Bellevue Hospital Center

City: Manhattan, New York
[CDC Authorized Lab]

Drone Routing Report

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Bellevue

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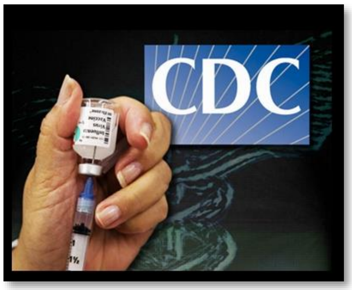


INTRODUCTION

Summary

Ever since the Center for Disease Control and Prevention (CDC) developed the power vaccination called NOSE for the relief of common cold, our main attention is that the vaccine must be transported immediately to the various nearby hospitals because of its short shelf life i.e. the vaccines produced in the lab need to be delivered to its nearby hospitals in the same hour. For that to happen, we need the help of drones to ship the vaccines as soon as the vaccine is produced by the laboratory.

Our main objective is to minimize the costs of scheduling the drones and the transportation costs of the drones.

Data Collection and Processing

Cities Location:

Cities were chosen based on the population of the city (min. 70,000 people in the city) and the distance from the lab (max 100 miles).

Hospital Location:

Major hospitals were selected based on a consistent and well-defined criterion i.e. the best-rated hospital and a centrally located hospital in the city.

Distance Collection:

Since, the drones are used to the transport the vaccines from the lab to the major hospitals (point-point trajectory), a straight-line distance (Google walking/driving distance were not used in this collection) was found to-and-fro the lab via the various major hospitals in the nearby cities and the distance between the various major hospitals in the cities. Within the distance matrix we generated, the diagonals were deliberately set at a very large value which essentially prohibits the program from forming a subtour of a drone from a certain hospital immediately to itself. https://distancecalculator.globefeed.com/US_Distance_Calculator.asp is the link we used to calculate the straight-line distances between all the hospitals.

Hour Calculation:

To calculate the hour i, the sum of the employee IDs (3639710770), modulo 4, and add 1.

Hour, i = 3

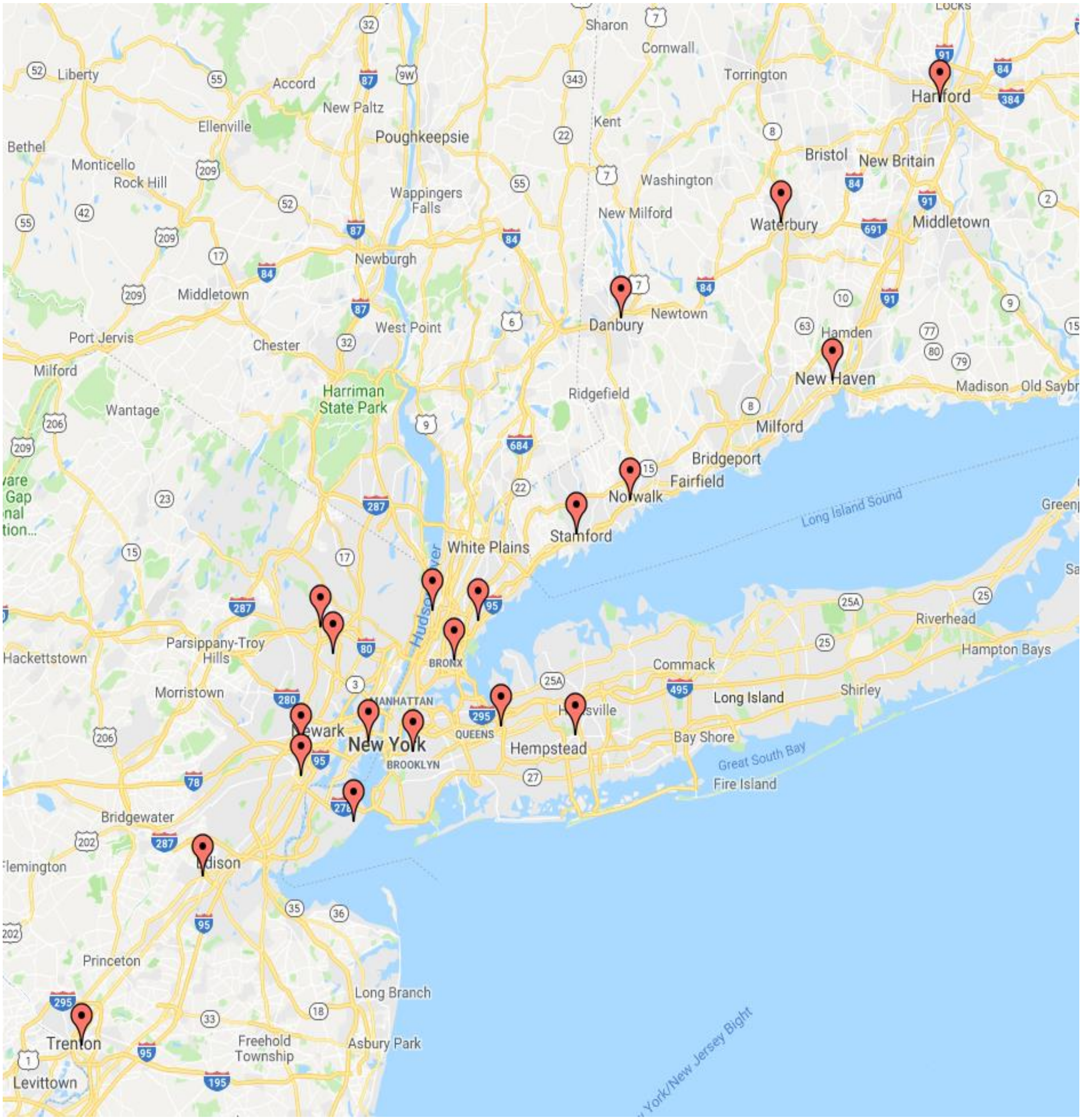
Hospital List

Hospital Name	City	State	Address	Population	Miles	Latitude	Longitude
NYC Health + Hospitals/Woodhull	Brooklyn	NY	760 Broadway, Brooklyn, NY 11206	2.649M	3.22	40.699336	-73.942747
Creedmoor Psychiatric Center	Queens	NY	79-25 Winchester Blvd, Queens Village, NY 11427	2.359 M	12.69	40.74115	-73.73067
Calvary Hospital	Bronx	NY	1740 Eastchester Rd, Bronx, NY 10461, USA	1.471M	10.21	40.848002	-73.843782
Staten Island University Hospital	Staten Island	NY	475 Seaview Ave, Staten Island, NY 10305, USA	479000	12.14	40.585263	-74.085023
St. Joseph's Medical Center	Yonkers	NY	127 S Broadway, Yonkers, NY 10701	202,019	13.76	40.910162	-73.896607
Montefiore New Rochelle Hospital	New Rochelle	NY	16 Guion Pl, New Rochelle, NY 10802, USA	80000	15.48	40.913144	-73.788059
Center for Primary Care	Hempstead	NY	2201 Hempstead Turnpike, East Meadow, NY 11554	774,959	22.09	40.7261764	-73.5537497
Jersey City Medical Center	Jersey City	NJ	355 Grand St, Jersey City, NJ 07302, USA	270,753	4.24	40.715257	-74.050824
Children's Hospital in New Jersey	Newark	NJ	201 Lyons Ave, Newark, NJ 07112	285,154	12.63	40.710274	-74.212585
RWJ University Hospital New Brunswick	New Brunswick	NJ	1 Robert Wood Johnson Pl, New Brunswick, NJ 08901	71000	30.1	40.495819	-74.448242
Fresenius Hospital Care	Passaic	NJ	350 Boulevard, Passaic, NJ 07055	71247	11.86	40.859469	-74.137986
St. Francis Medical Center	Trenton	NJ	601 Hamilton Ave, Trenton, NJ 08629, USA	85000	54.06	40.2166342	-74.741205
St. Joseph's Medical Center	Paterson	NJ	703 Main St, Paterson, NJ 07503, USA	148000	15.06	40.903608	-74.165726
Trinitas Regional Medical Center	Elizabeth	NJ	225 Williamson St, Elizabeth, NJ 07202, USA	148000	13.69	40.658751	-74.214224
Stamford Hospital - Bennett Medical Center	Stamford	CT	1 Hospital Plaza, Stamford, CT 06902	130000	31.04	41.0558536	-73.5538648
Waterbury Hospital	Waterbury	CT	64 Robbins St, Waterbury, CT 06708, USA	108000	74.06	41.55843	-73.058719
Yales New Haven Children's Hospital	New Haven	CT	1 Park St, New Haven, CT 06504	131000	66.77	41.3043831	-72.936538
Hartford Hospital	Hartford	CT	80 Seymour St, Hartford, CT 06102, USA	123000	97.22	41.7545616	-72.6792339
Norwalk Hospital	Norwalk	CT	34 Maple St, Norwalk, CT 06850, USA	89000	38.66	41.110862	-73.421968
Danbury Hospital	Danbury	CT	24 Hospital Ave, Danbury, CT 06810, USA	85246	53.66	41.4050857	-73.446289
Bellevue Hospital Center	Manhattan	NY	462 1st Avenue, New York, NY 10016	1.665M	0	40.739267	-73.97536

Distance Between Hospitals

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
	NYC Hea	Creedmo	Calvary H	Staten Isl	St. Josepl	Montefior	Center fo	Jersey Ci	Children's	RWJ Univ	Fresenit	St. Franci	St. Josepl	Trinitas R	Stamford	Waterbur	Yales Ne	Hartford H	Norwalk	Danbury	Bellevue
NYC Health + Hospitals/Woodhull	10000	11.42	11.49	10.98	16.05	16.84	20.45	5.76	14.21	30.1	15	53.62	18.24	14.48	31.92	75.09	67.07	98.1	39.34	55.24	3.22
Creedmoor Psychiatric Center	11.42	10000	9.43	21.56	15.62	12.21	9.36	16.79	25.34	41.32	22.73	64.27	25.3	25.9	23.62	66.42	56.83	88.81	30.21	48.2	12.69
Calvary Hospital	11.49	9.43	10000	22.23	6.28	5.37	17.36	14.12	21.55	40.01	15.34	64.2	17.24	23.33	20.87	63.84	56.82	87.05	28.54	43.75	10.21
Staten Island University Hospital	10.98	21.56	22.23	10000	25.83	27.61	29.6	9.35	10.99	20.02	19.2	42.78	22.39	8.51	42.89	86.02	77.97	109.08	50.32	65.83	12.14
St. Joseph's Medical Center	16.05	15.62	6.28	25.83	10000	5.87	22.82	16.74	22.4	41.64	13.41	66.2	14.13	24.91	19.96	61.55	56.35	85.13	27.79	40.4	13.76
Montefiore New Rochelle Hospital	16.84	12.21	5.37	27.61	5.87	10000	17.78	19.34	26.33	45.13	18.65	69.43	19.79	28.36	15.67	58.49	51.89	81.76	23.44	38.34	15.48
Center for Primary Care	20.45	9.36	17.36	29.6	22.82	17.78	10000	26.01	34.57	49.62	31.87	71.65	34.25	34.9	22.78	63.02	51.29	84.33	27.47	47.24	22.09
Jersey City Medical Center	5.76	16.79	14.12	9.35	16.74	19.34	26.01	10000	8.55	25.91	10.86	50.09	14.21	9.41	34.98	77.76	70.88	101.11	42.65	57.07	4.24
Children's Hospital in New Jersey	14.21	25.34	21.55	10.99	22.4	26.33	34.57	8.55	10000	19.34	11.03	43.96	13.5	3.49	41.93	83.93	78.22	107.52	49.74	62.53	12.63
RWJ University Hospital New Brunsw	30.1	41.32	40.01	20.02	41.64	45.13	49.62	25.91	19.34	10000	29.99	24.62	31.8	16.79	60.8	103.19	96.79	126.72	68.54	81.81	30.1
Fresenius Hospital Care	15	22.73	15.34	19.2	13.41	18.65	31.87	10.86	11.03	29.99	10000	54	3.34	14.34	33.34	74	70.01	97.74	41.46	52.62	11.86
St. Francis Medical Center	53.62	64.27	64.2	42.78	66.2	69.43	71.65	50.09	43.96	24.62	54	10000	56.17	41.3	85.07	127.73	120.69	151.17	92.72	106.48	54.06
St. Joseph's Medical Center	18.24	25.3	17.24	22.39	14.13	19.79	34.25	14.21	13.5	31.8	3.34	56.17	10000	16.94	33.65	73.24	69.78	97.05	41.38	51.08	15.06
Trinitas Regional Medical Center	14.48	25.9	23.33	8.51	24.91	28.36	34.9	9.41	3.49	16.79	14.34	41.3	16.94	10000	44.04	86.43	80.15	109.93	51.8	65.21	13.69
Stamford Hospital - Bennett Medical (	31.92	23.62	20.87	42.89	19.96	15.67	22.78	34.98	41.93	60.8	33.34	85.07	33.65	44.04	10000	43.2	36.4	66.22	7.85	24.8	31.04
Waterbury Hospital	75.09	66.42	63.84	86.02	61.55	58.49	63.02	77.76	83.93	103.19	74	127.73	73.24	86.43	43.2	10000	18.72	23.84	36.21	22.66	74.06
Yales New Haven Children's Hospital	67.07	56.83	56.82	77.97	56.35	51.89	51.29	70.88	78.22	96.79	70.01	120.69	69.78	80.15	36.4	18.72	10000	33.83	28.56	27.34	66.77
Hartford Hospital	98.1	88.81	87.05	109.08	85.13	81.76	84.33	101.11	107.52	126.72	97.74	151.17	97.05	109.93	66.22	23.84	33.83	10000	58.8	46.35	97.22
Norwalk Hospital	39.34	30.21	28.54	50.32	27.79	23.44	27.47	42.65	49.74	68.54	41.46	92.72	41.38	51.8	7.85	36.21	28.56	58.8	10000	20.35	38.66
Danbury Hospital	55.24	48.2	43.75	65.83	40.4	38.34	47.24	57.07	62.53	81.81	52.62	106.48	51.08	65.21	24.8	22.66	27.34	46.35	20.35	10000	53.66
Bellevue Hospital Center	3.22	12.69	10.21	12.14	13.76	15.48	22.09	4.24	12.63	30.1	11.86	54.06	15.06	13.69	31.04	74.06	66.77	97.22	38.66	53.66	10000

