DEEP LEARNING SPECIALIZATION

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Course - 1

Neural Networks & Deep Learning

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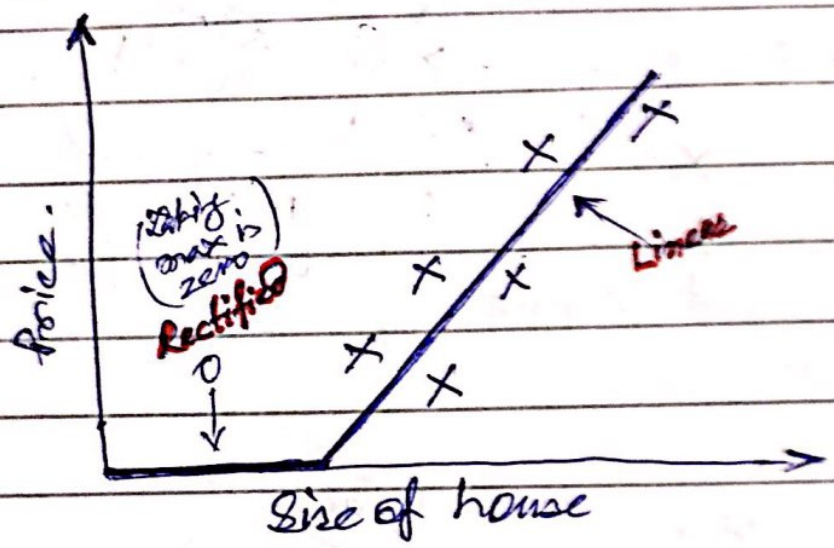
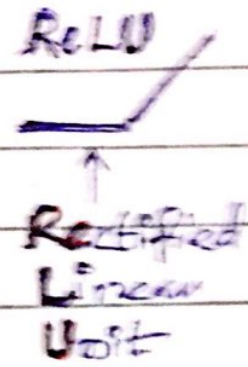
Deep Learning Specialization

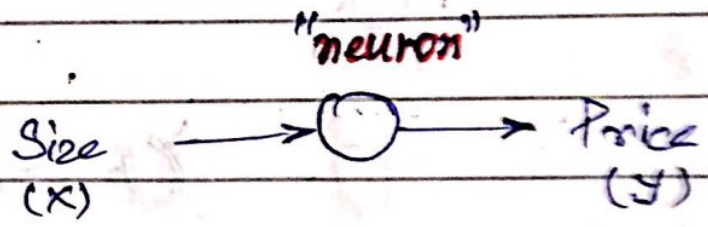
1. Neural Networks & Deep Learning

*. AI is the new electricity.
(will bring an equally big transf^n)

*. eg. Housing Price Prediction.







The above is a single "neuron" if we stack together multiple neurons then we will get a NEURAL NETWORK.

eg.

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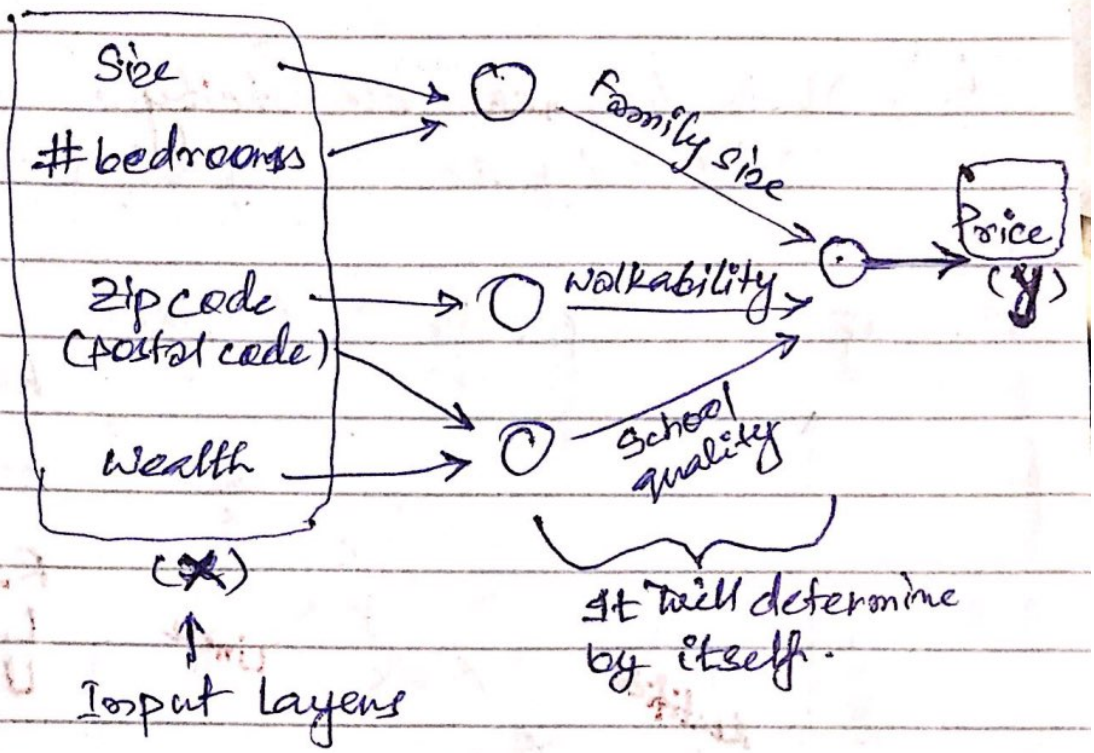
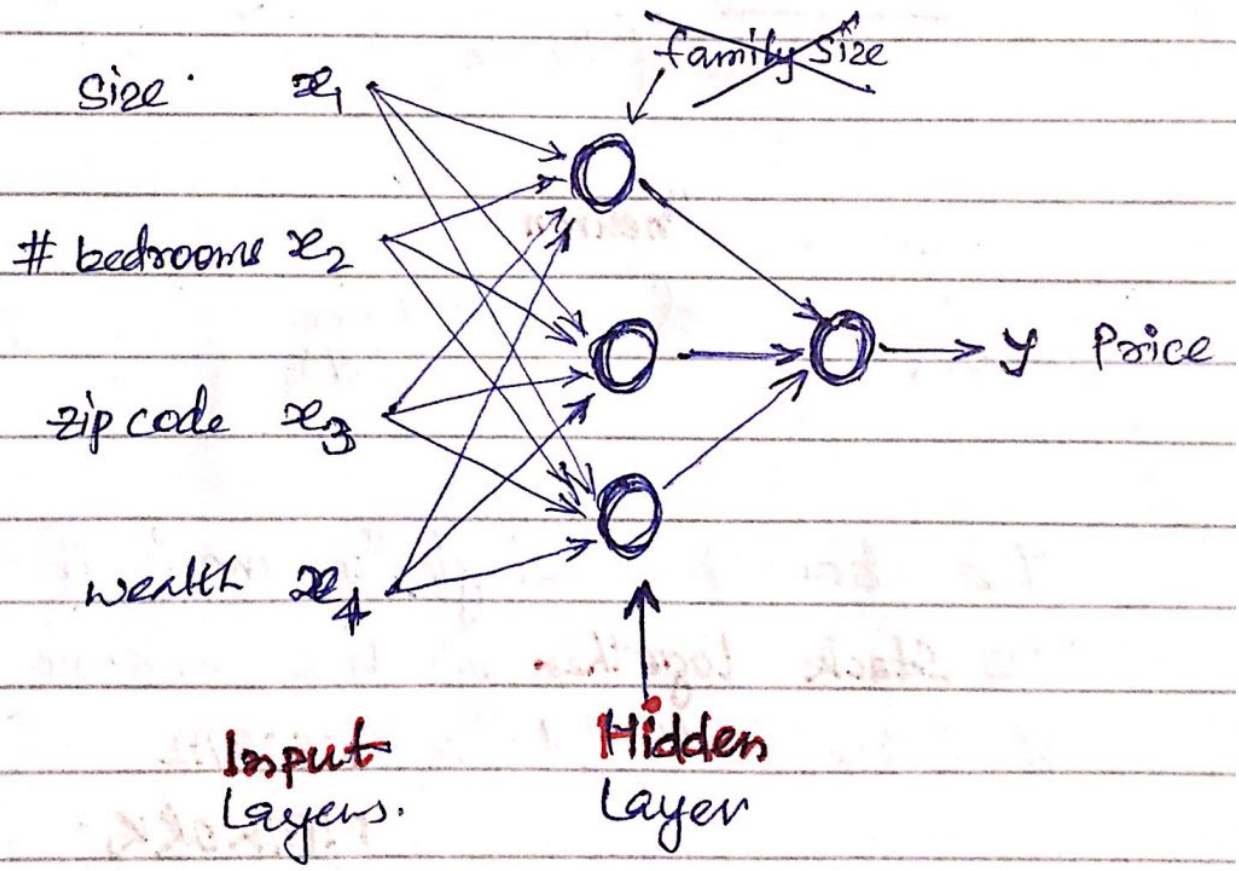
Example of Neural Network (Basic)

eg





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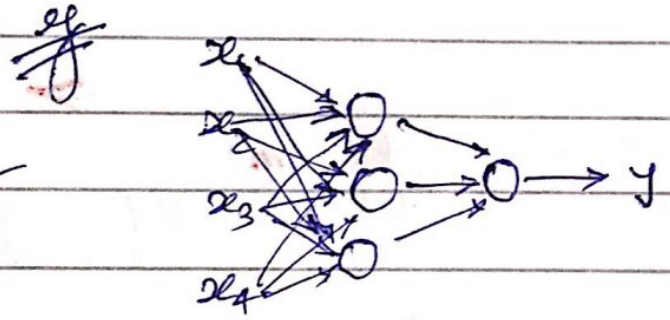
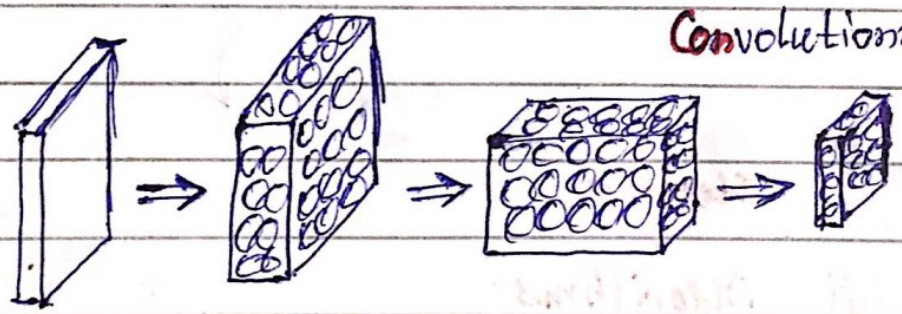
A. Supervised learning

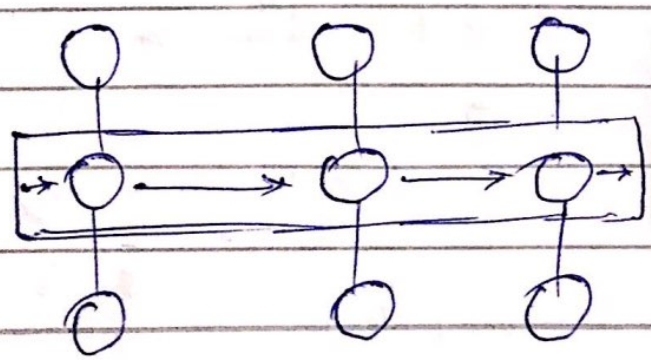
INPUT (X)	OUTPUT (Y)	APPLICATION	
Home features	Price	Real Estate	Std. NN
Ad, user info	Click on ad (0/1)?	Onl. Adv.	Std. NN
Image	Object (1,---1000)	Photo tagging	CNN
Audio	Text transcript	Speech Recog.	RNN
English	Chinese	M/c Trans^{n}	RNN
Image, Radar Info	Position of other cars	Aut. Driving	Hybrid or custom NN



