Risk-constrained electricity procurement for a large consumer

A.J. Conejo and M. Carrión

Abstract: A solution technique is formulated and provided for the electricity procurement problem faced by a large consumer. The objective of this consumer is to minimise procurement cost while limiting the risk of cost fluctuation due to pool price volatility. Electricity sources include the pool, bilateral contracts and self-production. Uncertainty is related to electricity pool prices. A realistic case study is analysed and results presented. Conclusions are duly drawn.

List of symbols

Optimisation continuous variables

c_t^{su} startup cost at hour t [€]
P_{Bt} power bought in spot market during hour t [MW]
P_{Cbt} power bought from contract b during hour t [MW]
P_{St} power generated by self-producing facility during hour t [MW]
P_{St}^C power self-produced and consumed during hour t [MW]
P_{St}^S power self-produced and sold in the pool during hour t [MW]
x_{bi} auxiliary variable for penalty calculations associated to the subset of hours i of contract b [MWh]
y_{bi} auxiliary variable for penalty calculations associated to subset of hours i of contract b [MWh]
z_{bi} auxiliary variable for penalty calculations associated to subset of hours i of contract b [MWh]

Optimisation binary variables

u_t binary variable that is equal to 1 if self-production unit is online in hour t and 0 otherwise
v_b binary variable that is equal to 1 if contract b is used and 0 otherwise

Other variables

C_B cost of buying energy from spot market throughout time horizon [€]
C_C cost of buying energy from all contracts throughout time horizon [€]
C_{Cb} cost of buying energy from contract b throughout time horizon [€]
C_S cost of self-producing energy throughout time horizon [€]
C^{exp} expected net cost of electricity procurement [€]
C^{risk} standard deviation of net cost of electricity procurement [€]
E_{bi} energy bought from contract b during subset of hours i [MWh]
R_S revenues from selling energy in the spot market throughout the time horizon [€]
S penalty cost associated with buying from all contracts [€]
S_b penalty cost associated with buying from contract b [€]
S_{bi} penalty cost associated with buying from the subset of hours i of contract b [€]

Constants

a quadratic cost coefficient of self-production unit [€/(MW)^2 h]
b linear cost coefficient of self-production unit [€/MWh]
c no-load cost coefficient of self-production unit [€]
c^{su} startup (constant) cost of self-production unit [€]
E_{bi}^{max} upper bound of energy associated to subset of hours i of contract b [MWh]
E_{bi}^{min} lower bound of energy associated to subset of hours i of contract b [MWh]
M_1 large enough positive constant [MW]
M_2 large enough positive constant [MWh]
P_{Dt} demand of large consumer during hour t [MW]
P_S^{max} capacity of self-production unit [MW]
P_S^{min} minimum power output of self-production unit [MW]
R^{dw} ramping down (shutdown) limit of self-production unit [MW/h]
R^{up} ramping up (startup) limit of self-production unit [MW/h]
V_{kl}^{exp} covariance matrix of prices spanning whole time horizon [(€/MW)^2]
α weighting positive constant to achieve appropriate tradeoff cost against risk [1/€]
λ_{Cbt} price for buying from contract b during hour t [€/MWh]
λ_t^{exp} expected value of the pool price in hour t [€/MWh]
σ_{bi}^{over} over-consumption penalty slope for subset of hours i of contract b [€/MWh]

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$\sigma_{bi}^{\text{under}}$ under-consumption penalty slope for subset of hours i of contract b [€/MWh]

Random variable

λ_t pool price at hour t [€/MWh]

Sets

H set of all hours of time horizon
H_{bi} subset of hours i of contract b

Numbers

n_b number of subsets of hours of contract b
n_C number of contracts
T number of hours of time horizon

Indices

b bilateral contract index
i index for the subsets of hours in each bilateral contract
k, l covariance matrix indices
t time (hour) index

1 Introduction

This paper considers a large consumer that should procure its electrical energy at minimum cost. We assume that electricity is an important input in the productive process of this consumer and that its acquisition constitutes a considerable part of the production cost of the consumer. We also consider that this consumer buys a significant part of its demand in the pool.

Owing to uncertainty in electricity pool prices, the target of the large consumer is, in fact, to minimise its expected cost of electricity procurement while properly limiting the volatility of this cost. We denominate this problem 'risk-constrained electricity procurement for a large consumer'. And by risk, we mean the volatility associated with the cost of electricity procurement, i.e. the risk of incurring a level of cost significantly different from its expected value.

Generally, the large consumer has available three sources of electricity, namely: (i) buying from the pool, (ii) buying from retailers or generators through bilateral contracts, and (iii) self-production. We consider simultaneously these three alternatives.

We assume that self-production can only supply a part of the electricity requirement, which is a realistic assumption. Additionally, it should be noted that the self-production facility can be used to sell electricity to the pool.

Uncertainty is only considered in the pool prices. Note that accurate price forecasts are required for the proposed technique to work efficaciously. Recent relevant references on price forecasting include [1–5].

We consider that the demand of the consumer is deterministic because it is directly related to the production process of the consumer and therefore it is easily predicted. However, if demand prediction is required, recent references on load forecasting include [6] and [7].

We consider that the consumer is a price-taker; that is, its actions do not modify pool prices. This is a reasonable assumption as in realistic markets the demand of a large consumer is generally small compared with the total demand of the system.

Based on the Markowitz approach [8], a risk-constrained electricity procurement problem is formulated as a medium-size mixed integer quadratic programming problem whose solution can be obtained using commercial optimisation software [9].

