Max Integral Operator: A Probabilistic Numerical Approach

Nishad Gothoskar
CSAIL MIT
2275 East Bayshore Road, Suite 160
Palo Alto, California 94303

Abstract

The Max Integral Operator is an expression of the form $\max_a \int f(a,s)ds$. Such expressions arise in many computational domains including machine learning, statistical inference, and optimization. Intuitively, maximizing over some variables and integrating out others is a useful operation to perform. In most practical settings, however, these integrals are non analytic and exact evaluation is computationally intractable. Much research has explored Bayesian quadrature, a method of numerical integration that provides estimates of uncertainty on an integral's value. In this work, we present a framework for efficiently evaluating the Max Integral Operation by incorporating this uncertainty information into the optimization procedure. We jointly optimize over the outer optimization variable and the selection of queries in estimating the inner integrals. As a result, our framework exhibits increased sample efficiency, allowing us to get accurate value estimates with fewer function evaluation queries. We investigate the effectiveness of our framework, the various applications, and the theoretical guarantees.

Introduction

Function maximization and numerical integration have received much focus from the optimization and applied mathematics communities, respectively. However, in performing these operations simultaneously there is added complexity and simply applying existing optimization and integration algorithms is computationally infeasible. We will take a deeper look at evaluation of the Max Integral Operator and,in particular, show how taking a probabilistic numerical perspective...

A reasonable method for evaluating an integral might be sampling-based Monte Carlo techniques, just as we would use to estimate an expectation in a discrete setting. However, in "Monte Carlo is Fundamentally Unsound", O'Hagan identified a series of inconsistencies with such methods. He then presents Bayes-Hermite quadrature, which treats the numerical integration problem as a statistical inference problem. By doing so, you may use the estimate of uncertainty of an integral's value as a convergence criterion and actively sample function evaluations that reduce uncertainty.

Bayesian quadrature involves fitting a Gaussian Process to the integrand and integrating the mean function of the GP. Under certain conditions of the kernel function and prior, this integral will have a closed form. Bayesian Monte Carlo and Gaussian Process Dynamic Programming state these closed forms and explore the applications of quadrature in computing marginal likelihood and RL action policies, respectively.

Other work on Bayesian quadrature has focused on cases where the integrand is non-negative (Osborne 2017) (Gunter 2016) and shown how using certain warping functions can induce cheap active learning schemes. Using these sampling procedures they show fast convergence compared to other numerical integration methods.

In both these works, however, they assume either a discrete optimization space or a relatively small continuous state space. We observe that if both these assumptions are violated, their methods of optimization over the integral are infeasible. In most use cases of the Max Integral Operator, we are interested in continuous high-dimensional spaces and therefore, it seems reasonable that our methods of evaluation should extend to these domains. For these reasons, we believe that methods for evaluating the Max Integral Operator merit further investigation.

The remainder of this paper is structured as follows: (1) We present the framework for optimization and three variants of implementation. Then, we do a thorough analysis of these variants on test functions. (2) We apply the optimization procedure to the Reinforcement Learning setting, specifically to Bellman Updates, and developed two extensions to existing algorithms for solving MDPs. The first is classical value iteration and the next is RTDP. We show how our Max Integral Optimization allows you to efficiently learn robust policies in stochastic continuous state-action domains. (3) We use our framework in computing maximum a posteriori estimates for a series of common Posterior Estimation problems (4) We investigate the theoretical guarantees we can make regarding optimality and convergence in Max Integral settings.

Preliminaries

In this section, we review Gaussian Process Regression, Bayesian Optimization and Quadrature, and Multi-armed Bandits. For further reference we recommend...

Gaussian Process Regression

A Gaussian process is a method of representing a distribution over functions. It can initially be defined by the prior mean function and a covariance function, which captures our smoothness assumptions on the functions we are modeling (see Rasmussen and Williams 2005). These priors can be engineered through existing domain knowledge, but in many cases the prior mean is assumed to be 0.

As we make additional function queries, we will condition on our set of observations using a Bayesian update method. By conditioning on our observation, we can derive the posterior distribution. This GP posterior is not a closed form function e.g. f(x) = sin(x), instead, at each input point the posterior is a Gaussian Distribution as follows:

$$\begin{aligned} & y|\mathbf{y}_* \sim \mathcal{N}(\mu, \sigma) \\ & \mu = m(X_*) + K(X_*, X)K_y^{-1}(y - m(X)) \\ & \sigma = m(X_*) + K(X_*, X)K_y^{-1}(y - m(X)) \end{aligned} \tag{1}$$

The core of a Gaussian process is the multivariate Gaussian. The model's assumption is that observed samples from the function are samples from a multivariate Gaussian. Then, for unobserved inputs we use the mean and covariance functions to infer a distribution. In a way, a Gaussian process is an extension of the multivariate Gaussian to the infinite-dimensional domain.

GPs are powerful tool in modeling unknown functions because the posterior distribution captures our uncertainty about the function's value. In the next subsection, we discuss how this can be leveraged in optimization and estimation procedures.