Standard NN



Convolutional NN



Recurrent NN

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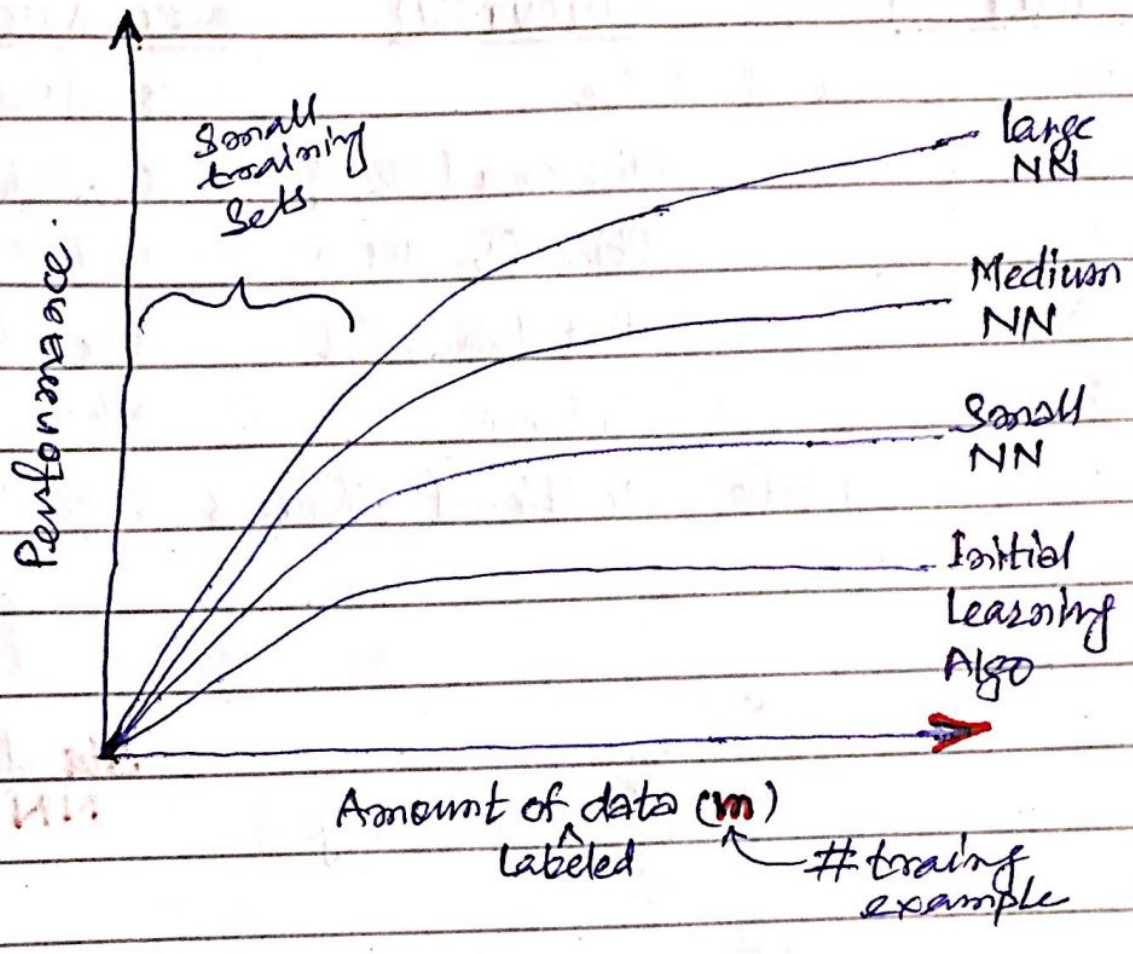
* Scale drives deep learning progress



(i) **Data.**

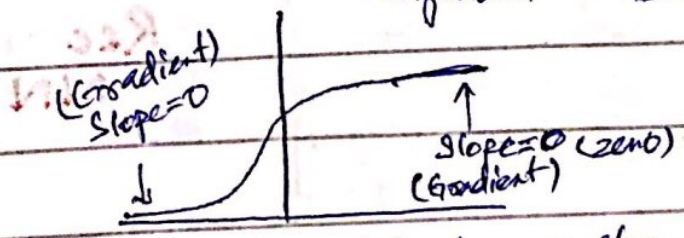
(ii) Computation (**Iterative**)

idea → Experiment → Code

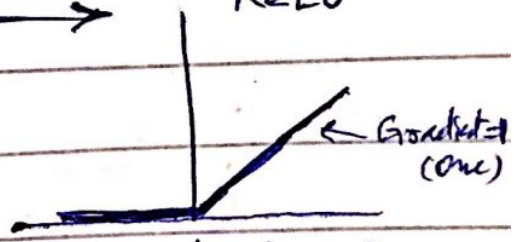
(iii) **Algorithms.**

"Gradient Descent" (work much faster)

Sigmoid → ReLU



As gradient = 0, learning becomes slow. parameters change very slow



Gradient = 1 for all positive values

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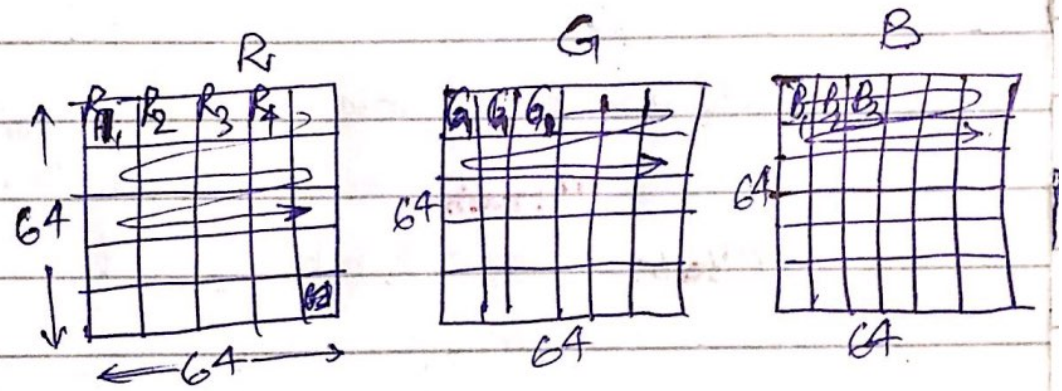
* Logistic Regression as a Neural Network

* Images are represented in R, G, B channel matrices in Computer.

Eg. If image is of 64×64 pixels.
Then it will be represented as.
3 (64×64) R, G, B matrices.



Input pixel values.

Change into by unrow.

Input feature Vector

$$X = \begin{bmatrix} R_1 \\ R_2 \\ \vdots \\ G_1 \\ G_2 \\ \vdots \\ B_1 \\ B_2 \\ \vdots \end{bmatrix}$$

$64 \times 64 \times 3 = 12288$

$n = n_x = 12288$

We have to predict $X \rightarrow Y$

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* Notation.

✓ (x, y) ; $x \in \mathbb{R}^{n_x}$ (n_x dimensional feature vector)

$$y \in \{0, 1\}$$

✓ m training example.

training set : $\{(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), \ldots (x^{(m)}, y^{(m)})\}$

✓ Sometimes we denote.

$m = m_{train}$ to denote number training example

$m_{test} \longrightarrow$ number of test examples

✓

$$X = \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \cdots & x^{(m)} \\ | & | & & | \end{bmatrix} \updownarrow n_x$$

$$\longleftarrow m \longrightarrow$$

$$X \in \mathbb{R}^{n_x \times m}$$

In python, $X.shape = (n_x, m)$

✓ $Y = [y^{(1)}, y^{(2)}, \ldots y^{(m)}]$

$Y \in \mathbb{R}^{1 \times m}$.

In python, $Y.shape = (1, m)$

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y will tell that image is a cat or not

If x is an Image

✓ Given X, want $\hat{y} = P(y=1|x)$

$(0 \le \hat{y} \le 1)$

$X \in \mathbb{R}^{n_x}$

Parameters $w \in \mathbb{R}^{n_x}$, $b \in \mathbb{R}$

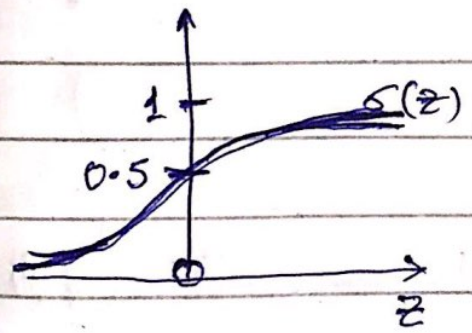
Real Number

✓ Output $\hat{y} = w^T x + b$ it is a linear fn.

here $\hat{y}$ can be much bigger than 1 or any value (-ve) not satisfying $(0 \le y \le 1)$

Therefore using sigmoid fn.

$$\text{Output } \hat{y} = \sigma(\underbrace{w^T x + b}_{z})$$



$$\sigma(z) = \frac{1}{1+e^{-z}}$$

If z is very large.

$\sigma(z) \approx 1$

If z is large -ve number

$$\sigma(z) = \frac{1}{1+\text{Big Number}}$$

$$\approx 0$$

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* Logistic Regression Cost Function

✓ To train the parameter w & b
we need to define a cost function.

We have

$$\hat{y} = \sigma(w^T x + b) \quad \text{where} \quad \sigma(z) = \frac{1}{1+e^{-z}}$$

given $\{(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), \ldots (x^{(m)}, y^{(m)})\}$ m training Ex.

we want $\hat{y}^{(i)} \approx y^{(i)}$

(predicted value closer to actual)

✓ We have x training example

$$\hat{y} = \sigma(\underbrace{w^T x + b}_{z}) \quad \text{where } \sigma(z) = \frac{1}{1+e^{-z}}$$

→ for i^{th} training example

$$\hat{y}^{(i)} = \sigma(w^T x^{(i)} + b) \quad \text{where } \sigma(z^{(i)}) = \frac{1}{1+e^{-z^{(i)}}}$$

$$z^{(i)} = w^T x^{(i)} + b$$

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computes the error for a single training example

Loss (error) function:

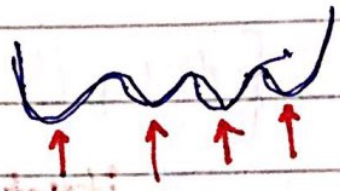
Square error

$$\mathcal{L}(\hat{y}, y) = \frac{1}{2}(\hat{y} - y)^2$$

loss fn.

to measure how good $\hat{y}$ as true label is y

By using this fn. optimization problem will become NON CONVEX



So gradient descent may not find GLOBAL OPTIMA

We use

$$\boxed{\mathcal{L}(\hat{y}, y)} = \boxed{-\left(y \log \hat{y} + (1-y)\log(1-\hat{y})\right)}$$

loss fn. applied to single training example

its informal justification. (We will look in details later)

If $y = 1$: $\mathcal{L}(\hat{y}, y) = -\log \hat{y} = \log\left(\frac{1}{\hat{y}}\right)$

To keep it maximum. $\hat{y}$ should be Large

so, $\log\left(\frac{1}{\text{Large}}\right) \approx \log(0) \approx 1$

If $y = 0$: $\mathcal{L}(\hat{y}, y) = -\log(1-\hat{y}) = \log\frac{1}{(1-\hat{y})}$

To keep it minimum. $\hat{y}$ should be small

so, $\log\frac{1}{(1-\text{small})} \approx \log(1) \approx 0$

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is the **average** of the loss functions of the entire training set.

→ **Cost function** → (cost of parameters)

↳ we need to **minimize** it

$$J(w,b) = \frac{1}{m}\sum_{i=1}^{m} \mathcal{L}(\hat{y}^{(i)}, y^{(i)})$$

$$= -\frac{1}{m}\left[y^{(i)} \log \hat{y}^{(i)} + (1-y^{(i)}) \log (1-\hat{y}^{(i)})\right]$$

Gradient Descent is an **iterative method** to minimize cost function by improving model parameters (weights & biases)

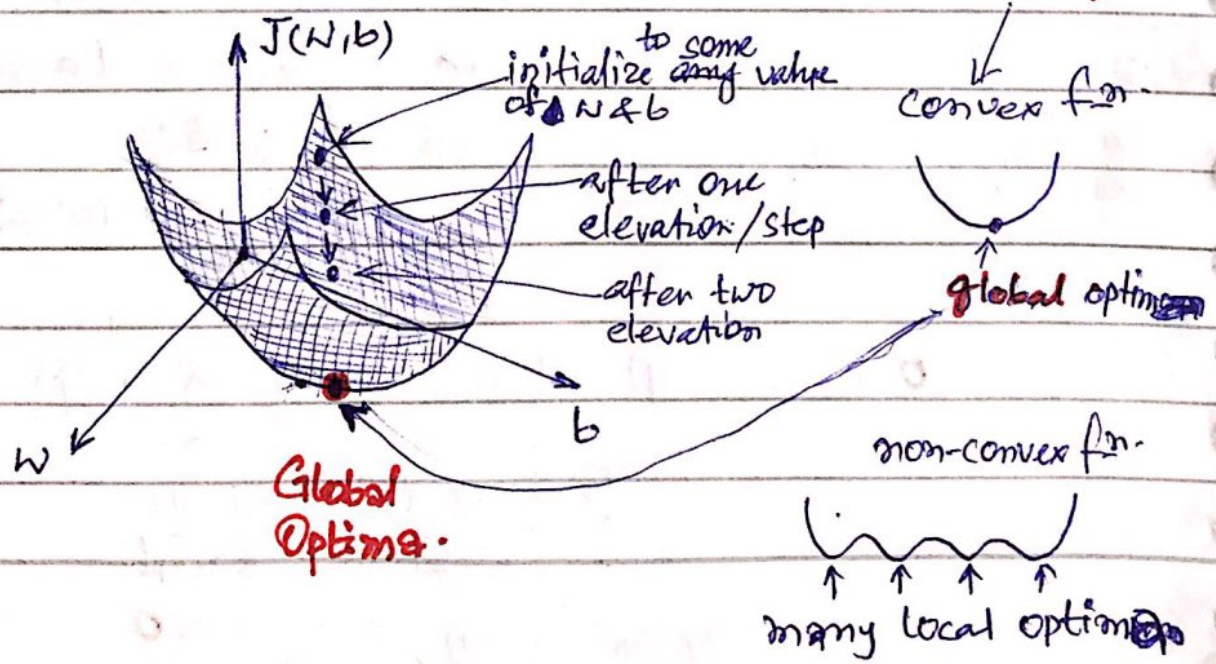
→ **Gradient Descent**

Recap:

$$\hat{y} = \sigma(w^T x + b), \quad \sigma(z) = \frac{1}{1+e^{-z}}$$

$$\boxed{\begin{aligned} J(w,b) &= \frac{1}{m}\sum_{i=1}^{m} \mathcal{L}(\hat{y}^{(i)}, y^{(i)}) \\ &= -\frac{1}{m}\sum_{i=1}^{m} y^{(i)} \log \hat{y}^{(i)} + (1-y^{(i)}) \log (1-\hat{y}^{(i)}) \end{aligned}}$$

want to **find w & b** to **minimize J(w,b)**



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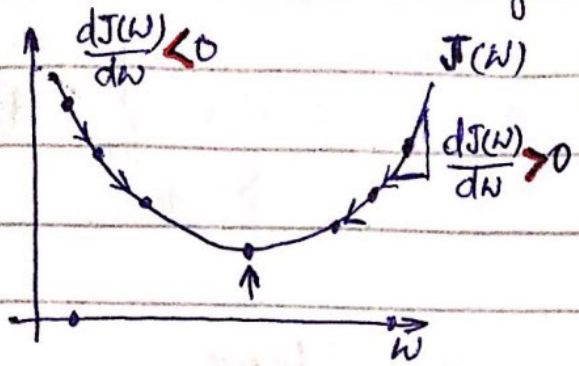
$w := \longrightarrow$ denotes updating w

for illustration we are ignoring b for now.



