ASSIGNMENT-4 (Problem Solving)

Q1. Maximize $Z = 5x_1 + 4x_2$

s.t. $6x_1 + 4x_2 \le 24$

$x_1 + 2x_2 \le 6$

$-x_1 + x_2 \le 1$

$x_2 \le 2$

for sensitive analysis –

$Z = 5x_1 + 4x_2$

$Z = c_1x_1 + c_2x_2$ where c_1 – exterior, c_2 – interior

Standard form – $Z = 5x_1 + 4x_2 + 0s_1 + 0s_2 + 0s_3 + 0s_4$

s.t. $6x_1 + 4x_2 + S_1 = 24$

$x_1 + 2x_2 + s_2 = 6$

$-x_1 + x_2 + s_3 = 1$

$x_2 + s_4 = 2$

$x_1, x_2, s_1, s_2, s_3, s_4 \ge 0$

On solving –

BV	C_B	X_B	x_1	x_2	S_1	S_2	S_3	S_4	
← s_1	0	24	6	4	1	0	0	0	$R_1 \leftarrow R_1/6$
s_2	0	6	1	2	0	1	0	0	$R_2 \leftarrow R_2 - R_1$
s_3	0	1	-1	1	0	0	1	0	$R_3 \leftarrow R_3 + R_1$
s_4	0	2	0	1	0	0	0	1	$R_4 \leftarrow R_4$
			-5 ↑	-4	0	0	0	0	

(x_1 enters, s_1 leaves)

x_1	5	4	1	2/3	1/6	0	0	0	$R_1 \leftarrow R_1 - \frac{2}{3}R_2$
← s_2	0	2	0	4/3	-1/6	1	0	0	$R_2 \leftarrow R_2 \times 3/4$
s_3	0	5	0	5/3	1/6	0	1	0	$R_3 \leftarrow R_3 - \frac{5}{3}R_2$
s_4	0	2	0	1	0	0	0	1	$R_4 \leftarrow R_4 - 2$
			0	-2/3 ↑	5/6	0	0	0	

(x_2 enters, s_2 leaves)

x_1	5	3	1	0	1/4	-1/2	0	0
x_2	4	3/2	0	1	-1/8	3/4	0	0
s_3	0	5/3	0	0	3/8	-5/4	1	0
s_4	0	1/2	0	0	1/8	-3/4	0	1
			0	0	3/4	1/2	0	0

optimality has reached.

$\therefore x_1 = 3,\ x_2 = 3/2$, $Z \Rightarrow 5 \times 3 + 4 \times 3/2 = 15 + 6$

$Z = 21$

(i) The ratio of unit revenue of exterior paint to interior →

$\frac{1}{2} \le c_1/c_2 \le \frac{6}{4}$ or $2/3 \le c_2/c_1 \le 2$

$\Rightarrow 0.5 \le c_1/c_2 \le 1.5$

(ii) given $c_1 = 6$ then

$6 \times 2/3 \le c_2 \le 6 \times 2 \Rightarrow 4 \le c_2 \le 12$

that is ₹4000 per ton to ₹12000 per ton.

(iii) So $c_1/c_2 = 5/2.5 \Rightarrow 2$

as range is $0.5 \le c_1/c_2 \le 1.5$ and thus 2 falls outside the range. Hence solution changes.

(iv) Dual prices of resource 1 & resource 2 and their feasibility ranges.

Q2. Maximize $z = 3x_1 + 2x_2 + 5x_3$

s.t. $x_1 + 2x_2 + x_3 \le 430$

$3x_1 + 2x_3 \le 460$

$x_1 + 4x_3 \le 420$

$x_1\ x_2\ x_3 \ge 0$

Standard: $Z = 3x_1 + 2x_2 + 5x_3 + 0s_1 + 0s_2 + 0s_3 + 0D_1 + 0D_2 + 0D_3$

s.t. $x_1 + 2x_2 + x_3 + s_1 = 430 + D_1$

$3x_1 + 2x_3 + s_2 = 460 + D_2$

$x_1 + 4x_3 + s_3 = 420 + D_3$

	BV	C_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3	D_1	D_2	D_3	
	s_1	0	430	1	2	1	1	0	0	1	0	0	$R_1 \to R_1 - R_2$
←	s_2	0	460	3	0	2	0	1	0	0	1	0	$R_2 \to R_2/2$
	s_3	0	420	1	4	0	0	0	1	0	0	1	$R_3 \to R_3$
		$Z=0$		-3	-2	$-5\uparrow$	0	0	0	0	0	0	