Although the technical literature is rich in papers addressing the viewpoint of the producer, i.e. addressing the self-scheduling problem of generating units, e.g. [10–16], few references are found on the large consumer perspective. The pioneering work of Daryanian [17], and the recent work of Gabriel *et al.* [18] and Kirschen [19] deserve special attention. In [17] the optimal response of a large consumer to electricity spot prices is derived, in [18] the medium-term risk-constrained profit maximisation problem faced by a retailer is analysed, and in [19] a detailed analysis of the decision-making tools that consumers and retailers need to participate in an electricity market is presented. In [20], a method for purchase allocation and demand bidding is provided. Not considering risk, the electric energy procurement problem by a large consumer is treated in [21]. The problem faced by an industrial consumer managing both electricity and heat (emphasising heat issues) is addressed by [22] and [23].

In the above context, the novel contribution of this paper is to provide a procedure that allows a large consumer to decide its electricity procurement taking into account the risk associated to cost volatility. Additionally, the optimal amount of self-produced energy to sell to the pool by the large consumer is determined. A detailed explanation of the formulation proposed to model contract penalties is included in the Appendix.

2 Modelling

2.1 Buying from and selling to the pool

The large consumer can buy energy from the pool. The cost incurred is a random variable whose relation to pool prices is

$$C_B = \sum_{t=1}^{T} \lambda_t P_{Bt} \tag{1}$$

where C_B is the cost of buying from the pool throughout the time horizon, λ_t is the pool price at hour t (original random variable), P_{Bt} is the power bought from the pool at hour t, and T is the number of hours of the considered time horizon.

The consumer can also sell self-produced electricity in pool. The corresponding revenue, which is also a random variable, is

$$R_S = \sum_{t=1}^{T} \lambda_t P_{St}^S \tag{2}$$

where R_S is the revenue from selling in the pool, and P_{St}^S is the power self-produced and sold to the pool during hour t.

It should be noted that time periods of 1 h are considered throughout the paper.

2.2 Buying from bilateral contracts

2.2.1 Buying cost: The cost from buying from a given bilateral contract b throughout the time horizon is given by

$$C_{Cb} = \sum_{t=1}^{T} \lambda_{Cbt} P_{Cbt} \tag{3}$$

and the total cost from buying from all bilateral contracts throughout the time horizon is

$$C_C = \sum_{b=1}^{n_C} \sum_{t=1}^{T} \lambda_{Cbt} P_{Cbt} \tag{4}$$

where C_{Cb} is the cost of buying from contract b, λ_{Cbt} is the price from buying from contract b at hour t (known constant), P_{Cbt} is the power bought from contract b at hour t, n_C is the number of bilateral contracts, and C_C is the total cost of buying from all bilateral contracts throughout the time horizon.

It should be noted that contracts last for the whole planning horizon.

2.2.2 Contract usage: The constraints below allow buying energy or not throughout the time horizon from a given contract b, i.e.

$$0 \leq P_{Cbt} \leq M_1 v_b \tag{5}$$

where v_b is a binary variable that equals 1 if contract b is used and 0 otherwise, and M_1 is a sufficiently large constant.

2.2.3 Penalty cost: Following real-world practice [18], a penalty cost is incurred if the energy bought from a contract within a given subset of hours (as defined below) is above or below pre-specified bounds.

The set of hours in the planning horizon is denoted by H. Note that the cardinality of H is T. This set is divided for each contract b in subsets of hours denoted by H_{bi}; then

$$\bigcup_{i=1,\ldots,n_b} H_{bi} = H \tag{6}$$

where n_b is the number of subsets of hours for contract b. Note that different contracts have different subsets of hours.

For instance, consider two daily (24 h) contracts. The first contract ($b = 1$) differentiates between offpeak ($i = 1$) and peak hours ($i = 2$). Then, $H_{11} = \{1\text{–}8, 14\text{–}16, 20\text{–}24\}$ and $H_{12} = \{9\text{–}13, 17\text{–}19\}$. The second contract ($b = 2$) differentiates between valley ($i = 1$), shoulder ($i = 2$) and peak ($i = 3$) hours. Then, $H_{21} = \{1\text{–}8, 20\text{–}24\}$, $H_{22} = \{14\text{–}16\}$ and $H_{23} = \{9\text{–}13, 17\text{–}19\}$. Note that, for this example, $n_C = 2$, $n_1 = 2$, $n_2 = 3$, $H = \{1\text{–}24\}$ and $T = 24$.

For each subset of hours H_{bt} of contract b, consumption bounds of energy are specified and denoted by $E_{bi}^{\min}$ and $E_{bi}^{\max}$. The total energy bought during a subset of hours i using contract b is computed as

$$E_{bi} = \sum_{t \in H_{bi}} P_{Cbt} \tag{7}$$

For penalty calculations, this energy can also be expressed as

$$E_{bi} = x_{bi} + y_{bi} + z_{bi} \tag{8}$$

where x_{bi}, y_{bi}, z_{bi} are auxiliary variables.

Penalties are incurred for under or over consumption during any subset of hours i using any contract b. The constraints below allow expressing these penalties mathematically:

$$x_{bi} \leq E_{bi}^{\min} \tag{9}$$

$$y_{bi} \leq E_{bi}^{\max} - E_{bi}^{\min} \tag{10}$$

$$z_{bi} \leq M_2 - E_{bi}^{\max} \tag{11}$$

$$0 \leq x_{bi}, y_{bi}, z_{bi} \tag{12}$$

where M_2 is a sufficiently large constant, e.g. $2E_{bi}^{\max}$.

