_{ni}(p)] \\
&= 1 - \prod_{i=1}^{N}\exp(-T_i(c_i d_{ni})^{-\theta}p^{\theta}) \text{ using (15)}, \\
&= 1 - \exp(-\Phi_n p^{\theta}).
\end{aligned}$$

Denote this distribution as $G_n(p)$, so that

$$G_n(p) \equiv \Pr[P_n \le p] = 1 - \exp(-\Phi_n p^\theta) \tag{18}$$

4.4

Next, we want to compute the probability distribution of goods prices that n *actually buys* from country i. That is, we want to compute the conditional probability distribution:

$$\Pr[P_n \le p | P_n = C_{ni}]$$

Now,

$$\begin{aligned} Pr[P_n \le p | P_n = C_{ni}] &= \Pr[P_n \le p | C_{ni} = \min\{C_{ns}; s \neq i\}] \\ &= \Pr[C_{ni} \le p | C_{ni} = \min\{C_{ns}; s \neq i\}] \\ &= \frac{1}{\pi_{ni}} \int_0^p \prod_{s \neq i} [1 - G_{ns}(q)] dG_{ni}(q) \\ &= \frac{1}{\pi_{ni}} \pi_{ni} \int_0^p \exp(-\Phi_n q^\theta) \Phi_n \theta q^{\theta - 1} dq, \end{aligned}$$

using our previous derivations. Thus,

$$\begin{aligned} \Pr[P_n \le p | P_n = C_{ni}] &= \int_0^p \exp(-\Phi_n q^\theta) \Phi_n \theta q^{\theta - 1} dq \\ &= \int_0^p dG_n(q) \\ &= G_n(p). \end{aligned}$$

So, for goods that are purchased in n, conditioning on the source does not affect the distribution of the good's price. This result seems at odds with reality, as we discussed in class. One would think that German cars bought in the US would have a different price distribution compared to Japanese cars bought in the US.

4.5

I'm not exactly sure how to do this problem, but here's my best shot.

(a) Given CES preferences over the unit mass of goods, n's demand for good j is given by

$$\begin{aligned} X_n(j) &= \left(\frac{P_n(j)}{P_n} \right)^{1-\sigma} X_n \\ &= \left(\frac{\min_{i=1,..,N}\{C_{ni}(j)\}}{P_n} \right)^{1-\sigma} X_n \end{aligned}$$

(b) Then, n's expected expenditure on good j, sourced from i is

$$\begin{aligned} X_{ni}(j) &= \mathbb{E}[X_n(j)|i*(j)=i]\Pr[i^*(j)=i] \\ &= \mathbb{E}\left[\left(\frac{\min_{i=1,..,N}\{C_{ni}(j)\}}{P_n}\right)^{1-\sigma} X_n|i^*(j)=i\right]\pi_{ni} \\ &= \mathbb{E}\left[\left(\frac{C_{ni}(j)}{P_n}\right)^{1-\sigma} X_n|i^*(j)=i\right]\pi_{ni} \\ &= \mathbb{E}\left[\left(\frac{P_n}{P_n}\right)^{1-\sigma} X_n|i^*(j)=i\right]\pi_{ni} \\ &= \pi_{ni}X_n. \end{aligned}$$

4.6

We've shown that π_{ni} is the probability that country n purchases good any good j from i. Since there is a unit measure of goods, it follows that π_{ni} is the total fraction of the $j \in [0,1]$ goods that are sourced from from i in country n. Then, we can split the the total unit measure $j \in [0,1]$ of goods purchased in n into the share supplied by each source country. Recall that conditioning on the source of a good does not affect the distribution of the good's price in n. Accordingly, the fraction of n's total expenditure that goes to country i is the same as the fraction of goods that n purchases from country i; namely, π_{ni}.

5 Quantitative analysis in the EK model

5.1 A peek at the data

See attached `R` code for the aggregation of the WIO data.

Table 1 (overleaf) shows the ratio of imports of intermediate goods to total imports for each country in the WIOD. For each country in the sample, imports of intermediate goods accounts for more than half of total imports; the mean intermediate import share is 0.64. These results imply that it is extremely important to incorporate and intermediate goods sector when modelling aggregate trade flows.

Table 2 presents the ratio of each country's trade deficit (imports minus exports) to total expenditure. For most countries, the trade deficit is tiny compared to total expenditure; the mean trade deficit to expenditure ratio is 0.002. These results suggest that the usual balanced trade assumption is fairly realistic.

Table 1: **Intermediate Goods Share of Total Imports by Country**

	Country	Intermediate trade share
1	AUS	0.55
2	AUT	0.60
3	BEL	0.67
4	BGR	0.67
5	BRA	0.68
6	CAN	0.60
7	CHN	0.75
8	CYP	0.46
9	CZE	0.69
10	DEU	0.57
11	DNK	0.61
12	ESP	0.62
13	EST	0.63
14	FIN	0.71
15	FRA	0.61
16	GBR	0.56
17	GRC	0.59
18	HUN	0.76
19	IDN	0.78
20	IND	0.81
21	IRL	0.76
22	ITA	0.63
23	JPN	0.61
24	KOR	0.76
25	LTU	0.58
26	LUX	0.78
27	LVA	0.48
28	MEX	0.70
29	MLT	0.64
30	NLD	0.68
31	POL	0.64
32	PRT	0.56
33	ROM	0.65
34	RUS	0.53
35	SVK	0.67
36	SVN	0.58
37	SWE	0.67
38	TUR	0.64
39	TWN	0.66
40	USA	0.53
41	RoW	0.70