Hospital Location



Demand Estimation

The estimated demand during hour i of city j, d_{ij}, is calculated via the following formula:

$$d_{ij} = p_i \times 35 \times \log^2(Pop_j),$$

where p_i is a parameter obtained from the function:

$$p_i = 1 - e^{-1/i}$$

	i = 1	i= 2	i= 3	i= 4
P_i	0.6321	0.3935	0.2835	0.2212

			Demand during hour i of city j (d_{ij})			
Hospital Name	**Population**	**$\log_2(Pop_j)$**	**i=1**	**i=2**	**i=3**	**i=4**
NYC Health + Hospitals/Woodhull	2,649,000	21.33701641	473	294	212	166
Creedmoor Psychiatric Center	2359000	21.16974399	469	292	211	164
Calvary Hospital	1471000	20.48836582	454	283	204	159
Staten Island University Hospital	479000	18.86966613	418	260	188	147
St. Joseph's Medical Center	202,019	17.62413146	390	243	175	137
Montefiore New Rochelle Hospital	80000	16.28771238	361	225	162	127
Center for Primary Care	774,959	19.56376046	433	270	195	152
Jersey City Medical Center	270,753	18.0466178	400	249	180	140
Children's Hospital in New Jersey	285,154	18.12138174	401	250	180	141
RWJ University Hospital New Brunswick	71000	16.1155314	357	222	160	125
Fresenius Hospital Care	71247	16.12054165	357	223	160	125
St. Francis Medical Center	85000	16.37517522	363	226	163	127
St. Joseph's Medical Center	148000	17.17523765	380	237	171	133
Trinitas Regional Medical Center	148000	17.17523765	380	237	171	133
Stamford Hospital - Bennett Medical Center	130000	16.9881521	376	234	169	132
Waterbury Hospital	108000	16.72067179	370	231	166	130
Yales New Haven Children's Hospital	131000	16.99920729	377	235	169	132
Hartford Hospital	123000	16.90829879	375	233	168	131
Norwalk Hospital	89000	16.44151772	364	227	164	128
Danbury Hospital	85246	16.37934452	363	226	163	127
Bellevue Hospital Center	1665000	20.66709075	0	0	0	0
Sum of demands in hour, i			7861	4897	3531	2756

Modeling

Mathematical Programming Formulation: We use a class of Integer Programming called Set Covering problems. Consider a set X consisting of the 'jobs' to be performed. Consider another set Y to be the set of 'means' by which the jobs can be performed. A means is a way in which one or more jobs can be accomplished. Our aim is to find the the subset of means which cover each job at least once and at minimum cost. The set of jobs is usually given, while the set of means is generated explicitly or implicitly (using a heuristic).

x_j = {Number of Drones Type 1 in shift j} nonnegative integer
y_k = { Number of Drones Type 2 in shift k} nonnegative integer

The objective function is,

$$\text{Min } z = \sum_j \boldsymbol{x_j * 350 + \sum_k y_k * 550}$$

Constraints are,

$$\sum_j \boldsymbol{x_j * 450 * T1_{ij}} + \sum_k \boldsymbol{y_k * 1000 * T2_{ij} \geq Demand_i}$$

$$\sum_j \boldsymbol{x_j \geq 3}$$

$$\sum_k \boldsymbol{y_k \geq 2}$$

First constraint ensures that total demand of the hospitals is satisfied for hour i

Second and third constraints takes into account the battery life of Type 1 and Type 2 drones, which cannot be used for more than 3 and 2 hours over all 4 hours respectively.

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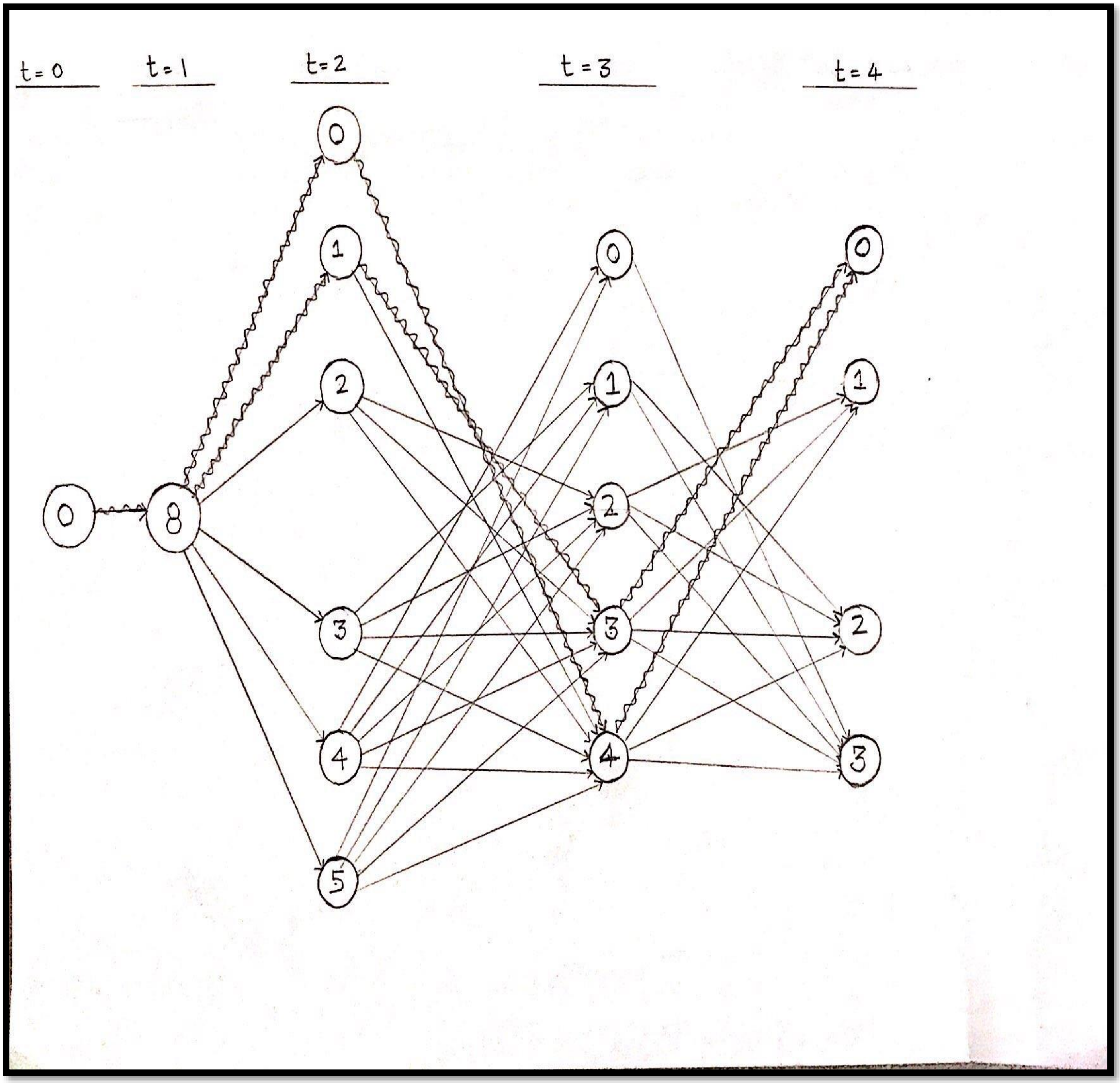


This is one of the methodologies that we use in order to make sure that we schedule the minimum number of drones required to satisfy the demand for vaccines across every hour. The main concept of dynamic programming is that it works backwards from the end of the problem towards the beginning, thus breaking up a large, cumbersome problem into smaller and more traceable subproblems.

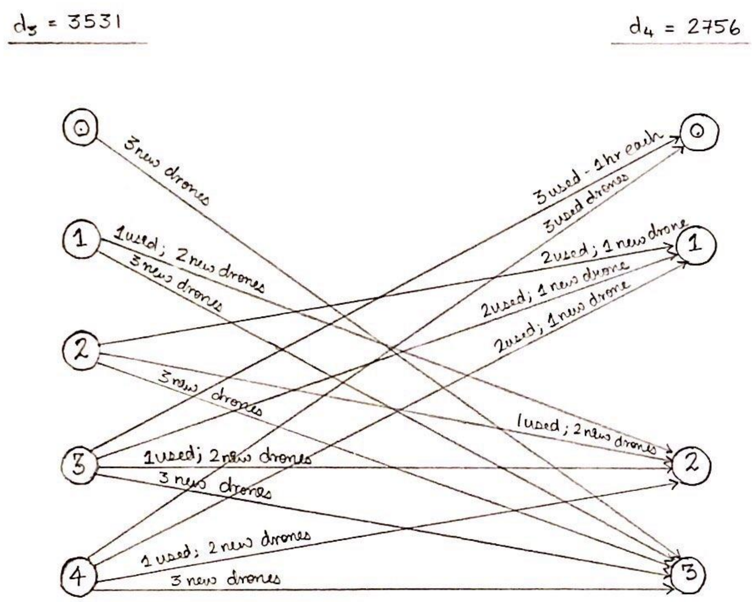
We divide the problem up into a set of stages and states. In the diagram above, the states (represented by the nodes) denote the number of drones scheduled from the previous hour which now have 1 hour of battery life left after satisfying the demand at that hour t. The stages are the hours t=0,1,2,3,4 where t=0 is a dummy stage. The lines in the diagram indicate all the possible decisions taken from that state, while the wavy lines above denote our two alternate optimal sets of decisions and solutions.

We know the states because we have the privilege of hindsight as the demand is deterministic in nature. Hence, in this case the answer(s) is straightforward, but this would not be the case if the demand had an element of stochastic variation.

Type 2:

We segmented out each subproblem and its respective calculations below. If the decision involves scheduling new drones from that stage, that drone (being Type 2) starts with 2 hours of battery life until the demand in the next hour is completely satisfied.

Below is an illustration of all the states and possible decisions that can be taken from hour 3 to hour 4:



DYNAMIC PROGRAMMING

$f_t(s)$ = minimum no. of drones from node S to node T in stage t

→ $\underline{t = 4}$:

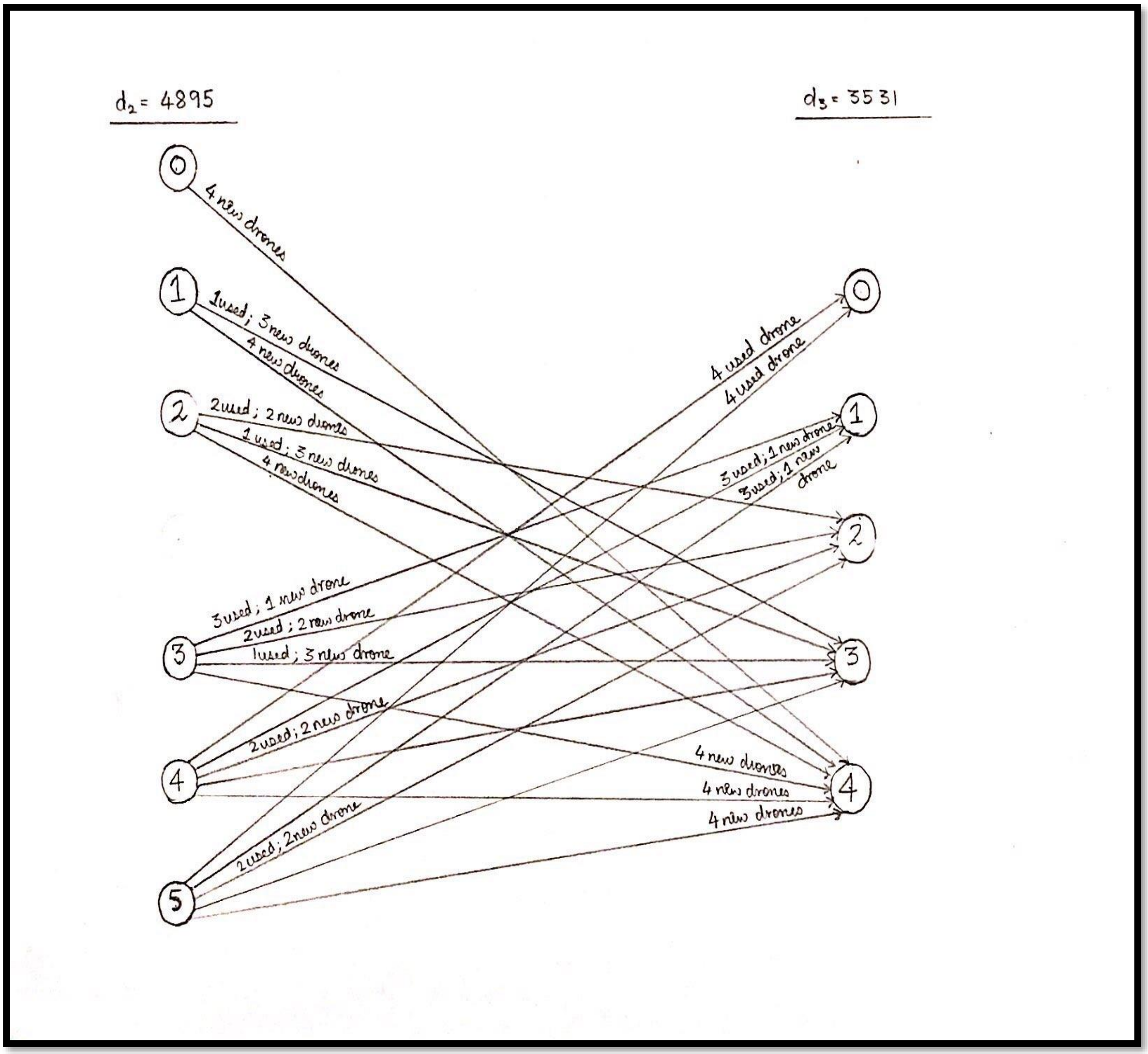
$f_4(0) = 0$

$f_4(1) = 1$

$f_4(2) = 2$

$f_4(3) = 3$

Similarly, below is an illustration of all the states and possible decisions that can be taken from hour 2 to hour 3:



$\longrightarrow$ $t = 3$:

$$f_3(0) = \min\{0 + f_4(3)\} = 3$$

$$f_3(1) = \min\{1 + f_4(3), 1 + f_4(2)\} = \{4, 3\} = 3$$

$$f_3(2) = \min\{2 + f_4(1), 2 + f_4(2), 2 + f_4(3)\} = \{3, 4, 5\} = 3$$

$$f_3(3) = \min\{3 + f_4(0), 3 + f_4(1), 3 + f_4(2), 3 + f_4(3)\} = \{3, 4, 5, 6\} = 3$$

$$f_3(4) = \min\{4 + f_4(0), 4 + f_4(1), 4 + f_4(2), 4 + f_4(3)\} = \{4, 5, 6, 7\} = 4$$

$\longrightarrow$ $t = 2$:

$$f_2(0) = \min\{0 + f_3(4)\} = 4$$

$$f_2(1) = \min\{1 + f_3(3), 1 + f_3(4)\} = \{4, 5\} = 4$$

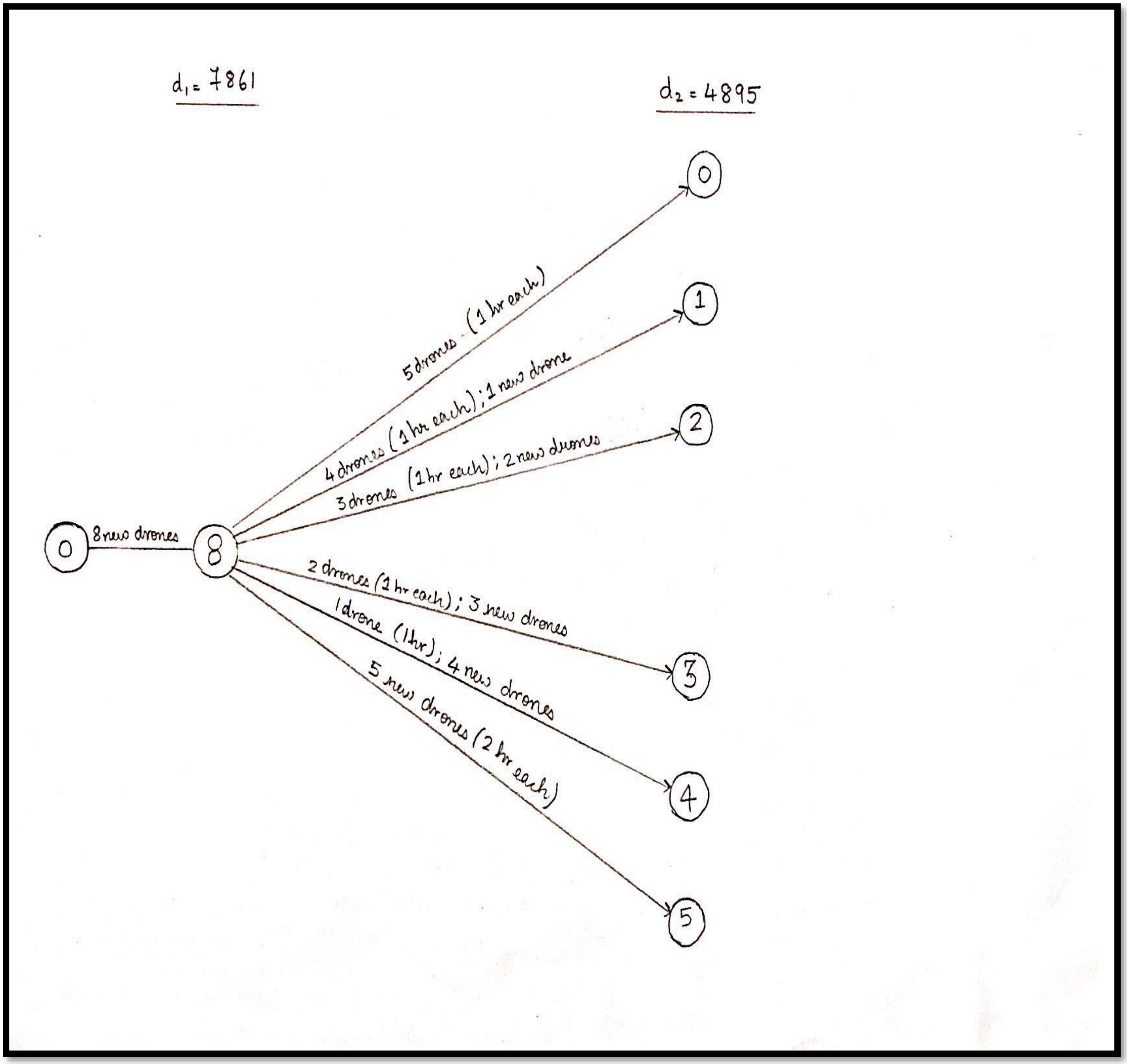
$$f_2(2) = \min\{2 + f_3(2), 2 + f_3(3), 2 + f_3(4)\} = \{5, 5, 6\} = 5$$

$$f_2(3) = \min\{3 + f_3(1), 3 + f_3(2), 3 + f_3(3), 3 + f_3(4)\} = \{6, 6, 6, 7\} = 6$$

$$f_2(4) = \min\{4 + f_3(0), 4 + f_3(1), 4 + f_3(2), 4 + f_3(3), 4 + f_3(4)\} = \{7, 7, 7, 7, 8\} = 7$$

$$f_2(5) = \min\{5 + f_3(0), 5 + f_3(1), 5 + f_3(2), 5 + f_3(3), 5 + f_3(4)\} = \{8, 8, 8, 8, 9\} = 8$$

Similarly, below is an illustration of all the states and possible decisions that can be taken from hour 1 to hour 2, as well as from the dummy stage:



$\rightarrow$ $t = 1$:

$$f_1(8) = \min\{8+f_2(0), 8+f_2(1), 8+f_2(2), 8+f_2(3), 8+f_2(4), 8+f_2(5)\} = \{12, 12, 13, 14, 15, 16\} = 12$$

$\rightarrow$ $t = 0$:

$$f_0(0) = \min\{0 + f_1(8)\} = 12$$

Thus, the minimum number of drones we can schedule while satisfying the demand is 12.

At Stage 1: We will have scheduled 8 drones

At Stage 2: We will have scheduled either 0 or 1 drones

At Stage 3: We will have scheduled 4 or 3 drones

At Stage 4: We will have scheduled 0 drones

The recursive function is $f_t(S) = \min \{x_t + f_{t+1}(x_t)\}$ subject to $d_t <= (x_t + S)*1000$

The objective function = $f_0(0)$

Where S <- number of drones with 1 hour of battery life

x_t <- the number of new drones scheduled at time t

Type 1

Type 1 drones can have up to 3 hours of battery life, thus enumerating the number of possible states and decisions increases the complexity of the task. We will have to consider the number of drones with 2 hours of battery life and the number of drones with 1 hour of battery life when considering the best decision at each state and at each stage. We have formulated the recursive and objective function below:

The recursive function is $f_t(S_1, S_2) = \min \{x_t + f_{t+1}(x_t, x_{t+1})\}$ subject to $d_t <= (x_t + S_1 + S_2)*1000$

The objective function = $f_0(0,0)$

Where S_1 <- number of drones with 1 hour of battery life

S_2 <- number of drones with 2 hours of battery life

x_t <- the number of new drones scheduled at time t

Modeling

The Vehicle Routing Problem (VRP) is one of the most studied among the NP-Hard combinatorial optimization problems, due both to its practical relevance and considerable difficulty.

In the Capacitated Vehicle Routing Problem (CVRP), all the customers correspond to deliveries, the demands are deterministic, known in advance and may not be split, the vehicles (drones of a type) are identical and are based at a single central depot (lab), only the capacity restrictions for the vehicles are imposed, and the objective is to minimize the total cost (i.e., the number of routes and/or their length of travel time) needed to serve all the customers. Generally, the travel cost between each pair of customer locations is the same in both directions, i.e., the resulting cost/distance matrix is symmetric.

We used the MTZ and Subtour Elimination Formulations to solve for the given problem.

Exact Formulation #1: MTZ

X_{ijk}= {1 if we go to the particular city i in Start to j in Dest; 0 if we don't go, k in Drone}
Y_{ik} = {1 if we enter city i in Start; 0 if not, k in Drone}

The objective function is,

$$\text{Min } \boldsymbol{z} = \sum_i \sum_j \sum_k \quad \boldsymbol{c_{ij}\, x_{ijk}} \ (1)$$

Constraints are,

$\sum_{i=1}^{i=N} \boldsymbol{d_i y_{ik} <= DC_k} \ (\boldsymbol{for\ j = 1, 2, 3 \ldots., N, for\ k\ =\ 1, .., m})$ *(2)*
$\sum_{k=1}^{k=m} \boldsymbol{y_{ik} = 1} \ (\boldsymbol{for\ i = 1, 2, 3 \ldots., N - 1})$ (3)
$\sum_{k=1}^{k=m} \boldsymbol{y_{ik} >= m} \ (\boldsymbol{for\ i = N})$ (4)
$\sum_{i=1}^{i=N} \sum_{k=1}^{k=m} \quad \boldsymbol{x_{ijk} >= 1} \ (\boldsymbol{for\ j = 1, 2, 3 \ldots., N})$ *(5)*
$\sum_{j=1}^{j=N} \boldsymbol{x_{ijk} = y_{ik}} \ (\boldsymbol{for\ i = 1, 2, 3 \ldots., N, k = 1, .., m})$ (6)
$\boldsymbol{u_i - u_j + N x_{ij} \leq N - 1} \ (\boldsymbol{for\ i \neq j;\ \ i = 1, 2, 3 \ldots., N; j = 1, 2, 3 \ldots., N})$ (7)
Where all $\boldsymbol{x_{ij} = 0\ or\ 1, all\ u_j >= 0}$
(1): is the objective function that minimizes the cost with respect to the arc traveled from i to j and the drone k used in this arc knowing that we have m available drones
(2): The demand d per station i should be less or equal to the capacity DC of the drone K
(3): Just one drone should leave the other hospitals.
(4): m vehicles should leave the lab

(5): ensures that the demand for each hospital is met by 1 drone
(6): The number of entering drones should be equal to the number of leaving drones per each hospital,i
(7): ensures that all sets of $\boldsymbol{x_{ij}}$ sub tours would be infeasible and all sets of $\boldsymbol{x_{ij}}$ that form a tour would be feasible

The above formulation was reconfigured from the textbook formulation [5].

Exact Formulation #2: Sub-Tour Elimination

X_{ijk}= {1 if we go to the particular city i in Start to j in Dest; 0 if we don't go, k in Drone}
Y_{ik} = {1 if we enter city i in Start; 0 if not, k in Drone}
The objective function is,

$$\text{Min } \boldsymbol{z} = \sum_i \sum_j \sum_k \quad \boldsymbol{c_{ij}\, x_{ijk}} \ (1)$$

Constraints are,

$$\sum_{i=1}^{i=N} \boldsymbol{d_i y_{ik} <= DC_k \ (for\ j = 1, 2, 3 \ldots., N, for\ k = 1, .., m)} \ \mathit{(2)}$$
$$\sum_{k=1}^{k=m} \boldsymbol{y_{ik} = 1 \ (for\ i = 1, 2, 3 \ldots., N - 1)} \ \mathit{(3)}$$
$$\sum_{k=1}^{k=m} \boldsymbol{y_{ik} >= m \ (for\ i = N)} \ \mathit{(4)}$$
$$\sum_{i=1}^{i=N} \sum_{k=1}^{k=m} \quad \boldsymbol{x_{ijk} = 1 \ (for\ j = 1, 2, 3 \ldots., N)} \ \mathit{(5)}$$
$$\sum_{j=1}^{j=N} \boldsymbol{x_{ijk} = y_{ik} \ (for\ i = 1, 2, 3 \ldots., N, k = 1, .., m)} \ \mathit{(6)}$$
$$\sum_{i,j\epsilon S} \boldsymbol{x_{ijk} <= |S| - 1 \ (for\ S\ C\ \{1, .., N\} = 1, 2, 3 \ldots., N, 2 \leq |S| \leq N - 2, k = 1, .., m)} \ \mathit{(7)}$$

(1): is the objective function that minimizes the cost with respect to the arc traveled from i to j and the drone k used in this arc knowing that we have m available drones
(2): The demand d per station i should be less or equal to the capacity DC of the drone K
(3): Just one drone should leave the other hospitals.
(4): m vehicles should leave the lab
(5): ensures that the demand for each hospital is met by 1 drone
(6): The number of entering drones should be equal to the number of leaving drones per hospital
(7): Sub tours elimination. Each hospital visited once cannot be visited again. |S| denotes cardinality of set S consisting of lab and hospitals

The above formulation was obtained from reconfigured from Sabiri's formulation [3].

Relaxation

To perform the relaxation for the exact formulation, we relax the requirement that hospital demand must only be satisfied by one drone. To that end, we can change the operator in equation (3) above from a hard equality to a greater than or equal to inequality as shown below (while the rest remains as is),

$\sum_{k=1}^{k=m} y_{ik} \geq 1 \ (for\ i = 1, 2, 3 \ldots., N-1)$

The relaxation problem can never be infeasible. If the initial problem is feasible, then any relaxation of it (an alteration which reduces the strictness of a constraint) will also be feasible. However, if we had
added an extra Type 1 drone in the AMPL formulations, then the relaxation on the initial solution obtained from Part 1 might prove to be infeasible.

It is also possible that the relaxation of the problem proves to be redundant, as is the case without problem. Because the objective function, the best solution may very well be that each hospital is visited by only 1 drone. However, this is not necessarily always the case.