Repeat {

$$w := w - \alpha \frac{d(J(w))}{dw}$$

}

Learning Rate.

notation

dw

$$w := w - \alpha dw$$

for

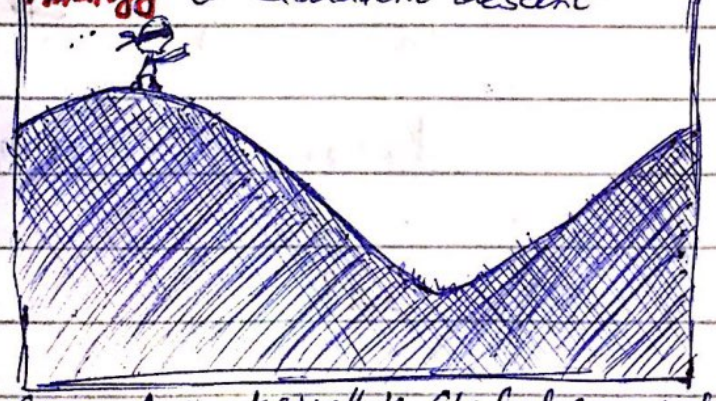
$$\boxed{\begin{aligned} J(w,b) = \quad & w := w - \alpha \frac{\partial J(w,b)}{\partial w} \\ & b := b - \alpha \frac{\partial J(w,b)}{\partial b} \end{aligned}}$$

partial derivative symbol.

because J is a fn. of two or more variable

Otherwise it will be denoted by "d" as derivative. (shown above) for One variable

Analogy to Gradient descent



Source: Andrej Karpathy's Stanford Cs231n Lecture 3

Rough terrain $\longrightarrow$ cost function

Minimizing Cost fn. $\longrightarrow$ trying to reach lower altitude

Feeling slope of terrain around you $\longrightarrow$ calculating Gradient

Taking a Step to the lowest altitude around you. $\longrightarrow$ One iteration of update to the model parameters (weights, biases)

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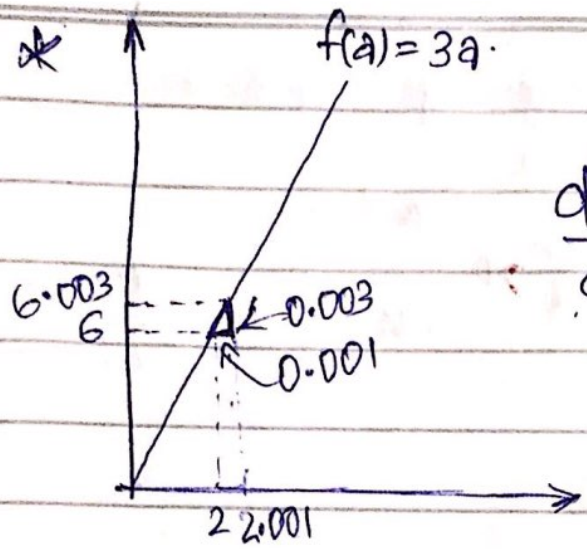


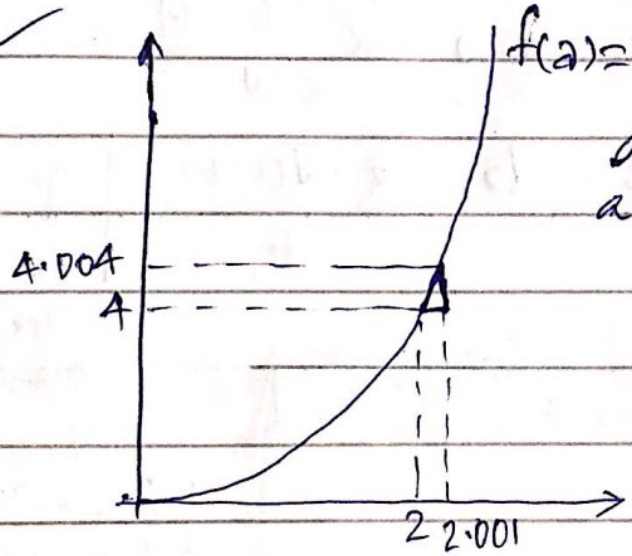
Derivative

$$\frac{df(a)}{da} = 3 = \frac{d}{da} f(a)$$

Slope

derivative of f(a) does not change irrespective of 'a' value.

Linear fn



$a = 2$, $f(a) = 4$

$a = 2.001$, $f(a) \approx 4.004$ (4.004001)

$$\frac{d}{da} f(a) = \frac{d}{da} a^2 = 2a$$

$\frac{d}{da} f(a)$ at $a = 5$ is 10

$f(a) = a^3$; $\frac{d}{da} f(a) = 3a^2$

if $a = 2.001$, $f(a) = 8.012$

$f(a) = \log_e(a)$; $\frac{d}{da} f(a) = \frac{1}{a}$.

if $a = 2$ $f(a) = 0.69315$

$a = 2.001$ $f(a) = 0.69365$

$\Delta a = 0.001$ $\Delta f(a) = 0.0005$

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How to compute derivatives of more complex functions.

Computation Graph.

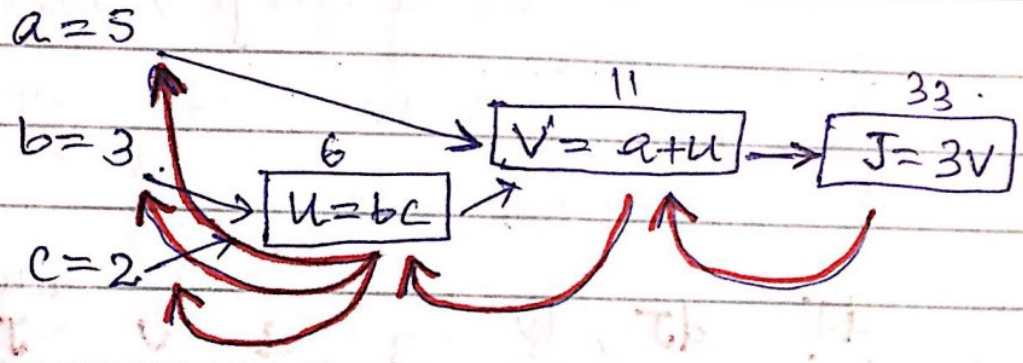
$$J(a,b,c) = \underbrace{3(\underbrace{a+\underbrace{bc}_{u}}_{v})}_{J}$$

computing this fn. includes 3 Steps:

$u = bc$

$v = a + u$

$J = 3v$



Computation graph **helps to optimize** some special output fn. $\boxed{J}$

In case of logistic Regn. it is $\boxed{\text{cost fn}}$

FORWARD

A **Left to Right** Pass.

we can compute the value of J **cost fn**

BACKWARD

In order to **compute the derivative**.

it will be.

Right to left Pass

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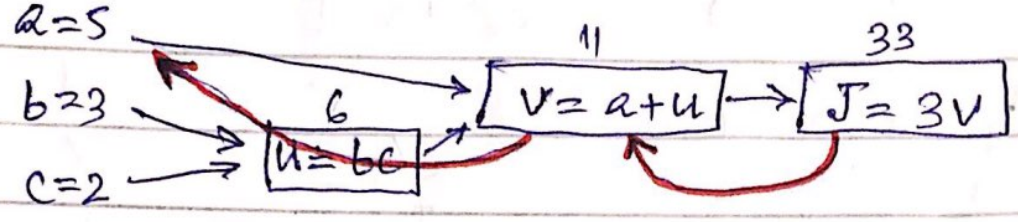
Derivatives with a Computation Graph.



✓ $\frac{dJ}{dv}$ ⟶ how the value of J change. when we change v value slightly

$\frac{dJ}{dv} = 3.$ $J = 3v$ $v = 11 \rightarrow 11.\underline{001}$
$J = 33 \rightarrow 33.\underline{003}$

✓ $\frac{dJ}{da} = \frac{dJ}{dv} \times \frac{dv}{da} = 3$ $\left\{ a \xrightarrow{\text{affects}} v \xrightarrow{\text{affects}} J \right.$ Chain Row.

$\frac{dJ}{dv} \rightarrow 3$, $\frac{dv}{da} \rightarrow 1$

$a = 5 \rightarrow 5.001$
$v = 11 \rightarrow 11.001$
$J = 33 \rightarrow 33.003$

✓ $\frac{d\ \text{finalOutputVar}}{d\ \text{var}}$ ⟶ "dvar"

Notation for coding.

$\frac{dJ}{db} \rightarrow db = 6$ $\frac{dJ}{dv} \rightarrow dv = 3$

$\frac{dJ}{dc} \rightarrow dc = 9$ $\frac{dJ}{da} \rightarrow da = 3.$

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Similarly,

$$\frac{dJ}{du} = 3 = \frac{dJ}{dv} \times \frac{dv}{du}$$

$u = 6 \rightarrow 6.001$
$v = 11 \rightarrow 11.001$
$J = 33 \rightarrow 33.003$

$$\frac{dJ}{db} = \underbrace{\frac{dJ}{du}}_{3} \times \underbrace{\frac{du}{db}}_{2} = 6$$

$$\frac{du}{db} = 2.$$

$b = 3 \rightarrow 3.001$
$u = bc = 6 \rightarrow 6.002$ as $c = 2$.

$J = 33 \rightarrow 33.006$

$$\frac{dJ}{dc} = \underbrace{\frac{dJ}{du}}_{3} \times \underbrace{\frac{du}{dc}}_{3.} = 9.$$

$$\frac{du}{dc} = 3$$

$c = 2 \rightarrow 2.001$
$u = bc = 6 \rightarrow 6.003$ as $b = 3$

$J = 33 \rightarrow 33.009$

finally see here

Derivative $\frac{dJ}{dv} = 3$; $\frac{dJ}{da} = 3$; $\frac{dJ}{db} = 6$; $\frac{dJ}{dc} = 9$

Notation $dv = 3$ $da = 3$ $db = 6$ $dc = 9$

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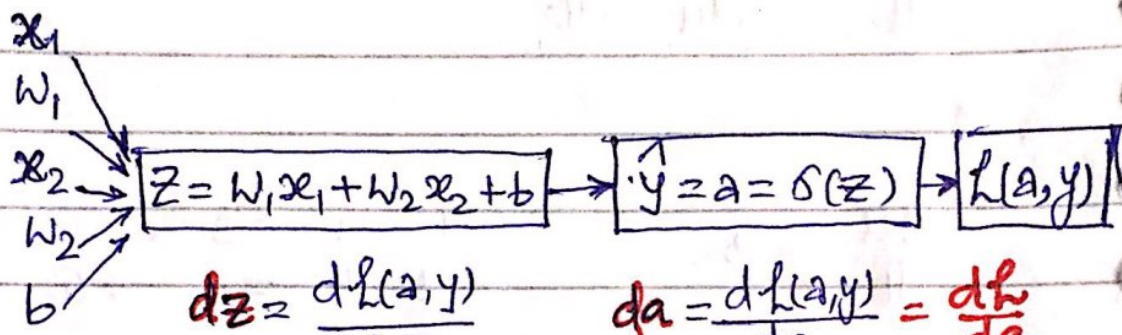
* Gradient Descent for Logistic Regression

Recap: $\hat{y} = a = \sigma(z)$

$z = w^T x + b$

$\mathcal{L}(a, y) = -(y \log(a) + (1-y)\log(1-a))$



$dz = \frac{d\mathcal{L}(a,y)}{dz}$

$da = \frac{d\mathcal{L}(a,y)}{da} = \frac{d\mathcal{L}}{da}$

$= -\frac{y}{a} + \frac{1-y}{1-a}$

Refer Pg 40

$= \frac{d\mathcal{L}}{da} \times \frac{da}{dz}$

$= \left[\frac{-y}{a} + \frac{1-y}{1-a}\right]\left[a(1-a)\right]$

$= a - y$

$\frac{\partial \mathcal{L}}{\partial w_1} = \text{"}dw_1\text{"} = x_1 dz$

$\text{"}dw_2\text{"} = x_2 dz$

$db = dz$

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Vectorization will help to omit explicit for loops in gradient descent. As for loop slows down execution in dealing with large datasets.

$$\boxed{\begin{aligned} w_1 &:= w_1 - \alpha dw_1 \\ w_2 &:= w_2 - \alpha dw_2 \\ b &:= b - \alpha db \end{aligned}}$$

