QOSF Mentorship - Screening Task 1

Contestant Details -

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Repo link - https://github.com/nishalkulkarni/QOSF_Screening_Task_1

In [1]:
```
from qiskit import *
from qiskit.circuit import Parameter,ParameterVector
from qiskit.quantum_info import random_statevector
from scipy.optimize import minimize
import matplotlib.pyplot as plt
import numpy as np
import time
```

Building the Circuit

For every layer we use N*2 parameters, where N = number of qubits in our circuit.

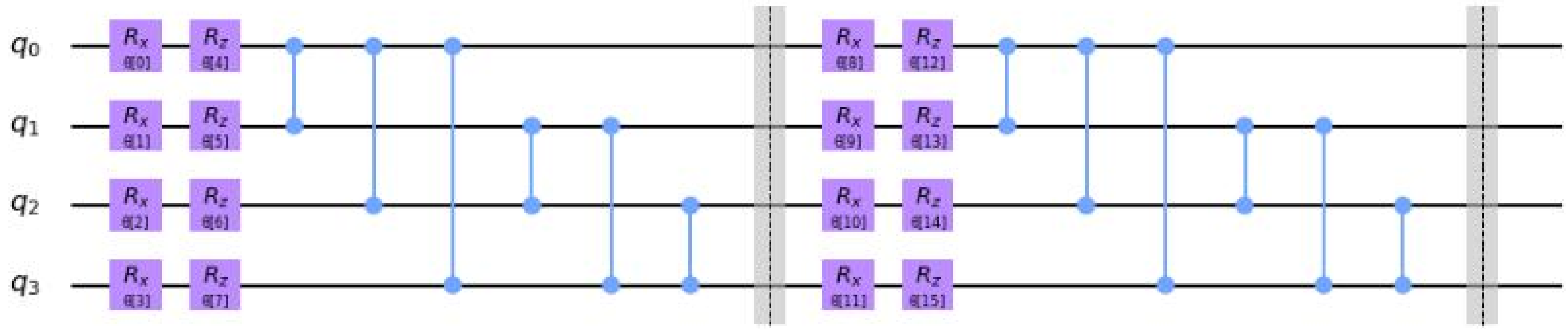
Therefore in total we have (N*2)*L parameters where L = number of layers (1 layers = 1 odd block + 1 even block)

In [2]:
```
def build_template(N, L, theta):
    circuit = QuantumCircuit(N)
    count = 0
    for _ in range(L):
        # Odd block
        for j in range(N):
            circuit.rx(theta[count],j)
            count+=1

        # Even block
        for j in range(N):
            circuit.rz(theta[count],j)
            count+=1
        for i in range(N):
            for j in range(i+1,N):
                    circuit.cz(i,j)

        circuit.barrier()
    return circuit
```

Here's the circuit built for 4 qubits and 2 Layers, we also pass a ParameterVector consisteing of (N*2)*L elements

In [3]:
```
build_template(N=4,L=2,theta=ParameterVector("θ",(4*2)*2)).draw()
```

Out[3]:



Helper functions

The get_statevector() function takes in a circuit and simulates it using qiskit's inbuilt statevector simulator backend and finally return the results as a list

In [4]:
```
def get_statevector(psi):
    backend = Aer.get_backend('statevector_simulator')
    result = execute(psi, backend).result()
    sv = result.get_statevector()
    return sv
```

To get a randomly generated vector on N qubits we use qiskit's inbuilt random_statevector() function which generates a statevector and return the list obtained. The statevector produced by N qubits should be of size 2^N.

In [5]:
```
def get_random_phi(N):
    random_phi = random_statevector(2**N).data
    return random_phi
```

To get initial parameters we use a random number generator which generates value between $[0, 1]$ and multiply it with 2π so as to get value in range of $[0, 2\pi]$

In [6]:
```
def get_initial_params(N,L):
    initial_params = list()
    for i in range((N*2)*L):
        initial_params.append(np.random.rand()*2*np.pi)
    return initial_params
```

The change_params() function creates and returns a new circuit based on an existing template and assigns all the parameters to the one's given in new_params.

In [7]:
```
def change_params(psi_template,theta,new_params):
    psi_assigned = psi_template
    for i in range(len(new_params)):
        psi_assigned = psi_assigned.assign_parameters({theta[i]:new_params[i]})
    return psi_assigned
```

To get the distance for a particular statevector we use the get_epsilon() function which takes in the statevector along with the randomly generated vector phi.