Problem #1: Mathematical Programming Formulation

hr:- set of hours i=1,2,3,4 where demands of all cities need to be satisfied
sx:- set of possible number of shifts for drone type 1
sy:- set of possible number of shifts for drone type 2
T1:- all possible shifts of drone type 1. 1 if drone works in hour i, 0 otherwise
T2:- all possible shifts of drone type 2. 1 if drone works in hour i, 0 otherwise
DEMAND:- total demand of all cities in hour i

```
set hr;
set sx;
set sy;

param DEMAND{i in hr}>=0;
param T1{i in hr,j in sx}>=0;
param T2{i in hr,k in sy}>=0;

var X{sx} integer>=0;
var Y{sy} integer>=0;

minimize COST:
            sum{j in sx}(X[j]*350)+sum{k in sy}(Y[k]*550);

subject to CAPACITY{i in hr}:
            sum{j in sx} X[j]*450*T1[i,j] + sum{k in sy} Y[k]*1000*T2[i,k] >= DEMAND[i];

subject to MIN_T1:
            sum{j in sx} X[j]>=3;

subject to MIN_T2:
            sum{k in sy} Y[k]>=2;
```

```
set hr:=1 2 3 4;

set sx:= 1 2 3 4;

set sy:= 1 2 3 4 5 6;

param DEMAND:=
1 7861
2 4897
3 3531
4 2756
;

param T1: 1 2 3 4 :=
1 1 1 0 1
2 1 1 1 0
3 1 0 1 1
4 0 1 1 1
;

param T2: 1 2 3 4 5 6 :=
1 1 1 1 0 0 1
2 0 1 0 0 1 0
3 1 0 0 1 0 0
4 0 0 1 1 1 1
;
```

We used a Set Covering formulation to enable AMPL to schedule the drones in non-consecutive hours within shifts. By encoding every combination of shifts for each type of drone ($^4C_2$ and $^4C_3$ combinations of shifts for Type 1 and Type 2 **drones** respectively), we generated the number of drones working in each hour (results in Results section).

Problem #2: MTZ Formulation

Start:- set of possible arrivals i
Dest:- set of possible destinations j
Drone:- set of drones k used
First:- set of all hospitals
Others:- set consisting of lab
N:- number of hospitals and lab
DC:- capacity of drone k
cost:- distance from starting point to destination point
demand:- demand of hospital

```
set Start;
set Dest;
set Drone;
set First;
set Others;

param N=21;
param DC{k in Drone};
param cost{i in Start, j in Dest};
param demand{i in Start};

var X{i in Start,j in Dest,k in Drone}binary;

var Y{i in Start,k in Drone}binary;

var U{i in Start}binary; #prob potential
var V{j in Dest}binary;



minimize z: sum{i in Start, j in Dest, k in Drone}cost[i,j]*X[i,j,k];

subject to mincost1{k in Drone} : sum{i in Start}demand[i]*Y[i,k]<=DC[k]; #demand[i] to demand[j]

subject to demandcov {j in Dest}: sum{i in Start, k in Drone}X[i,j,k]>=1;


#subject to inddrones{i in Start, j in Dest} :sum{k in Drone1]X[i,j,k]+sum{k in Drone2}X2[i,j,k] <=1;

subject to route1{i in First}:sum{k in Drone}Y[i,k]>=6;

subject to route2{i in Others}:sum{k in Drone}Y[i,k]=1;


subject to entleav1{i in Start, k in Drone}:sum{j in Dest}X[i,j,k]=Y[i,k];


subject to Subtourconstraint{i in Others, j in Others, k in Drone:i<>j}: U[i]-V[j] + N*X[i,j,k]<= N-1;
```

```
set Start:=1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21;
set Dest:=1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21;
set First:=21;
set Others:=1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20;

set Drone:=1 2 3 4 5 6;

param DC :=
1 450
2 450
3 450
4 450
5 1000
6 1000;

param demand:=
1 212
2 211
3 204
4 188
5 175
6 162
7 195
8 180
9 180
10 160
11 160
12 163
13 171
14 171
15 169
16 166
17 169
18 168
19 164
20 163
21 0
;

param cost: 1   2   3   4   5   6   7   8   9   10  11  12  13  14  15  16  17  18  19  20  21:=
1   10000   11.42   11.49   10.98   16.05   16.84   20.45   5.76    14.21   30.1    15  53.62   18.24   14.48   31.92   75.09   67.07   98.1    39.34   55.24   3.22
2   11.42   10000   9.43    21.56   15.62   12.21   9.36    16.79   25.34   41.32   22.73   64.27   25.3    25.9    23.62   66.42   56.83   88.81   30.21   48.2    12.69
3   11.49   9.43    10000   22.23   6.28    5.37    17.36   14.12   21.55   40.01   15.34   64.2    17.24   23.33   20.87   63.84   56.82   87.05   28.54   43.75   10.21
4   10.98   21.56   22.23   10000   25.83   27.61   29.6    9.35    10.99   20.02   19.2    42.78   22.39   8.51    42.89   86.02   77.97   109.08  50.32   65.83   12.14
5   16.05   15.62   6.28    25.83   10000   5.87    22.82   16.74   22.4    41.64   13.41   66.2    14.13   24.91   19.96   61.55   56.35   85.13   27.79   40.4    13.76
6   16.84   12.21   5.37    27.61   5.87    10000   17.78   19.34   26.33   45.13   18.65   69.43   19.79   28.36   15.67   58.49   51.89   81.76   23.44   38.34   15.48
7   20.45   9.36    17.36   29.6    22.82   17.78   10000   26.01   34.57   49.62   31.87   71.65   34.25   34.9    22.78   63.02   51.29   84.33   27.47   47.24   22.09
8   5.76    16.79   14.12   9.35    16.74   19.34   26.01   10000   8.55    25.91   10.86   50.09   14.21   9.41    34.98   77.76   70.88   101.11  42.65   57.07   4.24
9   14.21   25.34   21.55   10.99   22.4    26.33   34.57   8.55    10000   19.34   11.03   43.96   13.5    3.49    41.93   83.93   78.22   107.52  49.74   62.53   12.63
10  30.1    41.32   40.01   20.02   41.64   45.13   49.62   25.91   19.34   10000   29.99   24.62   31.8    16.79   60.8    103.19  96.79   126.72  68.54   81.81   30.1
11  15  22.73   15.34   19.2    13.41   18.65   31.87   10.86   11.03   29.99   10000   54  3.34    14.34   33.34   74  70.01   97.74   41.46   52.62   11.86
12  53.62   64.27   64.2    42.78   66.2    69.43   71.65   50.09   43.96   24.62   54  10000   56.17   41.3    85.07   127.73  120.69  151.17  92.72   106.48  54.06
13  18.24   25.3    17.24   22.39   14.13   19.79   34.25   14.21   13.5    31.8    3.34    56.17   10000   16.94   33.65   73.24   69.78   97.05   41.38   51.08   15.06
14  14.48   25.9    23.33   8.51    24.91   28.36   34.9    9.41    3.49    16.79   14.34   41.3    16.94   10000   44.04   86.43   80.15   109.93  51.8    65.21   13.69
15  31.92   23.62   20.87   42.89   19.96   15.67   22.78   34.98   41.93   60.8    33.34   85.07   33.65   44.04   10000   43.2    36.4    66.22   7.85    24.8    31.04
16  75.09   66.42   63.84   86.02   61.55   58.49   63.02   77.76   83.93   103.19  74  127.73  73.24   86.43   43.2    10000   18.72   23.84   36.21   22.66   74.06
17  67.07   56.83   56.82   77.97   56.35   51.89   51.29   70.88   78.22   96.79   70.01   120.69  69.78   80.15   36.4    18.72   10000   33.83   28.56   27.34   66.77
18  98.1    88.81   87.05   109.08  85.13   81.76   84.33   101.11  107.52  126.72  97.74   151.17  97.05   109.93  66.22   23.84   33.83   10000   58.8    46.35   97.22
19  39.34   30.21   28.54   50.32   27.79   23.44   27.47   42.65   49.74   68.54   41.46   92.72   41.38   51.8    7.85    36.21   28.56   58.8    10000   20.35   38.66
20  55.24   48.2    43.75   65.83   40.4    38.34   47.24   57.07   62.53   81.81   52.62   106.48  51.08   65.21   24.8    22.66   27.34   46.35   20.35   10000   53.66
21  3.22    12.69   10.21   12.14   13.76   15.48   22.09   4.24    12.63   30.1    11.86   54.06   15.06   13.69   31.04   74.06   66.77   97.22   38.66   53.66   10000
;
```

Subtour Elimination Formulation

Start:- set of possible arrivals i
Dest:- set of possible destinations j
Drone:- set of drones k used
First:- set of all hospitals
Others:- set consisting of lab
N:- number of hospitals and lab
DC:- capacity of drone k
n:- cardinality of number of starting points
S:- set of hospitals 2, 3,…,n-2
cost:- distance from starting point to destination point
demand:- demand of hospital

```
set Start ordered;
set Dest;
set Drone;
set First;
set Others;
param n := card{Start};
set S; #Number of Cities


param N=21;
param DC{k in Drone};
param cost{i in Start, j in Dest};
param demand{i in Start};

var X{i in Start,j in Dest,k in Drone}binary;

var Y{i in Start,k in Drone}binary;


minimize z: sum{i in Start, j in Dest, k in Drone}cost[i,j]*X[i,j,k];

subject to mincost1{k in Drone} : sum{i in Start}demand[i]*Y[i,k]<=DC[k]; #demand[i] to demand[j]
#subject to mincost1{k in Drone} : sum{i in Start, j in Dest}demand[i]*X[i,j,k]<=DC[k]; #demand[i] to demand[j]

subject to demandcov {j in Dest}: sum{i in Start, k in Drone}X[i,j,k]>=1;

#subject to inddrones{i in Start, j in Dest} :sum{k in Drone1]X[i,j,k]+sum{k in Drone2}X2[i,j,k] <=1;

subject to route1{i in First}:sum{k in Drone}Y[i,k]>=6;

subject to route2{i in Others}:sum{k in Drone}Y[i,k]=1;


#subject to entleav1{i in Start, k in Drone}:sum{j in Dest}X[i,j,k]=Y[i,k];

subject to Subtour{l in S,k in Drone}:sum{i in Start, j in Start}X[i,j,k]<=n-1;
```

```
set Start:=1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21;
set Dest:=1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21;
set First:=21;
set Others:=1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20;
set S:=2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19;
set Drone:=1 2 3 4 5 6;

param DC :=
1 450
2 450
3 450
4 450
5 1000
6 1000;

param demand:=
1 212
2 211
3 204
4 188
5 175
6 162
7 195
8 180
9 180
10 160
11 160
12 163
13 171
14 171
15 169
16 166
17 169
18 168
19 164
20 163
21 0
;


param cost: 1   2   3   4   5   6   7   8   9   10  11  12  13  14  15  16  17  18  19  20  21:=
1   10000   11.42   11.49   10.98   16.05   16.84   20.45   5.76    14.21   30.1    15  53.62   18.24   14.48   31.92   75.09   67.07   98.1    39.34   55.24   3.22
2   11.42   10000   9.43    21.56   15.62   12.21   9.36    16.79   25.34   41.32   22.73   64.27   25.3    25.9    23.62   66.42   56.83   88.81   30.21   48.2    12.69
3   11.49   9.43    10000   22.23   6.28    5.37    17.36   14.12   21.55   40.01   15.34   64.2    17.24   23.33   20.87   63.84   56.82   87.05   28.54   43.75   10.21
4   10.98   21.56   22.23   10000   25.83   27.61   29.6    9.35    10.99   20.02   19.2    42.78   22.39   8.51    42.89   86.02   77.97   109.08  50.32   65.83   12.14
5   16.05   15.62   6.28    25.83   10000   5.87    22.82   16.74   22.4    41.64   13.41   66.2    14.13   24.91   19.96   61.55   56.35   85.13   27.79   40.4    13.76
6   16.84   12.21   5.37    27.61   5.87    10000   17.78   19.34   26.33   45.13   18.65   69.43   19.79   28.36   15.67   58.49   51.89   81.76   23.44   38.34   15.48
7   20.45   9.36    17.36   29.6    22.82   17.78   10000   26.01   34.57   49.62   31.87   71.65   34.25   34.9    22.78   63.02   51.29   84.33   27.47   47.24   22.09
8   5.76    16.79   14.12   9.35    16.74   19.34   26.01   10000   8.55    25.91   10.86   50.09   14.21   9.41    34.98   77.76   70.88   101.11  42.65   57.07   4.24
9   14.21   25.34   21.55   10.99   22.4    26.33   34.57   8.55    10000   19.34   11.03   43.96   13.5    3.49    41.93   83.93   78.22   107.52  49.74   62.53   12.63
10  30.1    41.32   40.01   20.02   41.64   45.13   49.62   25.91   19.34   10000   29.99   24.62   31.8    16.79   60.8    103.19  96.79   126.72  68.54   81.81   30.1
11  15  22.73   15.34   19.2    13.41   18.65   31.87   10.86   11.03   29.99   10000   54  3.34    14.34   33.34   74  70.01   97.74   41.46   52.62   11.86
12  53.62   64.27   64.2    42.78   66.2    69.43   71.65   50.09   43.96   24.62   54  10000   56.17   41.3    85.07   127.73  120.69  151.17  92.72   106.48  54.06
13  18.24   25.3    17.24   22.39   14.13   19.79   34.25   14.21   13.5    31.8    3.34    56.17   10000   16.94   33.65   73.24   69.78   97.05   41.38   51.08   15.06
14  14.48   25.9    23.33   8.51    24.91   28.36   34.9    9.41    3.49    16.79   14.34   41.3    16.94   10000   44.04   86.43   80.15   109.93  51.8    65.21   13.69
15  31.92   23.62   20.87   42.89   19.96   15.67   22.78   34.98   41.93   60.8    33.34   85.07   33.65   44.04   10000   43.2    36.4    66.22   7.85    24.8    31.04
16  75.09   66.42   63.84   86.02   61.55   58.49   63.02   77.76   83.93   103.19  74  127.73  73.24   86.43   43.2    10000   18.72   23.84   36.21   22.66   74.06
17  67.07   56.83   56.82   77.97   56.35   51.89   51.29   70.88   78.22   96.79   70.01   120.69  69.78   80.15   36.4    18.72   10000   33.83   28.56   27.34   66.77
18  98.1    88.81   87.05   109.08  85.13   81.76   84.33   101.11  107.52  126.72  97.74   151.17  97.05   109.93  66.22   23.84   33.83   10000   58.8    46.35   97.22
19  39.34   30.21   28.54   50.32   27.79   23.44   27.47   42.65   49.74   68.54   41.46   92.72   41.38   51.8    7.85    36.21   28.56   58.8    10000   20.35   38.66
20  55.24   48.2    43.75   65.83   40.4    38.34   47.24   57.07   62.53   81.81   52.62   106.48  51.08   65.21   24.8    22.66   27.34   46.35   20.35   10000   53.66
21  3.22    12.69   10.21   12.14   13.76   15.48   22.09   4.24    12.63   30.1    11.86   54.06   15.06   13.69   31.04   74.06   66.77   97.22   38.66   53.66   10000
;
```

