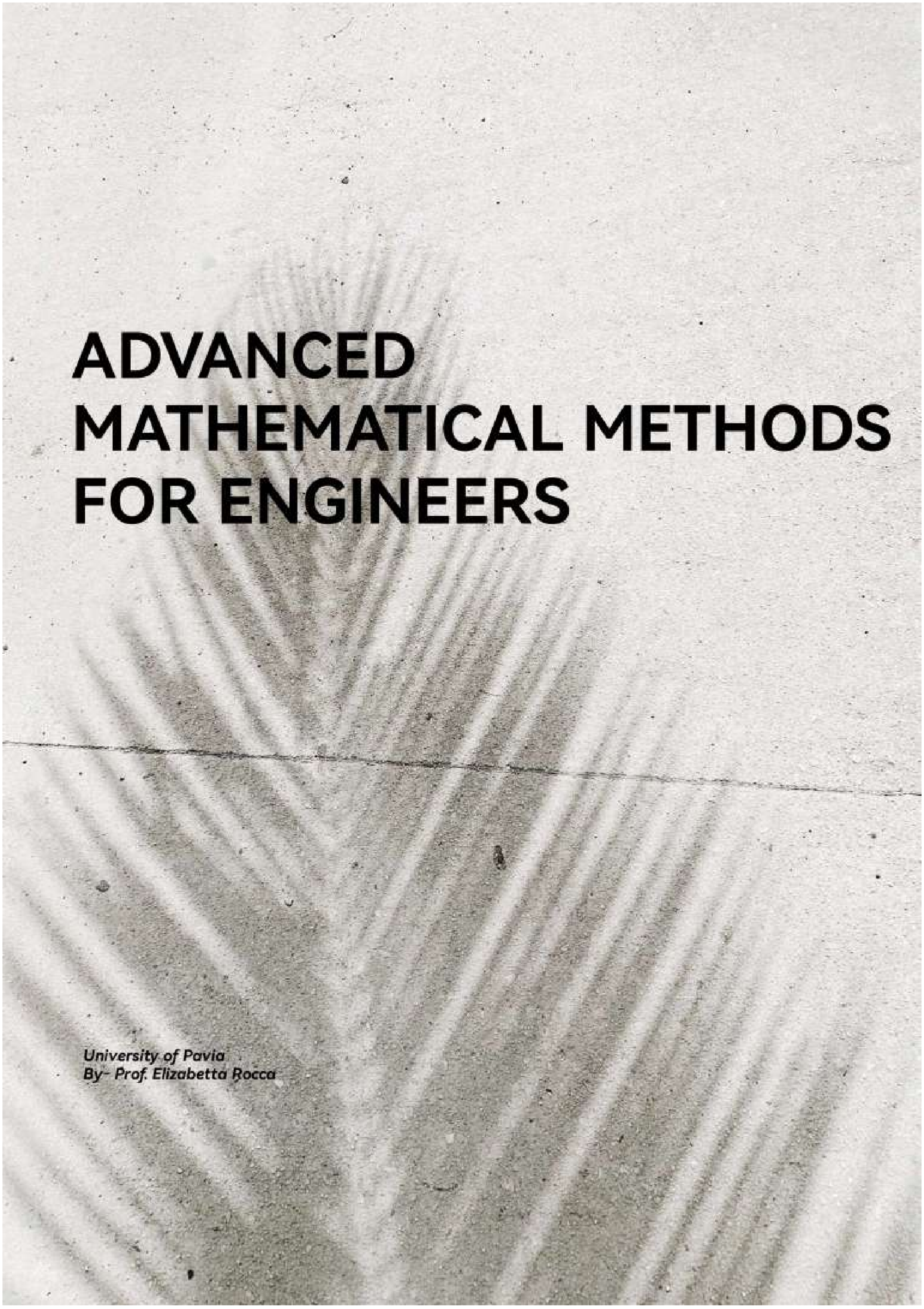

ADVANCED MATHEMATICAL METHODS FOR ENGINEERS

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① Peano Theorem

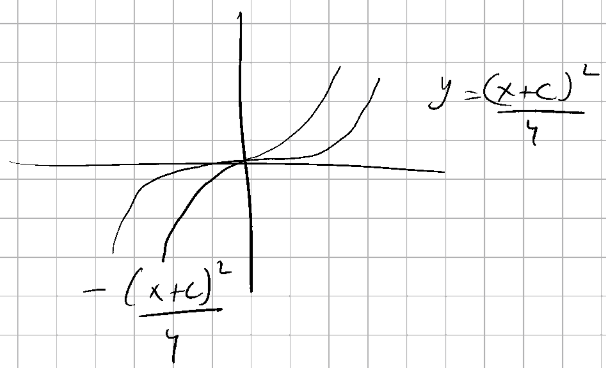
Consider a function $f : \Omega \subset \mathbb{R}^{n+1} \longrightarrow \mathbb{R}$