Bayesian Optimization

Bayesian optimization is a function optimization procedure that relies on priors and sequential updates of a posterior. The posterior will guide the queries made to the function, through an acquisition function. Bayesian Optimization is a black box optimizer beceause it relies on only the function inputs and outputs to create its' representation. Bayesian Optimization does not relate on derivatives, either.

In continuous domains, Gaussian processes are used as the posterior model because they allow us to easily condition on our previous observations and generalize to the infinite domain, according to our specificied prior mean and kernel function.

The key component of Bayesian Optimization is the acquisition function that defines how we select function evaluations. While there are many different acqusition functions used in practice, they all essentially balance what is commonly referred to as "exploration vs exploitation". Exploration is the tendency to query areas in the domain where there is uncertainty about value, while Exploitation is querying areas that have highest value.

The commonly used acquisition function is Upper Confidence Bound, because of its simplicity as well as it theoretical guarantees. The UCB function is as follow:

$$\begin{aligned} & UCB(x) = \mu + \beta\sigma \\ & argmax_x UCB(x) \end{aligned} \tag{2}$$

In UCB, as β tends to 0, the acquisition policy becomes exploitation and as it increases, the acquistion policy increasingly weights uncertainty.

Bayesian Quadrature

Just as Bayesian Optimzation takes a sequential query approach to optimizing a function, Bayesian quadrature takes a similar approach in estimating the integral of a black box function. These methods were first proposed as Bayes-Hermite quadrature (O'Hagan) and then reformulated in the Bayes Monte Carlo (Rasmussen). Most work has focused on integrals of the form:

$$\int f(x)p(x)dx$$

In Bayesian Quadrature, the acquisition function is chosen to prefer queries that can reduce the uncertainty about the value of the integral. This acquisition would be:

$$argminV(f|D) \tag{3}$$

An alternative acquistion function would be Uncertainty sampling

$$argminV(f|D) \tag{4}$$

Under certain conditions, the integral will have a closed form. One such case, is if the integrand f is modeled by a GP with an RBF kernel and the prior p is a Gaussian. For completeness we present the derivation of this closed form below:

$$\begin{aligned} & E_{f|\mathcal{D}}[f_p] = z^T K^{-1} f \\ & z = |A^{-1}B + I|^{-1/2}\, exp[-0.5(a-b)^T(A+B)^{-1}(a-b)] \end{aligned}$$

Multi Armed Bandits

In many real-world scenarios, we seek to maximize some reward or utility through our series of actions. These actions can, in addition to providing reward, give us more information about our problem scenario and inform following decision making. The multi armed bandits problem, originally presented by Robins, is a mathematical formalization of such a scenario. The problem has a number of "arms" which when pulled give certain reward. Over a series of "pulls" the player seeks to maximize the reward gained.

...

Bayesian optimization is a variant of the multi armed bandits problem with an additional assumption on the dependence of arms. Though we have already presented an overview of Bayesian optimization, ...

Problem Formulation and Method

In this paper we present an apporach to efficiently evaluate the Max Integral Operator (MIO) of a function. The MIO applied to a multivariable function f is as follows:

$$\max_a \int f(a,s,v)ds$$

a,s,v may represent sets of variables. s should consist of continuous values because this is the integration variable. The values in a and v may be either continuous or discrete as our method handles both cases. f must be a continuous function along the dimensions in s. In addtion, f must be integrable along the dimensions in s, for fixed inputs of a and v. We will assume these are infinite integrals (from negative infinity to positive infinity). So, more specifically, f must be a convergent integral, meaning the following limit must exist.

$$\lim_{x\to-\infty}\lim_{y\to\infty}\int_x^y f(a,s,v)ds$$

The methods we will present are derivative-free optimization techniques, so the target applications are blackbox functions where the derivatives are inaccessible.

In this paper, we will make the assumption that the function f takes the form:

$$f(a,s,v) = g(a,s,v) \cdot p(s|a,v)$$

for some function g, where $p(x)$ is a probability distribution. While this restricts the class of functions, in many applications, f will already have the above form. This is because g is now a larger class of functions than f. A simple example of this is that $\int 1ds = \infty$ but $\int 1 \cdot p(s)ds$ = 1.

Method

We first present the general optimization framework in Algorithm 1. The structure of the framework is agnostic to continuous or discrete inputs. The main function in Algorithm 1 is $OptMio$ which iteratively selects an action and refines the integral estimate for that action. This interleaves the Bayesian Optimization with the Bayesian Quadrature.

The $Init$ procedure will make the initial estimates for the integral. The procedure for refining the integral is shown in EXTEND. In order to improve the estimate of the integral, we select more inputs to add to our dataset and evaluate function f on these new inputs. Then, we refit the Gaussian Process and integrate using the procedure described in the previous section, $Integrate$.

The other important functions are $SubselectActions$ which will select initial actions to explore and $AddAction$ which will add to this set of actions when necessary.

