Distributed Multi-Temporal Risk Management Approach to Designing Dynamic Pricing

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***Abstract*—Due to the nature of the power system and market, there always exist physical and financial risks both in the long run at the planning stage, and in the short run at the operation stage. Many attempts to minimize these risks from the demand side so far have failed because of the lack of information on the true value of demand resources to the system. In this paper, we attempt to tackle these problems by proposing a framework where the system, load serving entities, and the end-users exchange the right information on the system condition (represented by the price) and the energy consumption with respect to it (represented by the demand function). This information should be exchanged through different layers of the market ranging from the end-users on the bottom to the system operator at the top. The information should also be presented in different timeframes so that it captures the right signals for different purposes, such as long-term planning on energy efficiency and short-term energy balance of supply and demand. We present the mathematical formulation of this framework as a decision making of each entity.**

NOMENCLATURE

Indices

h	hourly time step $(h = 1, \cdots, H)$
m	monthly time step within a year $(m = 1, \cdots, 12)$
n	yearly time step $(n = 1, \cdots, N)$
N_h	hourly time steps within N years
Y_h	hourly time steps within a year
s	end-users' group $(s = 1, \cdots, S)$

Variables

$r[\cdot]$	rate charged to each group of end-users per unit of energy over an hour, a month, or a year $\in \mathbf{R}^S$
$\hat{u}[\cdot]$	anticipated total energy usage by each group of end-users during a month, or a year $\in \mathbf{R}^S$
$y_{\text{a}}[\cdot]$	load serving entity's long-term energy procurement via annual bilateral contracts
$y_{\text{m}}[\cdot]$	load serving entity's long-term energy procurement via monthly bilateral contracts
$y_{\text{sp}}[h]$	load serving entity's spot market purchase amount for hour h
ζ	dummy variable for CVaR optimization

Parameters

d_h	hourly total load to be served by load serving entity with a known probability distribution
p^{lt}	long-term contract price for energy over a month, or a year
$\hat{p}_{\text{sp},h}$	anticipated spot market price at hour h
σ_h^{sp}	standard deviation of the anticipated spot market price at hour h
Σ_{sp}	covariance matrix of the anticipated spot market price
$C_{\text{inv},n}$	long-term investment cost in energy efficiency in year $n \in \mathbf{R}^S$
β	weight on risk term (CVaR) compared to the average cost
α	lower percentile of the probability distribution of the LSE's cost with respect to demand and price

I. INTRODUCTION

While there have been much effort in utilizing the most of the demand resources in power systems for the last decades, there are still issues to be resolved. The demand response programs in a lot of cases either lack the right incentive for the demand entities to participate in, or have the wrong information on the *baseline* load which the compensation is based on. This results in over- or under-investment of resources and inappropriate compensations to the wrong participants. The most fundamental cause of this is that the demand entities are not exposed to the right signal that reflects the true system condition, while the system operator does not see the true value of energy to the end-users. On each side, there lacks information on the other. This issue is more prominent when it comes to the very end of the demand side of the system, or the end-users, in relation to the entire system conditions.

The basic idea of our proposed framework of adaptive load management (ALM) [1] is along the line with the contract-based approaches such as *demand subscription* by Chao [2] or *buy the baseline* by Bushnell et al. [3] They argue that the demand entities should procure their energy in advance either in the day-ahead market or at a fixed price prior to the actual consumption, and the difference between this *ex ante* contract and the actual consumption should be cleared at the real-time market price or a different rate that better reflects the real-time market price. We expand the idea even further both temporally to the planning stage and spatially to the small end-users aggregated by various load serving entities (LSEs).

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Practically, it is impossible for the system operator or the load serving entity(LSE) to have perfect knowledge and information on the demand or the physical/financial system condition. The fundamental reason for this is because the system operation needs to be planned ahead for reliability ahead of the time the actual consumption of energy occurs. One effective way to tackle this difficulty is for both the parties of demand and supply (or the system) to agree on the optimal operation and price with the best possible forecast of information at certain points of time ahead of actual operation and consumption. After this agreement, however, as it approaches the time of actual consumption, the "best possible forecast of information" can change. The discrepancy between the older and newer forecast and plans, which becomes various forms of *risks* on different parties, can be reconciled by another form of contract at a shorter time scale than the previous ones, before it reaches the real-time clearance. This idea on the supply side is realized in some sense in the form of markets with different time scales such as capacity, future, and real-time markets.

