CS 6150: HW4

Due Date:

This assignment has 5 questions, for a total of 100 points. Unless otherwise specified, complete and reasoned arguments will be expected for all answers.

Question	Points	Score
Cycle Cover	20	
Attend all lectures	20	
Monotone path	20	
Vertex-disjoint paths	20	
Uniqueness of mincut	20	
Total:	100	

Question 1: Cycle Cover .... [20]

A cycle cover C of a directed graph $G(V, E)$ is a collection of vertex-disjoint directed cycles so that each vertex in V belongs to some cycle in C.

Give a polynomial time algorithm to compute a cycle cover of a given directed graph, or correctly report that one does not exist.

Question 2: Attend all lectures .... [20]

There are n lectures with start time s_i, end time e_i and t_{ij} being the time spent in going from lecture i to j. Now a group of students wants to attend all of the lectures by sending no more than one student to each lecture. Assume s_i, e_i, t_{ij} are all positive integers.

Find an algorithm that minimizes number of students covering all the lectures.

Question 3: Monotone path .... [20]

Solve all parts of question 2 in `http://web.engr.illinois.edu/~jeffe/teaching/algorithms/notes/25-maxflowext.pdf`

Breakdown is $5 + 5 + 10$.

Question 4: Vertex-disjoint paths .... [20]

Given a square grid (n x n) like the following figure, and a set of $m \leq n^2$ distinct points marked in black, devise an algorithm to determine whether there exists m vertex-disjoint paths starting at those marked points and ending at points at the boundary of the grid.
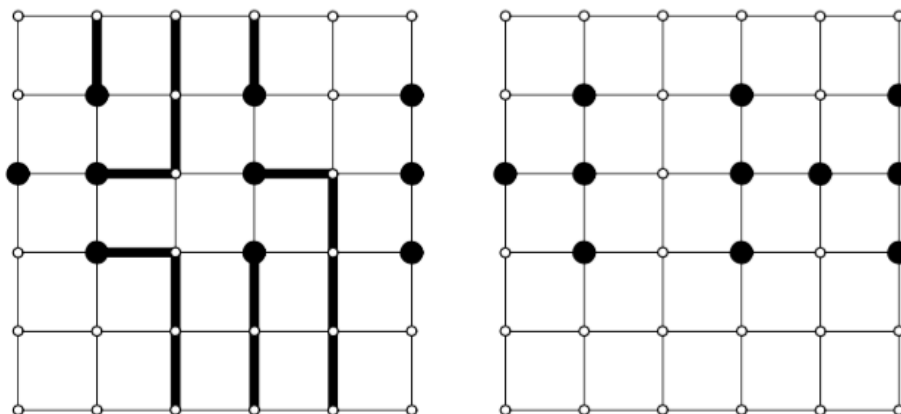

Figure 1: In the left grid there exists such a path, but in the right grid there isn't

Question 5: Uniqueness of mincut .... [20]

Given an s-t flow network, find a polynomial time algorithm to determine whether the mincut is unique.