Equation of Circles Let $\mathcal{C}$ be the circle with center at $C(h, k)$ and radius r. Then an equation for $\mathcal{C}$ is

$$(x - h)^2 + (y - k)^2 = r^2. \tag{0.8.1}$$

Proof Let $P(x, y)$ be any point on the circle. Since the distance from P to the center C is r, using the distance formula, we get

$$\sqrt{(x - h)^2 + (y - k)^2} = r.$$

Squaring both sides yields (0.8.1). □

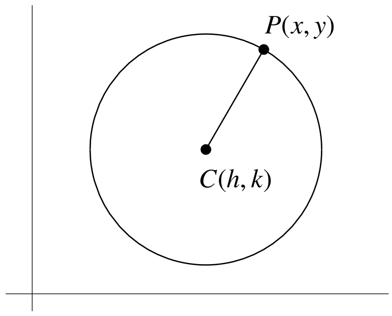

Figure 0.4

Example Find the center and radius of the circle given by

$$x^2 - 4x + y^2 + 6y - 12 = 0.$$

Solution Using the completing square method, the given equation can be written in the form (0.8.1).

$$\begin{aligned} x^2 - 4x + y^2 + 6y &= 12 \\ (x^2 - 4x + 4) + (y^2 + 6y + 9) &= 12 + 4 + 9 \\ (x - 2)^2 + (y + 3)^2 &= 25 \\ (x - 2)^2 + (y - (-3))^2 &= 5^2 \end{aligned}$$

The center is $(2, -3)$ and the radius is 5. □

FAQ How do we get the number "9" etc (the numbers added to both sides)?

Answer We want to find a number (denoted by a) such that $(y^2 + 6y + a)$ is a complete square. That is,

$$y^2 + 6y + a = (y + b)^2 \tag{0.8.2}$$

for some number b. Expanding the right-side of (0.8.2) (do this in your head) and comparing the coefficients of y on both sides, we get $2b = 6$, that is, $b = 3$. Hence comparing the constant terms on both sides, we get $a = b^2 = 9$.

Summary a = square of half of the coefficient of y. □

Exercise 0.8

1. For each of the following pairs of points, find the distance between them.
   (a) $(-3, 4)$ and the origin (b) $(4, 0)$ and $(0, -7)$
   (c) $(7, 5)$ and $(12, 17)$ (d) $(-2, 9)$ and $(3, -1)$
2. For each of the following circles, find its radius and center.
   (a) $x^2 + y^2 - 4y + 1 = 0$ (b) $x^2 + y^2 + 4x - 2y - 4 = 0$
   (c) $2x^2 + 2y^2 + 4x - 2y + 1 = 0$
3. For each of the following, find the distance from the given point to the given line.
   (a) $(-2, 3)$ and the y-axis
   (b) the origin and $x + y = 1$
   (c) $(1, 2)$ and $2x + y - 6 = 0$