The contracts between the end-users and the load serving entities should be decided in a similar way. End-users should be given an option to hedge against or take advantage of the risks of the volatile spot market price. End-users should be able to, through their LSEs, differentiate between the more *risky* but less expensive energy and its counterpart, and choose to purchase energy based on these differentiated levels of energy service. For example, the end-users can be rewarded with lower energy bills by providing information on their plans to use energy in advance so that the load serving entities can better hedge against the demand uncertainty. The more risk-prone end-users could also have an option to have contracts with the load serving entities to pay at a rate that reflects more of the spot market price. We suggest, in this paper, the framework of this idea of dynamic pricing, relating the contracts between the LSE and the suppliers or market, and the end-users.

Because there are always risks in making decisions with this imperfect information in advance, there needs an agreeable measure of these risks. Conditional value-at-risk (CVaR) is a risk measure that is widely used in various fields due to its superior mathematical properties and ease of optimization applications [4]–[6]. It has also been applied to risk management in electricity markets [7]–[9]. We also adopt CVaR to formulate the risk of an LSE from the uncertainty of demand and price.

An LSE can procure energy on behalf of its end-users through bilateral contracts with the best available information on the load it should serve, while trying to minimize the risk from the uncertainty of demand and price. At the same time, the LSE will decide on the end-user rate charged for this particular energy procurement at this level, the best forecast of the demand, and its price sensitivity. The end-users and the LSE reach an agreement on which price and what level of energy amount should be traded between them, based on these particular bilateral contracts. Once the long-term end-user rate and energy amount are locked in this way, there may be

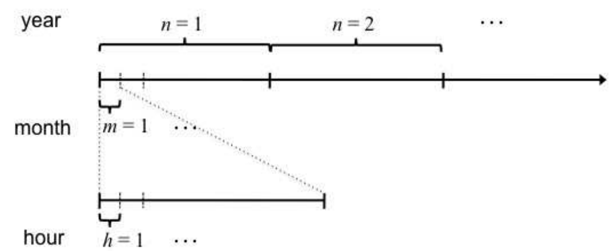


Fig. 1: The timeline of adaptive load management

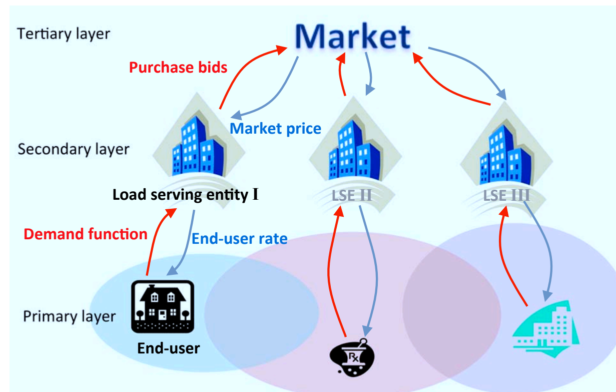


Fig. 2: Information exchange in adaptive load management

updated information on the demand and/or the market/system condition. The LSE can have another form of contracts that can adjust for this change within a shorter time scale than the previous ones, and it locks another end-user rate and energy amount with the end-users. This procedure can be repeated until they reach the point of actual consumption and clearance in the real-time market.

More specifically, we first run the longest timescale optimization by deciding on the yearly procurement of the energy for an LSE. This decision is made based on the end-users' demand sensitivity to the yearly rate and on the sensitivity to the investment in energy efficiency measures. After a yearly energy procurement has settled, we move on to the monthly contract offered by a supplier and the anticipated spot market price. Here again, the end-users' demand with respect to the monthly charge, or the demand function, is considered in making the decision. This procedure of yearly and monthly energy procurement can trickle down to the hourly energy procurement procedure in the same way. When it gets to the hourly energy procurement for days ahead, then the physical dynamics of the end-users' loads can also be included as a function of time and energy usage. The timeline of this procedure is shown in Fig. 1, and the diagram of information exchange is depicted in Fig. 2. Next section presents the detailed mathematical formulation of each decision making.

