1. a) Addition and multiplication are commutative.

$\forall x \in \mathbb{Z}, \forall y \in \mathbb{Z} \quad (x+y = y+x)$

$\forall x, y \in \mathbb{Z} \quad (x \times y = y \times x)$

b) $\mathbb{Z}$ is closed under addition and multiplication.

~~$\forall x \in \mathbb{Z}$~~

$\forall x \in \mathbb{Z}, \forall y \in \mathbb{Z} \quad (x+y \in \mathbb{Z})$

$\forall x, y \in \mathbb{Z} \quad (x \times y \in \mathbb{Z})$

c) Multiplication distributes over addition.

$\forall x, y, z \in \mathbb{Z} \quad (x+y) \times z = x \times z + y \times z$

d) Multiplication and addition are ~~additive~~ associative.

$\forall x, y, z \in \mathbb{Z} \quad (x+y)+z = x+(y+z)$

$\forall x, y, z \in \mathbb{Z} \quad (x \times y) \times z = x \times (y \times z)$

e) Multiplication and addition have the identity property.

$\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z} \quad (x+y = x)$

$\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z} \quad (x \times y = x)$

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2. $\alpha = \forall x\,(P(x) \vee Q(x))$ and
$\beta = \forall x\, P(x) \vee \forall x\, Q(x)$

Let P be set ~~of~~ of imaginary numb.
Q be set of real number.

$\therefore P \vee Q$ will be set of all possible number.

$\therefore \alpha = \forall x\,(P(x) \vee Q(x)) = \text{True}$
as every x will be ~~eigh~~ either in P or Q.

But $\forall x\, P(x)$ is not true as ~~so~~ Some number can be imaginary as well.

Same for $\forall x\, Q(x)$ as $\exists x, x \in P$
$\therefore \forall x\, P(x) = \text{False}$, $\forall x\, Q(x) = \text{False}$
$\therefore \beta = \forall x\, P(x) \vee \forall x\, Q(x)$
$= \text{False} \vee \text{False} = \text{False}$.

$\therefore$ For model of imaginary and real numbers
α is true but β is not.

Hence $\boxed{\alpha \not\models \beta \text{ is not true}}$

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3. ~~a)~~ Predicates in FOL & Knowledge base

a) own (Han, MF)

b) unhappy (Princess)

c) loves (Princess, Han)

d) $\forall x$ (own (x, MF) $\vee$ unhappy (x)) $\rightarrow$ visit (x, Obi)

e) $\forall x$, visit (x, Obi) $\rightarrow$ wise(x)

f) $\forall x$, (own (x, MF) $\wedge$ visit (x, Obi)) $\rightarrow$ teach (Obi, x)

g) $\forall x$, [(unhappy (x) $\vee$ own(x, MF)) $\wedge$ teach (Obi, x) $\rightarrow$ join (x, Rebel)

h) $\forall x, \forall y$ unhappy (x) $\wedge$ loves (x, y) $\rightarrow$ declare (x, y)

i) $\forall x, \forall y$ teach (Obi, x) $\wedge$ declare (y, x) $\wedge$ wise(y) $\rightarrow$ friend (x, Chew)