In [8]:
```
def get_epsilon(result,random_phi):
    e_theta = result - random_phi
    epsilon = 0
    for i in e_theta:
        epsilon += (i*np.conj(i)).real
    return np.sqrt(epsilon)
```

The circuit_to_epsilon function creates a circuit and assigns all the parameters, it calculates and returns epsilon for given parameters(params). This will be used by the find_minima function to arrive at the optimal parameters

In [9]:
```
def circuit_to_epsilon(params,N,L,phi):
    theta = ParameterVector("theta",(N*2)*L)
    psi_template = build_template(N,L,theta)
    psi_circuit = change_params(psi_template,theta,params)
    epsilon = get_epsilon(get_statevector(psi_circuit),phi)
    return epsilon
```

The find_minima function is used to find the minimum distance for a particular circuit. It requires **4 parameters**

- **N** - Number of qubits
- **L** - Number of layers
- **phi** - the random generated vetor on N qubits
- **current_params** - initial parameters from which the minimization is to be performed

And has 4 hyperparameters -

- **learning_rate** (default = 0.05)
- **delta** (default = 0.01)
- **tolerance** (default = 10^{-4})
- **momentum** (default = 0.8)

We use **Gradient Descent with Nesterov Accelerated Momentum** to optimize the parameters and find the minimum epsilon [distance].

Note: It is possible to get stuck at a local minima.

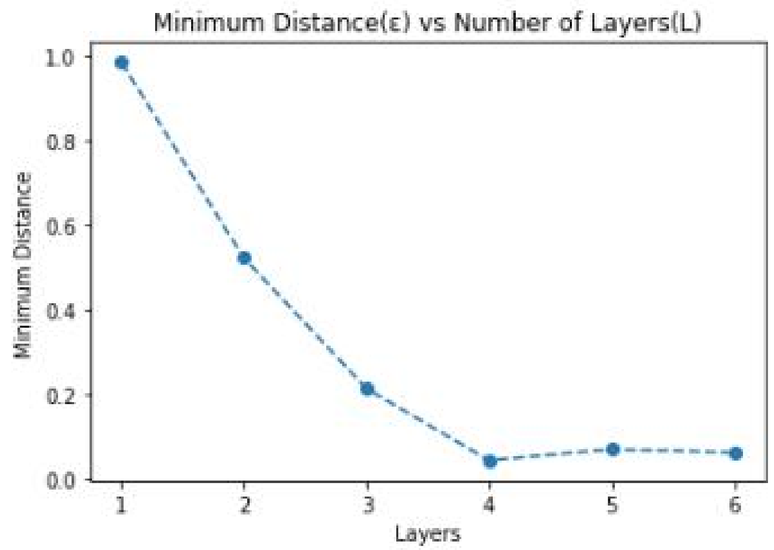
In [68]:
```
def find_minima(N,L,phi,current_params,learning_rate=0.05,delta=0.01,tolerance=1e-4,momentum=0.8):
    diff = 1
    iterations = 0
    params_len = len(current_params)
    momentum_update = [0 for i in range (params_len)]

    while(abs(diff)>tolerance):
        iterations +=1
        # Getting actual value for epsilon
        epsilon_prev = circuit_to_epsilon(current_params,N,L,phi)

        gradient = [0 for i in range (params_len)]

        # We find partial derivative by first principle, delta being the small update.
        for i in range(len (current_params)):
            new_params = current_params[:]
            new_params[i] = (current_params[i] + delta + momentum*momentum_update[i])%(2*np.pi) # mod by 2*pi (stick

            epsilon_next = circuit_to_epsilon(new_params,N,L,phi)

            gradient[i] = (epsilon_next-epsilon_prev)/delta

        # Updating Parameters to get predicted value of epsilon
        momentum_update = momentum*np.array(momentum_update) - learning_rate*np.array(gradient)
        final_params = [current_params[i] + momentum_update[i] for i in range(params_len)]

        epsilon_final = circuit_to_epsilon(final_params,N,L,phi)

        current_params = final_params[:]
        diff = epsilon_final - epsilon_prev


    return epsilon_final,final_params,iterations
```

Testing & Results

We find minimum distance for upto 6 layered circuits. As the optimizer used is not efficient this might take some time.