Relaxation Formulation

```
set Start ordered;
set Dest;
set Drone;
set First;
set Others;
param n := card{Start};
set S; #Number of Cities


param N=21;
param DC{k in Drone};
param cost{i in Start, j in Dest};
param demand{i in Start};

var X{i in Start,j in Dest,k in Drone}binary;

var Y{i in Start,k in Drone}binary;


minimize z: sum{i in Start, j in Dest, k in Drone}cost[i,j]*X[i,j,k];

subject to mincost1{k in Drone} : sum{i in Start}demand[i]*Y[i,k]<=DC[k]; #demand[i] to demand[j]

subject to demandcov {j in Dest}: sum{i in Start, k in Drone}X[i,j,k]>=1;

#subject to inddrones{i in Start, j in Dest} :sum{k in Drone1]X[i,j,k]+sum{k in Drone2}X2[i,j,k] <=1;

subject to route1{i in First}:sum{k in Drone}Y[i,k]>=6;

subject to route2{i in Others}:sum{k in Drone}Y[i,k]>=1;


subject to entleav1{i in Start, k in Drone}:sum{j in Dest}X[i,j,k]=Y[i,k];

subject to Subtour{l in S,k in Drone}:sum{i in Start, j in Start}X[i,j,k]<=n-1;
```

```
set Start:=1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21;
set Dest:=1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21;
set First:=21;
set Others:=1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21;
set S:=2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19;
set Drone:=1 2 3 4 5 6;

param DC :=
1 450
2 450
3 450
4 450
5 1000
6 1000;

param demand:=
1 212
2 211
3 204
4 188
5 175
6 162
7 195
8 180
9 180
10 160
11 160
12 163
13 171
14 171
15 169
16 166
17 169
18 168
19 164
20 163
21 0
;

param cost: 1   2   3   4   5   6   7   8   9   10  11  12  13  14  15  16  17  18  19  20  21:=
1  10000  11.42  11.49  10.98  16.05  16.84  20.45  5.76  14.21  30.1  15  53.62  18.24  14.48  31.92  75.09  67.07  98.1  39.34  55.24  3.22
2  11.42  10000  9.43  21.56  15.62  12.21  9.36  16.79  25.34  41.32  22.73  64.27  25.3  25.9  23.62  66.42  56.83  88.81  30.21  48.2  12.69
3  11.49  9.43  10000  22.23  6.28  5.37  17.36  14.12  21.55  40.01  15.34  64.2  17.24  23.33  20.87  63.84  56.82  87.05  28.54  43.75  10.21
4  10.98  21.56  22.23  10000  25.83  27.61  29.6  9.35  10.99  20.02  19.2  42.78  22.39  8.51  42.89  86.02  77.97  109.08  50.32  65.83  12.14
5  16.05  15.62  6.28  25.83  10000  5.87  22.82  16.74  22.4  41.64  13.41  66.2  14.13  24.91  19.96  61.55  56.35  85.13  27.79  40.4  13.76
6  16.84  12.21  5.37  27.61  5.87  10000  17.78  19.34  26.33  45.13  18.65  69.43  19.79  28.36  15.67  58.49  51.89  81.76  23.44  38.34  15.48
7  20.45  9.36  17.36  29.6  22.82  17.78  10000  26.01  34.57  49.62  31.87  71.65  34.25  34.9  22.78  63.02  51.29  84.33  27.47  47.24  22.09
8  5.76  16.79  14.12  9.35  16.74  19.34  26.01  10000  8.55  25.91  10.86  50.09  14.21  9.41  34.98  77.76  70.88  101.11  42.65  57.07  4.24
9  14.21  25.34  21.55  10.99  22.4  26.33  34.57  8.55  10000  19.34  11.03  43.96  13.5  3.49  41.93  83.93  78.22  107.52  49.74  62.53  12.63
10  30.1  41.32  40.01  20.02  41.64  45.13  49.62  25.91  19.34  10000  29.99  24.62  31.8  16.79  60.8  103.19  96.79  126.72  68.54  81.81  30.1
11  15  22.73  15.34  19.2  13.41  18.65  31.87  10.86  11.03  29.99  10000  54  3.34  14.34  33.34  74  70.01  97.74  41.46  52.62  11.86
12  53.62  64.27  64.2  42.78  66.2  69.43  71.65  50.09  43.96  24.62  54  10000  56.17  41.3  85.07  127.73  120.69  151.17  92.72  106.48  54.06
13  18.24  25.3  17.24  22.39  14.13  19.79  34.25  14.21  13.5  31.8  3.34  56.17  10000  16.94  33.65  73.24  69.78  97.05  41.38  51.08  15.06
14  14.48  25.9  23.33  8.51  24.91  28.36  34.9  9.41  3.49  16.79  14.34  41.3  16.94  10000  44.04  86.43  80.15  109.93  51.8  65.21  13.69
15  31.92  23.62  20.87  42.89  19.96  15.67  22.78  34.98  41.93  60.8  33.34  85.07  33.65  44.04  10000  43.2  36.4  66.22  7.85  24.8  31.04
16  75.09  66.42  63.84  86.02  61.55  58.49  63.02  77.76  83.93  103.19  74  127.73  73.24  86.43  43.2  10000  18.72  23.84  36.21  22.66  74.06
17  67.07  56.83  56.82  77.97  56.35  51.89  51.29  70.88  78.22  96.79  70.01  120.69  69.78  80.15  36.4  18.72  10000  33.83  28.56  27.34  66.77
18  98.1  88.81  87.05  109.08  85.13  81.76  84.33  101.11  107.52  126.72  97.74  151.17  97.05  109.93  66.22  23.84  33.83  10000  58.8  46.35  97.22
19  39.34  30.21  28.54  50.32  27.79  23.44  27.47  42.65  49.74  68.54  41.46  92.72  41.38  51.8  7.85  36.21  28.56  58.8  10000  20.35  38.66
20  55.24  48.2  43.75  65.83  40.4  38.34  47.24  57.07  62.53  81.81  52.62  106.48  51.08  65.21  24.8  22.66  27.34  46.35  20.35  10000  53.66
21  3.22  12.69  10.21  12.14  13.76  15.48  22.09  4.24  12.63  30.1  11.86  54.06  15.06  13.69  31.04  74.06  66.77  97.22  38.66  53.66  10000
;
```

Add (Greedy Approach)

An initial feasible solution was found using the greedy approach. From the solution obtained in Problem #1, 2 Type-II drones were added first, then 4 Type-I drones were added, then an extra Type-I drones was needed so that the drones travel from the laboratory to the closest unassigned hospital initially and then from its current location to another closest unassigned hospital.
New drones were added when the capacity of the drone was maxed out and stopped when all the demands were satisfied.

Hour 3: (Greedy approach)						Total
	NYC Health + Hospitals/Woodhull	Jersey City Medical Center	Children's Hospital in New Jersey	Trinitas Regional Medical Center	Staten Island University Hospital	
Drone 1(II)	212	180	180	171	188	931
Drone 2(II)	Calvary Hospital	Montefiore New Rochelle Hospital	St. Joseph's Medical Center(5)	Fresenius Hospital Care	St. Joseph's Medical Center(13)	
	204	162	175	160	171	872
Drone 1(I)	Creedmoor Psychiatric Center	Center for Primary Care				
	211	195				406
Drone 2(I)	RWJ University Hospital New Brunswich	St. Francis Medical Center				
	160	163				323
Drone 3(I)	Stamford Hospital - Bennett Medical Center	Norwalk Hospital				
	169	164				333
Drone 4(I)	Danbury Hospital	Waterbury Hospital				
	163	166				329
Drone 5(I)	Yales New Haven Children's Hospital	Hartford Hospital				
	169	168				337
						3531

Drone 1(II): 0-1-8-9-14-4-0	41.67	miles			
Drone 2(II): 0-3-6-5-11-13-0	53.26	miles			
Drone 1(I): 0-2-7-0	44.14	miles	Cost of the drones ($1000):	2850	dollars
Drone 2(I): 0-10-12-0	108.78	miles			
Drone 3(I): 0-15-19-0	77.55	miles			
Drone 4(I): 0-20-16-0	150.38	miles			
Drone 5(I): 0-17-18-0	197.82	miles			
Distance travelled by the drone:	673.6	miles			

Swap (Local Search Approach)

A 1-exchange swap operation was done to improve the objective function value by finding the best swap from the initial feasible solution (Add Heuristic). The swap heuristic was done until an improvement was no longer possible.

Hour 3: (Local Search approach)						Total
	NYC Health + Hospitals/Woodhull	Jersey City Medical Center	Children's Hospital in New Jersey	Trinitas Regional Medical Center	Staten Island University Hospital	
Drone 1(II)	212	180	180	171	188	931
Drone 2(II)	Calvary Hospital	Montefiore New Rochelle Hospit	St. Joseph's Medical Center(5)	St. Joseph's Medical Center(13)	Fresenius Hospital Care	
	204	162	175	171	160	872
Drone 1(I)	Creedmoor Psychiatric Center	Center for Primary Care				
	211	195				406
Drone 2(I)	RWJ University Hospital New Brunswich	St. Francis Medical Center				
	160	163				323
Drone 3(I)	Stamford Hospital - Bennett Medical Center	Norwalk Hospital				
	169	164				333
Drone 4(I)	Danbury Hospital	Waterbury Hospital				
	163	166				329
Drone 5(I)	Yales New Haven Children's Hospital	Hartford Hospital				
	169	168				337
						3531

	Distance Travelled				
Drone 1(II): 0-1-8-9-14-4-0	41.67	miles			
Drone 2(II): 0-3-6-5-13-11-0	50.78	miles			
Drone 1(I): 0-2-7-0	44.14	miles	Cost of the drones ($1000):	2850	dollars
Drone 2(I): 0-10-12-0	108.78	miles			
Drone 3(I): 0-15-19-0	77.55	miles			
Drone 4(I): 0-20-16-0	150.38	miles			
Drone 5(I): 0-17-18-0	197.82	miles			
Distance travelled by the drone:	671.12	miles			

SWAP 1

DRONE1 (II): 0 – 1 – 8 – 9 – 14 – 4 – 0

$\Rightarrow$ $1 \leftrightarrow 8$

$$\Delta f = -C_{91} - C_{89} + C_{08} + C_{19}$$
$$= -3.22 - 8.55 + 4.24 + 14.21$$
$$= 6.68$$

$\Rightarrow$ $8 \leftrightarrow 9$

$$\Delta f = -C_{18} - C_{9,14} + C_{19} + C_{8,14}$$
$$= -5.76 - 3.49 + 14.21 + 9.41$$
$$= 14.37$$

$\Rightarrow$ $9 \leftrightarrow 14$

$$\Delta f = -C_{89} - C_{14,4} + C_{8,14} + C_{94}$$
$$= -8.55 - 8.51 + 9.41 + 10.99$$
$$= 3.34$$

$\Rightarrow$ $14 \leftrightarrow 4$

$$\Delta f = -C_{9,14} - C_{4,0} + C_{9,4} + C_{14,0}$$
$$= -3.49 - 12.14 + 10.99 + 13.69$$
$$= 9.05$$

Since, all the values the of the $\Delta f > 0$, there won't be any change to the initial route for the drone 1

DRONE 2: $0-3-6-5-11-13-0$
(II)

$\Rightarrow$ $3 \leftrightarrow 6$

$$\Delta f = -C_{0,3} - C_{6,5} + C_{0,6} + C_{3,5}$$
$$= -10.21 - 5.87 + 15.48 + 6.28$$
$$= 5.68$$

$\Rightarrow$ $6 \leftrightarrow 5$

$$\Delta f = -C_{3,6} - C_{5,11} + C_{3,5} + C_{6,11}$$
$$= -5.37 - 13.41 + 6.28 + 18.65$$
$$= 6.15$$

$\Rightarrow$ $5 \leftrightarrow 11$

$$\Delta f = -C_{6,5} - C_{11,13} + C_{6,11} + C_{5,13}$$
$$= -5.87 - 3.34 + 18.65 + 14.13$$
$$= 23.57$$

$\Rightarrow$ $11 \leftrightarrow 13$

$$\Delta f = -C_{5,11} - C_{13,0} + C_{5,13} + C_{11,0}$$
$$= -13.41 - 15.06 + 14.13 + 11.86$$
$$= -2.48$$

This reduces the total distance traveled by drone.

The fourth 1- exchange swap for the route of drone 2 gives, $\Delta f < 0$. Hence a better value for the distance travelled by the drone 2 is achieved.

SWAP 2

DRONE 2: (II) $0-3-6-5-13-11-0$

$\Rightarrow$ $3 \leftrightarrow 6$

$$\Delta f = -C_{0,3} - C_{6,5} + C_{0,6} + C_{3,5}$$
$$= -10.21 - 5.87 + 15.48 + 6.28$$
$$= 5.68$$

$\Rightarrow$ $6 \leftrightarrow 5$

$$\Delta f = -C_{3,6} - C_{5,11} + C_{3,5} + C_{6,11}$$
$$= -5.37 - 13.41 + 6.28 + 18.65$$
$$= 6.15$$

$\Rightarrow$ $5 \leftrightarrow 13$

$$\Delta f = -C_{6,5} - C_{13,11} + C_{6,13} + C_{5,11}$$
$$= -5.87 - 3.34 + 19.79 + 13.41$$
$$= 23.99$$

$\Rightarrow$ $13 \leftrightarrow 11$

$$\Delta f = -C_{5,13} - C_{11,0} + C_{5,11} + C_{13,0}$$
$$= -14.13 - 11.86 + 13.41 + 15.06$$
$$= 2.48$$

After the routes of the drone 2 has been swapped, we calculated all the values of the $\Delta f > 0$. Hence, we found out that the optimal route for the drone 2 by the local search heuristic.