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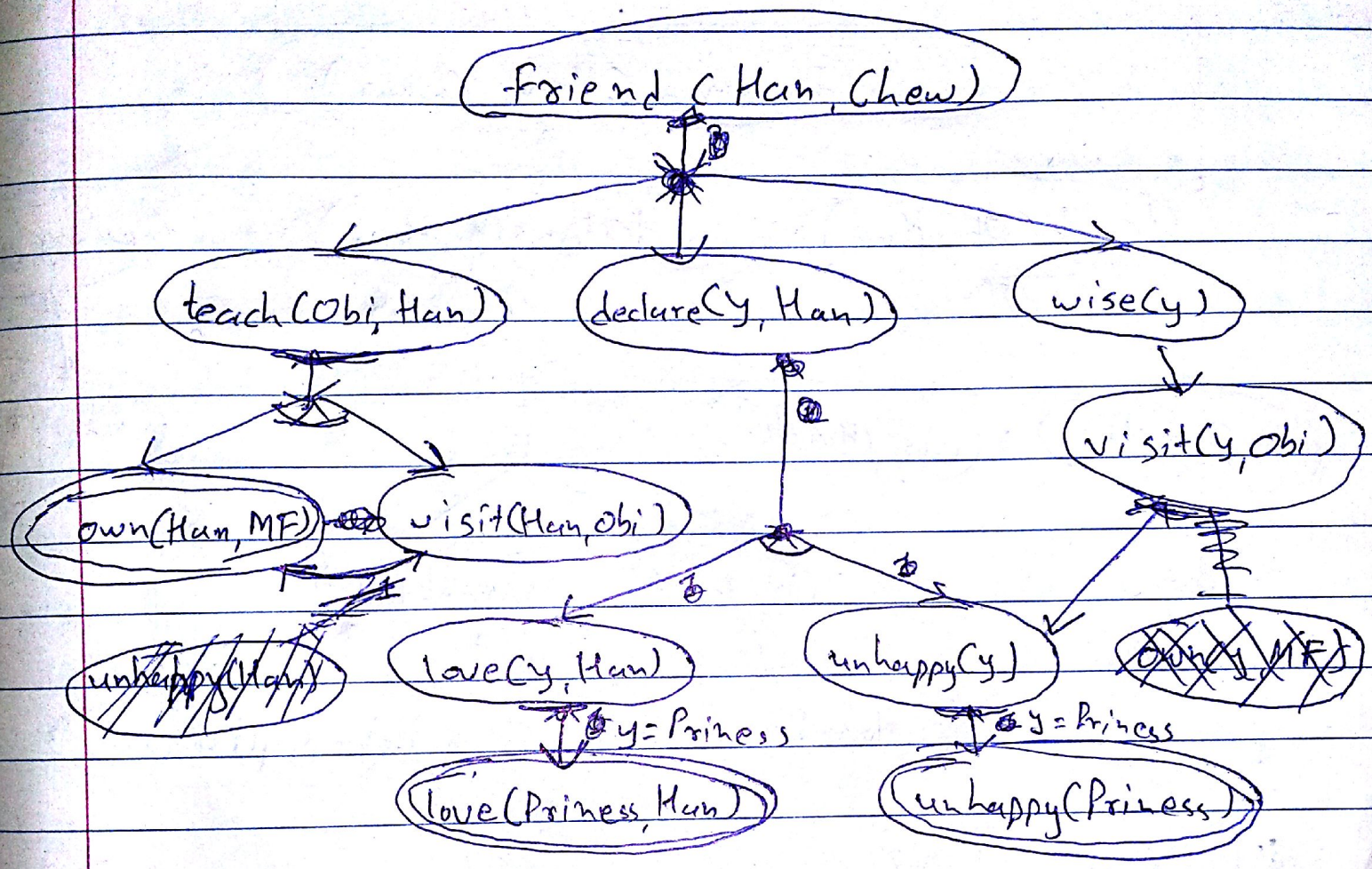
i) Backward Chaining:



Here all ~~leaves~~ leaf nodes are in KB
So it can proove root node to True.

Hence, it's proved that Han Solo has Chewbacca as a friend.