The penalty for a subset of hours i of contract b is then formulated as

$$S_{bi} = \sigma_{bi}^{\text{under}} (E_{bi}^{\min} v_b - x_{bi}) + \sigma_{bi}^{\text{over}} z_{bi} \tag{13}$$

where $\sigma_{bi}^{\text{under}}$ and $\sigma_{bi}^{\text{over}}$ are the penalty slopes (€/MWh) for under and over consumption, respectively.

The total penalty incurred using contract b is then

$$S_b = \sum_{i=1}^{n_b} \sigma_{bi}^{\text{under}} (E_{bi}^{\min} v_b - x_{bi}) + \sigma_{bi}^{\text{over}} z_{bi} \tag{14}$$

and the total penalty considering all contracts throughout the time horizon is

$$S = \sum_{b=1}^{n_C} \sum_{i=1}^{n_b} \sigma_{bi}^{\text{under}} (E_{bi}^{\min} v_b - x_{bi}) + \sigma_{bi}^{\text{over}} z_{bi} \tag{15}$$

Figure 1 shows the penalty associated with over or under consumption from a bilateral contract b during a given subset of hours i, H_{bi}.

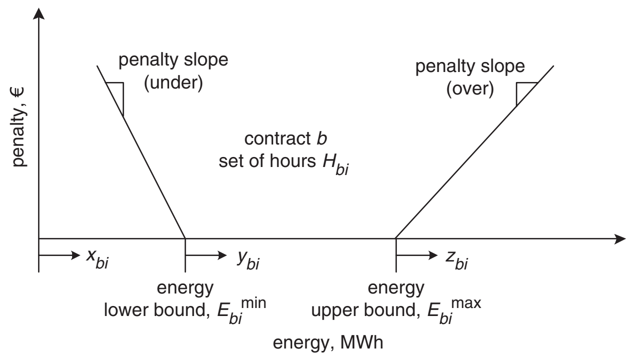

Fig. 1 *Penalty for under or over consumption from contract b during subset of hours i*

2.3 Self-production cost

The self-production cost C_S is expressed as

$$C_S = \sum_{t=1}^{T} (cu_t + bP_{St} + aP_{St}^2 + c_t^{\text{su}}) \tag{16}$$

where a, b and c are quadratic, linear and no-load cost coefficients, respectively, u_t is a binary variable used to indicate the status of the self-production unit, i.e. 0 if offline and 1 otherwise, P_{St} is the power generated by the self-production facility during hour t, and c_t^{su} is the startup cost at hour t.

Self-production costs include quadratic operating cost and constant startup cost. For the sake of clarity, no shutdown cost is considered. However, a more detailed cost modelling can be straightforwardly included in the proposed model, see [10] for details.

The modelling of the startup cost c_t^{su} at hour t requires the following constraints below

$$c_t^{\text{su}} \geq c^{\text{su}} (u_t - u_{t-1}) \tag{17}$$

$$c_t^{\text{su}} \geq 0 \tag{18}$$

where c^{su} is the constant startup cost.

The operating constraints of the self-production unit also include minimum and maximum power outputs:

$$P_S^{\min} u_t \leq P_{St} \leq P_S^{\max} u_t \tag{19}$$

where $P_S^{\max}$ and $P_S^{\min}$ are the maximum and minimum power outputs of the self-production unit, respectively.

Finally, ramping limits for the self-production units are expressed as

$$P_{St} - P_{St-1} \le R^{\mathrm{up}} u_t \tag{20}$$

$$P_{St-1} - P_{St} \le R^{\mathrm{dw}} u_{t-1} \tag{21}$$

where R^{up} and R^{dw} are ramping up (startup) and ramping down (shutdown) limits, respectively. For the sake of clarity, ramping up and ramping down limits are considered equal to startup and shutdown limits, respectively. Note that the minimum output power should be smaller than or equal to the ramping up (startup) or ramping down (shutdown) limits.

3 Expected net cost

The expected total net cost for energy procurement is computed as the expected value of the difference of acquisition costs and selling revenues, i.e.

$$\begin{aligned} C^{\mathrm{exp}} &= \mathrm{Exp}_{\lambda_1,\ldots,\lambda_T}\{C_B - R_S + C_C + S + C_S\} \\ &= \mathrm{Exp}_{\lambda_1,\ldots,\lambda_T}\{C_B - R_S\} + C_C + S + C_S \\ &= \mathrm{Exp}_{\lambda_1,\ldots,\lambda_T}\{C_B\} - \mathrm{Exp}_{\lambda_1,\ldots,\lambda_T}\{R_S\} \\ &\quad + C_C + S + C_S \end{aligned} \tag{22}$$

Note in the derivation above that C_C, S and C_S are deterministic variables.

Taking into account (1) and (2),

$$C^{\mathrm{exp}} = \sum_{t=1}^{T} \lambda_t^{\mathrm{exp}} (P_{Bt} - P_{St}^S) + C_C + S + C_S \tag{23}$$

It should be noted that price forecasts $(\lambda_t^{\mathrm{exp}}, t = 1, \ldots, T)$ are obtained using appropriate prediction techniques, such as those reported in [1] and [2].