II. Formulation

A. Decision making on energy years

An LSE is given the long-term bilateral contract offer for the years to come; a price per MWh on certain blocks of time during the year, e.g. peak hours on weekdays from March to July. The LSE also has an estimate of what its load would look like for the period based on historic data, and the estimate of the spot market price along with the monthly contract offers. With this price information of the system *and* the information of the energy consumption of the end-users that it serves, the LSE decides on how much energy to procure from the yearly contract; we call this amount of energy *energy year*[1]. The information on the demand of the end-users is formed as a function with respect to the yearly charge of energy to the users. In a long-term optimization, the demand is also a function of investment in long-term energy efficiency measures such as insulating a building or replacing an old refrigerator with a more energy-efficient one.

Then the LSE's optimization problem can be formulated as minimizing the energy cost and the risk from the uncertainty of demand and price less the revenue from the end-users:

$$\min_{y,r,\zeta} \sum_{h=1}^{N_h} \{p_n^{\text{lt}} y_{\text{a}}[h] + \hat{p}_m^{\text{lt}} y_m[h] + \hat{p}_{\text{sp}} y_{\text{sp}}[h]\} + \beta F_\alpha(y_{\text{a}}, \zeta) - \sum_{n=1}^{N} r^T[n] \hat{u}_n(r[n], C_{\text{inv},n}) \quad (1)$$

subject to

$$E\{d_h\} = y_{\text{a}}[h] + y_{\text{m}}[h] + y_{\text{sp}}[h] \text{ for } h = 1, \cdots, N_h$$
$$y_{\text{a,min}} \leq y_{\text{a}}[h] \leq y_{\text{a,max}} \text{ for } h = 1, \cdots, N_h$$
$$y_{\text{m,min}} \leq y_{\text{m}}[h] \leq y_{\text{m,max}} \text{ for } h = 1, \cdots, N_h$$
$$E\{\sum_{h=h_{\text{n,start}}}^{h_{\text{n,end}}} d_h\} = \hat{u}_n(r[n], C_{\text{inv},n}) \text{ for } n = 1, \cdots, N$$
$$\hat{u}_{n,\min} \leq \hat{u}_n \leq \hat{u}_{m,\max} \text{ for } n = 1, \cdots, N$$

where

$$F_\alpha(y_{\text{a}}, \zeta) = \zeta + \frac{1}{1-\alpha} E\{[p_n^{\text{lt}} y_{\text{a}}[h] + \hat{p}_m^{\text{lt}} y_m[h] + \hat{p}_{\text{sp}} y_{\text{sp}}[h] - \zeta]^+\}$$

is a term for risk from the uncertainty of demand d_h and price $\hat{p}_m^{\text{lt}}$ and $\hat{p}_{\text{sp}}$.

We assume a linear demand function throughout this paper. Therefore, it can be defined as:

$$\hat{u}(r[\cdot]) = Ar[\cdot] + b$$

where A is a S-dimensional square matrix and b is a column vector of length S. The diagonal elements of A are usually negative, or in other words, the higher the price the lower the demand. If we assume that the behavior of each group of end-users does not have any influence on each other, then A is diagonal. In this yearly time scale including the investment in energy efficiency, the demand function is

$$\hat{u}_n(r[n], C_{\text{inv}}) = A'_n r[n] + b'_n.$$

If we define the demand function with no investment as $\hat{u}_n(r[n], 0) = A_n r[n] + b_n$, then the elements of A'_n are likely to be greater than those of A_n. This is because if the end-user's premise is highly energy-efficient, then their energy usage is likely to be low without regard to the price, and thus less sensitive to the price change. Also, b'_n is likely to be smaller than b_n because of the reduction in energy consumption overall.

Consumer surplus from investing in energy efficiency is the difference of the energy cost from the case where they do not invest. If they do invest, it will incur them a certain capital cost, but the energy cost is likely to reduce. If they decide not to, then they have no initial capital cost, but the energy cost will be higher than the case where they improve their energy efficiency. Consumer surplus with $r[n]$ as a variable now can be defined mathematically as

$$CS(r[n]) = r^T[n]\hat{u}_n(r[n], 0) - r^T[n]\hat{u}_n(r[n], C_{\text{inv}}).$$

It only makes sense for an end-user to invest in energy efficiency when this CS is greater than zero over the course of the optimization time horizon. Note that this optimization is done by the end-users with $r[n]$ given from the LSE. The resulting optimal demand with respect to the given $r[n]$ will be sent back to the LSE so that it decides on the energy year it needs to purchase. This procedure is repeated until both the LSE and the end-user agree on the level of price and energy amount.