Table 2: **Ratio of Trade Deficit to Total Expenditure by Country**

	Country	Total Imports ($ b)	Total Exports ($ b)	Total Expenditure ($ b)	Deficit Ratio*	Trade deficit ($ b)
1	AUS	82183	89119	773044	-0.009	-6936
2	AUT	74129	76573	326787	-0.007	-2444
3	BEL	145488	159808	476141	-0.030	-14320
4	BGR	5687	5496	25011	0.008	191
5	BRA	70230	63215	1102372	0.006	7015
6	CAN	262124	318038	1274587	-0.044	-55914
7	CHN	233466	278005	3079150	-0.014	-44539
8	CYP	3720	1846	15698	0.119	1874
9	CZE	34011	31977	140246	0.015	2034
10	DEU	546757	612622	3331636	-0.020	-65865
11	DNK	54834	66476	256046	-0.045	-11642
12	ESP	174061	139171	1136802	0.031	34890
13	EST	3395	3174	12286	0.018	221
14	FIN	37210	50286	221058	-0.059	-13076
15	FRA	327210	348182	2392238	-0.009	-20972
16	GBR	371112	376887	2679956	-0.002	-5775
17	GRC	39250	16839	223624	0.100	22411
18	HUN	31956	28746	106129	0.030	3210
19	IDN	46340	64460	322243	-0.056	-18120
20	IND	65785	66604	900499	-0.001	-819
21	IRL	71357	87495	188948	-0.085	-16138
22	ITA	263034	269745	2128874	-0.003	-6711
23	JPN	392472	511342	8569322	-0.014	-118870
24	KOR	171804	197894	1127198	-0.023	-26090
25	LTU	4512	3723	19312	0.041	789
26	LUX	22195	25249	44725	-0.068	-3054
27	LVA	2702	2540	14573	0.011	162
28	MEX	172069	169818	1088325	0.002	2251
29	MLT	3216	2494	7928	0.091	722
30	NLD	181619	212380	710528	-0.043	-30761
31	POL	53523	44796	341961	0.026	8727
32	PRT	42004	28517	225600	0.060	13487
33	ROM	12402	10830	73977	0.021	1572
34	RUS	48576	97684	396030	-0.124	-49108
35	SVK	12680	12514	48242	0.003	166
36	SVN	9557	8933	39114	0.016	624
37	SWE	88073	108985	443365	-0.047	-20912
38	TUR	57821	41445	518382	0.032	16376
39	TWN	154033	170777	650183	-0.026	-16744
40	USA	1314500	981035	18636780	0.018	333465
41	RoW	1178338	1079715	6694889	0.015	98623

*Note**: the deficit ratio is trade deficit divided by total expenditure.

6 Gravity

6.1

The gravity specification from AvW's (2003) Armington model (with symmetric trade costs) is given by

$$X_{ni} = \frac{Y_n Y_i}{Y_w} \left(\frac{d_{ni}}{P_n P_i} \right)^{1-\sigma} \tag{19}$$

Taking logs

$$\log X_{ni} = -\log Y_w + \log Y_n + \log Y_i + (1-\sigma) \log d_{ni} - (1-\sigma) P_n - (1-\sigma) P_i$$

Given the parametric form for trade costs, the estimating equation becomes

$$\begin{aligned}\log \left(\frac{X_{ni}}{Y_n Y_i} \right) =& \alpha_0 + \alpha_1 (1-\sigma) \text{log distance}_{ni} + \alpha_2 (1-\sigma) \text{no contiguity}_{ni} \\ & + \alpha_3 (1-\sigma) \text{no common language}_{ni} + \alpha_4 (1-\sigma) \text{no colonial ties}_{ni} \\ & - (1-\sigma) m_n - (1-\sigma) m_i + \nu_{ni}, \end{aligned} \tag{20}$$

where m_n and m_i are importer and exporter fixed effects, respectively. Note that AvW impose unit income elasticities by using $\log \left(\frac{X_{ni}}{Y_n Y_i} \right)$ as the dependent variable. Santos-Silva and Tenreyro (SST) (2006) note that this is probably not the best approach (see fn 29), but I'll stick with it anyway. For simplicity, I estimate (20) only for the subsample of observations where $X_{ni} > 0$. HM say this is not a good idea.

6.2

(a) Clearly a zero trade flow for a country-pair observation poses a problem for the estimation of (20) because the dependent variable would be equal to $-\infty$. As SST (2006) note, most studies simply drop the zero observations when estimating gravity equations of the form (20), but Head and Meyer (HM) show that this results in selection bias (because zeros account for a large share of total country-pair observations). Another approach is to estimate the model using $X_{ni} + 1$ instead of X_{ni}, but SST and HM note that this procedure is bad: it yields inconsistent estimators of the gravity parameters and the results depend on the units of measurement of X_{ni}. Another approach is to assume that the observed X_{ni} is truncated at its lowest observed value and estimate (20) using Tobit regression. SST's proposed PPML method allows for zeros.

(b) As SST (2006) mention, for parameters in (20) to be unbiased, ν_{ni} must be statistically independent of the regressors.

(c) The estimated coefficient on log distance is $\alpha_1 \widehat{(1-\sigma)}$. That is, we cannot separately identify α_1 and $(1-\sigma)$.

(d)

(e)