STA258: Statistics with Applied Probability – Formula Sheet

Probability Distributions

Distribution	Distribution function	Support	Mean	Variance
Bernoulli	$f(x) = p^x(1-p)^{1-x}$	$x = 0, 1$	p	$p(1-p)$
Binomial	$f(x) = \binom{n}{x} p^x(1-p)^{n-x}$	$x = 0, 1, 2, \dots, n$	np	$np(1-p)$
Normal	$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$	$-\infty \leq x \leq +\infty$	μ	σ^2
Gamma	$f(x) = \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{-\frac{x}{\beta}}$	$x > 0$	$\alpha\beta$	$\alpha\beta^2$
Chi-Square	$f(x) = \frac{1}{2^{k/2}\Gamma(k/2)} x^{(k/2)-1} e^{-x/2}$	$x > 0$	k	$2k$

Transformations

Let X be a normally distributed random variable with a mean μ and variance σ^2.

$$Z = \frac{x-\mu}{\sigma}$$

Let $\bar{x}$ represent the sample mean of n independent observations from a distribution with finite mean μ and finite variance σ^2. For normal populations or sufficiently large n:

$$Z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$$

Let $\bar{x}$ and s^2 represent the sample mean and sample variance of n independent observations from a distribution with finite mean μ and some unknown finite variance. For normal populations or sufficiently large n:

$$T = \frac{\bar{x}-\mu}{s/\sqrt{n}}$$

Inference Procedures for the Population Mean μ

One-Sample

Confidence interval for μ when σ is known:

$$\bar{x} \pm Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

Confidence interval for μ when σ is unknown:

$$\bar{x} \pm t_{n-1,\alpha/2}\frac{s}{\sqrt{n}}$$

Continuity Correction for the Normal Approximation

Binomial Probability	Continuity Correction	Normal Approximation
$P(X = x)$	$P(x - 0.5 \leq X \leq x + 0.5)$	$P\left(\frac{x - 0.5 - \mu}{\sigma} \leq Z \leq \frac{x + 0.5 - \mu}{\sigma}\right)$
$P(X \leq x)$	$P(X \leq x + 0.5)$	$P\left(Z \leq \frac{x + 0.5 - \mu}{\sigma}\right)$
$P(X < x)$	$P(X \leq x - 0.5)$	$P\left(Z \leq \frac{x - 0.5 - \mu}{\sigma}\right)$
$P(X \geq x)$	$P(X \geq x - 0.5)$	$P\left(Z \geq \frac{x - 0.5 - \mu}{\sigma}\right)$
$P(X > x)$	$P(X \geq x + 0.5)$	$P\left(Z \geq \frac{x + 0.5 - \mu}{\sigma}\right)$
$P(a \leq X \leq b)$	$P(a - 0.5 \leq X \leq b + 0.5)$	$P\left(\frac{a - 0.5 - \mu}{\sigma} \leq Z \leq \frac{b + 0.5 - \mu}{\sigma}\right)$

Some Relationships Between Random Variables

Let $Z_1, Z_2, \ldots, Z_n \overset{\text{iid}}{\sim} N(0,1)$. Then

$$V = \sum_{i=1}^{n} Z_i^2 \sim \chi_n^2$$

Let $X_1, X_2, \ldots, X_n \overset{\text{iid}}{\sim} N(\mu, \sigma^2)$. Let $\bar{x}$ and S^2 represent the sample mean and variance of $X_1, X_2, \ldots, X_n$ respectively. Then

$$\frac{(n-1)S^2}{\sigma^2} = \frac{1}{\sigma^2} \sum_{i=1}^{n} (X_i - \bar{X})^2 \sim \chi_{n-1}^2$$

Let $Z \sim N(0,1)$ and let $W \sim \chi_n^2$. Then

$$T = \frac{Z}{\sqrt{\frac{W}{n}}} \sim t_n$$

Let $X_1, X_2, \ldots, X_{n_x} \overset{\text{iid}}{\sim} N(\mu_x, \sigma^2)$ with sample variance S_x^2, and let $Y_1, Y_2, \ldots, Y_{n_y} \overset{\text{iid}}{\sim} N(\mu_y, \sigma^2)$ with sample variance S_y^2. with the $X_i's$ and the $Y_j's$ all being independent. Then

$$F = \frac{\left(\frac{n_x - 1}{\sigma^2}\right) S_x^2 / (n_x - 1)}{\left(\frac{n_y - 1}{\sigma^2}\right) S_y^2 / (n_y - 1)} \sim F_{n_x - 1,\, n_y - 1}$$