We won’t do the swap operation for the type 1 drones since the drones’ travel between only 2 cities as the distance between the 2 cities will be the same

Results

Schedule:

```
:   X   Y    :=
1   0   2
2   0   4
3   2   1
4   2   0
5   .   0
6   .   0
;
```

The above denotes the number of Type 1 and Type 2 drones working each type of shift.

.

Type 1							
	1	2	3	4	No of drones/hr		Total Cost
1	1	1	0	1	2		5250
2	1	1	1	0	2		
3	1	0	1	1	4		
4	0	1	1	1	4		
	0	0	2	2			

Type 2							
	1	2	3	4	5	6	No of drones/hr
1	1	1	1	0	0	1	7
2	0	1	0	0	1	0	4
3	1	0	0	1	0	0	2
4	0	0	1	1	1	1	1
	2	4	1	0	0	0	

```
CPLEX 12.8.0.0: optimal integer solution; objective 5250
112 MIP simplex iterations
83 branch-and-bound nodes
```

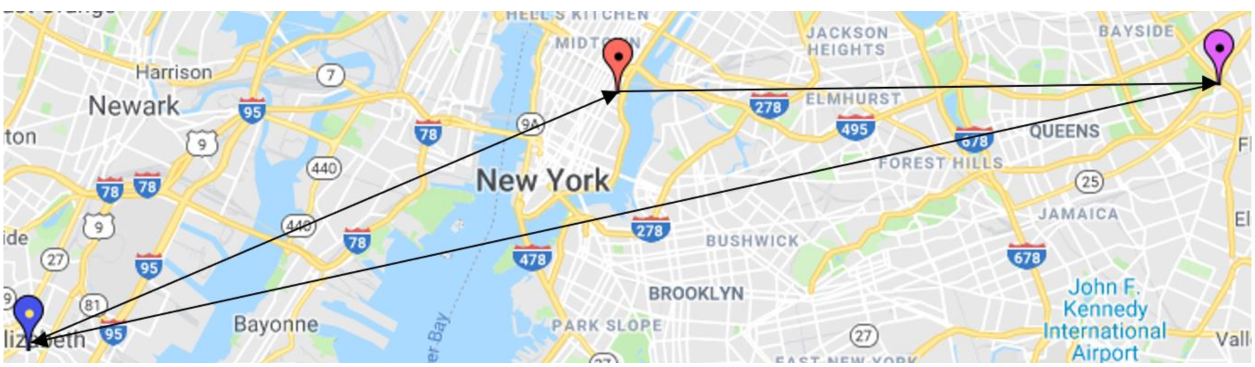
MTZ:

```
CPLEX 12.8.0.0: optimal integer solution; objective 258.44
144 MIP simplex iterations
0 branch-and-bound nodes
```

Drone 1:

Red Legend - Bellevue Hospital Center (LAB)
Purple Legend - NYC Health + Hospitals/Woodhull
Blue Legend - Children's Hospital in New Jersey

Drone 2:

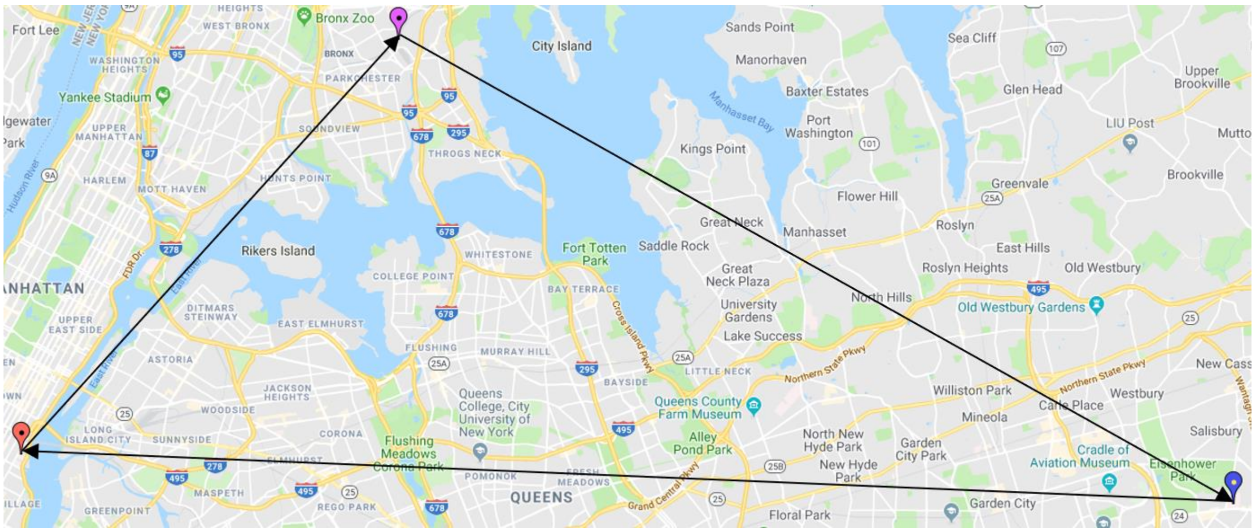


Red Legend - Bellevue Hospital Center (LAB)
Purple Legend - Creedmoor Psychiatric Center
Blue Legend - Trinitas Regional Medical Center

Drone 3:

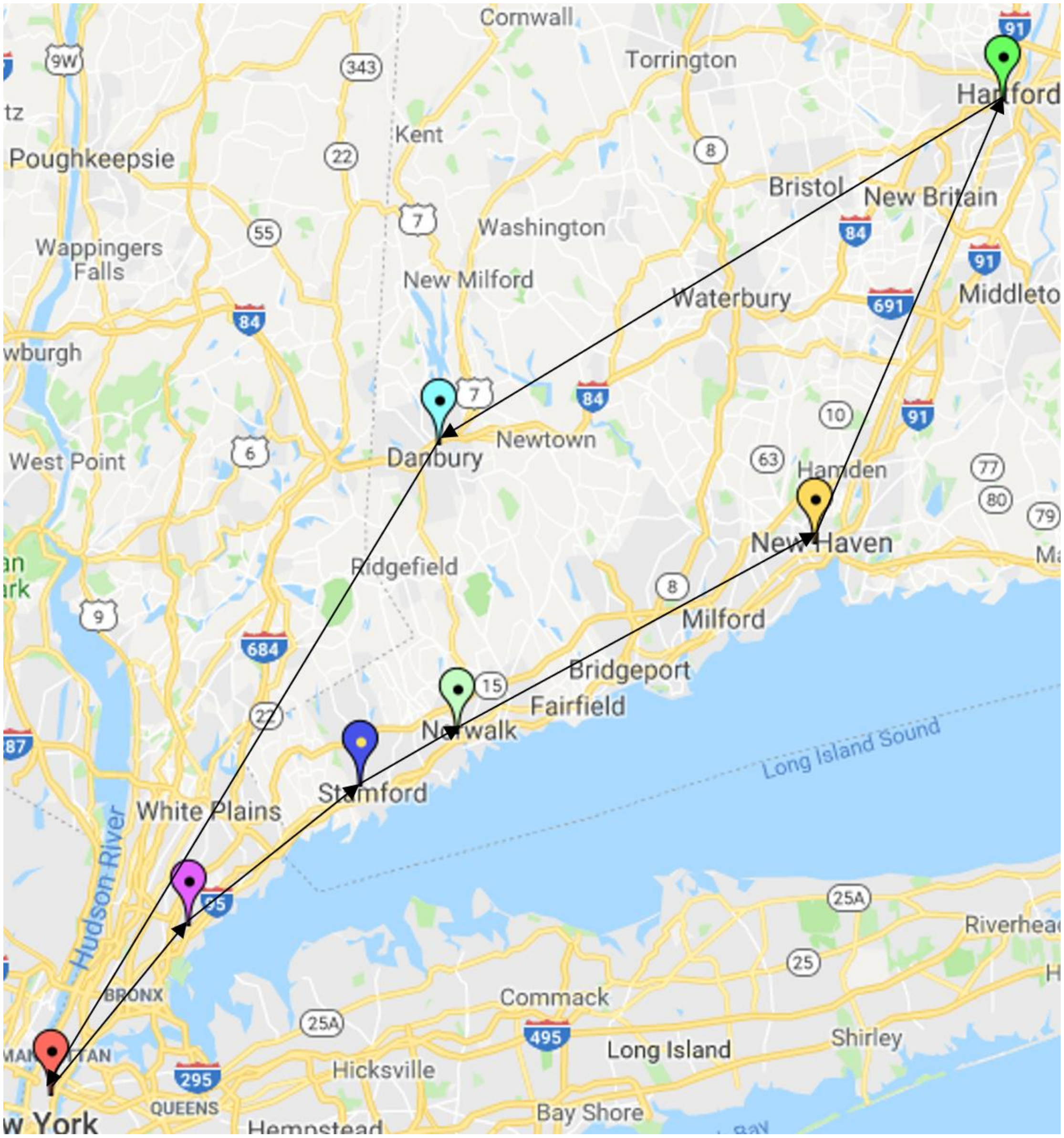
Red Legend - Bellevue Hospital Center (LAB)
Purple Legend – Staten Island University Hospital
Blue Legend - St. Joseph's Medical Center

Drone 4:



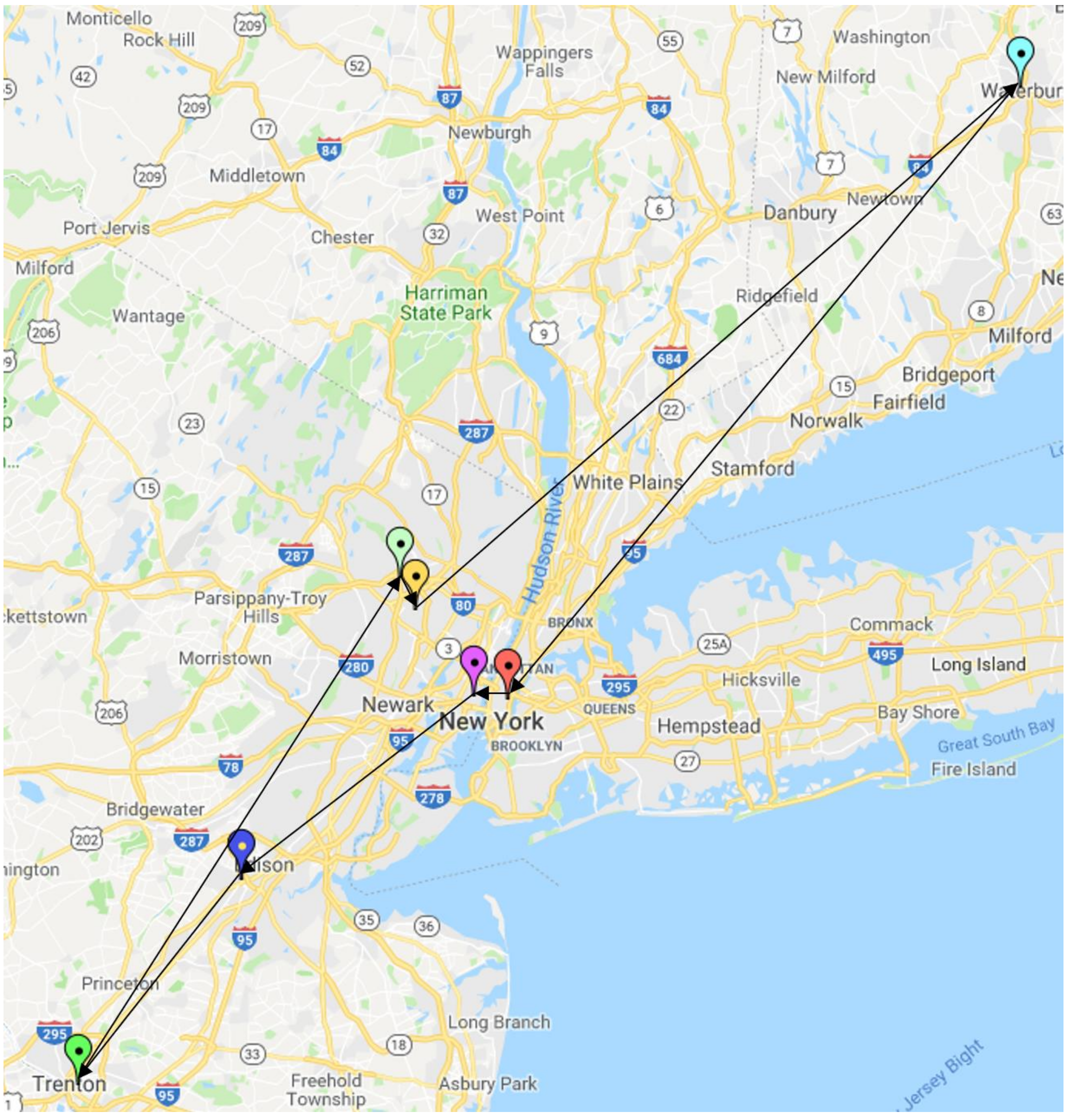
Red Legend - Bellevue Hospital Center (LAB)
Purple Legend – Calvary Hospital
Blue Legend - Center for Primary Care

Drone 5:



Red Legend – Bellevue Hospital Center (LAB)
Purple Legend – Montefiore New Rochelle Hospital
Blue Legend – Stamford Hospital - Bennett Medical Center
Yellow Legend – Yales New Haven Children's Hospital
Green Legend – Hartford Hospital
Light Green Legend – Norwalk Hospital
Light Blue Legend – Danbury Hospital

Drone 6:



Red Legend – Bellevue Hospital Center (LAB)
Purple Legend – Jersey City Medical Center
Blue Legend – RWJ University Hospital New Brunswich
Yellow Legend – Fresenius Hospital Care
Green Legend – St. Francis Medical Center
Light Green Legend – St. Joseph's Medical Center
Light Blue Legend – Waterbury Hospital

Subtour:

```
CPLEX 12.8.0.0: optimal integer solution; objective 206.92
22 MIP simplex iterations
0 branch-and-bound nodes
absmipgap = 2.84217e-14, relmipgap = 1.37356e-16
```