In [69]:
```
N = 4 # Number of qubits
MAX_LAYERS = 6 # Total number of layers
min_distances = list()
optimal_parameters = list()
iterations_per_layer = list()
time_per_layer = list()
phi = get_random_phi(N) # Same phi used for all L layered circuits

for L in range(1,MAX_LAYERS+1):
    initial_params = get_initial_params(N,L)

    start_time = time.time()
    final_epsilon,final_params,iterations = find_minima(N,L,phi,initial_params)
    end_time = time.time()

    min_distances.append(final_epsilon)
    optimal_parameters.append(final_params)
    iterations_per_layer.append(iterations)
    time_per_layer.append(end_time - start_time)
    print("Number of Layers - %d, Final epsilon - %.12f, Time taken - %.2fs"%(L,final_epsilon,time_per_layer[-1]))

# Plotting Results for Minimum Distance
plt.plot(list(range(1,MAX_LAYERS+1)),min_distances,'o--')
plt.xlabel("Layers")
plt.ylabel("Minimum Distance")
plt.title("Minimum Distance(ε) vs Number of Layers(L)")
plt.show()
```

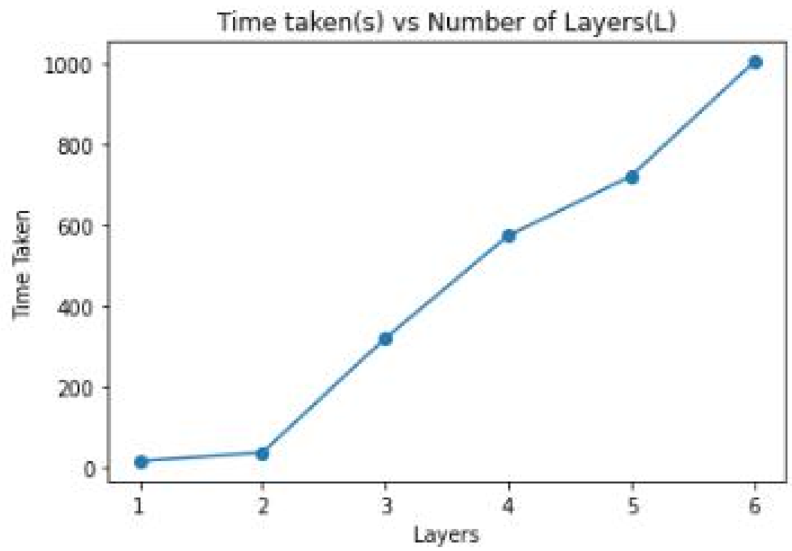
```
Number of Layers - 1, Final epsilon - 0.984235612540, Time taken - 16.67s
Number of Layers - 2, Final epsilon - 0.522358508610, Time taken - 37.38s
Number of Layers - 3, Final epsilon - 0.214597465789, Time taken - 320.84s
Number of Layers - 4, Final epsilon - 0.045164434209, Time taken - 575.10s
Number of Layers - 5, Final epsilon - 0.071549587393, Time taken - 722.75s
Number of Layers - 6, Final epsilon - 0.063657849315, Time taken - 1005.56s
```



We can infer from the graph that the minimum distance between ψ and ϕ is directly proportional to the number of layers, hence it is directly proportional to the number of parameters. Initially the minimum distance ϵ decreases rapidly as the number of layers increase, after a certain point we see stabilization and no significant decrease in distance is observed.

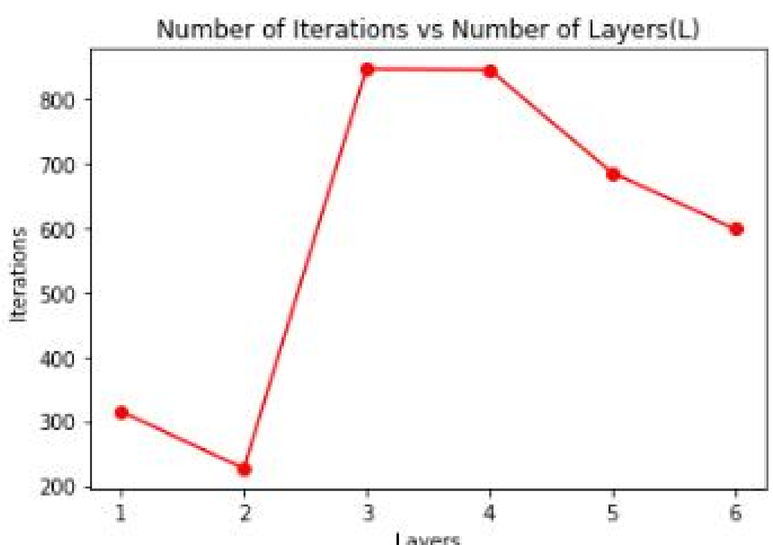
We try finding relation between time taken, number of iterations and number of layers below -

