Code : 100311

B.Tech 3rd Semester Exam., 2019
(New Course)

MATHEMATICS—III

(Differential Calculus)

Time : 3 hours

Full Marks : 70

Instructions :

(i) The marks are indicated in the right-hand margin.

*(ii) There are **NINE** questions in this paper.*

*(iii) Attempt **FIVE** questions in all.*

(iv) Question No. 1 is compulsory.

1. Choose the correct answer (any *seven*) : 2×7=14

(a) The function

$$f(x)=\begin{cases}(1+2x)^{\frac{1}{x}}, & x\neq 0\\ e^2, & x=0\end{cases}$$

is

(i) differentiable at $x=0$

(ii) continuous at $x=0$

(iii) discontinuous at $x=0$

(iv) not differentiable at $x=0$

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(b) The function

$$f(x,y)=\begin{cases}\dfrac{x^2-2xy+y^2}{x-y}, & (x,y)\neq(1,-1)\\ 0, & (x,y)=(1,-1)\end{cases}$$

is

(i) continuous at $(1,-1)$

(ii) discontinuous everywhere

(iii) discontinuous at $(1,-1)$

(iv) continuous everywhere

(c) If $w=\sin^{-1}u$, $u=\left(\dfrac{x^2+y^2+z^2}{x+y+z}\right)$, then $x\dfrac{\partial w}{\partial x}+y\dfrac{\partial w}{\partial y}+z\dfrac{\partial w}{\partial z}$ is equal to

(i) $\sin w$

(ii) $\cos w$

(iii) $\tan w$

(iv) $\cot w$

(d) The minimum value of

$$f(x,y,z)=x^2+y^2+z^2$$

such that $xyz=k^3$, is

(i) k^2

(ii) $9k^2$

(iii) $3k^2$

(iv) k^3

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(e) The solution of the differential equation

$$y\,dx - x\,dy + e^{\frac{1}{x}}\,dx = 0$$

is

(i) $y - xe^{\frac{1}{x}} = cx$

(ii) $y + xe^{-\frac{1}{x}} = cx$

(iii) $ye^{\frac{1}{x}} + x = cx$

(iv) $y + xe^{\frac{1}{x}} = cx$

(f) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, $r = |\vec{r}|$ and $\hat{r}$ is the unit vector of $\vec{r}$, then $\nabla \cdot \hat{r}$ is

(i) r

(ii) $2r$

(iii) $\frac{1}{r}$

(iv) $\frac{2}{r}$

(g) If C is any path from $(1, 0, 0)$ to $(2, 1, 4)$, then $\int_C [yz\,dx + (zx+1)\,dy + xy\,dz]$ is equal to

(i) 1

(ii) 2

(iii) 8

(iv) 9

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(h) If $P_n(x)$ is the Legendre polynomial of first kind, then the incorrect statement is

(i) $(2n+1)\,x\,P_n(x) = (n+1)\,P_{n+1}(x) + n\,P_{n-1}(x)$

(ii) $n\,P_n(x) = xP'_n(x) - P'_{n-1}(x)$

(iii) $(2n+1)\,P_n(x) = P'_{n+1}(x) + P'_{n-1}(x)$

(iv) $(n+1)\,P_n(x) = P'_{n+1}(x) - x\,P'_n(x)$

(i) The partial differential equation which satisfies the arbitrary functions

$$z = f\left(\frac{xy}{z}\right)$$

is

(i) $px + qy = 0$

(ii) $px - qy = 0$

(iii) $px + qy = z$

(iv) None of the above

(j) The singular solution of the differential equation

$$9\left(\frac{dy}{dx}\right)^2 (2-y)^2 = 4(3-y)$$

is

(i) $y^2(3-y) = 0$

(ii) $(2-y)^2(3-y) = 0$

(iii) $y = 3$

(iv) None of the above

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2. (a) If $y = A\cos(\log x) + B\sin(\log x)$, show that

$$x^2 y_{n+2} + (2n+1)x y_{n+1} + (n^2+1)y_n = 0$$

where $y_n = \dfrac{d^n y}{dx^n}$.

(b) Find :

$$\lim_{x\to 0} \frac{\sin x \cdot \sin^{-1} x - x^2}{x^6}$$ 7+7=14

3. (a) Show that the following function is continuous at the point (0, 0) :

$$f(x, y) = \begin{cases} \dfrac{2x^4 + 3y^4}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

(b) If $z(x+y) = x^2 + y^2$, show that

$$\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = 4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)$$ 7+7=14

4. (a) Transform the equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

into polar coordinates.

(b) Find the extreme values of $f(x, y, z) = 2x + 3y + z$, such that $x^2 + y^2 = 5$ and $x + z = 1$. 7+7=14

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5. (a) Evaluate

$$\int_{(0,\,0)}^{(2,\,1)} [(10x^4 - 2xy^3)\,dx - 3x^2y^2\,dy]$$

along the path $x^4 - 6xy^3 = 4y^2$.

(b) Evaluate $\iint_S F.ndS$, where $F = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$ and S is the surface of the cube bounded by $x=0$, $x=1$, $y=0$, $y=1$, $z=0$, $z=1$, by using Gauss divergence theorem. 7+7=14

6. (a) Evaluate

$$\frac{d}{d\theta}\{A \times (B \times C)\}$$

at $\theta = 0$, where $A = \sin\theta\hat{i} + \cos\theta\hat{j} + \theta\hat{k}$, $B = \cos\theta\hat{i} - \sin\theta\hat{j} - 3\hat{k}$, $C = 2\hat{i} + 3\hat{j} - \hat{k}$.

(b) A particle moves along the curve $x = t^3 + 1$, $y = t^2$, $z = 2t + 5$ where t is the time. Find the components of the velocity and acceleration at $t = 1$, in the direction $\hat{i} + \hat{j} + 3\hat{k}$. 7+7=14

7. (a) Solve :

$$(x + y + a)\frac{dy}{dx} = y^2 + b$$

(b) Solve :

$$x^2 + p^2 x = yp \quad \left(p = \frac{dy}{dx}\right)$$ 7+7=14

8. (a) Solve by the method of variation of parameters

$$\frac{d^2y}{dx^2} + n^2 y = \sec nx$$

(b) Solve :

$$\frac{d^2y}{dx^2} - 2\tan x \frac{dy}{dx} + 5y = \sec x \, e^x$$

7+7=14

9. (a) Solve :

$$z\left(\frac{\partial z}{\partial x}\right)^2 - z\left(\frac{\partial z}{\partial y}\right)^2 = (x - y)$$

(b) Evaluate :

$$\int_{-1}^{1} x \, P_n(x) P_{n-1}(x) \, dx$$

where P_n is the Legendre polynomial.

7+7=14

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