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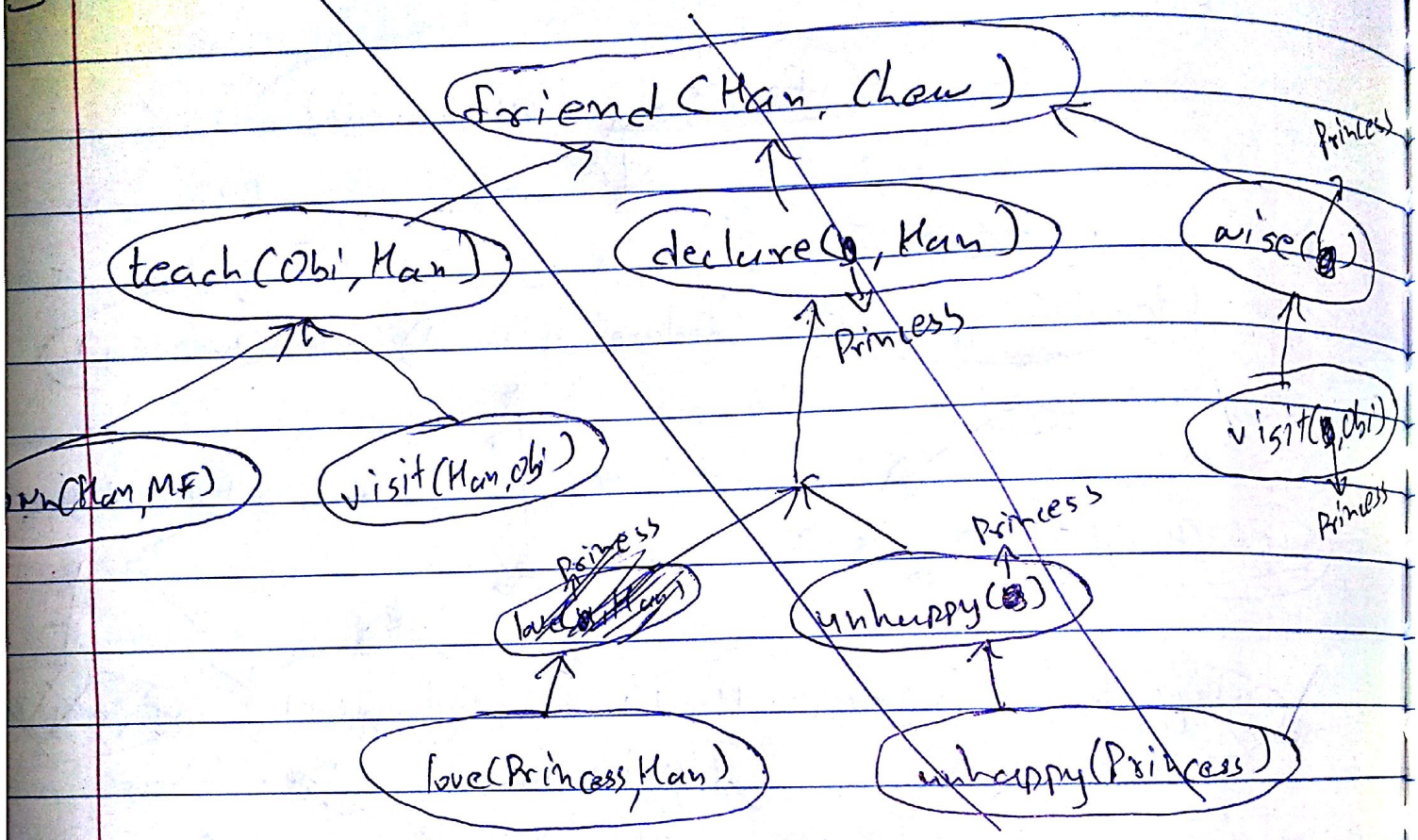
