Notes on MNS (2016)

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1 Overview and results

Main results:

1. In standard NKM there is an outsized response of output/consumption to forward guidance.
2. In a NKM with idiosyncratic labor income shocks and borrowing constraints, the response of output/consumption to forward guidance is far lower.
3. Something about ZLB [fill in]

1.1 Intuition [for my own understanding]

2 Result #1

I've replicated this in AIM. Some notes on the AIM code

- Note that the variable r_t in the code is the deviation of the real rate from the natural rate: $\tilde{r}_t = i_t - \mathbb{E}_t \pi_{t+1} - r_t^n$. With this note, you can easily map my code back to the exposition in the paper.

3 Result # 2

This is the main result of the paper, and it requires solving MNS's heterogeneous agent model. This is a hard problem. Based MNS's online appendix, these are the main steps in solving for equilibrium:

1. Solve the households' problems using the endogenous grid point method (Carrol, 2006).
2. Simulate the distribution of the households' asset holdings using Young's (2006) nonstochastic histogram method.
3. Checking the equilibrium
4. Updating the initial guess using results from a 'simpler' economy.

Useful resources:

- https://sites.google.com/a/nyu.edu/glviolante/teaching/quantmacro [which points to some books too]

3.1 Solving the households' problems

I need to use the endogenous grid point method (EGM) (which is a numerical method for implementing policy function iteration). Note that value function iteration is too slow for this problem. Also note that this method requires approximating the policy function for consumption using a 'shape preserving cubic spline'.

3.1.1 EGM

Here's roughly how EGM works. [I need to understand how exactly this relates to 'time iteration'; and the fact that MNS assume that the economy returns to steady state after 250 periods.]

Consider the simplified consumer's problem from MNS (i.e. I've assumed exogenous income rather than a wage and a labor choice)

$$\begin{aligned}
&\max \sum_t \beta^t u(c_t) \\
&\text{s.t. } c_t + \frac{b_{t+1}}{1+r_t} = b_t + z_t \\
&\& \ b_{t+1} \geq 0.
\end{aligned}$$

The Euler equation is (you can check handwritten notes for the details):

$$u'(c_t) \geq \beta(1+r_t) \sum_{z_{t+1}} \Pr(z_{t+1}|z_t) u'(c_{t+1})$$

For simplicity assume that the solution is interior (i.e. $b_{t+1} > 0$) so that the Euler holds with equality (note that I'll need to figure out how to deal with the constraint too!). I want to implement EGM, which is a policy function iteration method (c.f. value function iteration). Let $g_0(b,z)$ denote our initial guess of the optimal (consumption) policy for a given state pair (b,z). (Note that we are trying to solve for the true policy function $g(b,z)$). Then, I can write the Euler as:

$$u'(c) = \beta(1+r) \sum_{z'} \Pr(z'|z) u'(g_0(b', z')).$$

Then, today's consumption is simply

$$c = u'^{-1}\left(\beta(1+r) \sum_{z'} \Pr(z'|z) u'(g_0(b', z'))\right) \tag{1}$$

Now, recall that the HH optimally chooses b_{t+1}, which b_t is predetermined. Thus, (1) gives today's consumption for someone who drew income z today, and optimally chose b'. So, we can write (1) as

$$\tilde{g}_0(b', z) = u'^{-1}\left(\beta(1+r)\sum_{z'} \Pr(z'|z)u'(g_0(b', z'))\right) \tag{2}$$

Next, define a grid over values of *tomorrow's* bond holdings b' and call it $B = \{b_1, ..., b_{n_b}\}$, with $b_1 = 0$. Also suppose we have a grid of possible income draws, $Z = \{z_1, ..., z_{n_z}\}$, and an associated transition matrix Γ. Then we can write (2) as

$$\tilde{g}_0(b_i, z_j) = u'^{-1}\left(\beta(1+r)\sum_{\ell} \Gamma_{j\ell}u'(g_0(b_i, z_\ell))\right), \tag{3}$$

which gives the optimal policy c, for an agent with current income z_j, and who optimally chooses tomorrow's bond holdings b_i. Thus, for each (b_i, z_j) pair we can get the associated optimal consumption today using (3).

Next, we can use the budget constraint to back out *today's* bond holdings b_t:

$$b_t = c_t + \frac{b_{t+1}}{1+r_t} - z_t$$

Using the grid indicies, we can write:

$$b^*_{i,j} = \tilde{g}_0(b_i, z_j) + \frac{b_i}{1+r} - z_j,$$

where $b^*_{i,j}$ defines bond holdings today for a HH who gets an income draw z_j today, and who optimally chooses tomorrow's level of bond holdings, b_j. Then, we can finally define the 'endogenous grid' of today's bond holdings (for each level of income z_j):

$$B^*_j = \{b^*_{1,j}, ..., b^*_{n_b,j}\}.$$

Next, note that for each grid point in B^*_j we have the updated guess of the optimal policy:

$$g_1(b^*_{i,j}, z_j) = \tilde{g}_0(b_i, z_j)$$

Note that the grid points in B^*_j need not (and will not, in general) coincide with the points in the original grid B. Thus, to get the updated guesses $g_1(b_i, z_j)$ for each $b_i \in B$ we need to interpolate $g_1(b^*_{i,j}, z_j)$ (see below).

Now, let's try to deal with the borrowing constraint, $b_{t+1} \geq 0$. Recall that $b^*_{i,j}$ is today's bond holdings for someone with current income z_j and who optimally chooses b_i. Suppose the borrowing constraint binds; then agent optimally chooses $b_{t+1} = b_1 = 0$. Thus,

$$b^*_{1,j} = \tilde{g}_0(b_1, z_j) - z_j.$$

Recall that $\tilde{g}_0(b_1, z_j)$ is our guess of optimal consumption today. Thus, $b^*_{1,j}$ represents the *highest* level of bond holdings *today* for someone with current income z_j, such that the borrowing constraint binds. Then, for all grid points $b_i \leq b^*_{1,j}$, when $z = z_j$, the borrowing constraint must bind (i.e. $b_{t+1} = 0$). Then, we can use the budget constraint to get the updated guess of today's optimal consumption for constrained agents:

$$g_1^{constrained}(b_i, z_j) = b_i + z_j \text{ for all } b_i \leq b^*_{1,j}$$

Putting all this together, the updated guess of the optimal policy function is

$$g_1(b^*_{i,j}, z_j) = \begin{cases} \tilde{g}_0(b_i, z_j) & \text{if } b_i > b^*_{1,j} \\ b_i + z_j & \text{otherwise.} \end{cases}$$

3.1.2 Approximating the policy function

- QuantEcon recommends the Interpolations package in Julia. Not sure if it implements shape preserving cubic splines.
- I think Rudd and MF have notes on approximations using splines.

3.2 Simulating the distribution of asset holdings

3.3 Checking equilibrium

3.4 Updating the guess