CS 6150: HW4

Due Date:

This assignment has 5 questions, for a total of 100 points. Unless otherwise specified, complete and reasoned arguments will be expected for all answers.

Question	Points	Score
Integer programs	20	
LP duals	20	
Max cardinality matching	20	
Generalized Duals	20	
Best fit line	20	
Total:	100	

Question 1: Integer programs .... **[20]**

Write down integer programs for the following problems.

(a) [**10**] Let U be a set, and let $\mathcal{C} = \{S_1, \ldots, S_n\}$ be a collection of subsets of U. Each set S has a weight w_S. Find a subcollection $\mathcal{C}' \subset \mathcal{C}$ of minimum total weight ($w(\mathcal{C}') = \sum_{S \in \mathcal{C}'} w_S$) such that the sets in $\mathcal{C}'$ *cover* U: i.e

$$\cup_{S \in \mathcal{C}'} S = U$$

(b) [**10**] Let U be a universe, and let $\mathcal{C} = \{S_1, \ldots, S_n\}$ be a collection of subsets of U. Each element $u \in U$ has a weight w_u. Find a subset $H \subset U$ of minimum total weight ($w(H) = \sum_{u \in H} w_u$) such that each set in $\mathcal{C}$ is *hit* by H: i.e

$$\forall S \in \mathcal{C}, H \cap S \neq \emptyset$$

Question 2: LP duals .... **[20]**

(a) [**10**] Consider the linear program

$$\begin{aligned} \max \quad & 5x + 3y - 2z \\ \text{such that} \quad & 3x - 2y \leq 6 \\ & 4y + 2z \leq 7 \\ & -3x + 2z \leq 3 \end{aligned}$$

Write down the dual of this LP.

(b) [**10**] Write down the dual of the linear program obtained by relaxing the integer program from Question 1(b) above.

Question 3: Max cardinality matching .... **[20]**

(a) [**10**] Write down a linear program for computing a maximum cardinality matching in a bipartite graph. Your linear program will have one variable for each edge.

(b) [**10**] Write down the dual of this LP. What well known problem does it capture ?

Question 4: Generalized Duals .... **[20]**

We've seen that any linear program can be written in the canonical form

$$\begin{aligned} \max \quad & c^\top x \\ \text{such that} \quad & Ax \leq b \\ & x \geq 0 \end{aligned}$$

which gives rise to the corresponding dual

$$\begin{aligned} \min \quad & y^\top b \\ \text{such that} \quad & y^\top A \geq c \\ & y \geq 0 \end{aligned}$$

It turns out that first transforming a general linear program with equality and $\geq$ constraints into canonical form, and then writing the dual, can be a little inconvenient, and that it's easier to write the dual directly.

But what would this dual look like ? Let's take a general linear program that looks like this:

$$\begin{aligned} \max \quad & ax + by + cz \\ \text{such that} \quad & Ax + By + Cz \leq d \\ & Dx + Ey + Fz = e \\ & Gx + Hy + Iz \geq f \\ & x \geq 0, z \leq 0 \end{aligned}$$

Note that x, y, z are *vectors* and y is unconstrained (i.e the coordinates of y could be more or less than zero).

Write down the dual of this linear program. You will do this by first transforming this into the canonical setting, writing the canonical dual, and then rewriting the dual in simplified form. It will help to remember that if a and b are two variables that are both greater than zero, then $a - b$ represents a variable that could be either more or less than zero.

Do you notice any pattern in the relation between primal constraint and dual variables (and vice versa) ?

Question 5: Best fit line .... **[20]**

this is question 7(b) from the Erickson notes on LPs

You are given n points (x_i, y_i) in the plane, and you wish to find a line of *best fit*. But instead of the standard squared error norm, you will be using the ℓ_1 error: namely, for any given line $y = ax + b$, the error is given by

$$\epsilon_1(a, b) = \sum_{i=1}^{n} |y_i - (ax_i + b)|.$$

Write down a linear program to find a line that minimizes ϵ_1.