if $f \in C^0(\Omega)$

then $\exists$ atleast one solution y: $I(x_0) = (x_0 - \delta, x_0 + \delta)$

Counter example

Peano brush



lipt cont. if $\exists$ L

such that $|f(x) - f(y)| \leq L|x - y|$

local.

↳ locally lipt continuous

↳ $(x_0, y_0) \in \Omega$

Global

for a $f : [a,b] \times \mathbb{R} \xrightarrow{C^0} \mathbb{R}^n$

dom f → is a vertical strip

$$\lim_{y \to \infty} \left| \frac{f(y_2) - f(y_1)}{y_2 - y_1} \right| \leq L$$

asymptote theorem

$$l = \lim_{x \to \infty} y(x)$$

$$m = \lim_{y \to \infty} y'(x)$$

if $l < \infty$ $m = 0$

<u>MCT</u>

$f_n : X \longrightarrow [0, \infty)$

↳ sequence of measurable function on a set X such that

→ $f_n \longrightarrow f$ pointwise a.e.

→ $f_1 \leq f_2 \leq \ldots$
sequence is non decreasing.

Then

$$\lim_{n \to \infty} \int_X f_n = \int_X \lim f_n$$

e.g.,

$$\lim_{n \to \infty} \int_X \log\left(1 + \frac{x}{n}\right)^n dx \qquad n > 0$$

$$= \log e^x = x = f\, x.$$

DCT

$f_n : R \to R$

seq of measurable functions

→ converge pointwise a.e.

→ $|f_n(x)| \leq g(x)$

↳ L^1

function → riemann not lebesgue

↳ $\frac{\sin x}{x}$

function → lebesgue → not riemann.

Dirichlet function

→ continuous function on bounded intervals are lebesgue integrable.

metric space

$d(x,y)$ → metric
X → set of points.
} — metric space

such that

$d(x,y) = 0$ if $x = y$

$d(x,y) > 0$

$d(x,y) = d(y,x)$

$d(x,y) \leq d(x,z) + d(y,z)$

Normed space

set of vectors
metric (Norm)

$\|\cdot\| : X \longrightarrow [0, \infty)$

$\rightarrow$ +ve
$\rightarrow$ $\|x\| = 0$ if $x = 0$
$\rightarrow$ Scaling linear
$\rightarrow$ Δ inequality.
$\|x+y\| \leq \|x\| + \|y\|$

Complete set

$X = (0, 3)$ and sequence $\frac{1}{n}$

$d(x_n, x_m) \xrightarrow{n, m \to \infty} 0$
doesn't converge to 0 as 0 not in X

↳ Cauchy sequence
(x_n) such that $d(x_n, x_m) < \varepsilon$

$\forall \varepsilon > 0$

Banach Space

If $(X, d_{\|\cdot\|})$ is complete metric space
then the normed space $(X, \|\cdot\|)$ is called a Banach space

$d_{\|\cdot\|}(x, y) = \|x - y\|$

e.g., Real number line
1D real vector space

norm + complete metric space = Banach space

Inner product

$\langle \cdot , \cdot \rangle : X \times X \longrightarrow \mathbb{C}$

norm for inner product space,

$$\| X \|_{\langle \cdot , \cdot \rangle} = \sqrt{\langle x , x \rangle}$$

Hilbert space $\longrightarrow$ Banach space + Norm

convergent (complete)

$(X , \| \cdot \|_{\langle \cdot , \cdot \rangle})$

$(X, \langle \cdot , \cdot \rangle)$ is H.S if $(X, \| \cdot \|_{\langle \cdot , \cdot \rangle})$ is BS

e.g inner product in $\mathbb{R}^n$, $\mathbb{C}^n$

$$\langle x, y \rangle = \sum_{i=1}^{n} x_i y_i$$

not Hilbert

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