The core components of the algorithm are $ActionAcquisitionFunction$ and $StateAcquisitionFunction$ which select which actions to evaluate and which states to add to better estimate the integral. In the following subsections, we will explore the variety of implementations of these acquisition functions for different settings

Algorithm 1 Max Integral Optimization Framework

function OPTMIO($\boldsymbol{f}, actionsSpace, n1, n2$)
  $D = []$
  $actions =$ SUBSELECTACTIONS($actionsSpace$)
  $D =$ INIT($\boldsymbol{f}, actions, n1$)
  while not converged **do**
    if extend action set **then**
      $actions =$ ADDACTIONS($actionsSpace$)
    end if
    $a =$ ACTIONACQUISITIONFUNCTION($D, actions$)
    $D[a] =$ EXTEND($D[a], f, s, a, n2$)
  end while
  return $argmax_a D[a], max_a D[a]$
end function

function INIT($f, actions, n$)
  $D = []$
  for a in actions **do**
    $s =$ STATEACQUISTIONFUNCTION($emptyDatum, n$)
    $datum.X = s$
    $datum.Y = f(a,s)$
    $datum.gp.fit()$
    $datum.\mu, datum.\sigma =$ INTEGRATE($datum$)
    $D[a] = datum$
  end for
  return D
end function

function EXTEND(*datum*, f, s, a, n)
  $s =$ STATEACQUISTIONFUNCTION($datum, n$)
  $datum.X.append(s)$
  $datum.Y.append(f(a,s))$
  $datum.gp.fit()$
  $datum.mu, datum.var =$ INTEGRATE($datum$)
end function

Discrete Action Space In the finite discrete action space setting, we can optimize over all possible actions. This implies that our $SubselectActions$ will return all actions. And we will never have to extend our action set with $AddActions$. Now, using the $ActionAcquisitionFunction$ we optimize over all actions to select one that maximizes the acquisition function.

Then ...

Continuous Action Space In the continuous action setting, we have to select certain actions to explore and iteratively add to this set. In a way, this is similar to Bayesian optimization, where we have some initial input output pairs and iteratively add to our dataset.

To do this we will maintain a GP that maps actions to the mean estimate of the integral. Using this GP we can asses our uncertainty about actions and select actions to reduce our uncertainty and maximize the integral.

But this approach only considers the mean estimates and ignores the fact that with each action and mean estimate of the integral, we also have a corresponding variance estimate of the integral. ...

- Your .tex file must compile in PDFLaTeX — **no .ps or .eps figure files.**
- All fonts must be embedded in the PDF file — **this includes your figures.**
- Modifications to the style sheet (or your document) in an effort to avoid extra page charges are NOT allowed.
- No type 3 fonts may be used (even in illustrations).
- Your title must follow US capitalization rules.
- LaTeX documents must use the Times or Nimbus font package (do not use Computer Modern for the text of your paper).
- No LaTeX 209 documents may be used or submitted.
- Fonts that require non-English language support (CID and Identity-H) must be converted to outlines or removed from the document (even if they are in a graphics file embedded in the document).
- Two-column format in AAAI style is required for all papers.
- The paper size for final submission must be US letter. No exceptions.
- The source file must exactly match the PDF.
- The document margins must be as specified in the formatting instructions.
- The number of pages and the file size must be as specified for your event.
- No document may be password protected.
- Neither the PDFs nor the source may contain any embedded links or bookmarks.
- Your source and PDF must not have any page numbers, footers, or headers.
- Your PDF must be compatible with Acrobat 5 or higher.
- Your LaTeX source file (excluding references) must consist of a **single** file (use of the "input" command is not allowed.
- Your graphics must be sized appropriately outside of LaTeX (do not use the "clip" command) .

If you do not follow the above requirements, it is likely that we will be unable to publish your paper.

What Files to Submit

You must submit the following items to ensure that your paper is published:

- A fully-compliant PDF file.
- Your LaTeX source file submitted as a **single** .tex file (do not use the "input" command to include sections of your paper — every section must be in the single source file). The only exception is the bibliography, which you may include separately. Your source must compile on our system, which includes the standard LaTeX support files.
- All your graphics files.
- The LaTeX-generated files (e.g. .aux and .bib file, etc.) for your compiled source.
- All the nonstandard style files (ones not commonly found in standard LaTeX installations) used in your document (including, for example, old algorithm style files). If in doubt, include it.

Your LaTeX source will be reviewed and recompiled on our system (if it does not compile, you may incur late fees). **Do not submit your source in multiple text files.** Your single LaTeX source file must include all your text, your bibliography (formatted using aaai.bst), and any custom macros. Accompanying this source file, you must also supply any nonstandard (or older) referenced style files and all your referenced graphics files.

Your files should work without any supporting files (other than the program itself) on any computer with a standard LaTeX distribution. Place your PDF and source files in a single tar, zipped, gzipped, stuffed, or compressed archive. Name your source file with your last (family) name.

Do not send files that are not actually used in the paper. We don't want you to send us any files not needed for compiling your paper, including, for example, this instructions file, unused graphics files, and so forth. A shell script (created by an AAAI member — it might not work without modification on your system) that might help you create the LaTeX source package is included in the Author Kit.