* Gradient Descent on m examples

$$J(w,b) = \frac{1}{m}\sum_{i=1}^{m} \mathcal{L}(a^{(i)}, y^{(i)})$$

$$a^{(i)} = \hat{y}^{(i)} = \sigma(z^{(i)}) = \sigma(w^T x^{(i)} + b)$$

$$\frac{\partial}{\partial w_1} J(w,b) = \frac{1}{m}\sum_{i=1}^{m} \underbrace{\frac{\partial}{\partial w_1} \mathcal{L}(a^{(i)}, y^{(i)})}_{dw_1^{(i)}}$$

If

$J = 0;\ dw_1 = 0;\ dw_2 = 0;\ db = 0$

for $i = 1$ to m

$z^{(i)} = w^T x^{(i)} + b$

$a^{(i)} = \sigma(z^{(i)})$

$J += -\left[y^{(i)} \log a^{(i)} + (1 - y^{(i)}) \log(1 - a^{(i)})\right]$

$dz^{(i)} = a^{(i)} - y^{(i)}$

$dw_1 += x_1^{(i)} dz^{(i)}$

$dw_2 += x_2^{(i)} dz^{(i)}$

$db += dz^{(i)}$

Two for loops

for $i = 1$ to 2

for $n_x = n = 2$

$J /= m$

$dw_1 /= m$

$dw_2 /= m$

$db /= m$

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*. VECTORIZATION: a method to omit explicit for loops. (wherever possible)

✓ In logistic regression,

$z = w^T x + b$; $w = \begin{bmatrix} \vdots \end{bmatrix}$, $x = \begin{bmatrix} \vdots \end{bmatrix}$

$w \leftarrow \mathbb{R}^{n_x}$ $x \leftarrow \mathbb{R}^{n_x}$

Non-vectorized	Vectorized
$z = 0$ for i in range(n-x): $z \mathrel{+}= w[i] * x[i]$ $z \mathrel{+}= b$	$z = \underbrace{np.dot(w^T x)}_{w^T x} + b$

✓ GPU, CPU } SIMD (Single input multiple data.)

→ GPU & CPU use parallelized instructions

SIMD operation

→ GPU are much faster on SIMD calculations

another example ✓

$u = Av$ $u_i = \sum_j A_{ij} v_j$ $u = np.zeros((n,1))$ for i ---- for j ---- $u[i] \mathrel{+}= A[i][j] * v[j]$	$u = np.dot(A, v)$ eliminates two explicit for loops. (much faster)

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✓ another example : applying exponential operation on every element of a matrix or vector.

$$v = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix} \qquad u = \begin{bmatrix} e^{v_1} \\ e^{v_2} \\ \vdots \\ e^{v_n} \end{bmatrix}$$

```
u = np.exp(v)
```

much faster & eliminates one for loop with use of numpy built-in fn.

```
u = np.zeros((n,1))
for i in range(n):
    u[i] = math.exp(v[i])
```

✓ Using Vectorization on Logistic Regression Derivatives

$J=0,\ \boxed{\cancel{dw_1=0,\ dw_2=0}},\ db=0$

for $i = 1$ to m:

$z^{(i)} = w^T x^{(i)} + b$

$a^{(i)} = \sigma(z^{(i)})$

$J \mathrel{+}= -[y^{(i)} \log \hat{y}^{(i)} + (1-y^{(i)}) \log(1-\hat{y}^{(i)})]$

$dz^{(i)} = a^{(i)}(1-a^{(i)})$

for $j=1$ to n_x

$dw_j \mathrel{+}= x^{(i)} dz^{(i)}$

$n_x = 2$

$\boxed{\cancel{dw_1 \mathrel{+}= x_1^{(i)} dz^{(i)}} \quad \cancel{dw_2 \mathrel{+}= x_2^{(i)} dz^{(i)}}}$

$dw \mathrel{+}= x^{(i)} * dz^{(i)}$

$db \mathrel{+}= dz^{(i)}$

$J = J/m,\ \boxed{\cancel{dw_1 = dw_1/m,\ dw_2 = dw_2/m}},\ db = db/m$

$\Downarrow$

$dw = np.zeros((n\text{-}x, 1))$

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* VECTORIZING LOGISTIC REGRESSION

Let's examine forward propagation step if we have 'm' training examples.

for 1st training ex	for 2nd ---	for 3rd ---
$z^{(1)} = w^T x^{(1)} + b$	$z^{(2)} = w^T x^{(2)} + b$	$z^{(3)} = w^T x^{(3)} + b$
$a^{(1)} = \sigma(z^{(1)})$	$a^{(2)} = \sigma(z^{(2)})$	$a^{(3)} = \sigma(z^{(3)})$

$$X = \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \cdots & x^{(m)} \\ | & | & & | \end{bmatrix}$$

training input stack together

(n_x, m)

$$Z = [z^{(1)}\ z^{(2)} \cdots z^{(m)}] = w^T X + \underset{(1\times m)}{[b\ b \cdots b]}$$

$$= \underset{(1\times m)}{[w^T x^{(1)} + b\ \ w^T x^{(2)} + b\ \ \cdots\ w^T x^{(m)} + b]}$$

$$A = [a^{(1)}\ a^{(2)} \cdots a^{(m)}] = \sigma(Z)$$

$$Z = \underbrace{\text{np.dot}(w.T, X)} + b$$

(1,1)

When you add this vector to this row number 'b' python automatically expands it to (1, m) from (1,1)

This is known as "Broadcasting"

"Python Broadcasting"

(1, m)

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Also

$$w := w - \alpha dw$$
$$b := b - \alpha db$$

* VECTORIZING LOGISTIC REGRESSION'S GRADIENT OUTPUT (examine Backward propagation.)

$dz^{(1)} = a^{(1)} - y^{(1)}$ $\quad dz^{(2)} = a^{(2)} - y^{(2)}$ ------ for m training examples

$$\boxed{dz = [dz^{(1)}\, dz^{(2)} \cdots dz^{(m)}]_{(1\times m)}}$$

$$A = [a^{(1)} \cdots a^{(m)}]$$

$$Y = [y^{(1)} \cdots y^{(m)}]$$

$$dz = A - Y = [a^{(1)} - y^{(1)} \quad a^{(2)} - y^{(2)} \cdots a^{(m)} - y^{(m)}]$$

→ Previously we omit one for loop. for second for loop, we do the following:-

Initialize $dw = 0$

$$\begin{cases} dw \mathrel{+}= x^{(1)} dz^{(1)} \\ dw \mathrel{+}= x^{(2)} dz^{(2)} \\ \vdots \\ dw \mathrel{+}= x^{(m)} dz^{(m)} \\ dw \mathrel{/}= m \end{cases}$$

for loop involved here

$db = 0$

$$\begin{cases} db \mathrel{+}= dz^{(1)} \\ db \mathrel{+}= dz^{(2)} \\ \vdots \\ db \mathrel{+}= dz^{(m)} \\ db \mathrel{/}= m \end{cases}$$

Let's vectorize it

$$dw = \frac{1}{m} X dz^T$$

$$= \frac{1}{m} \begin{bmatrix} | & & | \\ x^{(1)} & \cdots & x^{(m)} \\ | & & | \end{bmatrix} \begin{bmatrix} dz^{(1)} \\ \vdots \\ dz^{(m)} \end{bmatrix}$$

$$= \frac{1}{m} [x^{(1)} dz^{(1)} + \cdots + x^{(m)} dz^{(m)}]_{(1\times n)}$$

Let's vectorize it

$$db = \frac{1}{m} \sum_{i=1}^{m} dz^{(i)}$$

$$= \frac{1}{m} np.sum(dz)$$

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* BROADCASTING IN PYTHON

✓ (m,n) matrix $\pm$ $(1,n) \rightsquigarrow (m,n)$

$*$ / $(m,1) \rightsquigarrow (m,n)$

✓ $(m,1) + \mathbb{R}$

$$\begin{bmatrix}1\\2\\3\end{bmatrix} + 100 = \begin{bmatrix}101\\102\\103\end{bmatrix}$$

$$[1,2,3] + 100 = [101, 102, 103]$$

Eg

$$A = \begin{array}{c} \\ \text{Carb} \\ \text{Protein} \\ \text{Fat} \end{array} \begin{array}{c} \begin{array}{cccc} \text{Apples} & \text{Beef} & \text{Eggs} & \text{Potatoes} \end{array} \\ \begin{bmatrix} 56.0 & 0.0 & 4.4 & 68.0 \\ 1.2 & 104.0 & 52.0 & 8.0 \\ 1.8 & 135.0 & 99.0 & 0.9 \end{bmatrix} \end{array}$$

$(3,4)$

cal = A.sum(axis=0)

axis = 0 ↓, 1 →

Percentage = 100* A/(cal.reshap(1,4))

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* AVOID "RANK-1" ARRAY

a = np.random.randn(5)

print(a.shape) = (5,)

"Rank-1 Array"

It is neither a row vector nor a column vector.

A **funny data structure** with non-intutive effects

use reshape to convert Rank-1 array into some vector

a = a.reshape((5,1))

OR, you can use "**assert**" to check on its shape

code: assert(a.shape == (5,1)), "error msg"

Otherwise **use**,

a = np.random.randn(5,1) → a column vector — print(a.shape) = (5,1)

or, a = np.random.randn(1,5) → a row vector — print(a.shape) = (1,5)

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* EXPLANATION OF LOGISTIC REGRESSION COST FUNCTION.

$\hat{y} = \sigma(w^T x + b)$ where $\sigma(z) = \frac{1}{1+e^{-z}}$

Interpret $\hat{y} = P(y=1 \mid x)$

$$\begin{cases} \text{If } y=1 : & P(y|x) = \hat{y} \\ \text{If } y=0 : & P(y|x) = 1-\hat{y} \end{cases}$$

Summarize it in one eqn.

$$\boxed{P(y|x) = \hat{y}^{y} (1-\hat{y})^{(1-y)}}$$

Taking log on both the sides,

maximize $\log P(y|x) = \log \hat{y}^{y} (1-\hat{y})^{(1-y)}$

$$= y \log \hat{y} + (1-y) \log (1-\hat{y})$$

$$= -\mathcal{L}(\hat{y}, y)$$

minimize = -ve of LOSS FUNCTION

Now for cost function,

cost on m examples

$$P(\text{labels in training set}) = \prod_{i=1}^{m} P(y^{(i)} | x^{(i)})$$

Taking log on both the sides,

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$$\log p(\text{----}) = \sum_{i=1}^{m} \underbrace{\log p(y^{(i)} | x^{(i)})}_{-\mathcal{L}(\hat{y}^{(i)}, y^{(i)})}$$

$$= -\sum_{i=1}^{m} \mathcal{L}(\hat{y}^{(i)}, y^{(i)})$$

The above justifies the cost
We have,

$$\text{Cost: } J(w, b) = \frac{1}{m} \sum_{i=1}^{m} \mathcal{L}(\hat{y}^{(i)}, y^{(i)})$$

(minimize)

For convenience that our quantities are better scaled we just add "$\frac{1}{m}$" extra scaling vector

Because we need to minimize the cost and due to maximize likelyhood, we get to rid off minus sign.

Assumption: Our m training examples are IID (Identically, Independently distributed)

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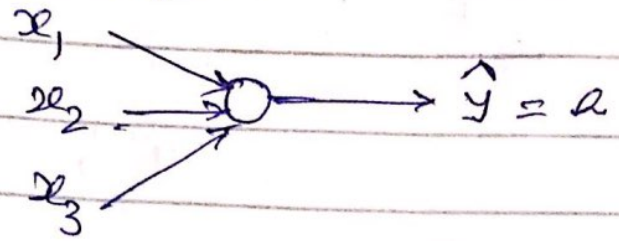
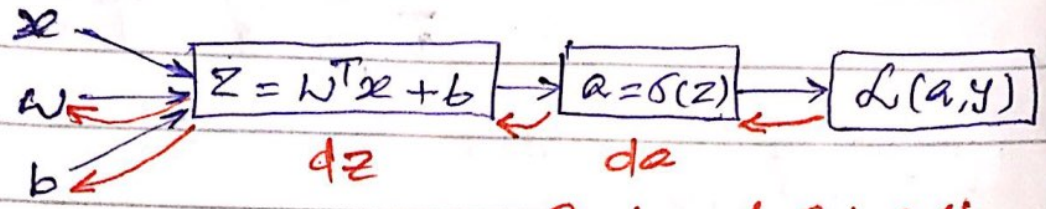
* NN Overview (One hidden Layer NN).

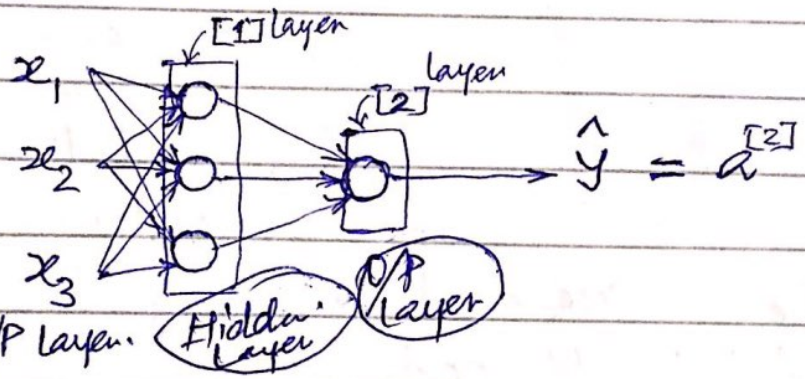
One layer.