4 Cost volatility

The variance of cost is computed as

$$\begin{aligned} (C^{\mathrm{risk}})^2 &= \mathrm{Var}_{\lambda_1,\ldots,\lambda_T}\{C_B - R_S + C_C + S + C_S\} \\ &= \mathrm{Var}_{\lambda_1,\ldots,\lambda_T}\{C_B - R_S\} \\ &= \mathrm{Var}_{\lambda_1,\ldots,\lambda_T}\left\{\sum_{t=1}^{T} \lambda_t (P_{Bt} - P_{St}^S)\right\} \\ &= \sum_{k=1}^{T}\sum_{l=1}^{T} (P_{Bk} - P_{Sk}^S) V_{kl}^{\mathrm{exp}} (P_{Bl} - P_{Sl}^S) \end{aligned} \tag{24}$$

where V_{kl}^{exp} is an estimate of the covariance matrix of pool prices throughout the considered time horizon. Note in the derivation above that C_C, S and C_S are deterministic variables.

The forecasting of the covariance matrix should be carried out using appropriate estimation techniques, such as that reported in [16].

To measure cost volatility while keeping simplicity and clarity, we use the variance of the cost. However, other risk measures, such as VaR and CVaR [24] can be used.

5 Risk-constrained energy procurement

5.1 Supplying the demand

Supplying the demand for every hour takes the form

$$P_{Dt} = P_{Bt} + \sum_{b=1}^{n_C} P_{Cbt} + P_{St}^C \tag{25}$$

where P_{Dt} is the consumer demand in hour t (considered a deterministic and known value) and P_{St}^C is the power self-produced and self-consumed in hour t.

Note that the power self-produced is divided into power sold to the market and power self-consumed, i.e.

$$P_{St} = P_{St}^S + P_{St}^C \tag{26}$$

5.2 General formulation

The risk constrained energy procurement problem for a large consumer is formulated below. Constant α allows weighting properly cost and risk [8]:

$$\begin{aligned} &\mathrm{minimise}_{P_{Bt},P_{St},P_{St}^S,P_{St}^C,u_t,c_t^{\mathrm{su}},\forall t;v_b,\forall b;P_{Cbt},\forall t,\forall b;x_{bi},y_{bi},z_{bi},\forall i,\forall b} \\ &\sum_{t=1}^{T} \lambda_t^{\mathrm{exp}} (P_{Bt} - P_{St}^S) \\ &+ \sum_{b=1}^{n_C}\sum_{t=1}^{T} \lambda_{Cbt} P_{Cbt} \\ &+ \sum_{b=1}^{n_C}\sum_{i=1}^{n_b} \sigma_{bi}^{\mathrm{under}} (E_{bi}^{\min} v_b - x_{bi}) + \sigma_{bi}^{\mathrm{over}} z_{bi} \\ &+ \sum_{t=1}^{T} (c u_t + b P_{St} + a P_{St}^2 + c_t^{\mathrm{su}}) \\ &+ \alpha \left[\sum_{k=1}^{T}\sum_{l=1}^{T} (P_{Bk} - P_{Sk}^S) V_{kl}^{\mathrm{exp}} (P_{Bl} - P_{Sl}^S)\right] \end{aligned} \tag{27}$$

subject to

$$P_{Dt} = P_{Bt} + \sum_{b=1}^{n_C} P_{Cbt} + P_{St}^C \quad \forall t \tag{28}$$

$$P_{St} = P_{St}^S + P_{St}^C \quad \forall t \tag{29}$$

$$0 \le P_{Cbt} \le M_1 v_b \quad \forall t, \forall b \tag{30}$$

$$\sum_{t \in H_{bi}} P_{Cbt} = x_{bi} + y_{bi} + z_{bi} \quad i = 1, \ldots, n_b; \forall b \tag{31}$$

$$x_{bi} \le E_{bi}^{\min} \quad i = 1, \ldots, n_b; \forall b \tag{32}$$

$$y_{bi} \le E_{bi}^{\max} - E_{bi}^{\min} \quad i = 1, \ldots, n_b; \forall b \tag{33}$$

$$z_{bi} \le M_2 - E_{bi}^{\max} \quad i = 1, \ldots, n_b; \forall b \tag{34}$$

$$c_t^{\mathrm{su}} \ge c^{\mathrm{su}} (u_t - u_{t-1}) \quad \forall t \tag{35}$$

$$P^{\min} u_t \le P_{St} \le P^{\max} u_t \quad \forall t \tag{36}$$

$$P_{St} - P_{St-1} \le R^{\mathrm{up}} u_t \quad \forall t \tag{37}$$

$$P_{St-1} - P_{St} \le R^{\mathrm{dw}} u_{t-1} \quad \forall t \tag{38}$$

$$P_{Bt}, P_{St}, P_{St}^S, P_{St}^C, c_t^{\mathrm{su}} \ge 0 \quad \forall t \tag{39}$$

$$x_{bi}, y_{bi}, z_{bi} \ge 0 \quad i = 1, \ldots, n_b, \forall b \tag{40}$$

$$P_{Cbt} \ge 0 \quad \forall t, \forall b \tag{41}$$

$$u_t \in \{0, 1\} \quad \forall t \tag{42}$$

$$v_b \in \{0, 1\} \quad \forall b \tag{43}$$

Note that, in the problem above, P_{S0} and u_0 are, respectively, the power output and the status of the self-production unit just before the first hour of the time horizon under study.