Using LaTeX to Format Your Paper

The latest version of the AAAI style file is available on AAAI's website. Download this file and place it in a file named "aaai.sty" in the TeX search path. Placing it in the same directory as the paper should also work. You must download the latest version of the complete author kit so that you will have the latest instruction set.

Document Preamble

In the LaTeX source for your paper, you **must** place the following lines as shown in the example in this subsection. This command set-up is for three authors. Add or subtract author and address lines as necessary, and uncomment the portions that apply to you. In most instances, this is all you need to do to format your paper in the Times font. The helvet package will cause Helvetica to be used for sans serif, and the courier package will cause Courier to be used for the typewriter font. These files are part of the PSNFSS2e package, which is freely available from many Internet sites (and is often part of a standard installation).

Leave the setcounter for section number depth commented out and set at 0 unless you want to add section numbers to your paper. If you do add section numbers, you must uncomment this line and change the number to 1 (for section numbers), or 2 (for section and subsection numbers). The style file will not work properly with numbering of subsubsections, so do not use a number higher than 2.

\documentclass[letterpaper]article
% Required Packages
\usepackage{aaai}

```
\usepackage{times}
\usepackage{helvet}
\usepackage{courier}
\setlength{\pdfpagewidth}{8.5in}
\setlength{\pdfpageheight}{11in}
%%%%%%%%%%
% PDFINFO for PDFLaTeX
% Uncomment and complete the following for metadata
(your paper must compile with PDFLaTeX)
\pdfinfo{
/Title (Input Your Paper Title Here)
/Author (John Doe, Jane Doe)
/Keywords (Input your paper's keywords in this optional
area)
}
%%%%%%%%%%
% Section Numbers
% Uncomment if you want to use section numbers
% and change the 0 to a 1 or 2
% \setcounter{secnumdepth}{0}
%%%%%%%%%%
% Title, Author, and Address Information
\title{Title}
\author{Author 1 \and Author 2\\
Address line\\
Address line\\
\And
Author 3\\
Address line\\
Address line}
%%%%%%%%%%
% Body of Paper Begins
\begin{document}
\maketitle
...
%%%%%%%%%%
% References and End of Paper
\bibliography{Bibliography-File}
\bibliographystyle{aaai}
\end{document}
```

Inserting Document Metadata with LaTeX

PDF files contain document summary information that enables us to create an Acrobat index (pdx) file, and also allows search engines to locate and present your paper more accurately. **Document Metadata for Author and Title are REQUIRED.**

If your paper includes illustrations that are not compatible with PDFTeX (such as .eps or .ps documents), you will need to convert them. The epstopdf package will usually work for eps files. You will need to convert your ps files to PDF however.

Important: Do not include *any* LaTeX code or nonascii characters (including accented characters) in the metadata. The data in the metadata must be completely plain ascii. It may not include slashes, accents, linebreaks, unicode, or any LaTeX commands. Type the title exactly as it appears on the paper (minus all formatting). Input the author names in the order in which they appear on the paper (minus all accents), separating each author by a comma. You may also include keywords in the Keywords field.

Preparing Your Paper

After the preamble above, you should prepare your paper as follows:

```
\begin{document}
\maketitle
...
\bibliography{Bibliography-File}
\bibliographystyle{aaai}
\end{document}
```

Incompatible Packages

The following packages are incompatible with aaai.sty and/or aaai.bst and must not be used (this list is not exhaustive — there are others as well):

- hyperref
- natbib
- geometry
- titlesec
- layout
- caption
- titlesec
- T1 fontenc package (install the CM super fonts package instead)

Illegal Commands

The following commands may not be used in your paper:

- \input
- \vspace (when used before or after a section or subsection)
- \addtolength
- \columnsep
- \top margin (or text height or addsidemargin or even side margin)

Paper Size, Margins, and Column Width

Papers must be formatted to print in two-column format on 8.5 x 11 inch US letter-sized paper. The margins must be exactly as follows:

- Top margin: .75 inches
- Left margin: .75 inches
- Right margin: .75 inches
- Bottom margin: 1.25 inches

The default paper size in most installations of LaTeX is A4. However, because we require that your electronic paper be formatted in US letter size, you will need to alter the default for this paper to US letter size. Assuming you are using the 2e version of LaTeX, you can do this by including the [letterpaper] option at the beginning of your file: \documentclass[letterpaper]article.

This command is usually sufficient to change the format. Sometimes, however, it may not work. Use PDFLaTeX and include \setlength{\pdfpagewidth}{8.5in} \setlength{\pdfpageheight}{11in} in your preamble.

Do not use the Geometry package to alter the page size. Use of this style file alters aaai.sty and will result in your paper being rejected.

Column Width and Margins. To ensure maximum readability, your paper must include two columns. Each column should be 3.3 inches wide (slightly more than 3.25 inches), with a .375 inch (.952 cm) gutter of white space between the two columns. The aaai.sty file will automatically create these columns for you.

