Q5.)

(a) CPTs for the Naive Bayes Model

C	P(C)
p	$\frac{(8+42+4+22+1+3+1+4)}{250} = 0.34$
n	$\frac{(2+1+31+6+11+3+90+21)}{250} = \frac{165}{250} = 0.66$

F_1	C	$P(F_1 \mid C)$
~~t~~	~~p~~	~~$(8+42+4+22)/\cancel{250}\ 85 = 76/250 = 0.304$~~
~~t~~	~~n~~	~~$(2+1+31+6)/250 = 40/250 = 0.16$~~
~~f~~	~~p~~	~~$(1+3+1+4)/250 = 9/250 = 0.036$~~
~~f~~	~~n~~	~~$(11+3+90+$~~

F_1	C	$P(F_1 \mid C)$
t	p	$(8+42+4+22)/85 = 76/85 \approx 0.894117$
t	n	$(2+1+31+6)/165 = 40/165 \approx 0.242424$
f	p	$(1+3+1+4)/85 = 9/85 \approx 0.10588$
f	n	$(11+3+90+21)/165 = 125/165 \approx 0.757$

≈ 0.75757

F_1	$P(F_1)$
t	$(8+2+42+1+4+31+22+6)/250 = 116/250 = 0.464$
f	$(1+11+3+3+1+90+4+21)/250 = 134/250 = 0.536$

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f_2	C	$P(F_2 \mid C)$
t	p	(8+42+1+3)/85 ≈ 0.63529
t	n	(2+1+11+3)/165 = 17/165 ≈ 0.103030
f	p	(4+22+1+4)/85 = 31/85 ≈ 0.3647
f	n	(31+6+90+21)/165 = 148/165 ≈ 0.8969

F_2	$P(F_2)$
t	(8+2+42+1+1+11+3+3)/250 = 0.284
f	(4+31+22+6+1+90+4+21)/250 = 0.716

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f_3	C	$P(F_3 \mid C)$
t	p	(8+4+1+1)/85 = 14/85 ≈ 0.16470
t	n	(2+31+11+90)/165 = 134/165 ≈ 0.81212
f	p	(42+22+3+4)/85 = 71/85 ≈ 0.83529
f	n	(1+6+3+21)/165 = 31/165 ≈ 0.187878

F_3	$P(F_3)$
t	(8+2+4+31+1+11+1+90)/250 = 0.592
f	(42+1+22+6+3+3+4+21)/250 = 0.408

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(b) The given conditional probability,
$P(C=p \mid F_1 = f, F_2 = t, F_3 = f)$ represents the probability of class variable C to be p (positive message) given that the event ~~word~~ $F_1 = f, F_2 = t, F_3 = f$ has already ~~occured~~ occured.

(b) $P(C=p \mid F_1=f, F_2=t, F_3=f)$

The probability of the given configuration can be calculated by.

$$P(C=p \mid F_1=f, F_2=t, F_3=f) =$$

$$= \frac{P(C=p)\, P(F_1=f, F_2=t, F_3=f \mid C=p)}{P(F_1=f, F_2=t, F_3=f)}$$

$$= \frac{P(C=p)\, P(F_1=f \mid C=p)\, P(F_2=t \mid C=p)\, P(F_3=f \mid C=p)}{P(F_1=f, F_2=t, F_3=f)}$$

$$= \frac{0.34 \times 0.10588 \times 0.63529 \times 0.83529}{[P(C=P) * P(F_1=f \mid C=P) * P(F_2=t \mid C=P) * P(F_3=f \mid C=P)] + [P(C=N) * P(F_1=f \mid C=n) * P(F_2=t \mid C=n) * P(F_3=f \mid C=n)]}$$

$$= \frac{0.0190}{0.0190 + [0.66 * 0.18 * 0.10 * 0.75]}$$

$$= \frac{0.0190}{0.0190 + 0.0096}$$

$$= \frac{0.0190}{0.02791} \approx 0.67$$

(c) For the given partial configuration
$(F_1 = f, F_2 = t, F_3 = f)$

~~$P(C = p \mid F_1 = f, F_2 = t, F_3 = f)$~~

For finding out the most likely value of the class variable C for partial configuration, we have to compute,

$P(C = p \mid F_1 = f, F_2 = t, F_3 = f)$
AND
$P(C = n \mid F_1 = f, F_2 = t, F_3 = f)$
and choose the maximum ~~of~~ out of them

It can be written as,

~~$\arg\max_{k \in \{p,n\}} P($~~

$$\arg\max_{k \in \{p,n\}} P(C = k \mid F_1 = f, F_2 = t, F_3 = f)$$

by definition

$$\Rightarrow \arg\max_{k} \frac{P(C = k, F_1 = f, F_2 = t, F_3 = f)}{P(F_1 = f, F_2 = t, F_3 = f)}$$

$P(F_1 = f, F_2 = t, F_3 = f)$ doesn't depend on K, hence,

$$= \arg\max_K P(C=K, F_1=f, F_2=t, F_3=f)$$

and by using Naive Bayes Assumption,

$$= \boxed{\arg\max_K P(C=K) \cdot P(F_1=f \mid C=K) \cdot P(F_2=t \mid C=K) \cdot P(F_3=f \mid C=K)}$$

$$= A(K)$$

In part b), we have already calculated that,

For $K=P$, $A(K=P) \approx 0.0190$

And for $K=n$, $A(K=N) \approx 0.0096$

$$\boxed{\text{And therefore, } \arg\max_K A(K) = P}$$

(d) On using the Joint Distribution Model,

$P(C=p \mid F_1=f, f_2=t, F_3=f)$

$$= \frac{P(C=p, F_1=f, f_2=t, F_3=f)}{P(F_1=f, F_2=t, F_3=f)}$$

$$= \frac{P(C=p, F_1=f, f_2=t, F_3=f)}{[P(C=p, F_1=f, F_2=t, F_3=f) + P(C=n, F_1=f, F_2=t, F_3=f)]}$$

$$\Rightarrow \frac{3/250}{(3/250)+(3/250)} \Rightarrow \frac{0.012}{0.024} = 0.5$$

e) By using Fully Independent Model,

$P(C=p \mid F_1 = f, F_2 = t, F_3 = f) = P(C=p)$

$= 85/250$

$= 0.34$