

# Max Integral Operator: A Probabilistic Numerical Approach

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## Abstract

The Max Integral Operator is an expression of the form  $\max_a \int f(a, s) ds$ . Such expressions arise in many computational domains including machine learning, statistical inference, and optimization. Intuitively, maximizing over some variables and integrating out others is a useful operation to perform. In most practical settings, however, these integrals are non analytic and exact evaluation is computationally intractable. Much research has explored Bayesian quadrature, a method of numerical integration that provides estimates of uncertainty on an integral's value. In this work, we present a framework for efficiently evaluating the Max Integral Operator by incorporating this uncertainty information into the optimization procedure. We jointly optimize over the outer optimization variable and the selection of queries in estimating the inner integrals. As a result, our framework exhibits increased sample efficiency, allowing us to get accurate value estimates with fewer function evaluation queries. We investigate the effectiveness of our framework, the various applications, and the theoretical guarantees.

## Introduction

Function maximization and numerical integration have received much focus from the optimization and applied mathematics communities, respectively. However, in performing these operations simultaneously there is added complexity and simply applying existing optimization and integration algorithms is computationally infeasible. We will take a deeper look at evaluation of the Max Integral Operator and, in particular, show how taking a probabilistic numerical perspective...

A reasonable method for evaluating an integral might be sampling-based Monte Carlo techniques, just as we would use to estimate an expectation in a discrete setting. However, in "Monte Carlo is Fundamentally Unsound", O'Hagan identified a series of inconsistencies with such methods. He then presents Bayes-Hermite quadrature, which treats the numerical integration problem as a statistical inference problem. By doing so, you may use the estimate of uncertainty of an integral's value as a convergence criterion and actively sample function evaluations that reduce uncertainty.

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Bayesian quadrature involves fitting a Gaussian Process to the integrand and integrating the mean function of the GP. Under certain conditions of the kernel function and prior, this integral will have a closed form. Bayesian Monte Carlo and Gaussian Process Dynamic Programming state these closed forms and explore the applications of quadrature in computing marginal likelihood and RL action policies, respectively.

Other work on Bayesian quadrature has focused on cases where the integrand is non-negative (Osborne 2017) (Gunter 2016) and shown how using certain warping functions can induce cheap active learning schemes. Using these sampling procedures they show fast convergence compared to other numerical integration methods.

In both these works, however, they assume either a discrete optimization space or a relatively small continuous state space. We observe that if both these assumptions are violated, their methods of optimization over the integral are infeasible. In most use cases of the Max Integral Operator, we are interested in continuous high-dimensional spaces and therefore, it seems reasonable that our methods of evaluation should extend to these domains. For these reasons, we believe that methods for evaluating the Max Integral Operator merit further investigation.

The remainder of this paper is structured as follows: (1) We present the framework for optimization and three variants of implementation. Then, we do a thorough analysis of these variants on test functions. (2) We apply the optimization procedure to the Reinforcement Learning setting, specifically to Bellman Updates, and developed two extensions to existing algorithms for solving MDPs. The first is classical value iteration and the next is RTDP. We show how our Max Integral Optimization allows you to efficiently learn robust policies in stochastic continuous state-action domains. (3) We use our framework in computing maximum a posteriori estimates for a series of common Posterior Estimation problems (4) We investigate the theoretical guarantees we can make regarding optimality and convergence in Max Integral settings.

## Preliminaries

In this section, we review Gaussian Process Regression, Bayesian Optimization and Quadrature, and Multi-armed Bandits. For further reference we recommend...

## Gaussian Process Regression

A Gaussian process is a method of representing a distribution over functions. It can initially be defined by the prior mean function and a covariance function, which captures our smoothness assumptions on the functions we are modeling (see Rasmussen and Williams 2005). These priors can be engineered through existing domain knowledge, but in many cases the prior mean is assumed to be 0.

As we make additional function queries, we will condition on our set of observations using a Bayesian update method. By conditioning on our observation, we can derive the posterior distribution. This GP posterior is not a closed form function e.g.  $f(x) = \sin(x)$ , instead, at each input point the posterior is a Gaussian Distribution as follows:

$$\begin{aligned} y|y_* &\sim \mathcal{N}(\mu, \sigma) \\ \mu &= m(X_*) + K(X_*, X)K_y^{-1}(y - m(X)) \\ \sigma &= m(X_*) + K(X_*, X)K_y^{-1}(y - m(X)) \end{aligned} \quad (1)$$

The core of a Gaussian process is the multivariate Gaussian. The model's assumption is that observed samples from the function are samples from a multivariate Gaussian. Then, for unobserved inputs we use the mean and covariance functions to infer a distribution. In a way, a Gaussian process is an extension of the multivariate Gaussian to the infinite-dimensional domain.