Overlength Papers

If your paper is too long, turn on \frenchspacing, which will reduce the space after periods. Next, shrink the size of your graphics. Use \centering instead of \begin{center} in your figure environment. If these two methods don't work, you may minimally use the following. For floats (tables and figures), you may minimally reduce \floatsep, \textfloatsep, \abovecaptionskip, and \belowcaptionskip. For mathematical environments, you may minimally reduce \abovedisplayskip, \belowdisplayskip, and \arraycolsep. You may also alter the size of your bibliography by inserting \fontsize{9.5pt}{10.5pt} \selectfont right before the bibliography.

Commands that alter page layout are forbidden. These include \columnsep, \topmargin, \topskip, \textheight, \textwidth, \oddsidemargin, and \evensizemargin (this list is not exhaustive). If you alter page layout, you will be required to pay the page fee *plus* a reformatting fee. Other commands that are questionable and may cause your paper to be rejected include \parindent, and \parskip. Commands that alter the space between sections are also questionable. The title sec package is not allowed. Regardless of the above, if your paper is obviously "squeezed" it is not going to to be accepted. Before using every trick you know to make your paper a certain length, try reducing the size of your graphics or cutting text instead or (if allowed) paying the extra page charge. It will be cheaper in the long run.

Figures

Your paper must compile in PDFLaTeX. Consequently, all your figures must be .jpg, .png, or .pdf. You may not use the .gif (the resolution is too low), .ps, or .eps file format for your figures.

When you include your figures, you must crop them **outside** of LaTeX. The command \includegraphics*[clip=true, viewport 0 0 10 10]... might result in a PDF that looks great, but the image is **not really cropped.** The full image can reappear when page numbers are applied or color space is standardized.

Type Font and Size

Your paper must be formatted in Times Roman or Nimbus. We will not accept papers formatted using Computer Modern or Palatino or some other font as the text or heading typeface. Sans serif, when used, should be Courier. Use Symbol or Lucida or Computer Modern for *mathematics only.*

Do not use type 3 fonts for any portion of your paper, including graphics. Type 3 bitmapped fonts are designed for fixed resolution printers. Most print at 300 dpi even if the printer resolution is 1200 dpi or higher. They also often cause high resolution imagesetter devices and our PDF indexing software to crash. Consequently, AAAI will not accept electronic files containing obsolete type 3 fonts. Files containing those fonts (even in graphics) will be rejected.

Fortunately, there are effective workarounds that will prevent your file from embedding type 3 bitmapped fonts. The easiest workaround is to use the required times, helvet, and courier packages with LaTeX2e. (Note that papers formatted in this way will still use Computer Modern for the mathematics. To make the math look good, you'll either have to use Symbol or Lucida, or you will need to install type 1 Computer Modern fonts — for more on these fonts, see the section "Obtaining Type 1 Computer Modern.")

If you are unsure if your paper contains type 3 fonts, view the PDF in Acrobat Reader. The Properties/Fonts window will display the font name, font type, and encoding properties of all the fonts in the document. If you are unsure if your graphics contain type 3 fonts (and they are PostScript or encapsulated PostScript documents), create PDF versions of them, and consult the properties window in Acrobat Reader.

The default size for your type should be ten-point with twelve-point leading (line spacing). Start all pages (except the first) directly under the top margin. (See the next section for instructions on formatting the title page.) Indent ten points when beginning a new paragraph, unless the paragraph begins directly below a heading or subheading.

Obtaining Type 1 Computer Modern for LaTeX. If you use Computer Modern for the mathematics in your paper (you cannot use it for the text) you may need to download type 1 Computer fonts. They are available without charge from the American Mathematical Society: http://www.ams.org/tex/type1-fonts.html.

Title and Authors

Your title must appear in mixed case (nouns, pronouns, and verbs are capitalized) near the top of the first page, centered over both columns in sixteen-point bold type (twenty-four point leading). This style is called "mixed case." Author's names should appear below the title of the paper, centered in twelve-point type (with fifteen point leading), along with affiliation(s) and complete address(es) (including electronic mail address if available) in nine-point roman type (the twelve point leading). (If the title is long, or you have many authors, you may reduce the specified point sizes by up to two points.) You should begin the two-column format when you come to the abstract.

Formatting Author Information Author information can be set in a number of different styles, depending on the number of authors and the number of affiliations you need to display. For several authors from the same institution, use \and:

\author{Author 1 \and ... \and Author *n*\\
Address line \\ ... \\ Address line}

If the names do not fit well on one line use:

\author{Author 1}\\
{\bf Author 2}\\ ... \\ {\bf Author *n*}\\
Address line \\ ... \\ Address line}

For authors from different institutions, use \And:

\author{Author 1\\ Address line \\ ... \\ Address line
\And ... \And Author *n*\\
Address line\\ ... \\ Address line}

To start a separate "row" of authors, use \AND:

\author{Author 1\\ Address line \\ ... \\ Address line\\
\AND
Author 2 \\ Address line \\ ... \\ Address line\\
\And
Author 3 \\ Address line \\ ... \\ Address line\\
}

If the title and author information does not fit in the area allocated, place \setlength\titlebox{*height*} after the \documentclass line where {*height*} is something like 2.5in.