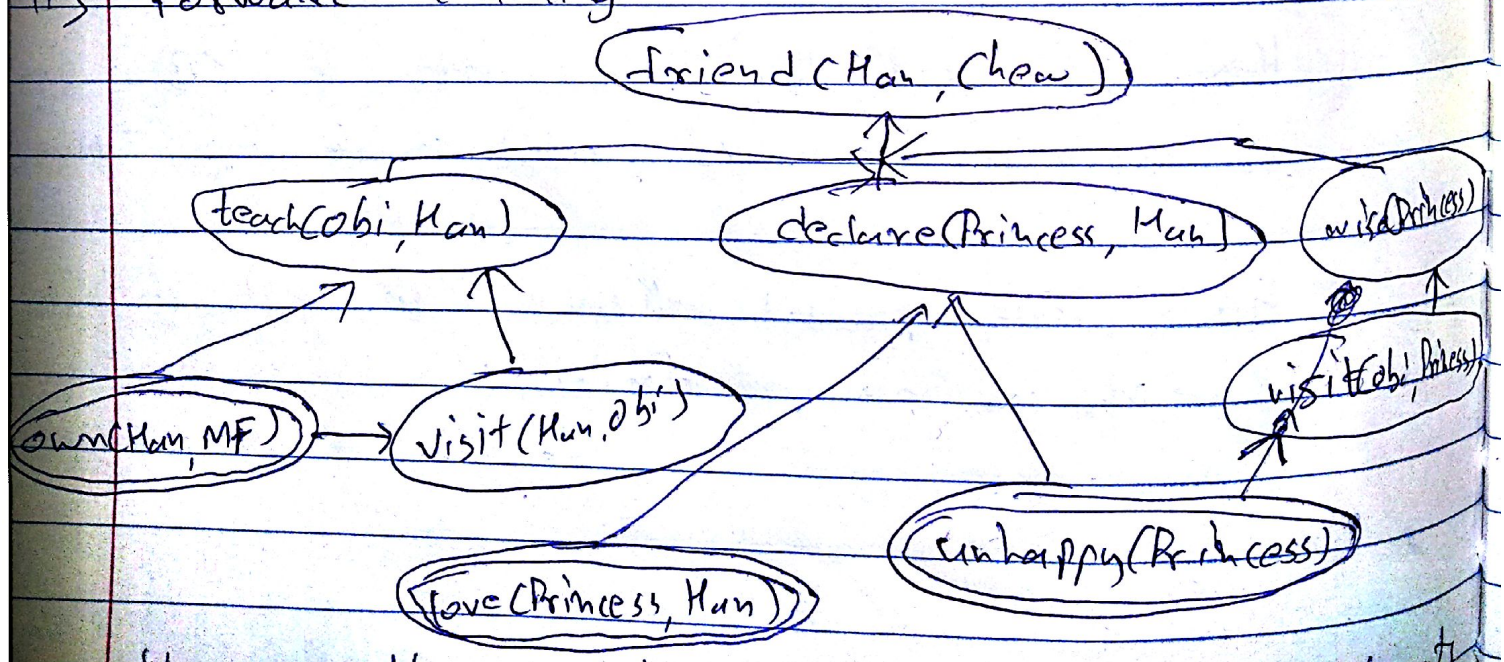
i) Forward Chaining :