(x_3 enters, s_2 leaves)

	BV	C_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3	D_1	D_2	D_3
←	s_1	0	200	$-1/2$	2	0	1	$-1/2$	0	1	$-1/2$	0
	x_3	5	230	$3/2$	0	1	0	$1/2$	0	0	$1/2$	0
	s_3	0	420	1	4	0	0	0	1	0	0	1
		$Z=1150$		$9/2$	$-2\uparrow$	0	0	$5/2$	0	0	$5/2$	0

(s_1 leaves ; x_2 enters)

BV	C_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3	D_1	D_2	D_3
x_2	2	100	$-1/4$	1	0	$1/2$	$-1/4$	0	$1/2$	$-1/4$	0
x_3	5	230	$3/2$	0	1	0	$1/2$	0	0	$1/2$	0
s_3	0	20	2	0	0	-2	1	1	-2	1	1
	$Z=1350$		4	0	0	1	2	0	1	2	0

So $Z = 1350$

$\therefore\ Z = 1350 + D_1 + 2D_2 + 0D_3$

$x_1 = 0,\quad x_2 = 100 + 1/2\,D_1 - 1/4\,D_2$

$x_3 = 230 + 1/2\,D_2,\quad s_3 = 20 - 2D_1 + D_2 + D_3$

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a) Now $D_1 = 440 - 430 = 10$

$D_2 = 490 - 460 = 30$

$D_3 = 400 - 420 = -20$

$\therefore x_1 = 0$

$x_2 = 100 + 1/2(10) - \frac{1}{4}(30) = 100 + 5 - 7.5 = 97.5 > 0$

$x_3 = 230 + 1/2(30) = 237.5 > 0$

$S_3 = 20 - 2(10) + (30) - 20 \Rightarrow 10 > 0$

(Hence feasible)

Dual prices are :-

Resource 1	1	$-200 \leq D_1 \leq 10$
Resource 2	2	$-20 \leq D_2 \leq 400$
Resource 3	0	$-20 \leq D_3 \leq \infty$

New profit $\Rightarrow 1350 + D_1 + 2D_2 + 0D_3$

$= 1350 + 10 + 2 \times 30 \Rightarrow 1350 + 10 + 60$

$\Rightarrow \underline{1420}$

b) $D_1 = 460 - 430 = +30$

$D_2 = 440 - 460 = -20$

$D_3 = 370 - 420 = -50$

$\therefore$ Hence values.

$x_1 = 0$

$x_2 = 100 + 1/2(30) - 1/4(-20) = 100 + 15 + 5 = 120 > 0$

$x_3 = 230 + 1/2(-20) = 230 - 10 = 220 > 0$

$S_3 = 20 - 2(30) - 20 + 50 = 20 - 60 - 20 + 50 = -10 < 0$

(Hence it is infeasible)

c) So Overtime cost $\Rightarrow \frac{50}{60} = \$0.83/min$

Revenue (Dual price) for operation 1 is \$1/min

as cost < Revenue. So it is beneficial

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d) Dual price of operation 2 = \$2/min

where $-20 \leq D_2 \leq 400$

& $D_1 = D_3 = 0$

$x_2 = 100 - 1/4 D_2 \geq 0 \Rightarrow D_2 \leq 400$

$x_3 = 230 \geq -1/2 D_2 \Rightarrow D_2 \geq -460$

$S_3 = 20 \geq -D_2 \Rightarrow D_2 \geq -20$

given for 2 hrs $\Rightarrow$ 120 minutes = D_2

Revenue increase = $2 \times 120 = \$240$

cost increase = $2 \times (15+10) = \$110$

as cost < Revenue $\Rightarrow$ acceptable.

e) No, operation 3 is already abundant. Hence it's dual price is 0. (also found in tableau)

f) Now $D_1 = 440 - 430 = 10$

also $-200 \leq D_1 \leq 10$.

cost $\Rightarrow \frac{40}{60} \times 10 = 6.67$ (Overtime)

