

Don't Try to Count - Detailed Revision Notes

Problem: Codeforces 1881B | Rating: 800 | TLE CP-31 Sheet Problem #7

Problem Statement

Given a string X of length n and a string S of length m such that $n \times m \leq 25$, you can perform the following operation any number of times:

- Append the current X to itself (i.e., $X = X + X$).

After each operation, X changes to the new concatenated string. Determine the **minimum** number of operations required for S to appear as a **substring** of X . If it never appears, output -1 .

Input Format:

- First line: T (number of test cases)
- For each test case:
 - Line: integers n, m
 - Line: string X
 - Line: string S

Output:

- For each test case, print the minimum operations or -1 if impossible.

Key Intuition & Observations

1. **Upper Bound on Operations:** Since $n \times m \leq 25$, worst-case $n=1$ and $m=25$ (e.g., $X="a"$, $S="aaaa...a"$ of length 25). Each operation doubles length of X :
 - After k operations, length = $n \times 2^k$.
 - To cover $m=25$, need smallest k such that $n \times 2^k \geq m$. For $n=1$, $2^k \geq 25 \rightarrow k=5$ (since $2^5=32$). Further operations only produce longer repeats of same pattern, so if not found by $k=5$, never found.
2. **Brute-Force Feasibility:** Maximum 6 versions of X (from 0 to 5 operations). Each check scans X for substring s in $O(|X| \times |S|)$ which is $O((n \times 2^k) \times m)$. At $k=5$, $n \times 2^5 \leq 32$, $m \leq 25 \rightarrow O(32 \times 25) = 800$ operations per test. With $T \leq 10^4$, worst-case $\sim 8 \times 10^6$ operations fits within limits.

Solution Approach

1. Precompute X variants:

- $X_0 = X$ (0 ops)
- For i in $[1..5]$: $X_i = X_{i-1} + X_{i-1}$

2. Check substring:

- For each i from 0 to 5:
 - If $|X_i| < m$, continue
 - Slide a window of size m over X_i and compare to S (or use built-in `find`).
 - If found, record i and break.

3. Output:

- If found at $i \leq 5$, print i ; otherwise print -1.

Time & Space Complexity

- **Time (per test):** $O(6 \times |X_5| \times m) = O((n \times 32) \times m) \leq O(32 \times 25) = 800 \rightarrow \text{total } O(T \times 800)$.
- **Space:** $O(n \times 2^5)$ to store largest X_5 , i.e. $O(32n)$.

Example Walkthroughs

Test Case	Operations	Answer
$X=a, S=aaaaa$ ($m=5$)	$X_0=a$ (len=1), $X_1=aa$ (2), $X_2=aaaa$ (4), $X_3=aaaaaaaa$ (8): found at $i=3$	3
$X=aba, S=abaaba$	$X_0=aba$ (3)<6, $X_1=abaaba$ (6): found at $i=1$	1
$X=xyz, S=xyzxyzxyz$ ($m=9$)	$X_0=3<9, X_1=6<9, X_2=12$: substrings include <code>xyzxyzxyz</code> at positions $0 \rightarrow 8 \rightarrow i=2$	2
$X=ab, S=c$	None of $X_0 \dots X_5$ contains 'c'	-1

Common Pitfalls

- **Infinite Loop Assumption:** Forgetting that doubling stops at 5 ops suffices by size constraints.
- **Off-by-One in Window:** Loop limit should be $i \leq |X_i| - m$ inclusive.
- **Ignoring $|X_i| < m$ check:** Skip checks when current X_i is shorter than S .

Competitive Programming Insights

- **Bounding Operations:** Use problem constraints ($n \times m \leq 25$) to cap brute force iterations.
- **Doubling Patterns:** Recognize exponential growth to meet length requirements quickly.
- **Window Sliding:** Efficient substring check via sliding window or built-in string search.

Final C++ Implementation Sketch

```
bool contains(const string &X, const string &S) {
    if (X.length() < S.length()) return false;
    return X.find(S) != string::npos;
}

int solve(string X, string S) {
    vector<string> V(6);
    V[0] = X;
    for (int i = 1; i <= 5; ++i)
        V[i] = V[i-1] + V[i-1];
    for (int i = 0; i <= 5; ++i) {
        if (contains(V[i], S)) return i;
    }
    return -1;
}

int main() {
    ios::sync_with_stdio(false);
    cin.tie(NULL);
    int T; cin >> T;
    while (T--) {
        int n, m; cin >> n >> m;
        string X, S; cin >> X >> S;
        cout << solve(X, S) << '\n';
    }
    return 0;
}
```