LaTeX Copyright Notice

The copyright notice automatically appears if you use aaai.sty. If you are creating a technical report, it is not necessary to include this notice. You may disable the copyright line using the \nocopyrightcommand. To change the entire text of the copyright slug, use: \copyrighttext {*text*}. Either of these must appear before \maketitle. Please be advised, however, that *if you disable or change the copyright line and transfer of copyright is required, your paper will not be published.*

Credits

Any credits to a sponsoring agency should appear in the acknowledgments section, unless the agency requires different placement. If it is necessary to include this information on the front page, use \thanks in either the \author or \title commands. For example:

\title{Very Important Results in AI\thanks{This work is supported by everybody.}}

Multiple \thanks commands can be given. Each will result in a separate footnote indication in the author or title with the corresponding text at the botton of the first column of the document. Note that the \thanks command is fragile. You will need to use \protect.

Please do not include \pubnote commands in your document.

Abstract

The abstract must be placed at the beginning of the first column, indented ten points from the left and right margins. The title Abstract should appear in ten-point bold type, centered above the body of the abstract. The abstract should be set in nine-point type with ten-point leading. This concise, one-paragraph summary should describe the general thesis and conclusion of your paper. A reader should be able to learn the purpose of the paper and the reason for its importance from the abstract. The abstract should be no more than two hundred words in length. (Authors who are submitting short one- or two-page extended extracts should provide a short abstract of only a sentence or so.) **Do not include references in your abstract!**

Page Numbers

Do not **ever** print any page numbers on your paper.

Text

The main body of the paper must be formatted in ten-point with twelve-point leading (line spacing).

Citations

Citations within the text should include the author's last name and year, for example (Newell 1980). Append lowercase letters to the year in cases of ambiguity. Multiple authors should be treated as follows: (Feigenbaum and Engelmore 1988) or (Ford, Hayes, and Glymour 1992). In the case of four or more authors, list only the first author, followed by et al. (Ford et al. 1997).

Extracts

Long quotations and extracts should be indented ten points from the left and right margins.

> This is an example of an extract or quotation. Note the indent on both sides. Quotation marks are not necessary if you offset the text in a block like this, and properly identify and cite the quotation in the text.

Footnotes

Avoid footnotes as much as possible; they interrupt the reading of the text. When essential, they should be consecutively numbered throughout with superscript Arabic numbers. Footnotes should appear at the bottom of the page, separated from the text by a blank line space and a thin, half-point rule.

Headings and Sections

When necessary, headings should be used to separate major sections of your paper. Remember, you are writing a short paper, not a lengthy book! An overabundance of headings will tend to make your paper look more like an outline than a paper.

First-level heads should be twelve-point Times Roman bold type, mixed case (initial capitals followed by lower case on all words except articles, conjunctions, and prepositions, which should appear entirely in lower case), with fifteen-point leading, centered, with one blank line preceding them and three additional points of leading following them. Second-level headings should be eleven-point Times Roman bold type, mixed case, with thirteen-point leading, flush left, with one blank line preceding them and three additional points of leading following them. Do not skip a line between paragraphs. Third-level headings should be run in with the text, ten-point Times Roman bold type, mixed case, with twelve-point leading, flush left, with six points of additional space preceding them and no additional points of leading following them.

Section Numbers The use of section numbers in AAAI Press papers is optional. To use section numbers in LaTeX, uncomment the setcounter line in your document preamble and change the 0 to a 1 or 2. Section numbers should not be used in short poster papers.

Section Headings. Sections should be arranged and headed as follows:

Acknowledgments. The acknowledgments section, if included, appears after the main body of text and is headed "Acknowledgments." This section includes acknowledgments of help from associates and colleagues, credits to sponsoring agencies, financial support, and permission to publish. Please acknowledge other contributors, grant support, and so forth, in this section. Do not put acknowledgments in a footnote on the first page. If your grant agency requires acknowledgment of the grant on page 1, limit the footnote to the required statement, and put the remaining acknowledgments at the back. Please try to limit acknowledgments to no more than three sentences.

Appendices. Any appendices follow the acknowledgments, if included, or after the main body of text if no acknowledgments appear.

References The references section should be labeled "References" and should appear at the very end of the paper (don't end the paper with references, and then put a figure by itself on the last page). A sample list of references is given later on in these instructions. Please use a consistent format for references. Poorly prepared or sloppy references reflect badly on the quality of your paper and your research. Please prepare complete and accurate citations.

Illustrations and Figures

Figures, drawings, tables, and photographs should be placed throughout the paper near the place where they are first discussed. Do not group them together at the end of the paper. If placed at the top or bottom of the paper, illustrations may run across both columns. Figures must not invade the top, bottom, or side margin areas. Figures must be inserted using the \usepackage{graphicx}. Number figures sequentially, for example, figure 1, and so on.

The illustration number and caption should appear under the illustration. Labels, and other text in illustrations must be at least nine-point type.