New revenue $\Rightarrow 1350 + 10 = \$1360$

Net revenue $\rightarrow 1360 - 6.67 \Rightarrow \$1353.33$

g) Dual price = \$2/min. $-20 \leq D_2 \leq 400$

$\because$ Decrease in cost $\Rightarrow \frac{15}{60} \times 30 \Rightarrow \$7.5$

$\therefore$ Lost revenue $\Rightarrow 2 \times 15 = \$30$

Lost revenue > Decrease in cost

Hence it is not advantageous. Not acceptable.

h) for function $z = x_1 + x_2 + 4x_3$

$z = 6x_1 + 3x_2 + 9x_3$

Max $Z = (3+d_1)x_1 + (2+d_2)x_2 + (5+d_3)x_3$

s.t $x_1 + 2x_2 + x_3 \leq 430$

$3x_1 + 2x_2 \leq 460$

$x_1 + 4x_2 \leq 420$

$x_1, x_2, x_3 \geq 0$

After solving using simplex method.

Or by this

B_V	C_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3	
s_1	0	430	1	2	1	1	0	0	$R_1 \leftarrow R_1 - R_2$
← s_2	0	460	3	0	(2)	0	1	0	$R_2 \leftarrow R_2/2$
s_3	0	420	1	4	0	0	0	1	$R_3 \leftarrow R_3$
	$z=0$		$-3-d_1$	$-2-d_2$	$-5-d_3$ ↑	0	0	0	

(S_2 leaves, x_3 enters)

← s_1	0	200	$-1/2$	(2)	0	1	$-1/2$	0	$R_1 \leftarrow R_1/2$
x_3	$5+d_3$	230	$3/2$	0	1	0	$1/2$	0	$R_2 \leftarrow R_2$
s_3	0	420	1	4	0	0	0	1	$R_3 \leftarrow R_3 - 4R_1$
	$z = (5+d_3) \times 230$		$\frac{9-d_1}{2} + \frac{3}{2}d_3$	$-2-d_2$ ↑	0	0	$\frac{9+d_3}{2}$	0	

(S_1 leaves, x_2 enters)

				d_1	d_2	d_3	0	0	0
d_2	x_2	$2+d_2$	100	$-1/4$	1	0	$1/2$	$-1/4$	0
d_3	x_3	$5+d_3$	230	$3/2$	0	1	0	$1/2$	0
0	s_3	0	20	2	0	0	-2	1	1
	$z = 1350 + 100d_2 + 230d_3$			$4 - \frac{1}{4}d_2 + \frac{3}{2}d_3 - d_1$	0	0	$1 + \frac{d_2}{2}$	$2 - \frac{1}{4}d_2 + \frac{1}{2}d_3$	0

$Z = 1350 + 100d_2 + 230d_3 + 0 \times 20$

Now, for feasibility; $4 - \frac{1}{4}d_2 + \frac{3}{2}d_3 - d_1 \geq 0$

$1 + d_2/2 \geq 0$

$2 - \frac{1}{4}d_2 + \frac{1}{2}d_3 \geq 0$

for $d_2 = d_3 = 0$

$-\infty \leq d_1 \leq 4$

for $d_1 = d_3 = 0$; $d_2 < 8$, $d_2 \geq -2$

$-2 \leq d_2 \leq 8$

for $d_1 = d_2 = 0$; $\infty \geq d_3 \geq -8/3$

(1) Now for $z = x_1 + x_2 + 4x_3$

$d_1 = -2$, $d_2 = -1$, $d_3 = -1$

x_1 ∴ $z_1 - c_1 = 4 - \frac{1}{4}(-1) + \frac{3}{2}(-1) + 2 \Rightarrow 19/4 \geq 0$

s_1 $z_4 - c_4 = 1 + \frac{d_2}{2} = 1 + (-1/2) = 1/2 > 0$

s_2 $z_5 - c_5 = 2 - \frac{1}{4}(-1) + \frac{1}{2}(-1) = 7/4 > 0$

So solution is unchanged.

(2) for $z = 6x_1 + 3x_2 + 9x_3$

$d_1 = 3$, $d_2 = 1$, $d_3 = 4$.

∴ x_1 ⇒ $4 - \frac{1}{4}(1) + \frac{3}{2}(4) - 3 \Rightarrow 27/4 > 0$

s_1 ⇒ $1 + \frac{1}{2}(1)$ ⇒ $1.5 > 0$

s_2 ⇒ $2 - \frac{1}{4}(1) + \frac{1}{2}(4)$ ⇒ $15/4 > 0$

So solution is unchanged.

