

Recursion★ Recursion -

A function calling itself again & again till it meets its base condition.

• Function call -

```
def add(x,y):
    return x+y
```

```
def mul(x,y):
    return x*y
```

```
def sub(x,y):
    return x-y
```

```
def print_result(x):
    print(x)
```

```
def main():
    x=10
    y=20
```

```
print_result(sub(mul(add(x,y),30),75))
```

~~point-result()~~

~~x=825~~

~~sub()~~

~~x=90 y=75~~

~~return 900-75~~

~~mul()~~

~~x=30 y=30~~

~~return 30*30~~

~~add()~~

~~x=10, y=20~~

~~return 10+20~~

~~main()~~

~~x=10, y=20~~

~~add 30~~

~~mul 900~~

~~sub 825~~

~~print-result~~

30

9000

825

• Recursion:

Function calling itself

sum of first 5 natural numbers:

$$\text{sum}(5) = 1 + 2 + 3 + 4 + 5$$

$$\text{sum}(4) = 1 + 2 + 3 + 4$$

$$\text{sum}(3) = 1 + 2 + 3$$

$$\text{sum}(2) = 1 + 2$$

$$\text{sum}(1) = 1$$

$$\text{sum}(5) = \text{sum}(4) + 5$$

↑
main problem

just smaller
sub problem

$$\text{sum}(n) = 1 + 2 + 3 + \dots + n - 1 + n$$

Relation :

$$\text{sum}(n) = \text{sum}(n-1) + n$$

• Steps to Recursive Code:

1. Assumption: Decide what your function does & assume it works
2. Main logic: Solve the problem with the help of the sub problem.
3. Base condition: Condition with smallest problem (stop)

* Problem: Sum of first n natural numbers.

1. Assumption: Assume "sum(n)" \rightarrow sum of first ' n ' natural numbers
2. Main logic: $\text{sum}(n) = \text{sum}(n-1) + n$
 - \uparrow problem
 - \uparrow just smaller sub problem
3. Base condition: (smallest problem is $n=1$)
 $\therefore \text{sum}(1) = 1$

if $n == 1$:
 return 1
 $\therefore \text{sum}(1) = 1$

* Code & Dry Run: $\text{sum}(5)$

def sum(N):

Base case of recursion

if $N == 1$:

 return 1

② # Main logic

 return $N + \text{sum}(N-1)$

$$15 \leftarrow 5 + 4 + 3 + 2 + 1$$

- Base condition: Recursion will not stop without base condition.

1. TLE: No. of iterations allowed $\rightarrow 10^8 \rightarrow 1 \text{ sec}$

2. MLE: (Memory Limit Exceeded):

In recursion \rightarrow stack overflow

If we create approximately more than 10^5 levels,
 it will throw stack overflow.

Q) Problem → Given value of N , calculate $N!$.

```
def fact(N):
```

Base condition

```
if n == 1 or n == 0:
```

return 1

Main Logic

```
return N * fact(n-1)
```

```
print(fact(4))
```

No. of calls

TC:

$\times 1$

$\times 2$

$\times 3$

$\times 4$

Time taken

$O(1)$

$O(N)$

$O(2)$

$O(3)$

$O(4)$

No. of calls

SC: $n + O(1)$

$O(N)$

Q) Problem → Print numbers in increasing order

printing(5) → 1 2 3 4 5 | output

printing(4) → 1 2 3 4

General - Main problem: printing(n)

smaller problem: printing($n-1$)

```

def printinc(N):
    # Base Cond
    if N == 1:
        print(N, end=" ")
        return
    # Main Logic
    printinc(N-1)
    print(N, end=" ")

```

O/P → 1 2 3 4 5

TC: O(N) SC: O(1)

Q) Problem → Print numbers in decreasing order.

printdec(5) → 5 4 3 2 1

General : Main problem printdec(n)

Smaller problem printdec(n-1)

```

def printdec(N):
    if N == 1:

```

```

        print(N, end=" ")
        return
    printdec(N-1)
    print(N, end=" ")

```

TC: O(N) SC: O(N)

* Time & Space Complexity of Recursion

Total TC : No. of funcⁿ call

* Time taken by each funcⁿ call

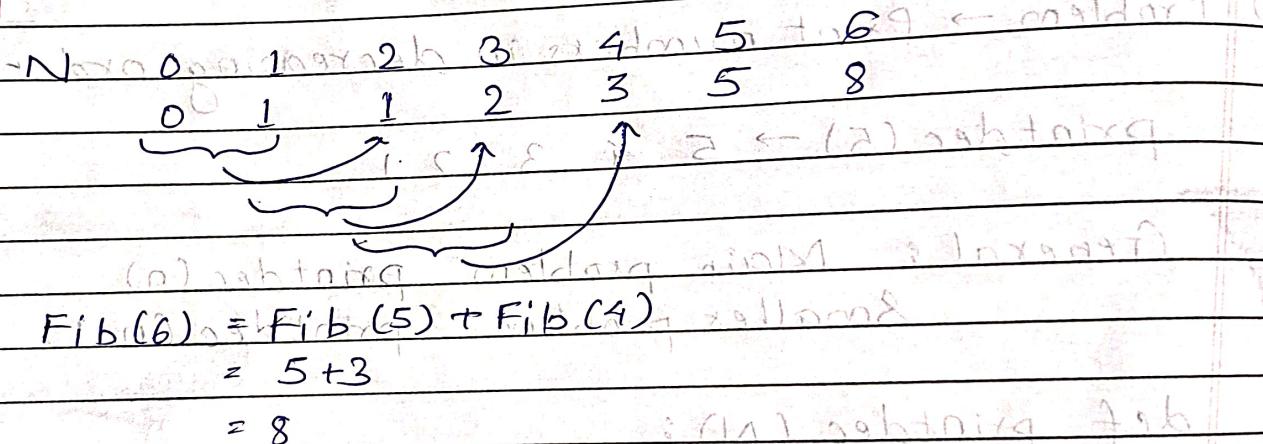
Total SC : Depth of the call stack

* (1-1) initiating

Space taken in each cell

* N^{th} number in the Fibonacci Series

The fibonacci series is a series of numbers



Q) If $N=7$, find the N^{th} number in the fib series

1. Assume $\text{Fib}(N)$

2. Main logic $\text{Fib}(n) = \text{Fib}(n-1) + \text{Fib}(n-2)$

3. Base Condⁿ $\text{Fib}(0)=0, \text{Fib}(1)=1$

if $n=0$ or $n=1$:

(1) 0 : 03 return n (1) 0 : 07

```
def Fib(n):
```

- 1. if $n = 0$ or $n = 1$:

return n

- 2. return $\underbrace{\text{Fib}(n-1)}_2 + \underbrace{\text{Fib}(n-2)}_3$

TC : level 1 : $1 \rightarrow 2^0$

2 : $2 \rightarrow 2^1$

3 : $4 \rightarrow 2^2$

GP : $a = 2^0$, $r = 2$, terms = N

Sum of GP = $a \times \frac{(r^n - 1)}{r - 1}$

$$= \frac{1 \times (2^n - 1)}{1}$$

$$= 2^n$$

2^n func calls

$O(1)$ in each call

TC : $O(2^n)$

SC : $O(N)$

Print the fibonacci from 1 to N

```
for i in range (1, n+1):
    print (Fib(i))
```