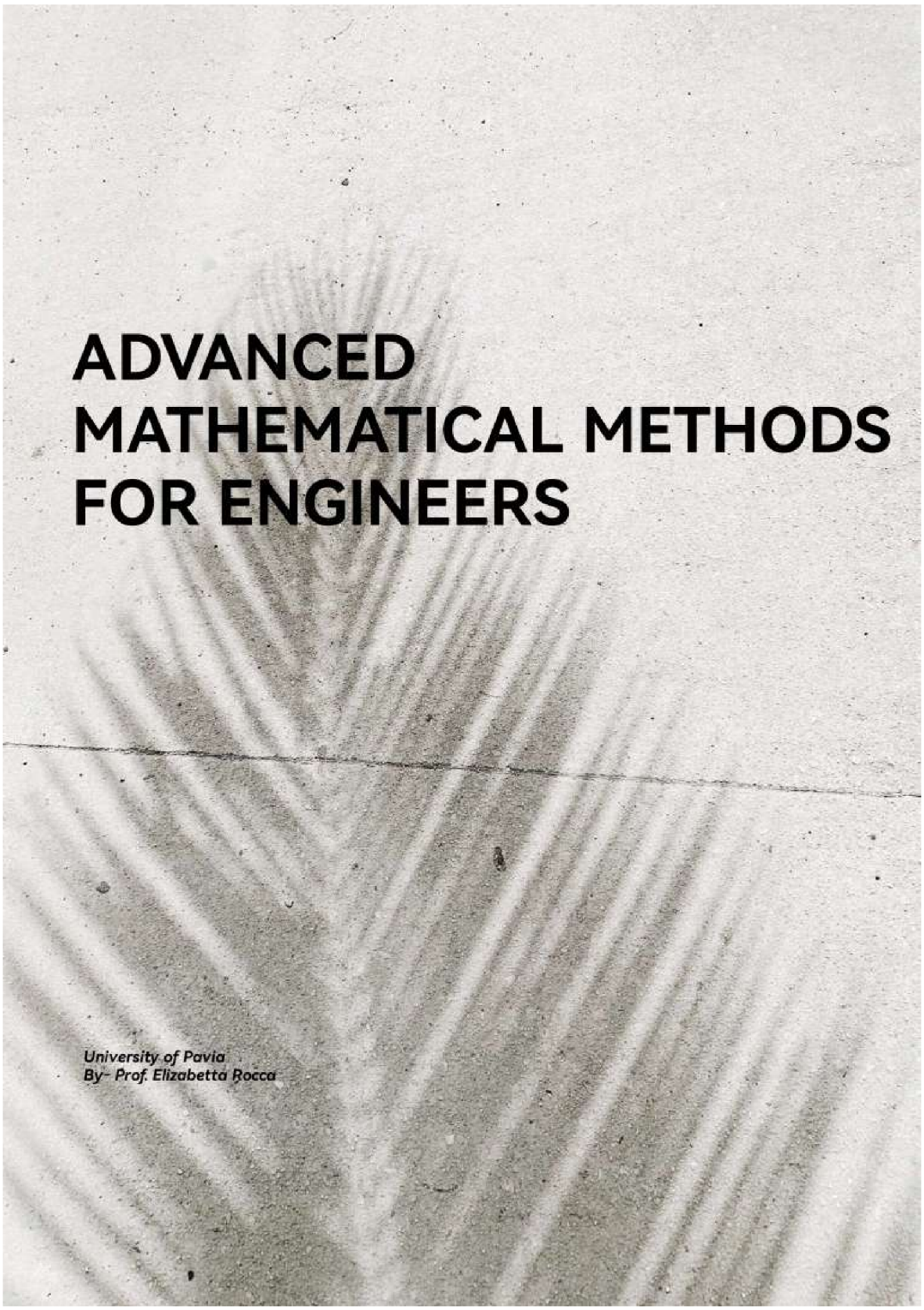


ADVANCED MATHEMATICAL METHODS FOR ENGINEERS

University of Pavia
By- Prof. Elizabetta Rocca

(1) State projection theorem
↳ everything (she squeezed every bit from the lady who asked this question)
↳ example
L^2 → space
→ set of constant functions
apply and show ? IDK exactly because I had not prepared this topic properly so I chose to shut my brain off for a moment to avoid activating my parasitic SCR and explode my head !

(2) Take a sequence $n\cos(nx)$ and explain in terms of distributions as $n \to \infty$.
the idea that the lady answered was very good !

Ans As $n \to \infty$ in any other space this function would blow up
L^1, L^2, Norm, Banach, Hilbert

But in case of distributions
say $f_n(x) = n\cos(nx)$

$\langle f_n, \varphi(x) \rangle$ where $\varphi(x) \in \mathcal{D}(R)$ → C^∞ with compact support

$$= \int_R (n\cos(nx))\, \varphi(x)\, dx$$

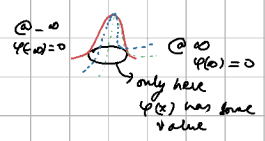
The idea is to make → $\frac{\cos nx}{n}$ somehow so that the whole thing goes to zero as $n \to \infty$

$n\cos nx$ can be written as $\frac{d}{dx}(\sin(nx))$ → $g(x)$

$$= \int_R g'(x)\, \varphi(x)\, dx$$

$$= \int_R [\sin nx]'\, \varphi(x)\, dx$$

↓ → apply by parts until you get
$\frac{\cos nx}{n}$
→ $\varphi(\infty) = 0$ as it has compact support.



(3) Local Cauchy Lipz. Theorem
↳ example where it holds
↳ example where it doesn't

I WAS LUCKY (MY SET OF QUESTIONS)

(1) GLOBAL CAUCHY LIPZ. → everything (she took 10+ minutes)
↳ example, counterexample
↓ $f(x,y) = g(x)\,y$ ↓ $f(x,y) = g(x)\,y^D$ (any power)
→ Solve using Lipz. theorem
→ where is g valid
→ where is y valid
} She helped me in this and I was able to shoot arrows in dark and somehow it hit target! idk what was target!

(2) Dominated Convergence Theorem → (1)
↳ every single detail.
+
↳ what is another improvement to this
↳ $|f_n - f| \xrightarrow{n\to\infty} 0$ → (2)
Prove this (Idk somehow I was able to, with her help)

(2) means → (1)
But (1) ⟶/ (2) (doesn't mean)

(3) Prove Heaviside step function is equal to δ. In sense of distribution. (this is very easy)

[So Yes, I was Lucky]

Pass in Maths ⇒ $\lim_{luck \to 0} Study(luck) = \infty$
work needed

$\lim_{luck \to \infty} Study(luck) =$ finite (huge) $\in \mathbb{R}$