Q3. X_1 = for raw ~~Product~~ P1

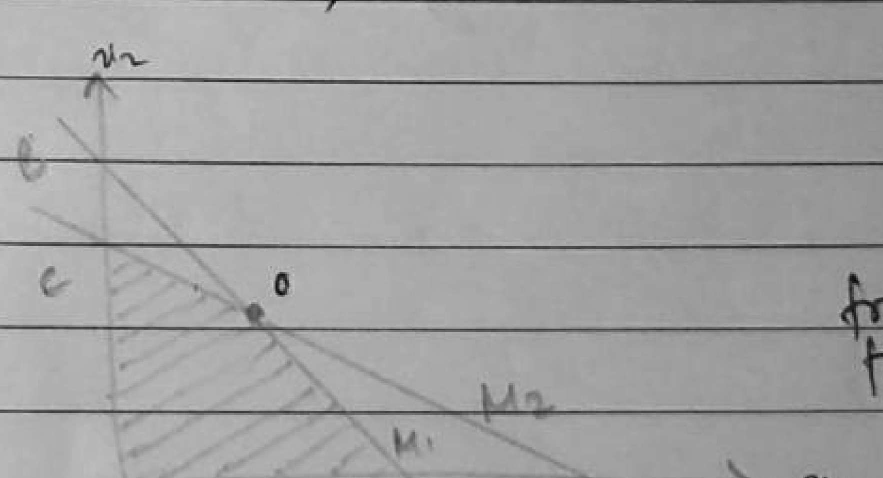
X_2 = for raw ~~Product~~ P2

Objective function - $Z = 2x_1 + 3x_2$

s.t. $2x_1 + 2x_2 \leq 8$ (Matial M_1)

$3x_1 + 6x_2 \leq 18$ (Material M_2)

$x_1, x_2 \geq 0$



By solving;

at $x_2 = 0$

for A $2x_1 = 8$, $x_1 = 4$

for B $3x_1 = 18 = x_1 = 6$

∴ A = (4,0)

B = (6,0)

for C : at $x_1 = 0$ $6x_2 = 18$, $x_2 = 3$
for B : at $x_1 = 0$ $2x_2 = 8$, $x_2 = 4$

$\therefore$ $B = (0, 4)$
$C = (0, 3)$

at optimum value $\Rightarrow$
$D \rightarrow (2, 2)$
$Z = \underline{\underline{10}}$

a) M_1 at $C \Rightarrow 2(0) + 2(3) = 6$
M_1 at $D \Rightarrow 2(6) + 2(0) = 12$
Z at $C \Rightarrow 2(0) + 3(3) = 9$
Z at $D \Rightarrow 2(6) + 3(0) = 12$

$\therefore$ dual price $\Rightarrow \dfrac{12-9}{12-6} = \$\, 0.50/\text{unit}$

range $\Rightarrow$ $(6 \le M_1 \le 12)$

Now M_2 at $A = 3(4) + 6(0) = 12$
M_2 at $B = 3(0) + 6(4) = 24$
Z at $A = 2(4) + 3(0) = 8$
Z at $B = 2(0) + 3(4) = 12$

$\therefore$ dual price $\Rightarrow \dfrac{12-8}{24-12} = \$\, 0.33/\text{unit}$.

range $\Rightarrow$ $(12 \le M_2 \le 24)$

(b) Dual price $= \$\, 0.50/\text{unit}$ and this lies in the range for M_1 i.e. $6 \le M_1 \le 12$

given increase in revenue $\Rightarrow 2 \times 0.5 = \$\, 1.0$
Increase in cost $\Rightarrow 2 \times 0.25 = \$\, 0.5$
cost < Revenue.

Hence purchase recommended.