Low-Resolution Bitmaps. You may not use low-resolution (such as 72 dpi) screen-dumps and GIF files—these files contain so few pixels that they are always blurry, and illegible when printed. If they are color, they will become an indecipherable mess when converted to black and white. This is always the case with gif files, which should never be used. The resolution of screen dumps can be increased by reducing the print size of the original file while retaining the same number of pixels. You can also enlarge files by manipulating them in software such as PhotoShop. Your figures should be a minimum of 266 dpi when incorporated into your document.

LaTeX Overflow. LaTeX users please beware: LaTeX will sometimes put portions of the figure or table or an equation in the margin. If this happens, you need to scale the figure or table down, or reformat the equation. Check your log file! You must fix any overflow into the margin (that means no overfull boxes in LaTeX). If you don't, the overflow text will simply be eliminated. **Nothing is permitted to intrude into the margins.**

Using Color. Your paper will be printed in black and white and grayscale. Consequently, because conversion to grayscale can cause undesirable effects (red changes to black, yellow can disappear, and so forth), we strongly suggest you avoid placing color figures in your document. Of course, any reference to color will be indecipherable to your reader.

Drawings. We suggest you use computer drawing software (such as Adobe Illustrator or, (if unavoidable), the drawing tools in Microsoft Word) to create your illustrations. Do not use Microsoft Publisher. These illustrations will look best if all line widths are uniform (half- to two-point in size), and you do not create labels over shaded areas. Shading should be 133 lines per inch if possible. Use Times Roman or Helvetica for all figure call-outs. **Do not use hairline width lines** — be sure that the stroke width of all lines is at least .5 pt. Zero point lines will print on a laser printer, but will completely disappear on the high-resolution devices used by our printers.

Photographs and Images. Photographs and other images should be in grayscale (color photographs will not reproduce well; for example, red tones will reproduce as black, yellow may turn to white, and so forth) and set to a minimum of 266 dpi. Do not prescreen images.

Resizing Graphics. Resize your graphics **before** you include them with LaTeX. You may **not** use trim or clip options as part of your \includgraphics command. Resize the media box of your PDF using a graphics program instead.

Fonts in Your Illustrations You must embed all fonts in your graphics before including them in your LaTeX document.

References

The aaai.sty file includes a set of definitions for use in formatting references with BibTeX. These definitions make the bibliography style fairly close to the one specified below.

To use these definitions, you also need the BibTeX style file "aaai.bst," available in the author kit on the AAAI web site. Then, at the end of your paper but before \enddocument, you need to put the following lines:

\bibliographystyle{aaai} \bibliography{bibfile1,bibfile2,...}

The list of files in the \bibliography command should be the names of your BibTeX source files (that is, the .bib files referenced in your paper).

The following commands are available for your use in citing references:

\cite: Cites the given reference(s) with a full citation. This appears as "(Author Year)" for one reference, or "(Author Year; Author Year)" for multiple references.
\shortcite: Cites the given reference(s) with just the year. This appears as "(Year)" for one reference, or "(Year; Year)" for multiple references.
\citeauthor: Cites the given reference(s) with just the author name(s) and no parentheses.
\citeyear: Cites the given reference(s) with just the date(s) and no parentheses.

Warning: The aaai.sty file is incompatible with the hyperref and natbib packages. If you use either, your references will be garbled.

Formatted bibliographies should look like the following examples.

Book with Multiple Authors
Engelmore, R., and Morgan, A. eds. 1986. *Blackboard Systems.* Reading, Mass.: Addison-Wesley.

Journal Article
Robinson, A. L. 1980a. New Ways to Make Microcircuits Smaller. *Science* 208: 1019–1026.

Magazine Article
Hasling, D. W.; Clancey, W. J.; and Rennels, G. R. 1983. Strategic Explanations in Consultation. *The International Journal of Man-Machine Studies* 20(1): 3–19.

Proceedings Paper Published by a Society
Clancey, W. J. 1983b. Communication, Simulation, and Intelligent Agents: Implications of Personal Intelligent Machines for Medical Education. In Proceedings of the Eighth International Joint Conference on Artificial Intelligence, 556–560. Menlo Park, Calif.: International Joint Conferences on Artificial Intelligence, Inc.

Proceedings Paper Published by a Press or Publisher
Clancey, W. J. 1984. Classification Problem Solving. In *Proceedings of the Fourth National Conference on Artificial Intelligence,* 49–54. Menlo Park, Calif.: AAAI Press.

University Technical Report
Rice, J. 1986. Poligon: A System for Parallel Problem Solving, Technical Report, KSL-86-19, Dept. of Computer Science, Stanford Univ.

Dissertation or Thesis
Clancey, W. J. 1979b. Transfer of Rule-Based Expertise through a Tutorial Dialogue. Ph.D. diss., Dept. of Computer Science, Stanford Univ., Stanford, Calif.

Forthcoming Publication
Clancey, W. J. 1986a. The Engineering of Qualitative Models. Forthcoming.

