STATISTICS WITH APPLIED PROBABILITY

Custom eBook for STA258

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Statistics with Applied Probability
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Chapter 0

Overview

Uncertainty is an inherent part of everyday life. We all face questions regarding uncertainty such as whether classes will go ahead as planned on any given day; will a flight leave on time; will a student pass a certain course? Uncertainties might also change depending on other factors, such as whether classes will still go ahead as planned when there is a snow warning in effect; if a flight is delayed can a person still manage to make their connection; will a student pass their course considering that the instructor is known to be a tough grader?

The ability to quantify uncertainty using rigorous mathematics is a powerful and useful tool. Calculating uncertainty on an intuitive level is something that is hard-wired in our DNA, such as the decision to fight or flight depending on a given set of circumstances. However we cannot always make such intuitive decisions based purely on hunches and gut feelings. Fortunes have been lost based on someone having a good feeling about something. If we have information available, we should make the best prediction possible using this information. For instance if we wanted to invest a lot of money in a company, we should use all available data such as past sales, market and industry trends, leadership ability of the CEO, forward looking statements etc. and with all this information we can then predict whether our investment will be profitable.

In order for companies to survive and remain competitive in todays environment it is essential to monitor industry trends and read markets properly. Companies that don't adapt and stick to an outdated business model tend to pay the price. At the other end of the spectrum, companies that understand the needs of the consumer, build their product around the consumer and keep evolving their product offerings based on consumer trends tend to perform well and remain competitive.

Statistics is the science of uncertainty and it is clearly a very useful subject for business. In this book you will be given an introduction to statistics and you will learn the framework as well as the language required at the introductory level. The material may be daunting at times, but the more you get familiar with the subject the more comfortable you will become with it. As business students, doing well in a statistics course will give you a competitive edge since the ability to interpret and perform quantitative analytics are skills that are highly desired by many employers.

Chapter 1

Introduction

1.1 Basics

Intuitively, statistics can be considered the science of uncertainty. Formally,

Definition 1.1 (Statistics).
Statistics is the science of collecting, classifying, summarizing, analyzing and interpreting data.

more information goes here
anything

Chapter 4

Normal Approximation to the Binomial Distribution

4.1 Definitions and Setup

Definition 4.1.1 (Statistic)

A *statistic* is a function of the observable random variables in a sample and known constants.
Since statistics are functions of the random variables observed in a sample, they themselves are random variables. As such, all statistics have a corresponding probability distribution, which we refer to as their *sampling distribution*.

Review from STA256

Bernoulli Distribution:
A Bernoulli trial is a single experiment with two outcomes:

- Success: $X = 1$ with probability p
- Failure: $X = 0$ with probability $1 - p$

$X = x$	0	1
$P(X = x)$	$1 - p$	p

The probability mass function (PMF) is:

$$f(x) = p^x (1 - p)^{1-x}, \quad x \in \{0, 1\}$$

Binomial Distribution:
A binomial distribution arises from n independent Bernoulli trials. Let:

$$X = \text{number of successes in } n \text{ trials}$$

Then:

$$X \sim \text{Binomial}(n, p)$$

where:

- Each trial results in either success (with probability p) or failure (with probability $1-p$)
- $X \in \{0, 1, \dots, n\}$

The PMF is:

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$$

4.2 Bernoulli Distribution

Consider the random experiment of rolling a die once. Define the random variable:

$$X_i = \begin{cases} 1 & \text{if the } i\text{-th roll is a six,} \\ 0 & \text{otherwise} \end{cases}$$

Then $X_i \sim \text{Bernoulli}(p)$, where $p = P(\text{rolling a six})$.
The expected value and variance of a Bernoulli random variable are:

$$\mu = \mathbb{E}(X) = p, \quad \sigma^2 = \text{Var}(X) = p(1-p)$$

4.3 Sampling Distribution of the Sum and MGF Derivation

Consider determining the sampling distribution of the sample total:

$$S_n = X_1 + X_2 + \cdots + X_n$$

Suppose $X_i \overset{iid}{\sim} \text{Bernoulli}(p)$. Then the moment-generating function of S_n is:

$$\begin{aligned} M_{S_n}(t) &= \mathbb{E}[e^{tS_n}] \\ &= \mathbb{E}\left[e^{t(X_1+X_2+\cdots+X_n)}\right] \\ &= \mathbb{E}\left[e^{tX_1}e^{tX_2}\dots e^{tX_n}\right] \quad \text{(independence)} \\ &= \mathbb{E}[e^{tX_1}] \cdot \mathbb{E}[e^{tX_2}] \cdots \mathbb{E}[e^{tX_n}] \\ &= M_{X_1}(t) \cdot M_{X_2}(t) \cdots M_{X_n}(t) \\ &= \left[pe^t + (1-p)\right]^n \end{aligned}$$

Since this is the MGF of a binomial random variable with parameters n and p, we conclude:

$$S_n \sim \text{Binomial}(n, p)$$

Example: Binomial Distribution from Die Rolls

We can think of rolling a die n times as an example of the binomial setting. Each roll gives either a six (a "success") or a number different from six (a "failure").
Knowing the outcome of one roll doesn't tell us anything about the others, so the n rolls are independent.
If we call a six a success, then:

- The probability of success on each trial is $p = P(\text{rolling a six}) = \frac{1}{6}$
- The probability of failure is $1 - p = \frac{5}{6}$

Let Y be the number of sixes rolled in n trials. Then $Y \sim \text{Binomial}(n, p)$, and the distribution of Y is called a **binomial distribution**.

4.4 Binomial Distribution Summary

A random variable Y is said to have a **binomial distribution** based on n trials with success probability p if and only if:

$$p(y) = \frac{n!}{y!(n-y)!} p^y (1-p)^{n-y}, \quad y = 0, 1, 2, \ldots, n \quad \text{and} \quad 0 \leq p \leq 1$$

The expected value and variance of a binomial random variable are:

$$\mu = \mathbb{E}(Y) = np, \quad \sigma^2 = \text{Var}(Y) = np(1-p)$$

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