B. Decision making on energy months

Now after settling on the yearly contract for the next years, LSE likes to decide on, over a course of one year, how much energy to procure for each month on a monthly contract, and how much to charge within each month to each group of end-users. Monthly long-term bilateral contract energy prices are given to the LSE by a supplier. We also assume that LSEs have information on the anticipated monthly energy usage of the end-users as a function of the rate charged for each month, in a similar way from the yearly optimization. It also has information on anticipated hourly spot market price. As opposed to Equation 1, the monthly contract price offer p_m^{lt} is given deterministically instead of as an expected value $\hat{p}_m^{\text{lt}}$. Also, now you have better estimate on the demand d_h than in the previous decision making as well.

$$\min_{y,r,\zeta} \sum_{h=1}^{Y_h} \{p_m^{\text{lt}} y_{\text{m}}[h] + \hat{p}_{\text{sp},h} y_{\text{sp}}[h]\} + \beta F_\alpha(y_{\text{m}}, \zeta) - \sum_{m=1}^{12} r^T[m]\hat{u}_m(r[m]) \quad (2)$$

[1]The concept of *energy minutes* was introduced by Professor Daniel Siewiorek, in the context of credit of energy bought by end-users that can be exchanged among each other.

$$\text{subject to } E\{d_h\} - y^{\star}_{\mathrm{a}}[h] = y_{\mathrm{m}}[h] + y_{\mathrm{sp}}[h] \text{ for } h = 1, \cdots, Y_h$$
$$y_{\mathrm{m,min}} \leq y_{\mathrm{m}}[h] \leq y_{\mathrm{m,max}} \text{ for } h = 1, \cdots, Y_h$$
$$E\{\sum_{h=h_{\mathrm{m,start}}}^{h_{\mathrm{m,end}}} d_h\} = \hat{u}_m(r[m]) \text{ for } m = 1, \cdots, 12 \tag{3}$$
$$\hat{u}_{m,\min} \leq \hat{u}_m \leq \hat{u}_{m,\max} \text{ for } m = 1, \cdots, 12$$

where

$$F_\alpha(y_{\mathrm{m}}, \zeta) = \zeta + \frac{1}{1-\alpha} E\{[p^{\mathrm{lt}}_m y_{\mathrm{m}}[h] + \hat{p}_{\mathrm{sp},h} y_{\mathrm{sp}}[h] - \zeta]^+\},$$

the term for risk from the uncertainty of demand d_h and price $\hat{p}_{\mathrm{sp}}$.

Note that solving for the hourly purchase amount $y_{\mathrm{m}}[h]$ and $y_{\mathrm{sp}}[h]$ is a linear programming problem and can be solved independently of $r[m]$, if there were not the coupling constraint Equation 3. This means that without the coupling relationship between the expected demand level and the monthly end-user rate, if $\hat{p}_{\mathrm{sp},h} - p^{\mathrm{lt}}_m > 0$ then the optimal $y_{\mathrm{sp}}[h]$ is its lower bound, and if $\hat{p}_{\mathrm{sp},h} - p^{\mathrm{lt}}_m < 0$ then $y_{\mathrm{sp}}[h]$ is the upper bound. This makes sense since when the spot market price is higher than the long-term contract price, then it is most profitable for the LSE to purchase all its energy from the contract, and vice versa. In reality, however, since the LSE does not have perfect knowledge on the anticipated price of the spot market in the future, the optimum will depend on the tradeoff between the expected cost that the LSE pays and the CVaR, the risk measure of the uncertainty.

Based on the optimal spot market purchase obtained from this formulation, we calculate the long-term monthly contract purchase amount and the monthly end-user rate, which is apart from the yearly end-user rate and energy amount locked in from the energy year optimization. In other words, the end-users will have a different rate and the amount limits on the energy that is purchased on the monthly contract, on top of the yearly contract that they made with the LSE.