GPs are powerful tool in modeling unknown functions because the posterior distribution captures our uncertainty about the function's value. In the next subsection, we discuss how this can be leveraged in optimization and estimation procedures.

## Bayesian Optimization

Bayesian optimization is a function optimization procedure that relies on priors and sequential updates of a posterior. The posterior will guide the queries made to the function, through an acquisition function. Bayesian Optimization is a black box optimizer because it relies on only the function inputs and outputs to create its' representation. Bayesian Optimization does not relate on derivatives, either.

In continuous domains, Gaussian processes are used as the posterior model because they allow us to easily condition on our previous observations and generalize to the infinite domain, according to our specified prior mean and kernel function.

The key component of Bayesian Optimization is the acquisition function that defines how we select function evaluations. While there are many different acquisition functions used in practice, they all essentially balance what is commonly referred to as "exploration vs exploitation". Exploration is the tendency to query areas in the domain where there is uncertainty about value, while Exploitation is querying areas that have highest value.

The commonly used acquisition function is Upper Confidence Bound, because of its simplicity as well as it theoretical guarantees. The UCB function is as follow:

$$\begin{aligned} UCB(x) &= \mu + \beta\sigma \\ \operatorname{argmax}_x UCB(x) \end{aligned} \quad (2)$$

In UCB, as  $\beta$  tends to 0, the acquisition policy becomes exploitation and as it increases, the acquisition policy increasingly weights uncertainty.

## Bayesian Quadrature

Just as Bayesian Optimization takes a sequential query approach to optimizing a function, Bayesian quadrature takes a similar approach in estimating the integral of a black box function. These methods were first proposed as Bayes-Hermite quadrature (O'Hagan) and then reformulated in the Bayes Monte Carlo (Rasmussen). Most work has focused on integrals of the form:

$$\int f(x)p(x)dx$$

In Bayesian Quadrature, the acquisition function is chosen to prefer queries that can reduce the uncertainty about the value of the integral. This acquisition would be:

$$\operatorname{argmin} V(f|D) \quad (3)$$

An alternative acquisition function would be Uncertainty sampling

$$\operatorname{argmin} V(f|D) \quad (4)$$

Under certain conditions, the integral will have a closed form. One such case, is if the integrand  $f$  is modeled by a GP with an RBF kernel and the prior  $p$  is a Gaussian. For completeness we present the derivation of this closed form below:

$$\begin{aligned} E_{f|D}[f_p] &= z^T K^{-1} f \\ z &= |A^{-1}B + I|^{-1/2} \exp[-0.5(a-b)^T(A+B)^{-1}(a-b)] \end{aligned}$$

## Multi Armed Bandits

In many real-world scenarios, we seek to maximize some reward or utility through our series of actions. These actions can, in addition to providing reward, give us more information about our problem scenario and inform following decision making. The multi armed bandits problem, originally presented by Robins, is a mathematical formalization of such a scenario. The problem has number of "arms" which when pulled give certain reward and over a series of "pulls" the player must maximize the reward gained.

...

Bayesian optimization is a variant of the multi armed bandits problem with an additional assumption on the dependence of arms. Though we have already presented an overview of Bayesian optimization, ...

## Problem Formulation and Method

### Problem Formulation and Method

In this paper we present an approach to efficiently evaluate the Max Integral Operator (MIO) of a function. The MIO applied to a multivariable function  $f$  is as follows:

$$\max_a \int f(a, s, v) ds$$

$a, s, v$  may represent sets of variables.  $s$  should consist of continuous values because this is the integration variable. The values in  $a$  and  $v$  may be either continuous or discrete as our method handles both cases.  $f$  must be a continuous function along the dimensions in  $s$ . In addition,  $f$  must be integrable along the dimensions in  $s$ , for fixed inputs of  $a$  and  $v$ . We will assume these are infinite integrals (from negative infinity to positive infinity). So, more specifically,  $f$  must be a convergent integral, meaning the following limit must exist.

$$\lim_{a \rightarrow -\infty} \lim_{b \rightarrow \infty} \int_a^b f(a, s, v) ds$$

The methods we will present are derivative-free optimization techniques, so the target applications are blackbox functions where the derivatives are inaccessible. In addition,

In this paper, we will make the assumption that the function  $f$  takes the form:

$$f(a, s, v) = g(a, s, v) \cdot p(s|a, v)$$

for some function

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**Using Color.** Your paper will be printed in black and white and grayscale. Consequently, because conversion to grayscale can cause undesirable effects (red changes to black, yellow can disappear, and so forth), we strongly suggest you avoid placing color figures in your document. Of course, any reference to color will be indecipherable to your reader.