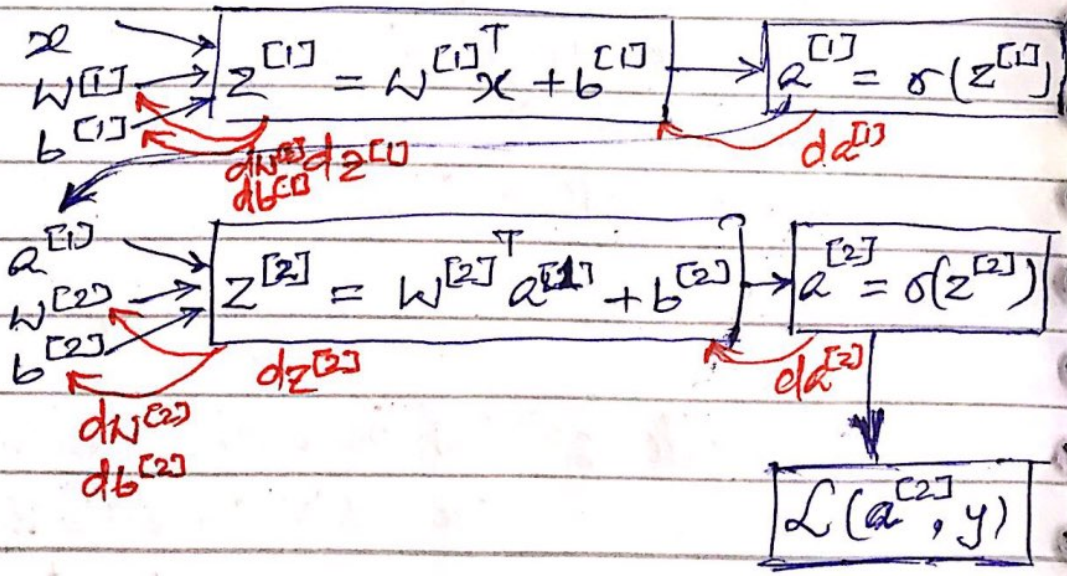


Backward Calculation for computing Derivatives

Two Layer.



do not count I/P layer. (layer 0)



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* NN Representation

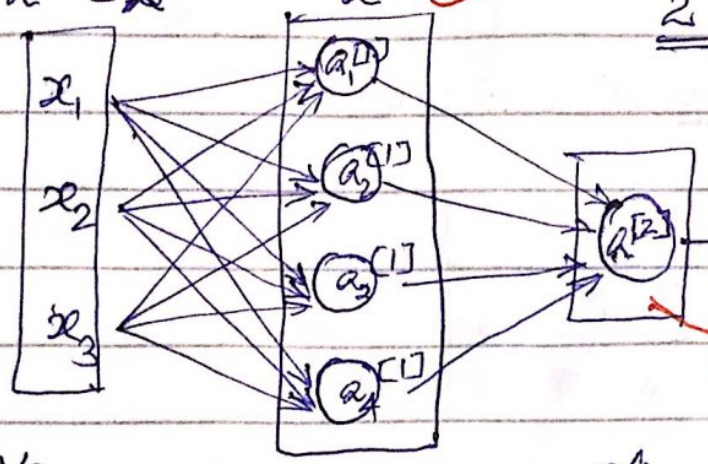
Parameters associated

$W^{[1]}$ (4,3), $b^{[1]}$ (4,1)

4 nodes of hidden layers

3 i/p features

2 Layer NN



$\hat{y} = a^{[2]}$

$W^{[2]}$ (1,4), $b^{[2]}$ (1,1)

1 node of layer

4 i/p features

i/p Layer $a^{[0]}$

Hidden Layer $a^{[1]}$

o/p Layer $a^{[2]}$

✓ → Notation "a" denotes "activations"
Here i/p layer passes the value X to the hidden layer (Subsequent layer)
Therefore $a^{[0]}$ = activation of i/p layer.

layer zero

✓ $a^{[1]} = \begin{bmatrix} a_1^{[1]} \\ a_2^{[1]} \\ \vdots \\ a_4^{[1]} \end{bmatrix}$ → 4 dimensional matrix
4 nodes

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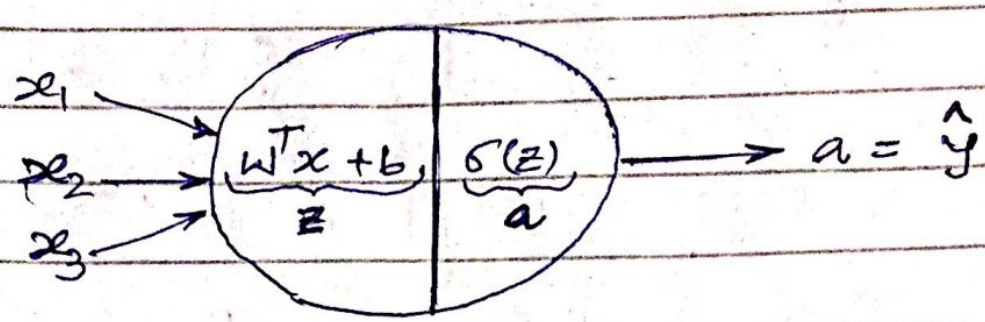
* Computing a NN's Output.

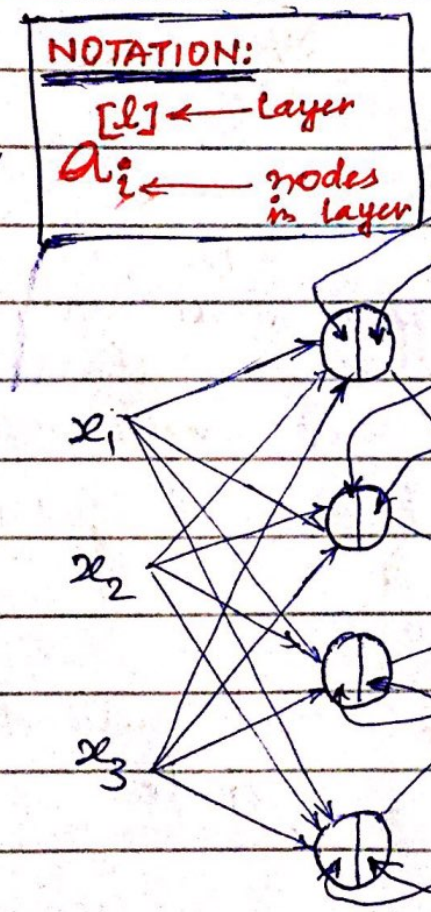
The circle in logistic Regression NN involves <u>two step</u> of calculation



NOTATION:

$[l] \leftarrow$ layer

$a_i \leftarrow$ nodes in layer



$z_1^{[1]} = W_1^{[1]T} x + b_1^{[1]}$

$a_1^{[1]} = \sigma(z_1^{[1]})$

$z_2^{[1]} = W_2^{[1]T} x + b_2^{[1]}$

$a_2^{[1]} = \sigma(z_2^{[1]})$

$z_1^{[2]} = W_1^{[2]T} x + b_1^{[2]}$

$a_1^{[2]} = \sigma(z_1^{[2]})$

$z_3^{[1]} = W_3^{[1]T} x + b_3^{[1]}$

$a_3^{[1]} = \sigma(z_3^{[1]})$

$z_4^{[1]} = W_4^{[1]T} x + b_4^{[1]}$

$a_4^{[1]} = \sigma(z_4^{[1]})$

i/p Layer	Hidden Layer	O/p Layer.
Layer-0	Layer-1	Layer-2
$a^{[0]}$	$a^{[1]}$	$a^{[2]}$

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$$z_1^{[1]} = w_1^{[1]T} x + b_1^{[1]}, \quad a_1^{[1]} = \sigma(z_1^{[1]})$$
$$z_2^{[1]} = w_2^{[1]T} x + b_2^{[1]}, \quad a_2^{[1]} = \sigma(z_2^{[1]})$$
$$z_3^{[1]} = w_3^{[1]T} x + b_3^{[1]}, \quad a_3^{[1]} = \sigma(z_3^{[1]})$$
$$z_4^{[1]} = w_4^{[1]T} x + b_4^{[1]}, \quad a_4^{[1]} = \sigma(z_4^{[1]})$$

$$z^{[1]} = \underbrace{\begin{bmatrix} - w_1^{[1]T} - \\ - w_2^{[1]T} - \\ \vdots \\ - w_4^{[1]T} - \end{bmatrix}}_{W^{[1]}}_{(4,3)} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}_{(3,1)} + \begin{bmatrix} b_1^{[1]} \\ b_2^{[1]} \\ \vdots \\ b_4^{[1]} \end{bmatrix}_{(4,1)}$$

$$= \begin{bmatrix} w_1^{[1]T} x + b_1^{[1]} \\ w_2^{[1]T} x + b_2^{[1]} \\ w_3^{[1]T} x + b_3^{[1]} \\ w_4^{[1]T} x + b_4^{[1]} \end{bmatrix}_{(4,1)} = \begin{bmatrix} z_1^{[1]} \\ z_2^{[1]} \\ z_3^{[1]} \\ z_4^{[1]} \end{bmatrix}_{(4,1)}$$

As input $x(x_1, x_2, x_3) = a^{[0]}$

= activation @ Layer 0

$$z^{[1]}_{(4,1)} = W^{[1]}_{(4,3)} a^{[0]}_{(3,1)} + b^{[1]}_{(4,1)}$$
$$a^{[1]}_{(4,1)} = \sigma(z^{[1]})_{(4,1)}$$

$$z^{[2]}_{(1,1)} = W^{[2]}_{(1,4)} a^{[1]}_{(4,1)} + b^{[2]}_{(1,1)}$$
$$a^{[2]}_{(1,1)} = \sigma(z^{[2]})_{(1,1)}$$

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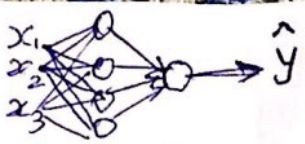

* Vectorizing across multiple ex$^{\underline{s}}$

→ for single training example.

for a given input feature vector x, we can generate $a^{[2]} = \hat{y}$

$$x \longrightarrow \hat{y} = a^{[2]}$$

→ for m training examples.

$$x^{(1)} \longrightarrow a^{[2](1)} = \hat{y}^{(1)}$$
$$x^{(2)} \longrightarrow a^{[2](2)} = \hat{y}^{(2)}$$
$$\vdots$$
$$x^{(m)} \longrightarrow a^{[2](m)} = \hat{y}^{(m)}$$

NOTATION:

Choose Carefully

$$\boxed{a^{[l](i)}}$$

l → Layer

i → training example i

→ As previously discussed, we have:

$$z^{[1]} = W^{[1]} \overset{a^{[0]}}{x} + b^{[1]}$$
$$a^{[1]} = \sigma(z^{[1]})$$
$$z^{[2]} = W^{[2]} a^{[1]} + b^{[2]}$$
$$a^{[2]} = \sigma(z^{[2]})$$

→ If we have un-vectorized implementation for m training example;

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Lower Case Vector axis $\xrightarrow{\text{Vectorize.}}$ Capital Case Matrix by stacking lower case vectors in columns.

lower Case

$a^{[0](i)}$

for $i=1$ to m

$$z^{[1](i)} = W^{[1]} x^{(i)} + b^{[1]}$$

$$a^{[1](i)} = \sigma(z^{[1](i)})$$

$a^{[1](i)}$

$$z^{[2](i)} = W^{[2]} a^{[1](i)} + b^{[2]}$$

$$a^{[2](i)} = \sigma(z^{[2](i)})$$

→ Vectorize the implementation of above for loop:

capital

$$Z^{[1]} = W^{[1]} X + b^{[1]}$$

$$A^{[1]} = \sigma(Z^{[1]})$$

$$Z^{[2]} = W^{[2]} A^{[1]} + b^{[2]}$$

$$A^{[2]} = \sigma(Z^{[2]})$$

Corresponds to different training examples

Horizontal Index

Where,

$$Z^{[1]} = \begin{bmatrix} | & | & & | \\ z^{[1](1)} & z^{[1](2)} & \cdots & z^{[1](m)} \\ | & | & & | \end{bmatrix}$$

Vertical Index Corresponds to hidden units

$$A^{[1]} = \begin{bmatrix} | & | & & | \\ a^{[1](1)} & a^{[1](2)} & \cdots & a^{[1](m)} \\ | & | & & | \end{bmatrix}$$

$$X = \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \cdots & x^{(m)} \\ | & | & & | \end{bmatrix}$$

different input features

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* Explanation for Vectorized Implementation:

We have

$$z^{[1](1)} = W^{[1]}x^{(1)} + \cancel{b^{[1]}} \to 0$$
$$z^{[1](2)} = W^{[1]}x^{(2)} + \cancel{b^{[1]}} \to 0$$
$$z^{[1](3)} = W^{[1]}x^{(3)} + \cancel{b^{[1]}} \to 0$$

Let's keep $b=0$ for convenience of calculation.