Producing Reliable PDF Documents with LaTeX

Generally speaking, PDF files are platform independent and accessible to everyone. When creating a paper for a proceedings or publication in which many PDF documents must be merged and then printed on high-resolution PostScript RIPs, several requirements must be met that are not normally of concern. Thus to ensure that your paper will look like it does when printed on your own machine, you must take several precautions:

- Use type 1 fonts (not type 3 fonts)
- Use only standard Times, Nimbus, and CMR font packages (not fonts like F3 or fonts with tildes in the names or fonts—other than Computer Modern—that are created for specific point sizes, like Times˜19) or fonts with strange combinations of numbers and letters
- Embed all fonts when producing the PDF
- Do not use the [T1]fontenc package (install the CM super fonts package instead)

Creating Output Using PDFLaTeX Is Required

By using the PDFTeX program instead of straight LaTeX or TeX, you will probably avoid the type 3 font problem altogether (unless you use a package that calls for metafont). PDFLaTeX enables you to create a PDF document directly from LaTeX source. The one requirement of this software is that all your graphics and images must be available in a format that PDFLaTeX understands (normally PDF).

PDFLaTeX's default is to create documents with type 1 fonts. If you find that it is not doing so in your case, it is likely that one or more fonts are missing from your system or are not in a path that is known to PDFLaTeX.

dvipdf Script Scripts such as dvipdf which ostensibly bypass the Postscript intermediary should not be used since they generally do not instruct dvips to use the config.pdf file.

dvipdfm Do not use this dvi-PDF conversion package if your document contains graphics (and we recommend you avoid it even if your document does not contain graphics).

Ghostscript

LaTeX users should not use GhostScript to create their PDFs.

Graphics

If you are still finding type 3 fonts in your PDF file, look at your graphics! LaTeX users should check all their imported graphics files as well for font problems.

Proofreading Your PDF

Please check all the pages of your PDF file. Is the page size A4? Are there any type 3, Identity-H, or CID fonts? Are all the fonts embedded? Are there any areas where equations or figures run into the margins? Did you include all your figures? Did you follow mixed case capitalization rules for your title? Did you include a copyright notice? Do any of the pages scroll slowly (because the graphics draw slowly on the page)? Are URLs underlined and in color? You will need to fix these common errors before submitting your file.

Improperly Formatted Files

In the past, AAAI has corrected improperly formatted files submitted by the authors. Unfortunately, this has become an increasingly burdensome expense that we can no longer absorb. Consequently, if your file is improperly formatted, it may not be possible to include your paper in the publication. If time allows, however, you will be notified via e-mail (with a copy to the program chair) of the problems with your file and given the option of correcting the file yourself (and paying a late fee) or asking that AAAI have the file corrected for you, for an additional fee. If you opt to correct the file yourself, please note that we cannot provide you with any additional advice beyond that given in your packet. Files that are not corrected after a second attempt will be withdrawn.

LaTeX 209 Warning

If you use LaTeX 209 we will not be able to publish your paper. Convert your paper to LaTeX2e.

Naming Your Electronic File

We request that you name your LaTeX source file with your last name (family name) so that it can easily be differentiated from other submissions. If you name your files with the name of the event or "aaai" or "paper" or "camera-ready" or some other generic or indecipherable name, you bear all risks of loss — it is extremely likely that your file may be overwritten.

Submitting Your Electronic Files to AAAI

Submitting your files to AAAI is a two-step process. It is explained fully in the author registration and submission instructions. Please consult this document for details on how to submit your paper.

Inquiries

If you have any questions about the preparation or submission of your paper as instructed in this document, please contact AAAI Press at the address given below. If you have technical questions about implementation of the aaai style file, please contact an expert at your site. We do not provide technical support for LaTeX or any other software package. To avoid problems, please keep your paper simple, and do not incorporate complicated macros and style files.

AAAI Press
2275 East Bayshore Road, Suite 160
Palo Alto, California 94303
Telephone: (650) 328-3123
E-mail: See the submission instructions for your particular conference or event.

Additional Resources

LaTeX is a difficult program to master. If you've used that software, and this document didn't help or some items were not explained clearly, we recommend you read Michael Shell's excellent document (testflow doc.txt V1.0a 2002/08/13) about obtaining correct PS/PDF output on LaTeX systems. (It was written for another purpose, but it has general application as well). It is available at www.ctan.org in the tex-archive.

Acknowledgments

AAAI is especially grateful to Peter Patel Schneider for his work in implementing the aaai.sty file, liberally using the ideas of other style hackers, including Barbara Beeton. We also acknowledge with thanks the work of George Ferguson for his guide to using the style and BibTeX files — which has been incorporated into this document — and Hans Guesgen, who provided several timely modifications, as well as the many others who have, from time to time, sent in suggestions on improvements to the AAAI style.

The preparation of the LaTeX and BibTeX files that implement these instructions was supported by Schlumberger Palo Alto Research, AT&T Bell Laboratories, Morgan Kaufmann Publishers, The Live Oak Press, LLC, and AAAI Press. Bibliography style changes were added by Sunil Issar. \pubnote was added by J. Scott Penberthy. George Ferguson added support for printing the AAAI copyright slug. Additional changes to aaai.sty and aaai.bst have been made by the AAAI staff.

Thank you for reading these instructions carefully. We look forward to receiving your electronic files!