**Drawings.** We suggest you use computer drawing software (such as Adobe Illustrator or, (if unavoidable), the drawing tools in Microsoft Word) to create your illustrations. Do not use Microsoft Publisher. These illustrations will look best if all line widths are uniform (half- to two-point in size), and you do not create labels over shaded areas. Shading should be 133 lines per inch if possible. Use Times Roman or Helvetica for all figure call-outs. **Do not use hairline width lines** — be sure that the stroke width of

all lines is at least .5 pt. Zero point lines will print on a laser printer, but will completely disappear on the high-resolution devices used by our printers.

**Photographs and Images.** Photographs and other images should be in grayscale (color photographs will not reproduce well; for example, red tones will reproduce as black, yellow may turn to white, and so forth) and set to a minimum of 266 dpi. Do not prescreen images.

**Resizing Graphics.** Resize your graphics **before** you include them with LaTeX. You may **not** use trim or clip options as part of your `\includgraphics` command. Resize the media box of your PDF using a graphics program instead.

**Fonts in Your Illustrations** You must embed all fonts in your graphics before including them in your LaTeX document.

## References

The `aaai.sty` file includes a set of definitions for use in formatting references with BibTeX. These definitions make the bibliography style fairly close to the one specified below. To use these definitions, you also need the BibTeX style file “`aaai.bst`,” available in the author kit on the AAAI web site. Then, at the end of your paper but before `\enddocument`, you need to put the following lines:

```
\bibliographystyle{aaai} \bibliography{bibfile1,bibfile2,...}
```

The list of files in the `\bibliography` command should be the names of your BibTeX source files (that is, the `.bib` files referenced in your paper).

The following commands are available for your use in citing references:

`\cite`: Cites the given reference(s) with a full citation. This appears as “(Author Year)” for one reference, or “(Author Year; Author Year)” for multiple references.

`\shortcite`: Cites the given reference(s) with just the year. This appears as “(Year)” for one reference, or “(Year; Year)” for multiple references.

`\citeauthor`: Cites the given reference(s) with just the author name(s) and no parentheses.

`\citeyear`: Cites the given reference(s) with just the date(s) and no parentheses.

**Warning:** The `aaai.sty` file is incompatible with the `hyperref` and `natbib` packages. If you use either, your references will be garbled.

Formatted bibliographies should look like the following examples.

### *Book with Multiple Authors*

Engelmore, R., and Morgan, A. eds. 1986. *Blackboard Systems*. Reading, Mass.: Addison-Wesley.

### *Journal Article*

Robinson, A. L. 1980a. New Ways to Make Microcircuits Smaller. *Science* 208: 1019–1026.

### *Magazine Article*

Hasling, D. W.; Clancey, W. J.; and Rennels, G. R. 1983. Strategic Explanations in Consultation. *The International Journal of Man-Machine Studies* 20(1): 3–19.

### *Proceedings Paper Published by a Society*

Clancey, W. J. 1983b. Communication, Simulation, and Intelligent Agents: Implications of Personal Intelligent Machines for Medical Education. In *Proceedings of the Eighth International Joint Conference on Artificial Intelligence*, 556–560. Menlo Park, Calif.: International Joint Conferences on Artificial Intelligence, Inc.

### *Proceedings Paper Published by a Press or Publisher*

Clancey, W. J. 1984. Classification Problem Solving. In *Proceedings of the Fourth National Conference on Artificial Intelligence*, 49–54. Menlo Park, Calif.: AAAI Press.

### *University Technical Report*

Rice, J. 1986. Poligon: A System for Parallel Problem Solving, Technical Report, KSL-86-19, Dept. of Computer Science, Stanford Univ.

### *Dissertation or Thesis*

Clancey, W. J. 1979b. Transfer of Rule-Based Expertise through a Tutorial Dialogue. Ph.D. diss., Dept. of Computer Science, Stanford Univ., Stanford, Calif.

### *Forthcoming Publication*

Clancey, W. J. 1986a. The Engineering of Qualitative Models. Forthcoming.