If we take different training examples and stack them in different columns.

$$W^{[1]} = \begin{bmatrix} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{bmatrix}, \quad W^{[1]}x^{(1)} = \begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix}$$

$$W^{[1]}x^{(2)} = \begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix}$$

$$W^{[1]}x^{(3)} = \begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix}$$

Then the corresponding results, also stacked up in columns. (as Z)

$$W^{[1]} \underbrace{\begin{bmatrix} | & | & | \\ x^{(1)} & x^{(2)} & x^{(3)} \\ | & | & | \end{bmatrix}}_{X} = \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix} = \begin{bmatrix} | & | & | \\ z^{[1](1)} & z^{[1](2)} & z^{[1](3)} \\ | & | & | \end{bmatrix}$$

$$= Z^{[1]}$$

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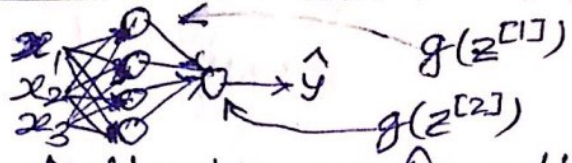
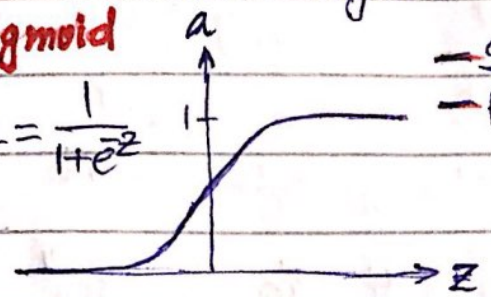


* Activation functions

What Activation f^{n} you will use in hidden layer(s) and o/p layer.

Sigmoid

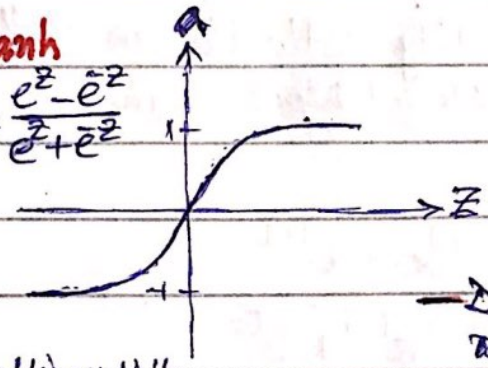
$a = \frac{1}{1+e^{-z}}$



- Suit when o/p is $\{0,1\}$
- For binary classification

tanh

$a = \frac{e^{z}-e^{-z}}{e^{z}+e^{-z}}$



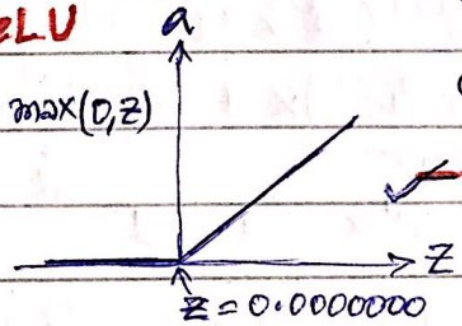
- tanh is almost superior than sigmoid except $\hat{y} \in \{0,1\}$

- Downsize of Sigmoid & tanh is when z is very large or small then the gradient (derivative or slope of this f^{n}) becomes very small (close to zero) → slow down Gradient Descent

Rectified Linear Unit

ReLU

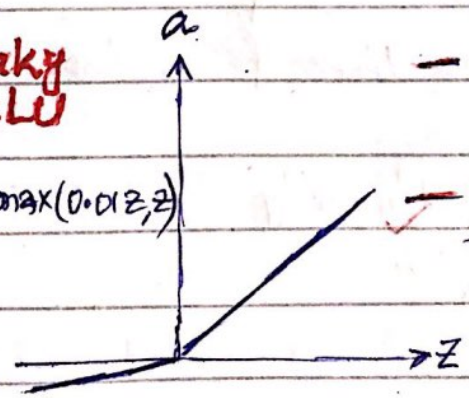
$a = \max(0, z)$



Technically, Derivative is not well defined when z is exactly zero. In practice you can pretend derivative as 1 or 0

Leaky ReLU

$a = \max(0.01z, z)$



- ~~As z is~~ for -ve z, derivative is zero for ReLU. Therefore you can use Leaky ReLU.

- In practice, maximum no. of training examples has $z > 0$ therefore ReLU works best.

For given x: (In a forward Propagation)

$z^{[1]} = W^{[1]}x + b^{[1]}$ $\qquad$ $z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$

$a^{[1]} = \cancel{\sigma(z^{[1]})}\ g(z^{[1]})$ $\qquad$ $a^{[2]} = \cancel{\sigma(z^{[2]})}\ g(z^{[2]})$

a non-linear function

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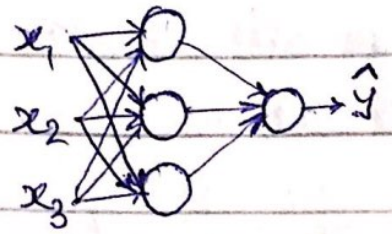
Is a critical part of NN's

* Why do we need **non-linear** activation functions



Forward Propg^n. Equation

for given x:

$$z^{[1]} = W^{[1]}x + b^{[1]}$$

$$a^{[1]} = \cancel{g^{[1]}(z^{[1]})}\ z^{[1]}$$

$$z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$$

$$a^{[2]} = \cancel{g^{[2]}(z^{[2]})}\ z^{[2]}$$

→ If $g(z) = z$ → "Linear activation function" or, "Identity activation function" (as we get Output (z) as given Input (z))

Then,

$$a^{[1]} = z^{[1]} = W^{[1]}x + b^{[1]}$$

$$a^{[2]} = z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$$

$$= W^{[2]}(W^{[1]}x + b^{[1]}) + b^{[2]}$$

$$= \underbrace{(W^{[2]}W^{[1]})}_{W'}x + \underbrace{W^{[2]}b^{[1]} + b^{[2]}}_{b'}$$

$$= W'x + b'$$

So, if we take linear activation f^n. then it turns out that output $\hat{y}$ is a linear f^n of input features

If we do not add non-linearity then we are not computing interesting f^{n}s.

Exception: You might use linear activation f^n. when you use m/c learning on regression problems.

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* Derivatives of Activation fns.

To implement Backward Propagation in a N.N, we need to compute derivative of Activation fns.

→ **Sigmoid**

$$g(z) = \frac{1}{1+e^{-z}} = a. \quad \left\{ \begin{array}{l} \text{In NN,} \\ \text{activation fn.} \\ a = g(z) \end{array} \right.$$

$$\therefore g'(z) = \frac{d}{dz} g(z) = \frac{d}{dz}\left(\frac{1}{1+e^{-z}}\right)$$

$$= \frac{d}{dz}\left((1+e^{-z})^{-1}\right)$$

$$= (1+e^{-z})^{-2} \cdot (-1) \cdot \frac{d}{dz}(1+e^{-z})$$

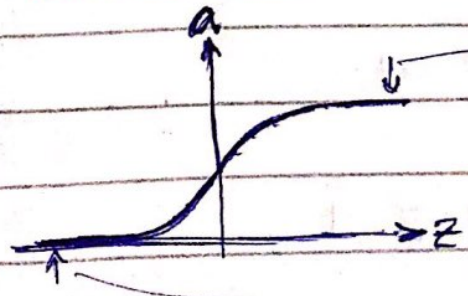
$$= (1+e^{-z})^{-2} \cdot (-1) \cdot \left[0 + (-1)(e^{-z})\right]$$

$$= (1+e^{-z})^{-2} \cdot e^{-z}$$

$$= \frac{e^{-z}}{(1+e^{-z})^2} = \frac{1}{(1+e^{-z})} \cdot \frac{e^{-z}}{(1+e^{-z})}$$

$$= \frac{1}{(1+e^{-z})} \cdot \left[1 - \frac{1}{(1+e^{-z})}\right]$$

$$\frac{d}{dz} g(z) = \boxed{g'(z) = g(z)\,(1 - g(z))} \boxed{= a(1-a)}$$



$z = 10 \quad g(z) \approx 1$

$\frac{d}{dz} g(z) \approx (1(1-1)) \approx 0$

$z = -10 \quad g(z) \approx 0$

$\frac{d}{dz} g(z) \approx 0(1-0) \approx 0$

$z = 0 \quad g(z) = \frac{1}{2}$

$\frac{d}{dz} g(z) = 1/2(1-1/2) = 1/4$

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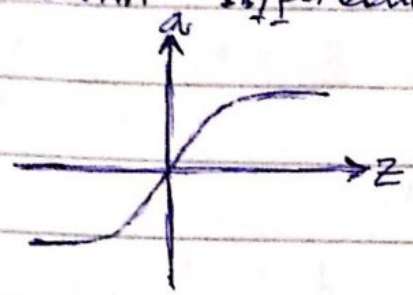
→ Tanh Hyperbolic Tangent Activation Fn.



$$g(z) = \tanh(z) = \frac{e^{z} - e^{-z}}{e^{z} + e^{-z}}$$

As per Quotient Rule, $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x) \cdot g(x) - g'(x) \cdot f(x)}{[g(x)]^2}$

$$\therefore g'(z) = \frac{d}{dz}(\tanh(z)) = \frac{d}{dz}\left(\frac{e^{z} - e^{-z}}{e^{z} + e^{-z}}\right)$$

$$= \frac{\frac{d}{dz}(e^{z} - e^{-z}) \cdot (e^{z} + e^{-z}) - (e^{z} - e^{-z}) \cdot \frac{d}{dz}(e^{z} + e^{-z})}{(e^{z} + e^{-z})^2}$$

$$= \frac{[e^{z} - (-1)e^{-z}](e^{z} + e^{-z}) - (e^{z} - e^{-z})[e^{z} + (-1)e^{-z}]}{(e^{z} + e^{-z})^2}$$

$$= \frac{(e^{z} + e^{-z})^2 - (e^{z} - e^{-z})^2}{(e^{z} + e^{-z})^2}$$

$$= 1 - \frac{(e^{z} - e^{-z})^2}{(e^{z} + e^{-z})^2}$$

$$\boxed{g'(z) = 1 - [\tanh(z)]^2 = 1 - a^2}$$

If $z = 10$, $\tanh(z) \approx 1$

$g'(z) \approx 0$

$z = -10$, $\tanh(z) \approx -1$

$g'(z) \approx 0$

$z = 0$, $\tanh(z) = 0$

$g'(z) = 1$

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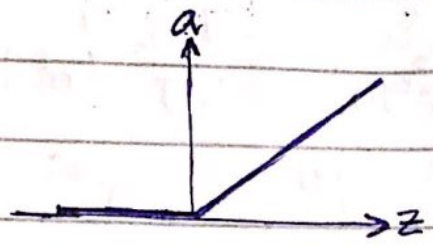
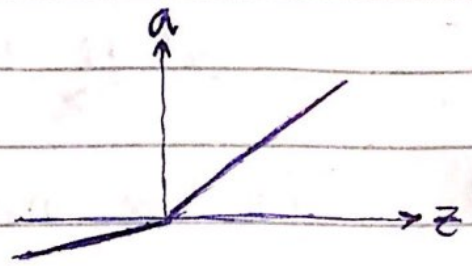
→ ReLU and Leaky ReLU





$g(z) = \max(0, z)$

$$g'(z) = \begin{cases} 0 & \text{if } z<0 \\ 1 & z>0 \\ \text{Undefined} & z=0 \end{cases}$$

$g(z) = \max(0.01z, z)$

$$g'(z) = \begin{cases} 0.01 & \text{if } z<0 \\ 1. & z>0 \\ \text{Undefined} & z=0 \end{cases}$$

Technically derivative of $g(z)$ is Undefined when z is exactly zero $(z=0)$

But,

Practically you can take

$$z = 0.0000000$$

Hence $\boxed{g'(z) = 1.}$

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* Gradient descent for NN's

For a single hidden layer NN:

→ Parameters: $W^{[1]}$, $b^{[1]}$, $W^{[2]}$, $b^{[2]}$

$(n^{[1]}, n^{[0]})$ $(n^{[1]}, 1)$ $(n^{[2]}, n^{[1]})$ $(n^{[2]}, 1)$

If we have, $n_x = n^{[0]}$, $n^{[1]}$, $n^{[2]} = 1$ {for this example}

Input Features, Hidden Units, Output Units

Then

→ Cost function:

$$J(W^{[1]}, b^{[1]}, W^{[2]}, b^{[2]}) = \frac{1}{m} \sum_{i=1}^{m} \mathcal{L}(\hat{y}, y)$$

$a^{[2]}$

for Binary Classifn.

→ Gradient Descent

Repeat {

Compute Predictions ($\hat{y}^{(i)}$, for $i = 1 \ldots m$)

$$dW^{[1]} = \frac{\partial J}{\partial W^{[1]}}, \quad db^{[1]} = \frac{\partial J}{\partial b^{[1]}}, \ldots$$

$$W^{[1]} = W^{[1]} - \alpha dW^{[1]}$$

$$b^{[1]} = b^{[1]} - \alpha db^{[1]}, \ldots$$

Similarly for $W^{[2]}$ & $b^{[2]}$

→ Summarizing Eqns.

Forward Propagation

$$Z^{[1]} = W^{[1]} X + b^{[1]}$$

$$A^{[1]} = g^{[1]}(Z^{[1]})$$

$$Z^{[2]} = W^{[2]} A^{[1]} + b^{[2]}$$

$$A^{[2]} = g^{[2]}(Z^{[2]}) = \sigma(Z^{[2]})$$

for Binary Classification Activation fn. is Sigmoid

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Backward Propagation

$$dZ^{[2]} = A^{[2]} - Y \qquad Y = [y^{(1)}, y^{(2)}, \ldots y^{(m)}]$$

$$dW^{[2]} = \frac{1}{m} dZ^{[2]} A^{[1]}$$

$$db^{[2]} = \frac{1}{m} np.sum(dZ^{[2]}, axis = 1, keepdims = True)$$

Summing horizontally

It prevents python for outputting funny Rank-1 arrays.

