Code : 100311

B.Tech 3rd Semester Exam., 2020
(New Course)

MATHEMATICS—III

(Differential Calculus)

Time : 3 hours *Full Marks* : 70

Instructions :

(i) The marks are indicated in the right-hand margin.

*(ii) There are **NINE** questions in this paper.*

*(iii) Attempt **FIVE** questions in all.*

*(iv) Question No. **1** is compulsory.*

1. Choose the correct answer of the following (any *seven*) : 2×7=14

(a) The value of $\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{1/x}$ is

(i) 0

(ii) 1

(iii) e

(iv) $1/e$

(b) Let $f(x)=|x|$ and $g(x)=|x^3|$, then

(i) $f(x)$ and $g(x)$ both are continuous at $x=0$

(ii) $f(x)$ and $g(x)$ both are differentiable at $x=0$

(iii) $f(x)$ is differentiable but $g(x)$ is not differentiable at $x=0$

(iv) $f(x)$ and $g(x)$ both are not differentiable at $x=0$

(c) The value of $\nabla^2[(1-x)(1-2x)]$ is equal to

(i) 2

(ii) 3

(iii) 4

(iv) 6

(d) If $v=xy^2\hat{i}-2x^2yz\hat{j}-3yz^2\hat{k}$, then the value of curl v at (1, −1, 1) is equal to

(i) $-(\hat{j}-2\hat{k})$

(ii) $(\hat{i}-3\hat{k})$

(iii) $-(\hat{i}-2\hat{k})$

(iv) $(\hat{i}-2\hat{j}-\hat{k})$

(e) The degree of the differential equation

$$y\frac{dx}{dy} + \left(\frac{dx}{dy}\right)^2 + \sin y\left(\frac{dx}{dy}\right)^3 - \cos x = 0$$

is

(i) 0

(ii) 1

(iii) 2

(iv) Cannot be determined

(f) The solution of the boundary value problem

$$(x - y^2 x)\,dx + (x^2 y - y)\,dy = 0,\ y(0) = 0$$

is

(i) $x^2 - y^2 = 0$

(ii) $2x - y = 0$

(iii) $x - 2y = 0$

(iv) None of the above

(g) Let $P_n(x)$ be the Legendre polynomial of degree $n \geq 0$. If

$$\int_{-1}^{1} P_{n-1}^2(x)\,dx = \frac{2}{(kn - l)}$$

then the value of (k, l) is

(i) (1, 1) (ii) (1, 2)

(iii) (2, 1) (iv) (2, 2)

(h) The general solution of Bessel differential equation

$$x^2 y''(x) + xy'(x) + (x^2 - 64)y(x) = 0$$

is

(i) $y = AJ_8(x) + BJ_{-8}(x)$, where A and B are arbitrary constants

(ii) $y = AJ_8(x) + BY_{-8}(x)$, where A and B are arbitrary constants

(iii) $y = AJ_8(x) + J_{-8}(x)$, where A is arbitrary constant

(iv) $y = J_{3/4}(x) + Y_{3/4}(x)$

(i) The equation $p\tan y + q\tan x = \sec^2 z$ is of order

(i) 1

(ii) 2

(iii) 0

(iv) None of the above

(j) The solution of $p\tan x + q\tan y = \tan z$ is

(i) $\sin x / \sin y = \varphi(\sin y / \sin z)$

(ii) $\sin x . \sin y = \varphi(\sin y / \sin z)$

(iii) $\sin x / \sin y = \varphi(\sin y,\ \sin z)$

(iv) $\sin x / \sin y = \varphi(\sin y . \sin z)$

2. *(a)* If $y = (\sin^{-1} x)^2$, then show that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0$$

Hence find $(y_n)_0$. 7

(b) Find the value of

$$\lim_{x \to 0} \left(\frac{\tan x}{x} \right)^{1/x^2}$$ 7

3. *(a)* Discuss the continuity of the following function $f(x, y)$ at point (0, 0) : 7

$$f(x, y) = \begin{cases} \dfrac{\sin \sqrt{|xy|} - \sqrt{|xy|}}{\sqrt{x^2 + y^2}}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

(b) For the function

$$f(x, y) = \begin{cases} \dfrac{xy(2x^2 3y^2)}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

check whether $f_{xy}(0, 0)$ and $f_{yx}(0, 0)$ are equal or not. 7

4. *(a)* Find the minimum value of $x^2 + y^2 + z^2$ subject to the condition $xyz = a^3$. 8

(b) Obtain the second-order Taylor's series approximation to the function

$$f(x, y) = xy^2 + y\cos(x - y)$$

about the point (1, 1). 6

5. *(a)* If $f = (x^2 + y^2 + z^2)^{-n}$, then find div grad f and determine n, if div grad $f = 0$. 6

(b) Verify Green's theorem for

$$\int_C [(xy + y^2)dx + x^2 dy]$$

where C is bounded by $y = x$, $y = x^2$. 8

6. *(a)* Find the value of n for which the vector $r^n \mathbf{r}$ is solenoidal, where $\mathbf{r} = x\hat{i} + y\hat{j} + z\hat{k}$. 7

(b) Solve the differential equation

$$(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)dy = 0$$ 7

7. Solve the following differential equations : 7+7=14

(a) $p = \sin(y - xp)$. Also find its singular solution.

(b) $x^2 \dfrac{d^2y}{dx^2} - 3x\dfrac{dy}{dx} + 5y = x\log x$

8. *(a)* Prove that

$$2nJ_n(x) = x(J_{n-1}(x) + J_{n+1}(x))$$ 6

(b) Prove that

$$\sum_{n=0}^{\infty} \frac{x^{n+1}}{n+1} P_n(1) = \frac{1}{2}\log\left(\frac{1+x}{1-x}\right)$$ 8

9. Solve the following differential equations : 7+7=14

(a) $x^2p + y^2q = (x+y)z$

(b) $(x+y)(p+q)^2 + (x-y)(p-q)^2 = 1$

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