ii) Forward Chaining :



We are able to prove root by given statements in KB.

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4. ~~b.~~ Resolution

Predicates

able(Zeus) [Zeus is able to prevent evil.]

willing(Zeus) [Zeus is willing to prevent evil.

impotent(Zeus) [Zeus is impotent.]

malevolent(Zeus) [Zeus is malevolent]

exists(Zeus) [Zeus exists]

prevent(Zeus) [Zeus prevents evil.]

a) able(Zeus) $\wedge$ willing(Zeus) $\rightarrow$ prevent(Zeus)

$= \neg$ (able(Zeus) $\wedge$ willing(Zeus) $\vee$ prevent(Zeus)

$= \neg$ able(Zeus) $\vee \neg$ willing(Zeus) $\vee$ prevent(Zeus)

b) $\neg$ able(Zeus) $\rightarrow$ impotent(Zeus)

$= \neg$ ($\neg$ able(Zeus)) $\vee$ impoten(Zeus)

$=$ able(Zeus) $\vee$ impotent(Zeus)

c) $\neg$ willing(Zeus) $\rightarrow$ malevolent(Zeus)

$=$ willing(Zeus) $\vee$ malevolent(Zeus)

d) $\neg$ prevent(Zeus)

e) exists(Zeus) $\rightarrow$ ($\neg$ impotent(Zeus) $\wedge \neg$ malevolent(Zeus))

$= \neg$ exists(Zeus) $\vee \neg$ impotent(Zeus) $\vee \neg$ malevolent(Zeus)

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Resolution

$\neg$ exists(Zeus) $\vee$ $\neg$ impotent(Zeus) $\vee$ $\neg$ malevolent(Zeus)

$\neg$ exists(Zeus)

willing(Zeus) $\vee$ malevolent(Zeus)

$\neg$ impotent(Zeus) $\vee$ $\neg$ malevolent(Zeus)

willing(Zeus) $\vee$ $\neg$ impotent(Zeus)

able(Zeus) $\vee$ impotent(Zeus)

$\neg$ willing(Zeus) $\vee$ able(Zeus)

$\neg$ able(Zeus) $\vee$ $\neg$ willing(Zeus) $\vee$ prevent(Zeus)

prevent(Zeus)

$\neg$ prevent(Zeus)

False

Hence $\neg$ [exists(Zeus)]

$\therefore$ Zeus does not exists.

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5. Proof by contradiction.

a) $\neg[\forall x((P(x)) \Rightarrow P(x))]$

$= \forall x(\neg P(x) \vee P(x))$

$=$ True

$\therefore \neg[\forall x(P(x) \Rightarrow P(x))] = \neg$ True $=$ False

It's contradiction.

Hence $\forall x(P(x) \Rightarrow P(x))$ is valid.

b) $\neg[(\neg \exists x P(x)) \Rightarrow \forall x \neg P(x)]$

$= \neg[\neg(\neg \exists x P(x)) \vee (\forall x \neg P(x))]$

$= \neg[(\exists x P(x)) \vee (\forall x(\neg P(x)))]$

$= \neg[(\exists x P(x)) \vee \neg(\neg \forall x P(x))]$

$= \neg[(\exists x P(x)) \vee \neg(\exists x P(x))]$

$= \neg$ (True) $=$ False.

Hence $\neg \exists x P(x) \Rightarrow \forall x \neg P(x)$ is valid.

c) $\neg[(\forall x((P(x) \vee Q(x)))) \Rightarrow (\forall x P(x) \vee \exists x Q(x))]$

$= \neg[\neg(\forall x(P(x) \vee Q(x))) \vee \forall x P(x) \vee \exists x Q(x)]$

$= \neg[\neg \forall x \neg(P(x) \vee Q(x)) \vee \forall x P(x) \vee \exists x Q(x)]$

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$= \neg [\exists x (\neg P(x) \wedge \neg Q(x)) \vee \forall x\, P(x) \vee \exists x\, Q(x)]$

$= \neg [\exists x ((\neg P(x) \vee Q(x)) \wedge (\neg Q(x) \vee Q(x)))$
$\vee \forall x\, P(x)]$

$= \neg [\exists x ((\neg P(x) \vee Q(x)) \wedge \text{True}) \vee \forall x\, P(x)]$

$= \neg [\exists x (\neg P(x) \vee Q(x)) \vee \forall x\, P(x)]$

$= \neg [\exists x (\neg P(x) \vee Q(x)) \vee P(x)]$

$= \neg (\exists x (\neg P(x) \vee Q(x))) \wedge \neg P(x)$

$= \neg \exists x \neg (\neg P(x) \vee Q(x)) \wedge \neg P(x)$

$= \forall x (P(x) \wedge \neg Q(x)) \wedge \neg P(x)$

$= P(x) \wedge \neg P(x) \wedge \neg Q(x)$

$= \text{False} \wedge \neg Q(x)$

$= \text{False}$

Hence $(\forall x (P(x) \vee Q(x))) \Rightarrow (\forall x\, P(x) \vee \exists x\, Q(x))$
is valid statement.

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