$dim = (n^{[2]}, 1)$

In this case, $= (1, 1)$

$$dZ^{[1]} = \underbrace{W^{[2]T} dZ^{[2]}}_{(n^{[1]}, m)} * \underbrace{g^{[1]\prime}(Z^{[1]})}_{(n^{[1]}, m)}$$

Refer. Pg. 40

element-wise product

$$dW^{[1]} = \frac{1}{m} dZ^{[1]} X^T$$

$$\underset{(n^{[1]},1)}{db^{[1]}} = \underbrace{\frac{1}{m} np.sum(dZ^{[1]}, axis = 1, keepdims = True)}_{\text{reshape it into dimension of } db^{[1]} \text{ i.e } (n^{[1]}, 1)}$$

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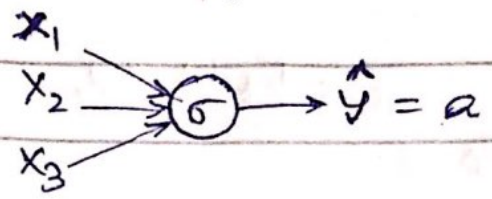
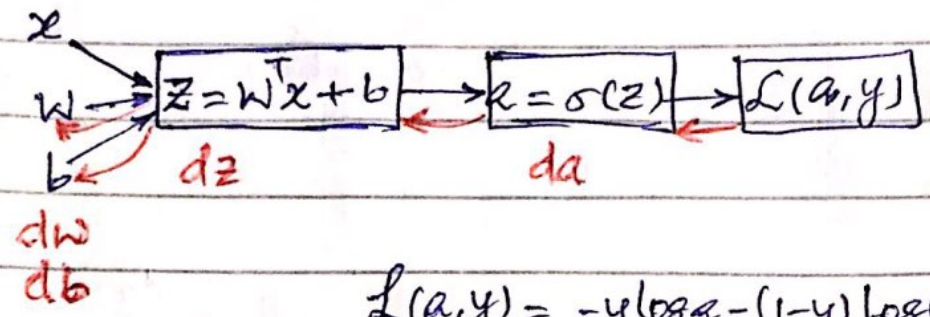
* Backpropagation intuition

→ Logistic Regression:



{ One Layer NN

Computation Graph:



$$\mathcal{L}(a,y) = -y\log a - (1-y)\log(1-a)$$

$$\boxed{da = \frac{d}{da}\mathcal{L}(a,y) = \frac{-y}{a} + \frac{(1-y)}{(1-a)}}$$

$$\boxed{dz = a - y = \frac{d}{dz}\mathcal{L}(a,y)}$$

Refer Pg.16

$$\boxed{dz = \frac{d\mathcal{L}}{da} \cdot \frac{da}{dz}}$$

$$dz = da \cdot dz$$

$$\boxed{dz = da \cdot g'(z)}$$

$a = \sigma(z)$

$a = g(z)$

$\therefore \frac{da}{dz} = \frac{d(g(z))}{dz}$

$= g'(z)$

$$dw = \frac{d\mathcal{L}(a,y)}{dw}$$

$$= \frac{d\mathcal{L}}{dz} \cdot \frac{dz}{dw}$$

$$\boxed{dw = dz \cdot x}$$

$$db = d\mathcal{L}(a,y)$$

$$= \frac{d\mathcal{L}}{dz} \cdot \frac{dz}{db}$$

$$= dz \cdot 1$$

$$\boxed{db = dz}$$

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$$x \xrightarrow{W^{[1]},\ b^{[1]}} \boxed{z^{[1]} = W^{[1]}x + b^{[1]}} \rightarrow \boxed{a^{[1]} = \sigma(z^{[1]})} \xrightarrow{W^{[2]},\ b^{[2]}} \boxed{z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}} \rightarrow \boxed{a^{[2]} = \sigma(z^{[2]})} \rightarrow \boxed{\mathcal{L}(a^{[2]}, y)}$$

$dW^{[1]}$, $db^{[1]}$, $dz^{[1]}$, $da^{[1]}$, $dW^{[2]}$, $db^{[2]}$, $dz^{[2]}$, $da^{[2]} = \frac{-y}{a^{[2]}} + \frac{(1-y)}{(1-a^{[2]})}$

Dimension is same for a vector and its derivative.

$$dz^{[1]} = W^{[2]T} dz^{[2]} * g^{[1]\prime}(z^{[1]})$$

$$dz^{[2]} = a^{[2]} - y$$

$$dW^{[2]} = dz^{[2]} \cdot a^{[1]T}$$

$$db^{[2]} = dz^{[2]}$$

similar as

$$dW = dz \cdot x$$

$a^{[0]}$

$$W^{[2]} \xrightarrow{\text{dimension}} (n^{[2]}, n^{[1]})$$

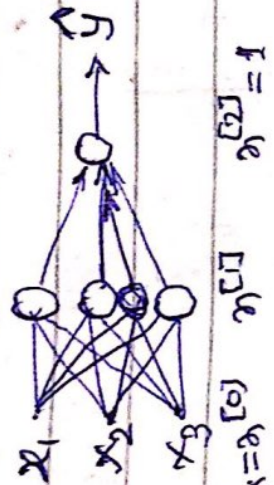
$$z^{[2]}, dz^{[2]} \rightarrow (n^{[2]}, 1) \rightarrow (1, 1)$$

$$z^{[1]}, dz^{[1]} \rightarrow (n^{[1]}, 1)$$

$$dW^{[1]} = dz^{[1]} \cdot x^T$$

$$db^{[1]} = dz^{[1]}$$

Two Layer NN



In case of Logistic Reg^n W is a column vector. $W^{[2]}$ is a row vector. Therefore Transpose is used.

$W^{[2]}$ [———]

→ Summary of Gradient Descent { Two layer NN

Vectorize

$$dz^{[2]} = a^{[2]} - y \qquad dZ^{[2]} = A^{[2]} - Y$$

$$dW^{[2]} = dz^{[2]} a^{[1]T} \qquad dW^{[2]} = \frac{1}{m} dZ^{[2]} A^{[1]T}$$

$$db^{[2]} = dz^{[2]} \qquad db^{[2]} = \frac{1}{m} np.sum(dZ^{[2]}, axis=1, keepdims = True)$$

$$dz^{[1]} = W^{[2]T} dz^{[2]} * g^{[1]\prime}(z^{[1]}) \qquad dZ^{[1]} = W^{[2]T} dZ^{[2]} * g^{[1]\prime}(Z^{[1]})$$

$$dW^{[1]} = dz^{[1]} x^T \qquad dW^{[1]} = \frac{1}{m} dZ^{[1]} X^T$$

$$db^{[1]} = dz^{[1]} \qquad db^{[1]} = \frac{1}{m} np.sum(dZ^{[1]}, axis=1, keepdims = True)$$

Gradient Derivatives

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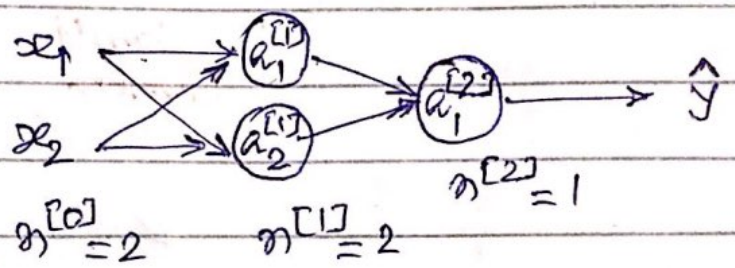
* Random Initialization

✓ for training NN $\xrightarrow[\text{Imp}]{}$ Initialize weights Randomly

✓ for Logistic Regression,

It was okay to initialize weights to zero

→ What happens if we initialize weights to zero



If we initialize, $W^{[1]} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $b^{[1]} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Therefore, $a_1^{[1]} = a_2^{[1]}$ $\qquad dz_1^{[1]} = dz_2^{[1]}$

Both of the hidden units are computing **same function**

So, $a_1^{[1]}$ & $a_2^{[1]}$ are identical or **symmetric**

$dW = \begin{bmatrix} u & v \\ u & v \end{bmatrix}$ $\qquad \boxed{W^{[1]} = W^{[1]} - \alpha dW}$ → after one elevation

Every row takes on the same value

$\therefore W^{[1]} = \begin{bmatrix} - & - & - \\ - & - & - \end{bmatrix}$

We want different hidden units to compute different functions **The solⁿ to this is to** Initialize your parameters **RANDOMLY**

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$W^{[1]} = np.random.randn((2,1)) * 0.01$

$b^{[1]} = np.zeros((2,1))$

$W^{[2]} = np.random.randn((1,2)) * 0.01$

$b^{[2]} = 0$

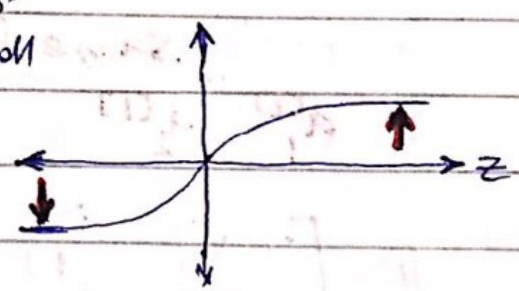
Small Number

If we take large weights value
then when we compute activation values

$$\uparrow z^{[1]} = \uparrow W^{[1]} x + b^{[1]}$$

$$a^{[1]} \uparrow = g^{[1]}(z^{[1]} \uparrow)$$

If $W^{[1]}$ is big then
$\therefore z^{[1]}$ is very big/small
$\therefore a^{[1]}$ is very big



In case we are doing Binary classification then activation fn. (Sigmoid or tanh) will be saturated (gradient/slope = 0)

So learning will be slow

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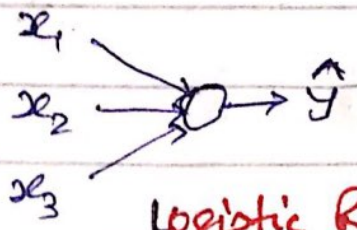
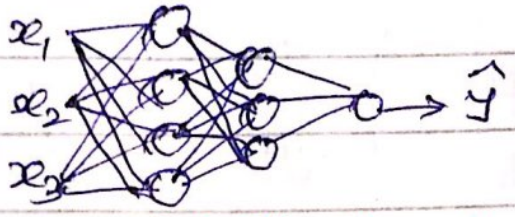
* Deep L-Layer NN

"Shallow"

1 layer NN

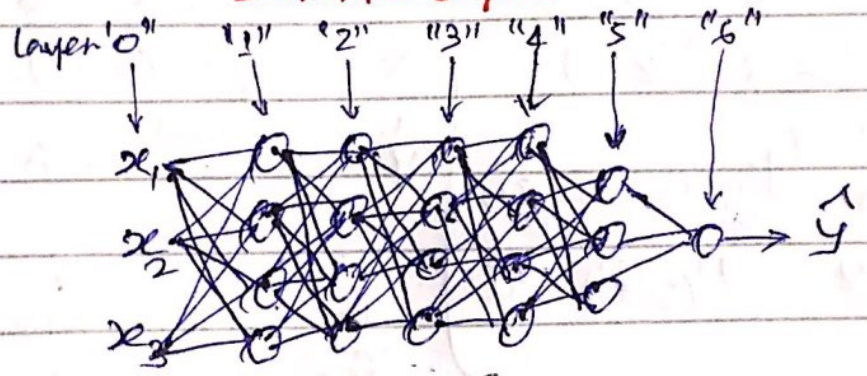


Logistic Regⁿ.



3 layer NN

2 hidden layers



5 hidden layers.

"deep"

6 layer NN

NOTATION:

$L = 6$ (# layers)

$n^{[l]}$ = # of units in layer 'l'

$n^{[1]} = n^{[2]} = n^{[3]} = n^{[4]} = 4$

$n^{[0]} = n_x = 3$

$n^{[5]} = 3$

$n^{[6]} = n^{[L]} = 1$

$a^{[0]} = x$, $a^{[L]} = \hat{y}$

$a^{[l]}$ = activation in layer 'l'

$a^{[l]} = g^{[l]}(z^{[l]})$ Similarly $w^{[l]}$ & $b^{[l]}$ — weight, bias

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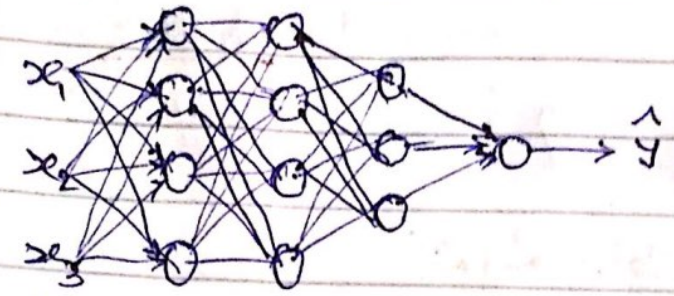
* Forward Propagation in a Deep Network.



