Probabilitic Numerical Methods for Evaluating the Max Integral Operator

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Max Integral Operator

In this work, we will explore techniques of estimating expressions of the following form:

$$\max_{a} \int f(a, s) \, \mathrm{d}s$$

In high dimensions and continuous spaces, exact evaluation is many times intractable. As a result we have to take a probabilistic numerical approach to accurately and efficiently estimate its value.

The Max Integral Operator is used in a variety of computational problems across many fields. In Reinforcement Learning it is used in MDPs for value update and POMDPs for belief updates. In inference problems we may use in computing maximum a posteriori estimate. Intuitively, maximizing over some parameters while integrating out others is a useful/important operation.

Bayesian Quadrature

Quadrature techniques, first presented as Bayes-Hermite Quadrature [1], are used to actively sample function evaluations in order to reduce uncertainty about an integral's value. This is usually done by maintaining a Gaussian Process to model the integrand.

Most work in BQ has focused on integrals of the form:

$$\int f(x)p(x)\,\mathrm{d}x$$

In our framework, we will take advantage of quadrature techniques and use the estimated distribution over value to guide the optimization procedure.

Bibliography

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[3] Rasmussen, C.E., Ghahramani, Z.: Bayesian Monte Carlo. In Becker, S., Obermayer, K., eds.: Advances in Neural Information Processing Systems. Volume 15. MIT Press, Cambridge, MA (2003)