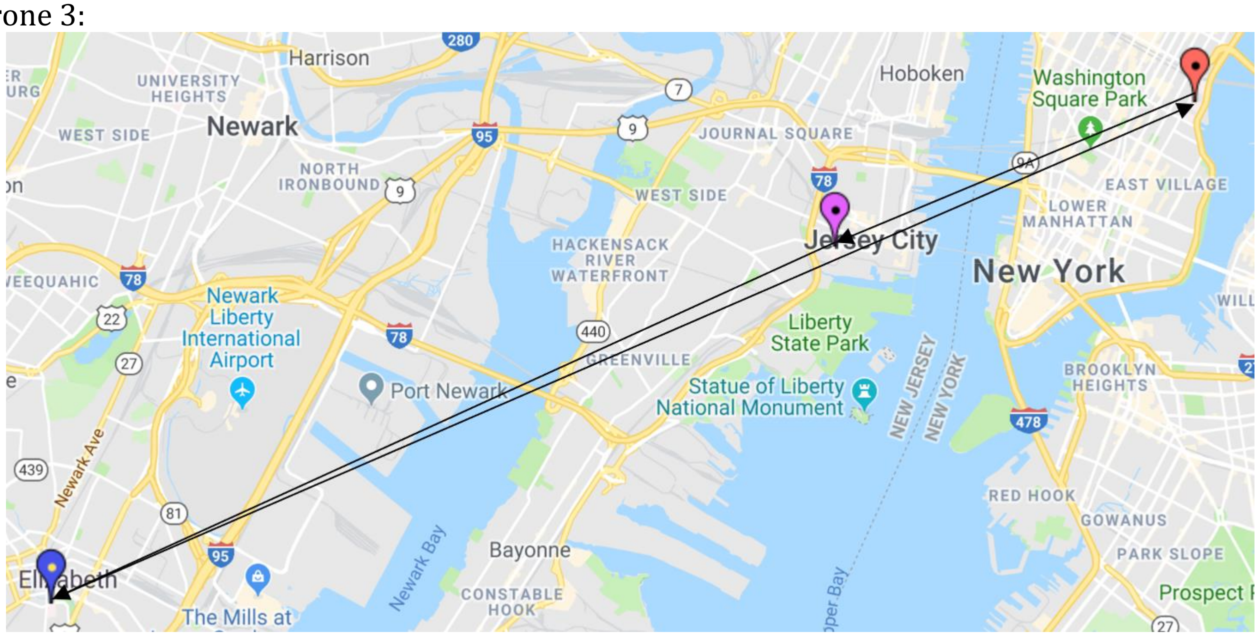
Drone 1

Red Legend – Bellevue Hospital Center (Lab)
Purple Legend – Calvary Hospital
Blue Legend – Staten Island University Hospital

Drone 2:

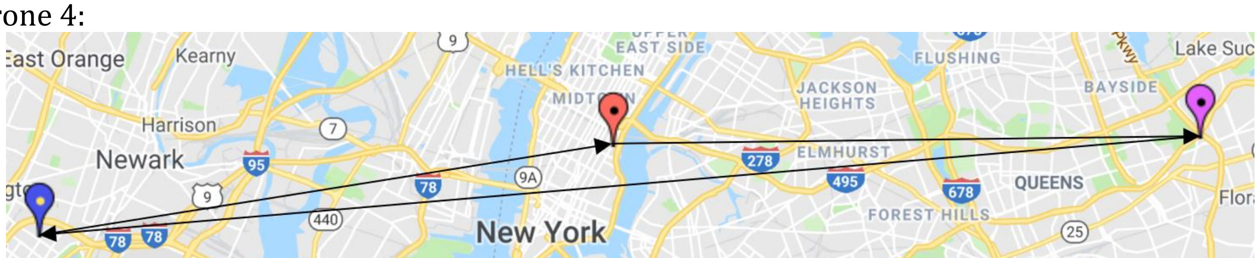
Red Legend – Bellevue Hospital Center (Lab)
Purple Legend – NYC Health + Hospitals/Woodhull
Blue Legend - Center for Primary Care

Drone 3:



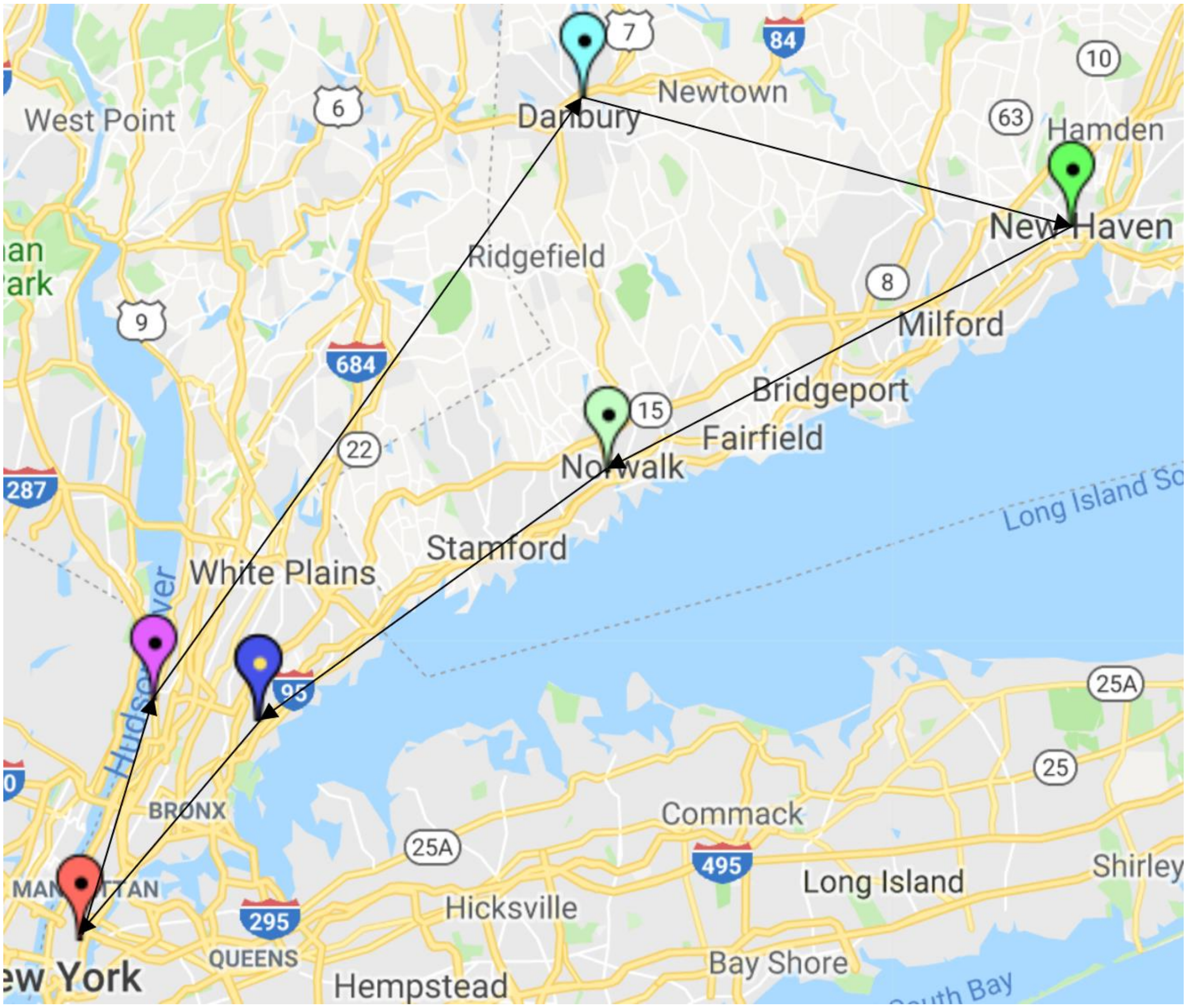
Red Legend – Bellevue Hospital Center (Lab)
Purple Legend – Jersey City Medical Center
Blue Legend - Trinitas Regional Medical Center

Drone 4:



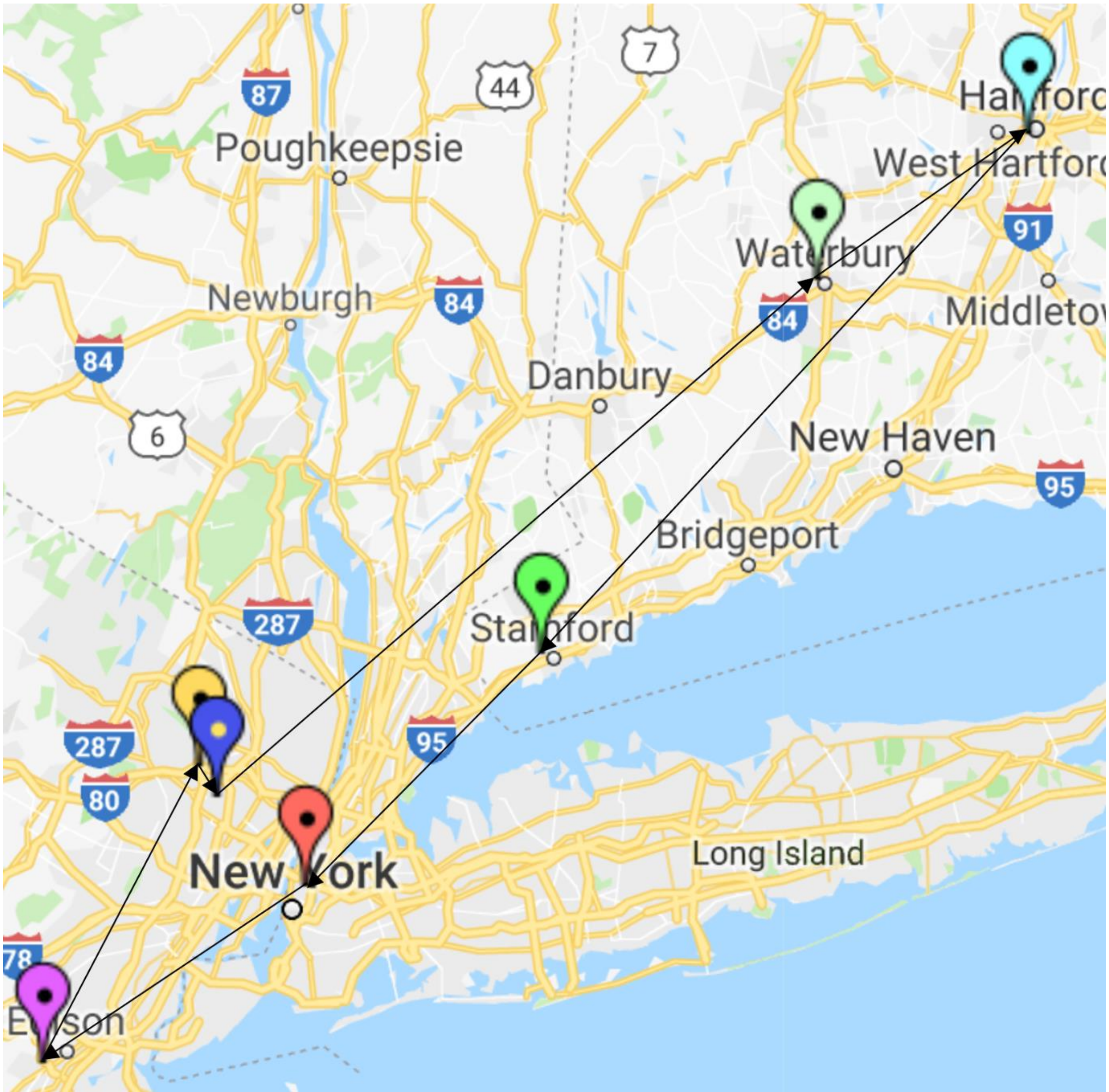
Red Legend – Bellevue Hospital Center (Lab)
Purple Legend – Creedmoor Psychiatric Center
Blue Legend – Children's Hospital in New Jersey

Drone 5:



Red Legend – Bellevue Hospital Center (Lab)
Purple Legend – St. Joseph's Medical Center
Blue Legend – Montefiore New Rochelle Hospital
Yellow Legend – St. Francis Medical Center
Green Legend – Yales New Haven Children's Hospital
Light Green Legend – Norwalk Hospital
Light Blue Legend – Danbury Hospital

Drone 6:

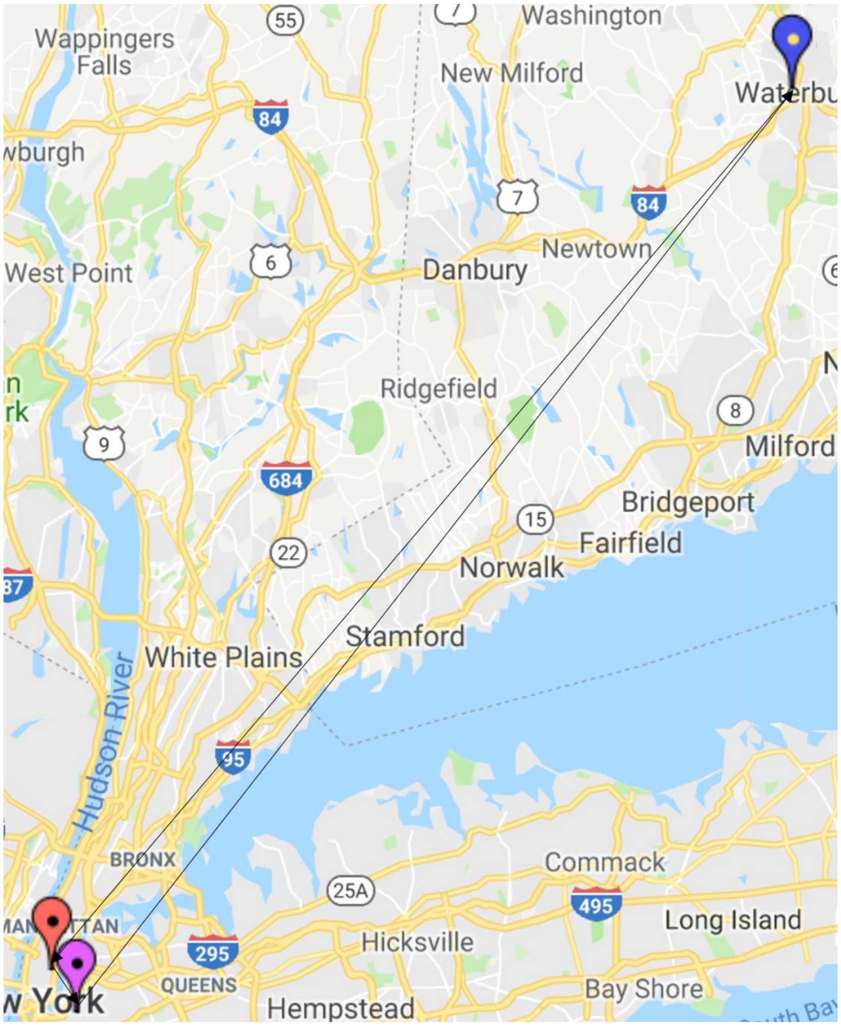


Red Legend – Bellevue Hospital Center (Lab)
Purple Legend – RWJ University Hospital New Brunswick
Blue Legend – Fresenius Hospital Care
Yellow Legend – St. Joseph's Medical Center
Green Legend – Stamford Hospital - Bennett Medical Center
Light Green Legend – Waterbury Hospital
Light Blue Legend – Hartford Hospital

