Bihar Engineering University, Patna

End Semester Examination - 2022

Course: B.Tech. | *Semester: III* | *Time: 03 Hours*
Code: 100311 | *Subject: Mathematics-III (Differential Calculus)* | *Full Marks: 70*

Instructions:-
(i) The marks are indicated in the right-hand margin.
*(ii) There are **NINE** questions in this paper.*
*(iii) Attempt **FIVE** questions in all.*
*(iv) Question No. **1** is compulsory.*

Q.1 Choose the correct answer of the following (Any seven question only): **[2 x 7 = 14]**

(a) The value of $\lim_{x \to 0} \left(\frac{\sin x}{x}\right)^{1/x}$ is
(i) 0 (ii) 1
(iii) e (iv) 1/e

(b) The value of the integral
$\int_C \{yzdx + (xz+1)dy + xydz\}$
Where C is any path from (1, 0, 0) to (2, 1, 4) is
(i) 6 (ii) 7
(iii) 8 (iv) 9

(c) The maximum value of $\sin x + \cos x$ is
(i) 1 (ii) 2
(iii) $\sqrt{2}$ (iv) 0

(d) The value of ∇^2 [(1-x) (1-2x)] is equal to
(i) 2 (ii) 3
(iii) 4 (iv) 6

(e) The degree of the differential equation
$y\frac{dx}{dy} - \left(\frac{dx}{dy}\right)^2 - \sin y\left(\frac{dx}{dy}\right)^3 - cosx = 0$ is
(i) 0 (ii) 1
(iii) 2 (iv) Cannot be determined

(f) If $= tan^{-1}\frac{y}{x}$, then div (grad f) is equal to
(i) 1 (ii) -1
(iii) 0 (iv) 2

(g) If P_n is the Legendre polynomial of first kind, then the value of $\int_{-1}^{1} xP_nP'_n\, dx$ is
(i) $\frac{2}{(2n+1)}$ (ii) $\frac{2n}{(2n+1)}$
(iii) $\frac{2}{(2n+3)}$ (iv) $\frac{2n}{(2n+3)}$

(h) If J_n is the Bessel's function of first kind, then the value of $J_{-\frac{1}{2}}$ is
(i) $\sqrt{\frac{2}{\pi x}}\left(\frac{\cos x}{x} - \sin x\right)$ (ii) $\sqrt{\frac{2}{\pi x}}\left(\frac{\sin x}{x} - \cos x\right)$
(iii) $\sqrt{\frac{2}{\pi x}}\sin x$ (iv) $\sqrt{\frac{2}{\pi x}}\cos x$

(i) The solution of $p\tan x + q\tan y = \tan z$ is
(i) $\sin x / \sin y = \varphi(\sin y/\sin z)$ (ii) $\sin x \,.\sin y = \varphi(\sin y/\sin z)$
(iii) $\sin x / \sin y = \varphi(\sin y, \sin z)$ (iv) $\sin x / \sin y = \varphi(\sin y.\sin z)$

(j) The vector $\vec{v} = e^x \sin y\hat{\imath} + e^x \cos y\hat{\jmath}$ is
(i) Solenoidal (ii) irrational (iii) rotational (iv) cannot be found

 P.T.O.

Q.2 (a) Form the partial differential equation $(x-a)^2 + (y-b)^2 + z^2 = 1$. [7]

(b) Solve $xp + yq = 3z$ [7]

Q.3 (a) Find the directional derivative of $\emptyset = z^2yz + 4xz^2$ at the point (1, -2, 1) in the direction of the vector $2\hat{\imath} - \hat{\jmath} - 2\hat{k}$. [7]

(b) Find a unit vector normal to the surface $x^3+y^3 + 3xyz = 3$ at the point (1, 2, -1) [7]

Q.4 Solve the following questions:-

(a) Solve partial differential equation $\frac{y^2z}{x}p + xzq = y^2$. [7]

(b) Show that the function $f(x,y) = \begin{cases} \frac{xy}{\sqrt{x^2+y^2}}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$ [7]

is continuous at origin.

Q.5 (a) If $f = (x^2 + y^2 + z^2)^{-n}$, then find div grad f and determine n, if div grad $f = 0$. [7]

(b) Verify Green's theorem for $\int_C \{(xy + y^2)dx + x^2dy\}$ [7]

Where C is bounded by $y = x$, $y = x^2$.

Q.6 (a) Evaluate the integral by changing the order of integration [7]

$$\int_0^\infty\int_0^x xe^{-\frac{x^2}{y}}dydx$$

(b) Solve the differential equation [7]

$(x^2 + y^2 + x)\,dx - (2x^2 + 2y^2 - y)\,dy = 0$

Q.7 Verify the stokes' theorem for [14]

$A = (y - z + 2)\,i + (yz + 4)\,j - xz\,k$

Where S is the surface of the cube $x = 0$, $y = 0$, $z = 0$, $x = 2$, $y = 2$ and $z = 2$ above the xy-plane.

Q.8 (a) Prove that [6]

$$2nJ_n(x) = x\,(J_{n-1}(x) + J_{n+1}(x))$$

(b) Prove that [8]

$$\sum_{n=0}^{\infty}\frac{x^{n+1}}{n+1}P_n(1) = \frac{1}{2}\log\left(\frac{1+x}{1-x}\right)$$

Q.9 Solve the following questions:

(a) Using Green's theorem, evaluate $\int_C[(y - \sin x)\,dx + \cos x dy]$ where C is the plane triangle enclosed by the lines $y = 0$, $x = \frac{\pi}{2}$ and $y = \frac{2x}{\pi}$ [7]

(b) Prove that $\mathrm{div}(r^n\vec{r}) = (n+3)r^n$. Hence show that div $\left(\frac{\vec{r}}{r^3}\right)$ is solenoidal. [7]