In [70]:
```
# Plotting Results for time
plt.plot(list(range(1,MAX_LAYERS+1)),time_per_layer,'o-')
plt.xlabel("Layers")
plt.ylabel("Time Taken")
plt.title("Time taken(s) vs Number of Layers(L)")
plt.show()
```



As we can see from the above plot, time taken increases for every additional layer. This is because of increasing number of parameters.

In [71]:
```
# Plotting Results for number of iterations
plt.plot(list(range(1,MAX_LAYERS+1)),iterations_per_layer,'ro-')
plt.xlabel("Layers")
plt.ylabel("Iterations")
plt.title("Number of Iterations vs Number of Layers(L)")
plt.show()
```



The reason we see no strong correlation between between the number of iterations and number of layers might be because of the poor optimizer.

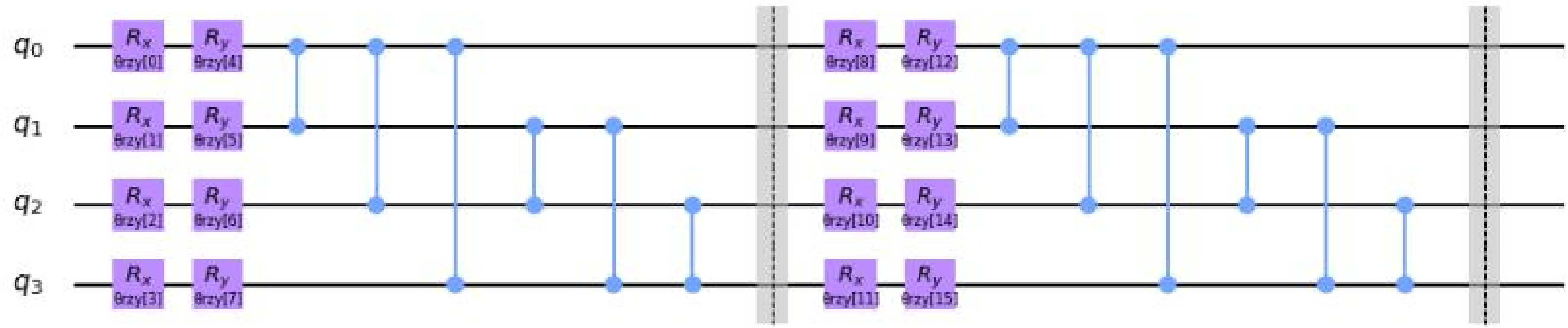
Bonus Question - Using other gates for the parametrized gates

Case #1 - Replacing Rz with Ry gates

In [73]:
```python
def build_circuit_rzy(N, L, theta):
    circuit = QuantumCircuit(N)
    count = 0
    for _ in range(L):
        # Odd block
        for j in range(N):
            circuit.rx(theta[count],j)
            count+=1

        # Even block
        for j in range(N):
            circuit.ry(theta[count],j)
            count+=1
        for i in range(N):
            for j in range(i+1,N):
                    circuit.cz(i,j)

        circuit.barrier()
    return circuit
```

In [74]:
```python
build_circuit_rzy(N=4,L=2,theta=ParameterVector("θrzy",(4*2)*2)).draw()
```

Out[74]:

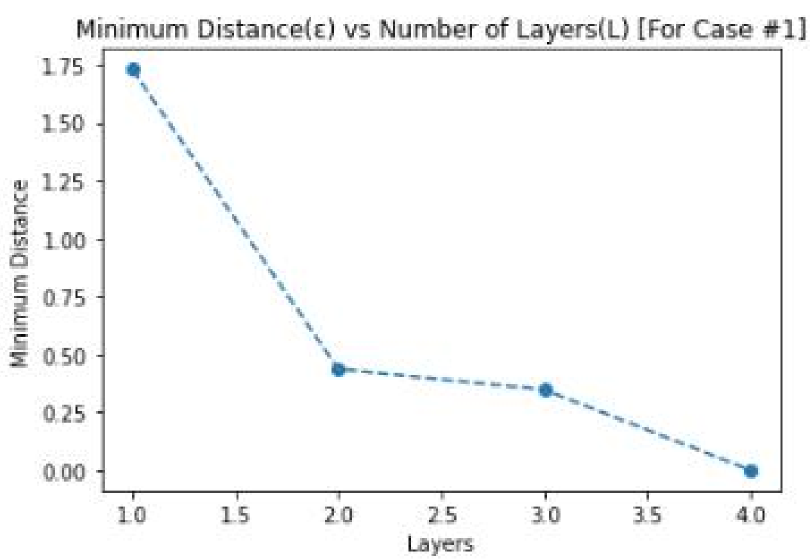


In [78]:
```python
def rzy_circuit_to_epsilon(params,N,L,phi):
    theta = ParameterVector("theta",(N*2)*L)
    psi_template = build_circuit_rzy(N,L,theta)
    psi_circuit = change_params(psi_template,theta,params)
    epsilon = get_epsilon(get_statevector(psi_circuit),phi)
    return epsilon
```

Unlike before where we used a custom optimizer to find the minimum distance, here we use **SciPy's inbuilt minimize function** to get the optimal parameters. Also we perform optimization upto 4 layers.

In [79]:
```
N = 4 # Number of qubits
MAX_LAYERS = 4 # Total number of layers
min_distances = list()
time_per_layer = list()
phi = get_random_phi(N) # Same phi used for all L layered circuits

for L in range(1,MAX_LAYERS+1):
    initial_params = get_initial_params(N,L)

    num_params = (N*2)*L
    bounds = [(0,2*np.pi)]*num_params

    start_time = time.time()
    result = minimize(rzy_circuit_to_epsilon,initial_params,args=(N,L,phi),bounds=bounds)
    end_time = time.time()

    min_distances.append(result.fun)
    time_per_layer.append(end_time - start_time)
    print("Number of Layers - %d, Final epsilon - %.12f, Time taken - %.2fs"%(L,result.fun,time_per_layer[-1]))

# Plotting Results for Minimum Distance
plt.plot(list(range(1,MAX_LAYERS+1)),min_distances,'o--')
plt.xlabel("Layers")
plt.ylabel("Minimum Distance")
plt.title("Minimum Distance(ε) vs Number of Layers(L) [For Case #1]")
plt.show()
```

```
Number of Layers - 1, Final epsilon - 1.730490758520, Time taken - 8.35s
Number of Layers - 2, Final epsilon - 0.436964020616, Time taken - 13.98s
Number of Layers - 3, Final epsilon - 0.347391487983, Time taken - 54.45s
Number of Layers - 4, Final epsilon - 0.000808265285, Time taken - 320.67s
```



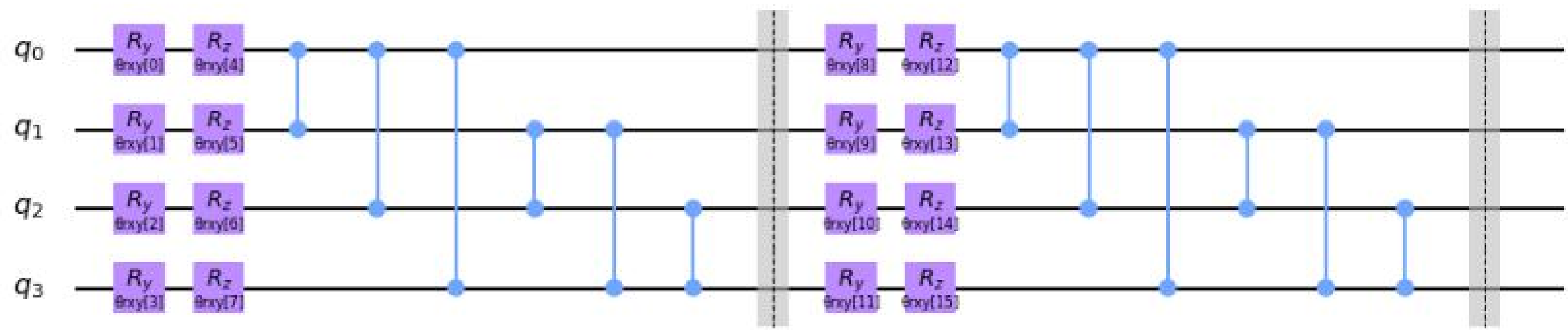
A similar trend is observed, the minimum distance decreases as the number of layers increase even after replacing all the Rz gates with Ry