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(c) Dual price for M_2 ⇒ \$ 0.33 / unit
which valid in range $12 \le M_2 \le 24$
So the purchase $\le$ \$ 0.33 / unit

(d) Dual price of $M2$ = \$ 0.33 / unit valid in range
$12 \le M_2 \le 24$.
Now it is increased by 3 unit ie from 18 to 21 units.
∴ Increase in revenue = $3 \times 0.33 \Rightarrow \$0.99$
Hence optimum revenue now becomes ⇒
original revenue + increase = $10 + 0.99 = \$10.99$

(e) Optimality condition for CA/CB
$z = 2x_1 + 3x_2$
$(C_A x_1 + C_B x_2)$

$$\frac{3}{6} \le \frac{CA}{CB} \le \frac{2}{2} \Rightarrow 0.5 \le \frac{CA}{CB} \le 1$$

(f) Maximize $z = 2x_1 + 3x_2$
$C_B = 3$, $3 \times 0.5 \le CA \le 3 \times 1 \Rightarrow 1.5 \le CA \le 3$
$CA = 2$, $2 \times 1 \le CB \le 2 \times 2 \Rightarrow 2 \le CB \le 4$

(g) If revenues for CA & CB are changed to \$ 5 & \$ 4
∴ $\frac{CA}{CB} = \frac{5}{4} = \$1.25$ which falls outside the range of $0.5 \le \frac{CA}{CB} \le 1$.
So solution changes ~~however~~ the point doesn't remains same –
$x_A = 4$, $x_B = 0$
$Z = 5x_A + 4x_B \Rightarrow 5(4) + 4(0) = \underline{\underline{20}}$

(h) Case I ⇒ $z = 5x_A + 3x_B$
CA ⇒ 5 is outside (1.5, 3). So optimum changes to $x_A = 4$, $x_B = 0$, z \$20.
Case 2 ⇒ $z = 2x_A + 4x_B$.
CB = 4 is in the range (2, 4). So optimum is same.
$Z = 2(2) + 4(2) = \$12$.

Q4. a) x_1 = No. of shifts / week
x_2 = No. of blouses / week

Maximize $Z = 8x_1 + 12x_2$

s.t. $20x_1 + 60x_2 \le 25 \times 8 \times 5 \times 60 = 60{,}000$

$70x_1 + 60x_2 \le 35 \times 8 \times 5 \times 60 = 84{,}000$

$12x_1 + 4x_2 \le 5 \times 8 \times 5 \times 60 = 12{,}000$

$x_1, x_2 \ge 0$

On solving :-

$Z = 8x_1 + 12x_2 + 0S_1 + 0S_2 + 0S_3$

s.t. $20x_1 + 60x_2 + S_1 = 60{,}000$

$70x_1 + 60x_2 + S_2 = 84000$

$12x_1 + 4x_2 + S_3 = 12{,}000$

$x_1, x_2, S_1, S_2, S_3 \ge 0$

BV	CB	XB	x_1	x_2	S_1	S_2	S_3	
← S_1	0	60000	20	60	1	0	0	$R_1 \leftarrow R_1/60$
S_2	0	84000	70	60	0	1	0	$R_2 \leftarrow R_2 - 6$
S_3	0	12000	12	4	0	0	1	$R_3 \leftarrow R_3 - 4$
		$Z = 0$	-8	-12 ↑	0	0	0	

(S_1 leaves, x_2 enters)

BV	CB	XB	x_1	x_2	S_1	S_2	S_3	
x_2	12	1000	1/3	1	1/60	0	0	$R_1 \leftarrow R_1 -$
← S_2	0	24000	50	0	-1	1	0	$R_2 \leftarrow R_2/50$
S_3	0	8000	32/3	0	$-1/5$	0	1	$R_3 \leftarrow R_3 -$
		$Z = 12000$	-4 ↑	0	1/5	0	0	

(S_2 leaves, x_1 enters)

BV	CB	XB	x_1	x_2	S_1	S_2	S_3
x_2	12	840	0	1	7/300	$-1/50$	0
x_1	8	480	1	0	$-1/50$	1/50	0
S_3	0	2880	0	0	29/45	$-16/75$	1
		$Z = 13920$	0	0	0.12	0.08	0

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(a) Hence $x_1 = 480$, $x_2 = 840$
& $z = \$ 13920/\text{week}$

(b) Let s_1, s_2, s_3 be slack variables.
So resultant Z will be
$z = 13920 + 0.12 s_1 + 0.08 s_2 + 0 s_3$.

		In hour
cutting	\$ 0.12 /min	\$ 7.2 /hr
Sewing	\$ 0.08/min	\$ 4.80/hr
Packaging	\$ 0/hr.	

(c) Maximum wages will be \$ 7.20/hr for cutting & \$ 4.8/hr for sewing.