Relaxation:

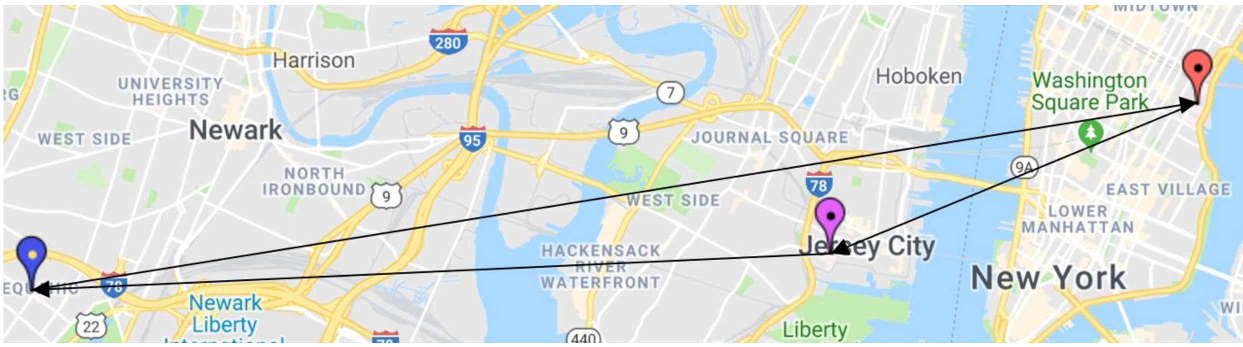
```
CPLEX 12.8.0.0: optimal integer solution; objective 258.44
118 MIP simplex iterations
0 branch-and-bound nodes
```

Drone 1:



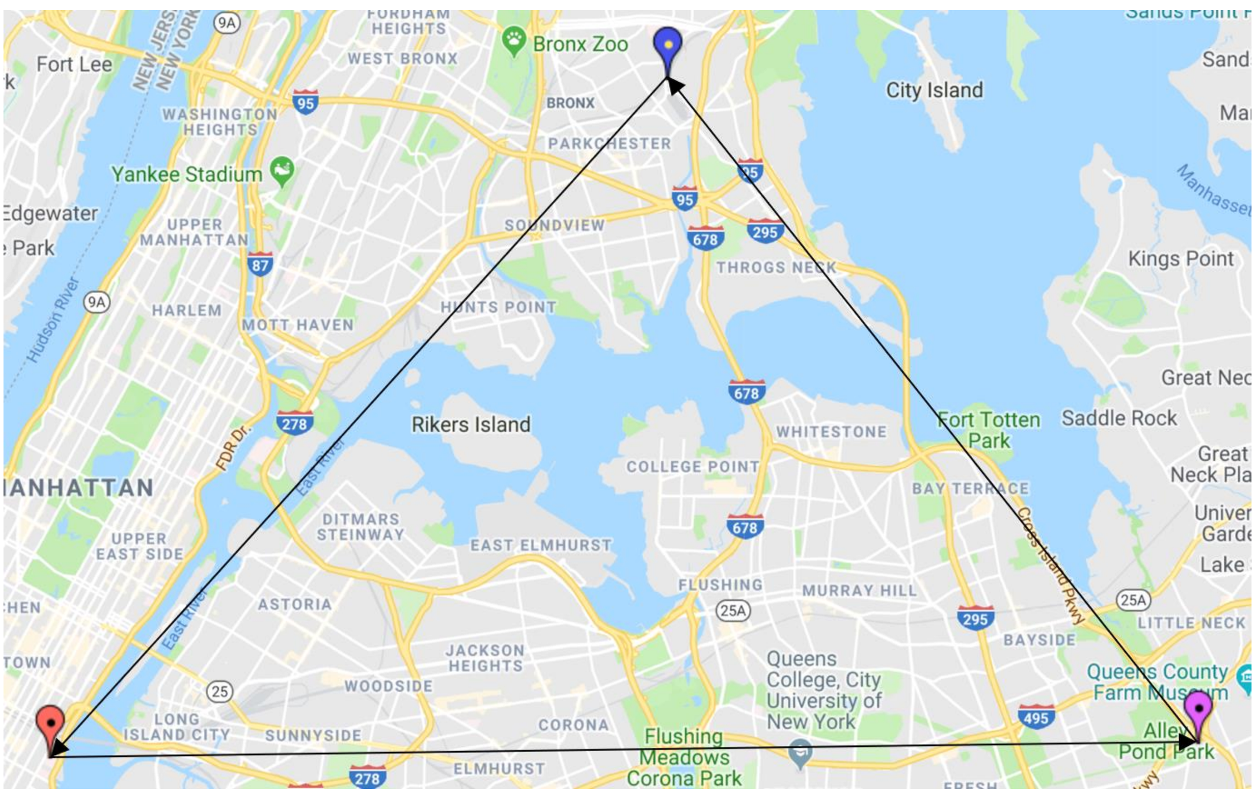
Red Legend - Bellevue Hospital Center
Purple Legend - NYC Health + Hospitals/Woodhull
Blue Legend - Waterbury Hospital

Drone 2:



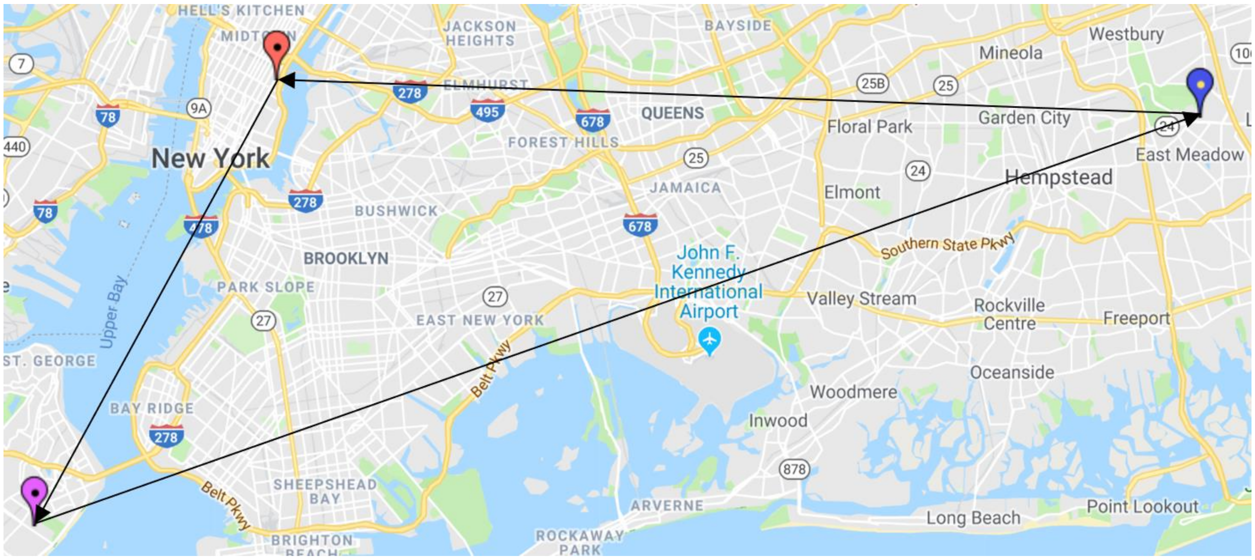
Red Legend - Bellevue Hospital Center
Purple Legend – Jersey City Medical Center
Blue Legend – Children's Hospital in New Jersey

Drone 3:



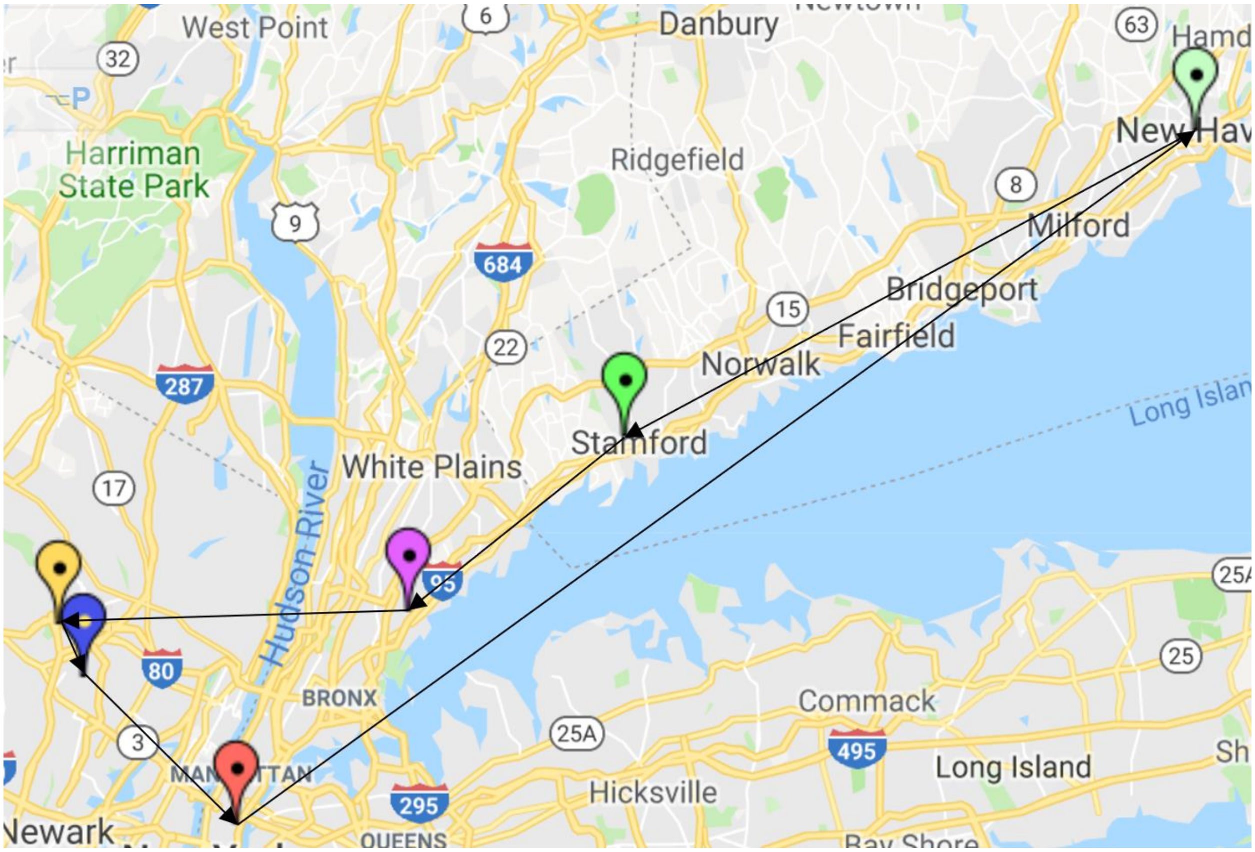
Red Legend - Bellevue Hospital Center
Purple Legend – Creedmoor Psychiatric Center
Blue Legend – Calvary Hospital

Drone 4:



Red Legend - Bellevue Hospital Center
Purple Legend – Staten Island University Hospital
Blue Legend – Center for Primary Care

Drone 5:



Red Legend - Bellevue Hospital Center
Purple Legend – Montefiore New Rochelle Hospital
Blue Legend – Fresenius Hospital Care
Yellow Legend – St. Joseph's Medical Center
Green Legend – Stamford Hospital - Bennett Medical Center
Light Green Legend – Yales New Haven Children's Hospital
Light Blue Legend – Hartford Hospital

Drone 6:

Red Legend - Bellevue Hospital Center
Purple Legend – St. Joseph's Medical Center
Blue Legend – RWJ University Hospital New Brunswick
Yellow Legend – St. Francis Medical Center
Green Legend – Trinitas Regional Medical Center
Light Green Legend – Norwalk Hospital
Light Blue Legend – Danbury Hospital

Recommendations

	Iteration	No. of variables	Constraints
Subtour	22	2	176
MTZ	137	4	2840

From the above statistics, we would highly recommend using Subtour Elimination constraints. It is less computationally expensive than using MTZ. This is keeping with the assumption that the demand from 1 hospital can only be satisfied by 1 drone.

If all of our laptops happen to get recalled again or another disaster of dire consequences occur, then the heuristic used is a viable alternative to the mathematical AMPL formulations that obtained our optimal solution. While it not as good a solution as the AMPL formulations, it is still a quick (yet inefficient) way to get a feasible solution that Management might be content with.

References

[1] Paola Toth and Daniele Vigo, *Models, Relaxations and Exact Approaches for The Capacitated Vehicle Routing Problem (2002)*

[2] Nathalie Helal, Frederic Pichon, Daniel Porumbel, David Mercier and Eric Lefevre. *The Capacitated Vehicle Routing Problem with Evidential Demands: a Belief-Constrained Programming Approach*

[3] Chayma Sabiri, *The Fleet Management (Capacitated Vehicle Routing Problem with Fixed Fleet of Delivery Vehicles of Uniform capacity) (2017)*

[4] Hipolito Hernandez-Perez and Juan-Jose Salazar-Gonzalez, *A branch-and-cut algorithm for a traveling salesman problem with pickup and delivery (2003)*

[5] Wayne L. Winston, *Operations Research - Applications and Algorithms*