Case #2 - Replacing Rx with Ry gates

In [81]:
```
def build_circuit_rxy(N, L, theta):
    circuit = QuantumCircuit(N)
    count = 0
    for _ in range(L):
        # Odd block
        for j in range(N):
            circuit.ry(theta[count],j)
            count+=1

        # Even block
        for j in range(N):
            circuit.rz(theta[count],j)
            count+=1
        for i in range(N):
            for j in range(i+1,N):
                circuit.cz(i,j)

        circuit.barrier()
    return circuit
```

In [82]:
```
build_circuit_rxy(N=4,L=2,theta=ParameterVector("θrxy",(4*2)*2)).draw()
```

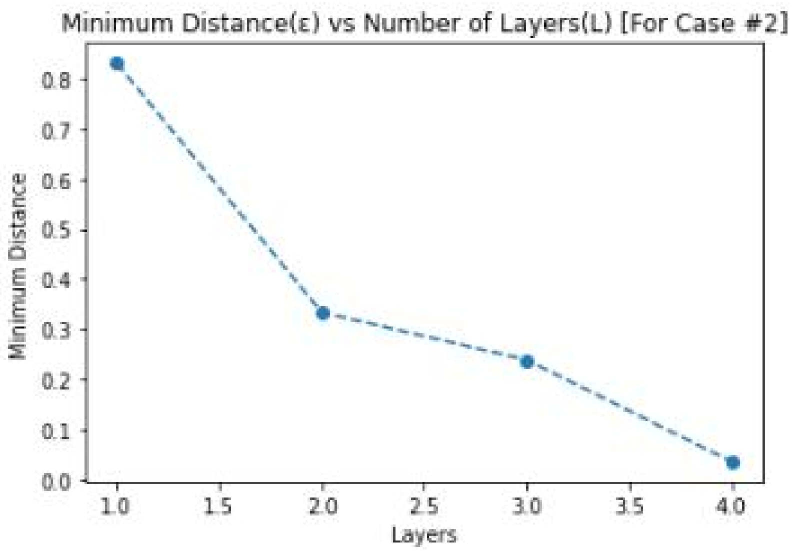
Out[82]:



In [84]:
```
def rxy_circuit_to_epsilon(params,N,L,phi):
    theta = ParameterVector("theta",(N*2)*L)
    psi_template = build_circuit_rxy(N,L,theta)
    psi_circuit = change_params(psi_template,theta,params)
    epsilon = get_epsilon(get_statevector(psi_circuit),phi)
    return epsilon
```

In [85]:
```python
N = 4 # Number of qubits
MAX_LAYERS = 4 # Total number of layers
min_distances = list()
time_per_layer = list()
phi = get_random_phi(N) # Same phi used for all L layered circuits

for L in range(1,MAX_LAYERS+1):
    initial_params = get_initial_params(N,L)

    num_params = (N*2)*L
    bounds = [(0,2*np.pi)]*num_params

    start_time = time.time()
    result = minimize(rxy_circuit_to_epsilon,initial_params,args=(N,L,phi),bounds=bounds)
    end_time = time.time()

    min_distances.append(result.fun)
    time_per_layer.append(end_time - start_time)
    print("Number of Layers - %d, Final epsilon - %.12f, Time taken - %.2fs"%(L,result.fun,time_per_layer[-1]))

# Plotting Results for Minimum Distance
plt.plot(list(range(1,MAX_LAYERS+1)),min_distances,'o--')
plt.xlabel("Layers")
plt.ylabel("Minimum Distance")
plt.title("Minimum Distance(ε) vs Number of Layers(L) [For Case #2]")
plt.show()
```

```
Number of Layers - 1, Final epsilon - 0.831293000917, Time taken - 0.88s
Number of Layers - 2, Final epsilon - 0.333375167315, Time taken - 9.60s
Number of Layers - 3, Final epsilon - 0.238243864475, Time taken - 190.01s
Number of Layers - 4, Final epsilon - 0.035627560321, Time taken - 174.63s
```



Again we notice a similar trend after replacing the Rx gates with Ry gate.

Conclusion: For the given circuit we see that distance decreases drastically as the number of layers increase, after a certain number of layers we don't notice such drastic decrease and stabilization of ϵ is observed. This trend hold true even after replacing the Rx or the Rz gates with Ry gates.