The problem above is a moderate size mixed-integer quadratic programming problem. The objective function

(27) includes (i) the expected net cost of buying from the pool (cost from buying minus revenue from selling), (ii) the cost of buying from bilateral contracts, (iii) the potential penalties associated with bilateral contracts, (iv) the cost incurred by the self-production facility, and (v) the penalised cost variance. Constraint (28) enforces that the load is supplied in all time periods. Constraint (29) states that the energy self-produced can be consumed or sold in the pool. Constraint (30) makes it possible to use or not any bilateral contract. Constraints (31)–(34) make it possible to include the possible penalties associated with contract usage. Constraint (35) allows computing correctly the startup cost in every hour. Constraints (36)–(38) enforce the technical constraints of the self-production facility. Constraints (39)–(43) constitute variable declarations.

It should be noted that the formulation above allows the consumer to buy energy simultaneously from several bilateral contracts. It should also be noted that the formulation proposed can be easily extended to consider the possibility of the self-production facility selling energy through bilateral contracts.

5.3 Problem size

The problem size of the mixed-integer nonlinear programming problem (27)–(43) expressed as the number of binary variables, real variables, constraints and bounds is given in Table 1.

Table 1: Computational size of problem (27)–(43)

Number of binary variables	$T + n_C$
Number of real variables	$T(n_C + 5) + 3\sum_{b=1}^{n_C} n_b$
Number of constraints	$T(n_C + 7) + \sum_{b=1}^{n_C} n_b$
Number of bounds	$Tn_C + 3\sum_{b=1}^{n_C} n_b$

T: number of time periods (hours)
n_C: number of bilateral contracts
n_b: number of types of hours in contract b

6 Case study

The time horizon considered spans five working days. Demand and spot price data are shown in Figs. 2 and 3, respectively. The structure of the estimate covariance matrix, which is positive definite and dense, is shown in Fig. 4.

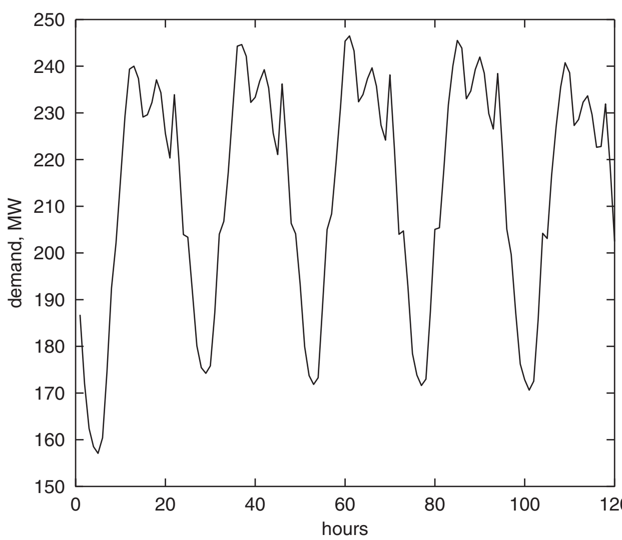

Fig. 2 *Demand data*

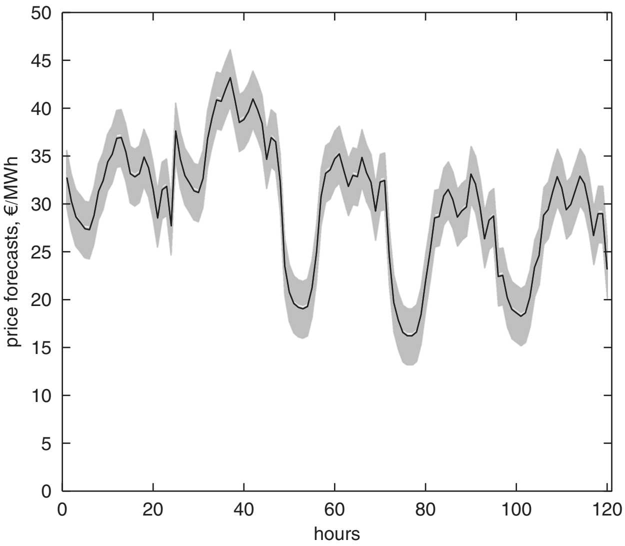

Fig. 3 *Price data*

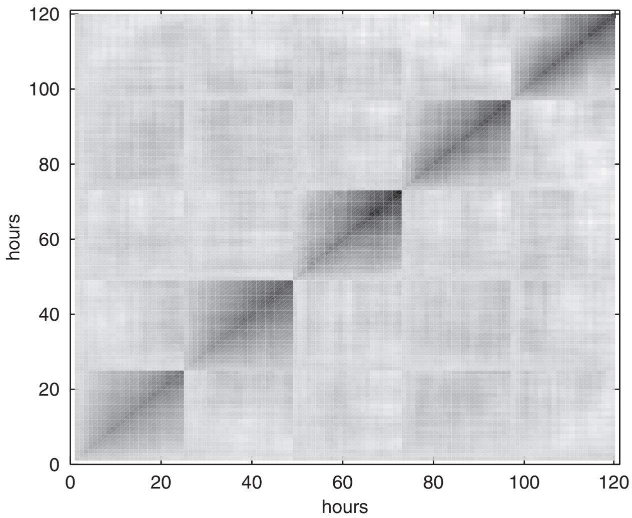

Fig. 4 *Structure of covariance matrix*

Two bilateral contracts spanning the five working days horizon are considered. Both contracts differentiate between peak (11:00–14:00 and 18:00–19:00 for the five days) and offpeak (01:00–10:00, 15:00–17:00 and 20:00–24:00 for the five days) hours. Table 2 gives price, bounds and penalty data for both contracts. Data for the self-production facility is given in Table 3. Note that the self-production unit is offline at the beginning of the planning horizon.