C. Decision making on energy hours

The formulation for decision making on energy hours differs from the previous ones. Since in this time scale, it is much closer to the actual consumption time point, we can also include the hourly state dynamics of the end-users' premises in terms of their energy usage and their utility of using energy.

$$\min_{x,y,u,\zeta} \sum_{h=1}^{H} \{\hat{p}_{\mathrm{sp},h} y_{\mathrm{sp}}[h] - r[h]^T \hat{u}[h] - \xi f(x[h], \hat{u}[h])\} + \beta F_\alpha(y_{\mathrm{h}}, \zeta) \tag{4}$$
$$\text{subject to } E\{d_h\} - y^{\star}_{\mathrm{a}}[h] - y^{\star}_{\mathrm{m}}[h] = y_{\mathrm{sp}}[h]$$
$$x[h+1] = g(x[h], \hat{u}[h], \theta_h)$$
$$x_{\min}[h] \leq x[h] \leq x_{\max}[h]$$
$$\hat{u}_{\min}[h] \leq \hat{u}[h] \leq \hat{u}_{\max}[h]$$
$$\text{for all } h = 1, \cdots, H$$

where we use

$$F_\alpha(y_{\mathrm{h}}, \zeta) = \zeta + \frac{1}{1-\alpha} E\{[\hat{p}_{\mathrm{sp},h} y_{\mathrm{sp}}[h] - \zeta]^+\}$$
$$f(x[h], \hat{u}[h]) = r[h]^T \hat{u}[h] + \alpha (x[h] - x_{\mathrm{set}}[h])^T (x[h] - x_{\mathrm{set}}[h]) \tag{5}$$
$$g(x[h], \hat{u}[h], \theta_h) = Ax[h] + B\hat{u}[h] + \theta_h$$

in our formulation.

Here, x is the state at end-users' premise by groups such as the indoor temperature. We assume that each group has their own setpoint of their state x_{set} at each hour. $f(x, u)$ represents the end-users' cost of using energy either monetary or non-monetary such as temperature discomfort, as a function of the energy usage u and the state x. In order to account for the weight on the end-users' cost relative to the LSE's, ξ is used as the weight factor. Also, within $f(x, u)$ denoted in Equation 5, the weight on the non-monetary discomfort (the second term) relative to the monetary cost (the first term) is captured by the parameter α. This information needs to come from the end-users and be sent to the LSE. $g(x, u, \theta)$ is the state dynamic function where it depends on the previous state and the energy usage, with the given external factor θ. Instead of representing the demand as a function of price in the other cases, here we assume that the end-user rate at each hour is predefined by the LSE and the end-users aim to minimize their energy bill while maximizing the utility from consuming energy.

As with the other decision making problems, this optimization also calls for the information exchange between the LSE and the end-users. The LSE needs to decide on the $r[h]$ and notify the end-users. The end-users need to calculate their optimal demand with the given price, taking into account their own physical state dynamics, and send the information to the LSE. The LSE may want to recalculate a new $r[h]$ after assessing the difference between the sum of $u[h]$ and the yearly and monthly contracted amount $y^{\star}_{\mathrm{n}}[h]$ and $y^{\star}_{\mathrm{m}}[h]$. Then the LSE should send the new signal back to the users so that they correct their optimal energy usage, until the two parties agree and settle on the optimal price and demand for the spot market purchase.

At this point of time when the actual consumption to be decided on is close, the end-users can give their demand information with respect to the price, or the demand function, with a simple decentralized optimization, given that the end-users know the physical dynamics of their loads $x[h+1] = f(x[h], u[h], r[h])$. With a given price vector value r, the end-users can calculate the optimal $u^\star$ satisfying the constraints. In order to calculate the price sensitivity of demand, the end-user can calculate for the optimal $u^\star_{\mathrm{pert}}$ for a slightly different price r_{pert}. By using a regression model for the relationship between $r^\star$ and $u^\star$, we can calculate the price sensitivity of demand [1]. This procedure is depicted in Fig. 2 between the primary and secondary layers.