## Producing Reliable PDF Documents with L<sup>A</sup>T<sub>E</sub>X

Generally speaking, PDF files are platform independent and accessible to everyone. When creating a paper for a proceedings or publication in which many PDF documents must be merged and then printed on high-resolution PostScript RIPs, several requirements must be met that are not normally of concern. Thus to ensure that your paper will look like it does when printed on your own machine, you must take several precautions:

- Use type 1 fonts (not type 3 fonts)
- Use only standard Times, Nimbus, and CMR font packages (not fonts like F3 or fonts with tildes in the names or fonts—other than Computer Modern—that are created for specific point sizes, like Times~19) or fonts with strange combinations of numbers and letters
- Embed all fonts when producing the PDF
- Do not use the [T1]fontenc package (install the CM super fonts package instead)

## Creating Output Using PDFL<sup>A</sup>T<sub>E</sub>X Is Required

By using the PDFL<sup>A</sup>T<sub>E</sub>X program instead of straight L<sup>A</sup>T<sub>E</sub>X or T<sub>E</sub>X, you will probably avoid the type 3 font problem altogether (unless you use a package that calls for metafont). PDFL<sup>A</sup>T<sub>E</sub>X enables you to create a PDF document directly from L<sup>A</sup>T<sub>E</sub>X source. The one requirement of this software is that all your graphics and images must be available in a format that PDFL<sup>A</sup>T<sub>E</sub>X understands (normally PDF).

PDFL<sup>A</sup>T<sub>E</sub>X's default is to create documents with type 1 fonts. If you find that it is not doing so in your case, it is likely that one or more fonts are missing from your system or are not in a path that is known to PDFL<sup>A</sup>T<sub>E</sub>X.



**dvipdf Script** Scripts such as dvipdf which ostensibly bypass the Postscript intermediary should not be used since they generally do not instruct dvips to use the config.pdf file.

**dvipdfm** Do not use this dvi-PDF conversion package if your document contains graphics (and we recommend you avoid it even if your document does not contain graphics).

## Ghostscript

L<sup>A</sup>T<sub>E</sub>X users should not use GhostScript to create their PDFs.

## Graphics

If you are still finding type 3 fonts in your PDF file, look at your graphics! L<sup>A</sup>T<sub>E</sub>X users should check all their imported graphics files as well for font problems.

## Proofreading Your PDF

Please check all the pages of your PDF file. Is the page size A4? Are there any type 3, Identity-H, or CID fonts? Are all the fonts embedded? Are there any areas where equations or figures run into the margins? Did you include all your figures? Did you follow mixed case capitalization rules for your title? Did you include a copyright notice? Do any of the pages scroll slowly (because the graphics draw slowly on the page)? Are URLs underlined and in color? You will need to fix these common errors before submitting your file.

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## L<sup>A</sup>T<sub>E</sub>X 209 Warning

If you use L<sup>A</sup>T<sub>E</sub>X 209 we will not be able to publish your paper. Convert your paper to L<sup>A</sup>T<sub>E</sub>X 2e.

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If you have any questions about the preparation or submission of your paper as instructed in this document, please contact AAAI Press at the address given below. If you have technical questions about implementation of the aaai style file, please contact an expert at your site. We do not provide technical support for L<sup>A</sup>T<sub>E</sub>X or any other software package. To avoid problems, please keep your paper simple, and do not incorporate complicated macros and style files.

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## Additional Resources

L<sup>A</sup>T<sub>E</sub>X is a difficult program to master. If you’ve used that software, and this document didn’t help or some items were not explained clearly, we recommend you read Michael Shell’s excellent document (testflow doc.txt V1.0a 2002/08/13) about obtaining correct PS/PDF output on L<sup>A</sup>T<sub>E</sub>X systems. (It was written for another purpose, but it has general application as well). It is available at [www.ctan.org](http://www.ctan.org) in the tex-archive.

## Acknowledgments

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The preparation of the L<sup>A</sup>T<sub>E</sub>X and BibTeX files that implement these instructions was supported by Schlumberger Palo Alto Research, AT&T Bell Laboratories, Morgan Kaufmann Publishers, The Live Oak Press, LLC, and AAAI Press. Bibliography style changes were added by Sunil Issar. \pubnote was added by J. Scott Penberthy. George Ferguson added support for printing the AAAI copyright slug. Additional changes to aaai.sty and aaai.bst have been made by the AAAI staff.

Thank you for reading these instructions carefully. We look forward to receiving your electronic files!