Figure 5 shows the expected procurement cost as a function of its standard deviation. The expected procurement cost is calculated as the sum of the terms in (27) that are not multiplied by factor α. On the other hand, the standard deviation is defined as the square root of the cost

Table 2: Data for contracts

	Contract 1		Contract 2		
	Peak	offpeak	peak	offpeak	
Hourly price	41.0	36.5	40.0	37.5	€/MWh
Energy upper bound	2800	2900	2600	2700	MWh
Energy lower bound	2600	2700	2300	2400	MWh
Penalty slope (over)	2.0	2.1	2.0	2.1	€/MWh
Penalty slope (under)	2.0	2.3	2.0	2.3	€/MWh

Table 3: Data for self-production unit

Capacity	130.00	MW
Minimum power output	20.00	MW
Ramping limit (all)	80.00	MW/h
Quadratic cost	0.01	€/(MW)^{2}h
Linear cost	28.00	€/MWh
No-load cost	400.00	€
Startup cost	200.00	€

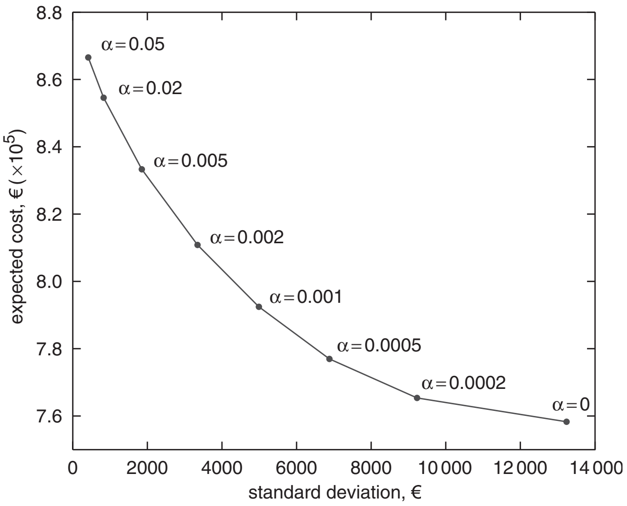

Fig. 5 *Expected cost against cost standard deviation*

variance. In (27), the cost variance is the term multiplied by factor α. As expected, observe that the expected cost increases as its standard deviation decreases. This Figure is obtained by solving problem (27)–(43) for different values of α. Parameter α lies in the range $[0, \inf)$ and its actual value materialises the tradeoff between expected procurement cost and risk; therefore, it depends on the preferences of the consumer. A conservative consumer prefers minimising risk while its demand is satisfied, so it chooses a large value of α to increase the weight of the risk measure in (27). On the other hand, another consumer may be prepared to assume higher risk in the hope of obtaining lower cost, so its selected value for α is close to 0. A detailed discussion on how to obtain appropriate values for the weighting factor α is outside of the scope of this paper.

Figure 6 illustrates the mix of electricity sources for supplying the demand of the consumer incurring the lowest cost, which implies the highest risk of cost volatility. This solution is obtained solving problem (27)–(43) for $\alpha = 0$.

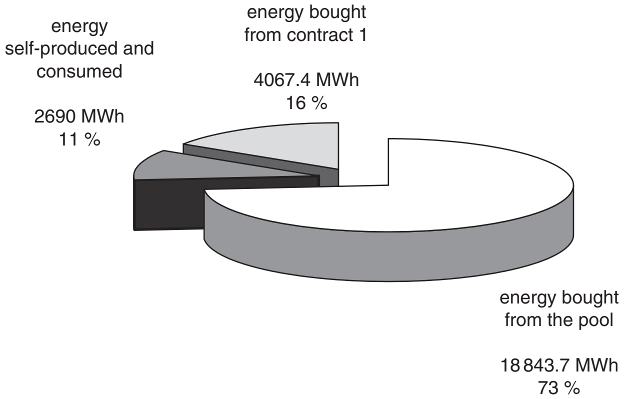

Fig. 6 *Procurement mix for high risk*

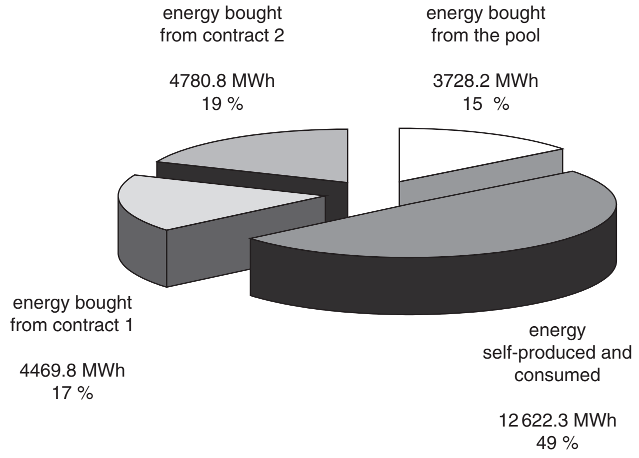

Fig. 7 *Procurement mix for moderate risk*

Analogously, Fig. 7 illustrates the mix of electricity sources for supplying the demand of the consumer incurring a reasonable level of risk. This solution is obtained solving problem (27)–(43) for $\alpha = 0.05$. With respect to the case in Fig. 6 ($\alpha = 0$), the procurement cost increases by 14%, while the energy bought in the spot market (riskiest source) decreases from 74% ($\alpha = 0$) to 21% ($\alpha = 0.05$). Therefore, as the cost volatility risk becomes more important, more expensive but less risky procurement sources are selected.

