interpreted as $s_2 + s_3 + s_4 + s_5 + s_6 + s_7$. Note that the index is used only to keep track of the terms to be added; it does not factor into the calculation of the sum itself. The index is therefore called a *dummy variable*. We can use any letter we like for the index. Typically, mathematicians use *i*, *j*, *k*, *m*, and *n* for indices.

Let's try a couple of examples of using sigma notation.

Example 5.1

Using Sigma Notation

a. Write in sigma notation and evaluate the sum of terms 3^i for $i = 1, 2, 3, 4, 5$.

b. Write the sum in sigma notation:

$$1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25}.$$

Solution

a. Write

$$\begin{aligned}\sum_{i=1}^{5} 3^i &= 3 + 3^2 + 3^3 + 3^4 + 3^5 \\ &= 363.\end{aligned}$$

b. The denominator of each term is a perfect square. Using sigma notation, this sum can be written as $\sum_{i=1}^{5} \frac{1}{i^2}$.

5.1 Write in sigma notation and evaluate the sum of terms 2^i for $i = 3, 4, 5, 6$.

The properties associated with the summation process are given in the following rule.

Rule: Properties of Sigma Notation

Let $a_1, a_2, \ldots, a_n$ and $b_1, b_2, \ldots, b_n$ represent two sequences of terms and let *c* be a constant. The following properties hold for all positive integers *n* and for integers *m*, with $1 \le m \le n$.

1.
$$\sum_{i=1}^{n} c = nc \tag{5.1}$$

2.
$$\sum_{i=1}^{n} ca_i = c\sum_{i=1}^{n} a_i \tag{5.2}$$

3.
$$\sum_{i=1}^{n} (a_i + b_i) = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i \tag{5.3}$$

4.
$$\sum_{i=1}^{n} (a_i - b_i) = \sum_{i=1}^{n} a_i - \sum_{i=1}^{n} b_i \tag{5.4}$$