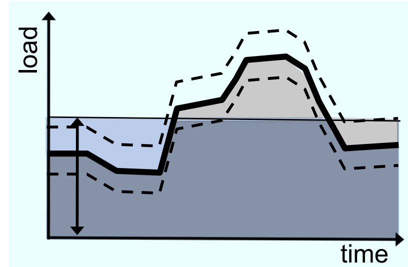
III. Implications on Dynamic Pricing

This proposed framework calls for a fundamental change in the current market structure on the demand side and the demand response programs. There needs a pricing structure between the LSEs and the end-users to communicate and choose for the proper rate that the both parties can agree on. The information needed from the end-users does not necessarily have to be calculated by the end-users themselves. With the communication and computation infrastructure rolled out and used more widely, the terminal devices (smart meters) can do the job for the end-users when needed. On the end-users's side, the interface should be intuitive and simplified so that the complicated and intelligent computation is conducted by the computing terminal. LSEs can also think of a way to interpolate the end-users' demand and price information from the historic data, if available. This assumption will make the implementation of this framework much more feasible even to the small end-users. In any case, the information of the demand and their desired level of end-user rate should be communicated to the LSEs.

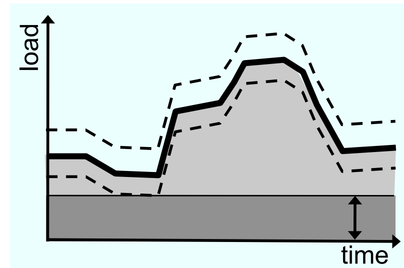
This extended *demand subscription* framework can change the demand market structure where the LSEs bear all the risk from the uncertainty of the demand and price, while the end-users in return pay for the high premium of avoiding this risk with a high flat rate. Since the end-users can opt to participate in procuring energy in advance with a possibly lower rate of bilateral contracts, the LSEs can relieve some of the risk from 1) the uncertainty from the demand by communicating the end-users' needs, and 2) the uncertainty from the market price by procuring the more desired level of energy.

In minimizing the risk, the LSEs and the end-users will have a diversity in how much risk they are willing to take. This can also open up more choices for the end-users to subscribe to different energy services with various risk-reliability profiles. For example, given the same expected demand profile for a year, an LSE that is financially risk-prone (i.e. willing to take the risks in price uncertainty) would choose to procure less amount of energy from the bilateral contracts than its counterpart who is financially risk-averse. However, if an LSE sees the uncertainty of demand as a bigger risk factor and likes to hedge more against demand uncertainty, then it would procure higher amount of energy from the *ex ante* bilateral contracts. In other words, an LSE's optimal portfolio of procuring energy will depend on not only its risk aversity, but also what *kind* of risk it is more interested in hedging against more than the others. This concept is depicted in Fig. 3, where the vertical arrows denote the level of energy procurement with long-term bilateral contracts and the rest of the load is fulfilled from the spot market.

The long-term contracts can take different forms, and it can affect the risk from the price uncertainty. For example, with the same risk aversity, the optimal portfolio will differ if the LSE is allowed to sell back the energy that they procured from the previous longer-term contracts. If they are not allowed to do so, then it would limit the LSE's transactions on long-term contracts and result in more inflexible and conservative (i.e. risk-averse) portfolios. Therefore, in designing the markets in various time scales, it should be considered which markets should bind the physical transactions and which should allow for financial sellbacks.



(a) Financially risk-prone or physically risk-averse LSE



(b) Financially risk-averse or physically risk-prone LSE

Fig. 3: An example of optimal portfolio of LSEs with different risk aversity

IV. Conclusions and Future Work

We present the basic idea and formulation of adaptive load management framework that enables the right signals to penetrate throughout the system and throughout the timeline. This is to make demand more adaptive to the condition of the system and to enable the system planning and operation to take the true value of demand on the very end of the demand side. We have shown through mathematical formulations that this concept requires multi-layered and multi-temporal optimizations and adaptations between the entities and the system. We like to point out that with the new communication and metering technologies and infrastructure becoming available, this framework can be implemented to the small end-users in the system, who should certainly be represented in the wholesale market through load serving entities. Especially with more data on end-users' energy usage and the price sensitivity of their demand becoming available, more information on end-users' energy usage can be better represented to the LSEs and to the system/market.

With the proposed framework, we conjecture that the risk of uncertainty of market and demand lessens because the entities make decisions iteratively based on their purpose and value of energy, and exchange that information until they reach an equilibrium. Because this framework proposes decision makings at many different time points, when the pre-agreed equilibrium is not valid any more for any party, they can reconcile the difference at the next decision-making time point on the time horizon.

Specific methodologies to solve each of these problems are the very next research topics of this work. Also, some numerical examples to illustrate this concept should follow.

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