Problem (27)–(43) has been solved using SBB-CONOPT under GAMS [9] on a Linux-based server with four processors clocking at 1.60 GHz and 2 GB of RAM. The CPU time requirement to solve problem (27)–(43) for a gap below 3% is given in Table 4. It should be noted that, with the solver used, the solution time increases as the quadratic (nonlinear) term of problem (27)–(43) becomes more dominant (α increases).

Table 4: CPU solution time (s)

Case	α	CPU time
1	0.0000	6
2	0.0001	95
3	0.0002	107
4	0.0005	116
5	0.0010	136
6	0.0020	163
7	0.0050	227
8	0.0100	245
9	0.0200	309
10	0.0300	329
11	0.0500	374

7 Conclusions

This paper has analysed the electricity procurement problem faced by a large consumer. The objective of this consumer is to minimise cost while limiting cost volatility. An appropriate procedure is developed to achieve this objective. Extensive computational simulation shows the appropriate behaviour of the proposed procedure and its ability to resolve the cost against risk tradeoff faced by the large consumer. Emphasis on minimising cost results in a comparatively small procurement cost but at a high risk of cost volatility, and the predominant use of the cheapest energy source (pool) that exhibits a volatile price. Conversely, emphasis on minimising the risk of cost volatility results in a comparatively large procurement cost while the volatility risk of this cost is low and the use of energy sources (bilateral contracts and self-production) comparatively expensive, but with stable prices. Computational requirements are moderate.

8 References

1 Nogales, F.J., Contreras, J., Conejo, A.J., and Espínola, R.: 'Forecasting next-day electricity prices by time series models', *IEEE Trans. Power Syst.*, 2002, **17**, (2), pp. 342–348
2 Contreras, J., Espínola, R., Nogales, F.J., and Conejo, A.J.: 'ARIMA models to predict next-day electricity prices', *IEEE Trans. Power Syst.*, 2003, **18**, (3), pp. 1014–1020
3 Conejo, A.J., Plazas, M.A., Espínola, R., and Molina, A.B.: 'Day-ahead electricity price forecasting using the wavelet transform and ARIMA models', *IEEE Trans. Power Syst.*, 2005, **20**, (2), pp. 1035–1042
4 Conejo, A.J., Contreras, J., Espínola, R., and Plazas, M.A.: 'Forecasting electricity prices for a day-ahead pool-based electric energy market', *Int. J. Forecast.*, 2005, **21**, (3), pp. 435–462
5 Nogales, F.J., and Conejo, A.J.: 'Electricity price forecasting through transfer function models', *J. Oper. Res. Soc.*, 2006, (to be published)
6 Hippert, H.S., Pedreira, C.E., and Souza, R.C.: 'Neural networks for short-term load forecasting: a review and evaluation', *IEEE Trans. Power Syst.*, 2001, **16**, (1), pp. 44–55
7 Huang, S.J., and Shih, K.R.: 'Short-term load forecasting via ARMA model identification including non-gaussian process considerations', *IEEE Trans. Power Syst.*, 2003, **18**, (2), pp. 673–679
8 Markowitz, H.: 'Portfolio selection: efficient diversification of investments' (Yale University Press, New Haven, 1959)
9 Brooke, A., Kendrick, D., Meeraus, A., and Raman, R.: 'GAMS: a user's guide' (GAMS Development Corporation, 1998)
10 Arroyo, J.M., and Conejo, A.J.: 'Optimal response of a thermal unit to an electricity spot market', *IEEE Trans. Power Syst.*, 2000, **15**, (3), pp. 1098–1104
11 de la Torre, S., Arroyo, J.M., Conejo, A.J., and Contreras, J.: 'Price-maker self-scheduling in a pool-based electricity market: a mixed-integer LP approach', *IEEE Trans. Power Syst.*, 2002, **17**, (4), pp. 1037–1042
12 Sheblé, G.B.: 'Decision analysis tools for GENCO dispatchers', *IEEE Trans. Power Syst.*, 1999, **14**, (2), pp. 745–750
13 Conejo, A.J, Contreras, J., Arroyo, J.M., and de la Torre, S.: 'Optimal response of an oligopolistic generating company to a competitive pool-based electric power market', *IEEE Trans. Power Syst.*, 2002, **17**, (2), pp. 424–430
14 Arroyo, J.M., and Conejo, A.J.: 'Optimal response of a power generator to energy, AGC and reserve pool-based markets', *IEEE Trans. Power Syst.*, 2002, **17**, (2), pp. 404–410
15 Conejo, A.J, Nogales, F.J., and Arroyo, J.M.: 'Price-taker bidding strategy under price uncertainty', *IEEE Trans. Power Syst.*, 2002, **17**, (4), pp. 1081–1088
16 Conejo, A.J., Nogales, F.J., Arroyo, J.M., and García-Bertrand, R.: 'Risk-constrained self-scheduling of a thermal power producer', *IEEE Trans. Power Syst.*, 2004, **19**, (3), pp. 1569–1574
17 Daryanian, B., Bohn, R.E., and Tabors, R.D.: 'Optimal demand-side response to electricity spot prices for storage-type customers', *IEEE Trans. Power Syst.*, 1989, **4**, (3), pp. 897–903
18 Gabriel, S.A., Genc, M.F., and Balakrishnan, S.: 'A simulation approach to balancing annual risk and reward in retail electrical power markets', *IEEE Trans. Power Syst.*, 2002, **17**, (4), pp. 1050–1057
19 Kirschen, D.S.: 'Demand-side view of electricity markets', *IEEE Trans. Power Syst.*, 2003, **18**, (2), pp. 520–527
20 Liu, Y., and Guan, X.: 'Purchase allocation and demand bidding in electric power markets', *IEEE Trans. Power Syst.*, 2003, **8**, (1), pp. 106–112
21 Conejo, A.J., Fernández-González, J.J., and Alguacil, N.: 'Energy procurement for large consumers in electricity markets', *IEE Proc. Gener. Transm. Distrib.*, 2005, **152**, (3), pp. 357–364
22 Gómez-Villalba, E., and Ramos, A.: 'Optimal energy management of an industrial consumer in liberalized markets', *IEEE Trans. Power Syst.*, 2003, **18**, (2), pp. 716–723
23 Illerhaus, S.W., and Verstege, J.F.: 'Optimal operation of industrial CHP-based power systems in liberalized energy markets'. Proc. IEEE Power Tech, BPT99-352-13, Budapest, Hungary, 1999
24 Rockafellar, R.T., and Uryasev, S.: 'Optimization of conditional value-at-risk', *J. Risk*, 2000, **2**, (3), pp. 21–41

9 Appendix

The objective of this Appendix is to explain the formulation used to calculate the penalties associated with over or under consumption from bilateral contracts. Depending on the different zones shown in Fig. 5, all possible cases are considered. Let be the contract b and the subset of hours i. The following cases can occur:

1 *Contract b is not selected*: $v_b = 0$
If contract b is not selected, $v_b = 0$, constraints (5)–(7) enforce that $E_{b,i} = 0$. If $E_{b,i} = 0$, constraints (8) and (12) establish that $x_{b,i}$, $y_{b,i}$ and $z_{b,i}$ are equal to zero. Finally, if v_b, $x_{b,i}$, $y_{b,i}$ and $z_{b,i}$ are zero, (13) enforces that the penalty for subset of hours i of contract b is also zero.

2 *Contract b is selected*: $v_b = 1$
When contract b is selected, three cases can occur, depending on the consumption during the subset of hours i:

(*a*) Under consumption zone: $E_{b,i} < E_{b,i}^{\min}$
To satisfy the constraint (8), $x_{b,i}$, $y_{b,i}$ and $z_{b,i}$ are fixed depending on their limits stated by constraints (9)–(12). Taking into account the expression of the objective function (27), it can be observed that the higher is $x_{b,i}$, the lower is the expected cost. The contrary effect can be observed for $z_{b,i}$. For that, in order to minimise the expected cost, $x_{b,i}$ and $z_{b,i}$ must be equal to their greatest and smallest possible values, respectively. This fact is a consequence of the signs of the coefficients of $x_{b,s}$ and $z_{b,s}$ (negative and positive, respectively) in the objective function (27). Because of this, and considering constraints (8) and (12), it can be stated that, in this case, $x_{b,i} = E_{b,i}$ and $y_{b,i} = z_{b,i} = 0$. It should be noted that $y_{b,s}$ does not appear in the objective function. Therefore, if $v_b = 1$, $x_{b,i} = E_{b,i}$ and $z_{b,i} = 0$, the penalty, calculated from (13) is:

$$S_{b,i} = \sigma_{b,i}^{\text{under}}(E_{b,i}^{\min} - E_{b,i}) \tag{44}$$

It should be noted that because $E_{b,i} < E_{b,i}^{\min}$ the penalty, $S_{b,i}$, is a positive value.

(*b*) No penalty zone: $E_{b,i}^{\min} \leq E_{b,i} \leq E_{b,i}^{\max}$
In this case, the energy bought during the subset of hours i using contract b is in the no penalty zone. In the same way as in the case above, constraint (8) can be satisfied if $z_{b,i}$ is fixed to 0. However, $x_{b,s}$ reaches its upper limit, $E_{b,i}^{\min}$, and $y_{b,i}$ is equal to $E_{b,i} - E_{b,i}^{\min}$. Finally, the value of the penalty in this case is:

$$S_{b,i} = \sigma_{b,i}^{\text{under}}(E_{b,i}^{\min} - E_{b,i}^{\min}) = 0 \tag{45}$$

(*c*) Over consumption zone: $E_{b,i} \geq E_{b,i}^{\max}$
In this case, penalties associated to over consumption are considered. Unlike the above cases, constraint (8) cannot be satisfied if $z_{b,i} = 0$. This is a consequence of $x_{b,i}$ and $y_{b,i}$ reaching their respective upper limits, $E_{b,i}^{\min}$ and $(E_{b,i}^{\max} - E_{b,i}^{\min})$. These limits are defined by constraints (9) and (10). Because of that, to satisfy constraint (8), $z_{b,i}$ is fixed to $(E_{b,i} - E_{b,i}^{\max})$. The expression of the penalty in the over consumption zone is then:

$$S_{b,i} = \sigma_{b,i}^{\text{over}}(E_{b,i} - E_{b,i}^{\max}) \tag{46}$$