$x = a^{[0]}$

$$Z^{[1]} = W^{[1]} \overset{a^{[0]}}{x} + b^{[1]}$$
$$a^{[1]} = g^{[1]}(Z^{[1]})$$
$$Z^{[2]} = W^{[2]} a^{[1]} + b^{[2]}$$
$$a^{[2]} = g^{[2]}(Z^{[2]})$$

Similarly,

$$Z^{[4]} = W^{[4]} a^{[3]} + b^{[4]}$$
$$a^{[4]} = g^{[4]}(Z^{[4]}) = \hat{y}$$

Vectorized Version:

$$Z^{[1]} = W^{[1]} A^{[0]} + b^{[1]} \quad l=1$$
$$A^{[1]} = g^{[1]}(Z^{[1]})$$
$$Z^{[2]} = W^{[2]} A^{[1]} + b^{[2]} \quad l=2$$
$$A^{[2]} = g^{[2]}(Z^{[2]})$$
$$\hat{y} = g^{[4]}(Z^{[4]}) = A^{[4]} \quad l=4$$

Seems that there is an explicit 'for' loop for forward propagation for layer. $l = 1$ to layer 'L'

This is one place where we can not get rid off **'for' loop**. It is OKAY & Acceptable

$$\boxed{\begin{aligned} Z^{[l]} &= W^{[l]} A^{[l-1]} + b^{[l]} \\ A^{[l]} &= g^{[l]}(Z^{[l]}) \end{aligned}}$$

Generalized Eq^n. (Vectorized)

$$Z^{[1]} = \begin{bmatrix} | & | & | & | \\ z^{[1](1)} & z^{[1](2)} & z^{[1](3)} & z^{[1](m)} \\ | & | & | & | \end{bmatrix}$$

Vectorized version comes by stacking vertically all z for 'm' training examples.

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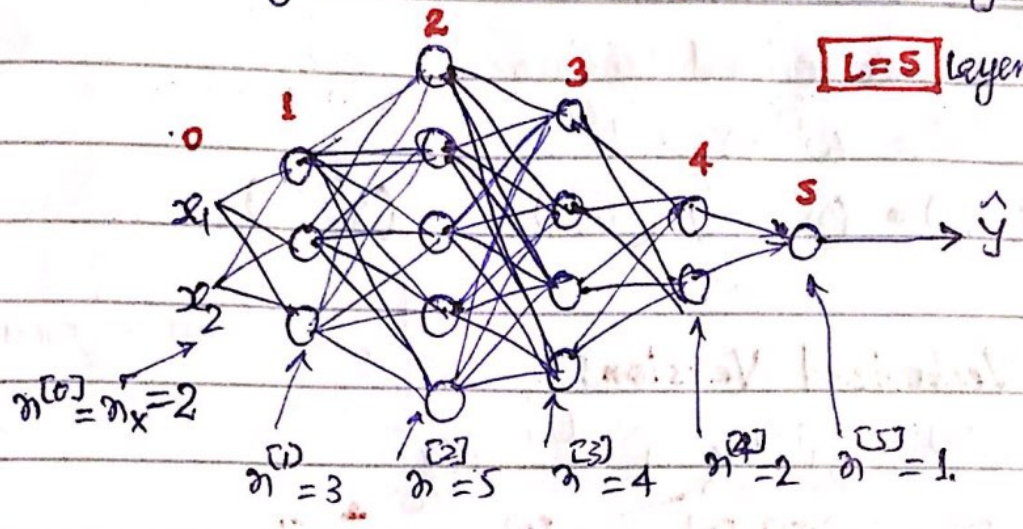
One of Debugging Tool

* Getting Your Matrix Dimensions Right

$\boxed{L=5}$ layer 5 NN.



As Z is vector of activations, a column vector.

$\therefore$ $Z^{[1]}$. $(3,1)$ → 3 hidden units in layer '1'

$$Z^{[1]} = W^{[1]} X + b^{[1]}$$

$(3,1) \leftarrow (3,2) \cdot (2,1) \quad (3,1) \rightarrow$ This should be

$(n^{[1]},1) \quad (n^{[1]},n^{[0]}) \quad (n^{[0]},1) \quad (n^{[1]},1)$

Therefore

$Z^{[l]}$	:	$(n^{[l]},1)$	: $dZ^{[l]}$
$W^{[l]}$	:	$(n^{[l]},n^{[l-1]})$	: $dW^{[l]}$
$a^{[l]}$	:	$(n^{[l]},1)$	: $da^{[l]}$
$b^{[l]}$	:	$(n^{[l]},1)$	: $db^{[l]}$

for Backward Propogation

Since $a^{[l]} = g^{[l]}(Z^{[l]})$

Both will have same dimension

See what happens when we Vectorized the given implementation.

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Non-Vectorized Version.

$$Z^{[1]} = W^{[1]} \cdot X + b^{[1]}$$

$$(n^{[1]}, 1) = (n^{[1]}, n^{[0]})\ (n^{[0]}, 1) \qquad (n^{[1]}, 1).$$

Vectorized Version:

$$\begin{bmatrix} \vdots & \vdots & & \vdots \\ Z^{[1](1)} & Z^{[1](2)} & \cdots & Z^{[1](m)} \\ \vdots & \vdots & & \vdots \end{bmatrix}$$

$$Z^{[1]} = W^{[1]} X + b^{[1]}$$

still same

$$(n^{[1]}, m) \quad (n^{[1]}, n^{[0]}) \quad (n^{[0]}, m) \quad (n^{[1]}, 1)$$

By Python Broadcasting It will get duplicated to

(Refer Pg. 22)

$$(n^{[1]}, m)$$

for vectorized version:

$Z^{[l]}$	: $(n^{[l]}, m)$	: $dZ^{[l]}$
$W^{[l]}$	: $(n^{[l]}, n^{[l-1]})$	: $dW^{[l]}$
$A^{[l]}$	: $(n^{[l]}, m)$	: $dA^{[l]}$
$b^{[l]}$	: $(n^{[l]}, 1)$	: $db^{[l]}$

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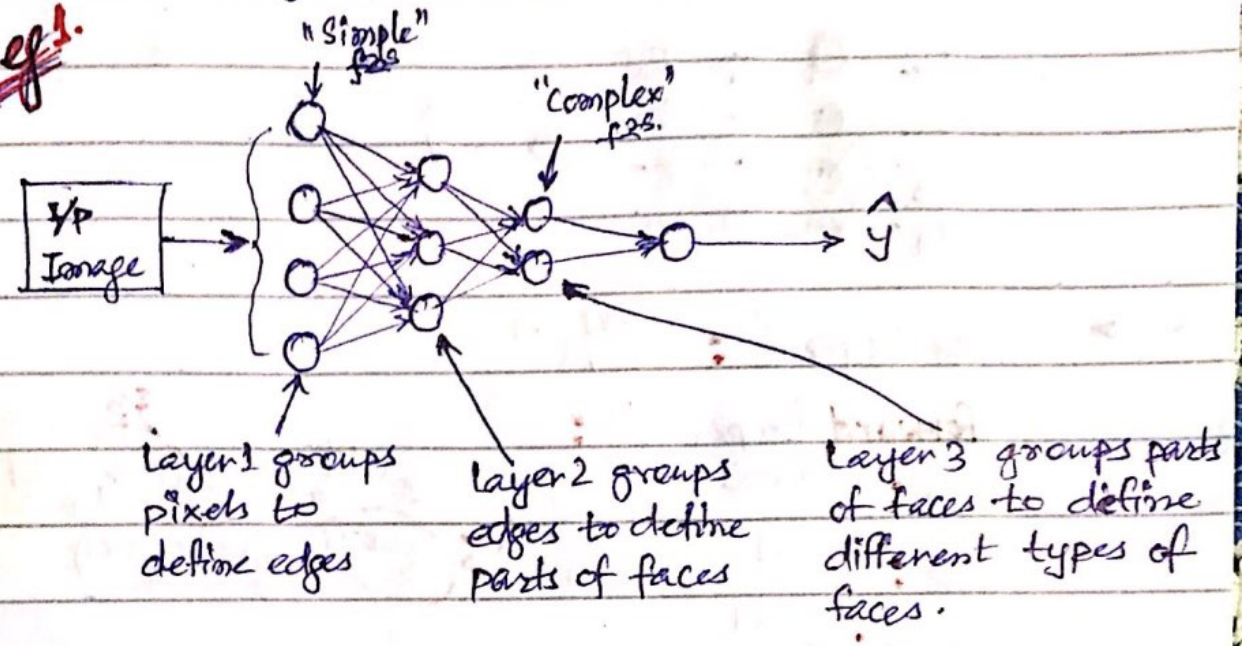
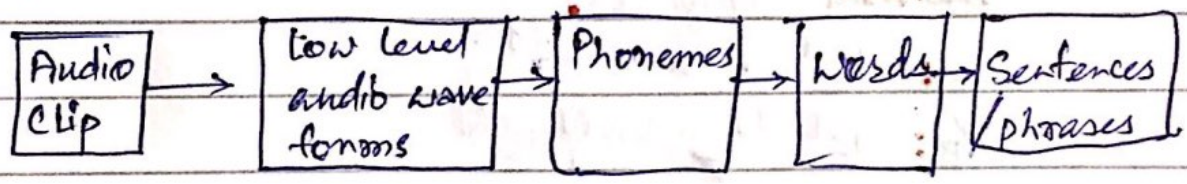
* Why Deep Representations

eg 1.



eg 2. Similarly for Speech Recognition:



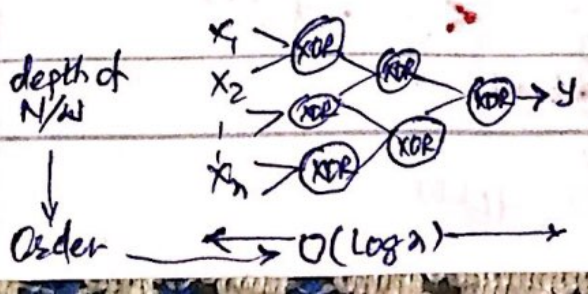
→ Circuit theory & Deep learning

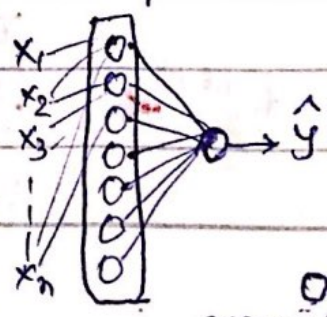
There are fns you can compute with a "Small" L-Layer deep neural network.

that

"Shallower" networks require exponentially more hidden units to compute.

$y = x_1 \text{ XOR } x_2 \text{ — XOR } x_n$





$O(2^n) \approx 2^{n-1}$ exponentially large

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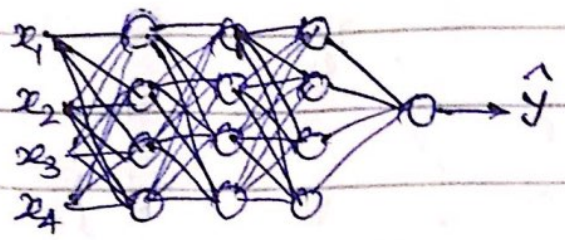
* Building Blocks of Deep NNs



→ for layer 'l' : $W^{[l]}, b^{[l]}$

Forward Propagation:

I/p : $a^{[l-1]}$

O/p : $a^{[l]}$

Cache : $z^{[l]}$

$$z^{[l]} = W^{[l]} \overset{I/p}{a^{[l-1]}} + b^{[l]}$$

$$\overset{O/p}{a^{[l]}} = g^{[l]}(z^{[l]})$$

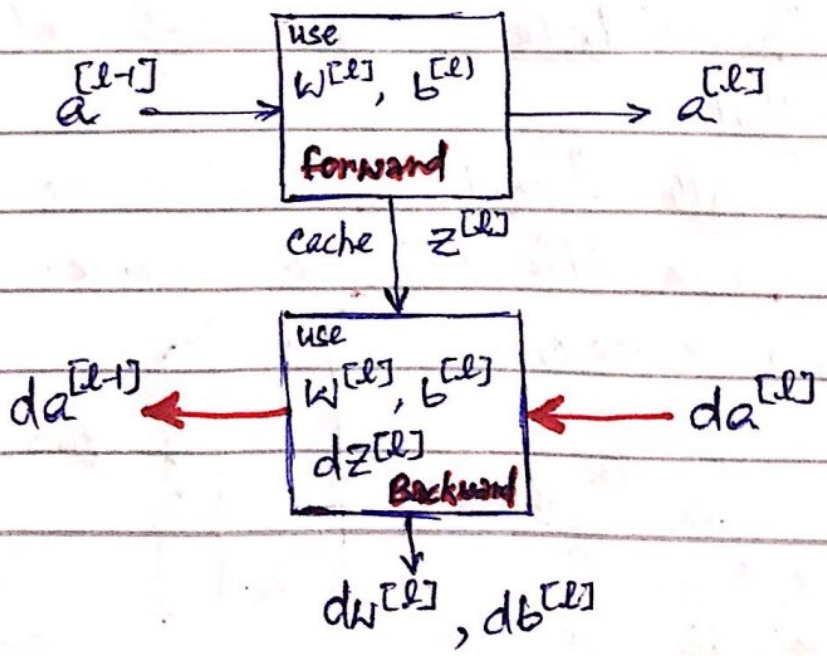
Backward Propagation:

I/p : $da^{[l]}$, cache $z^{[l]}$

O/p : $da^{[l-1]}$, $dW^{[l]}, db^{[l]}$

cache :

→ for layer 'l'



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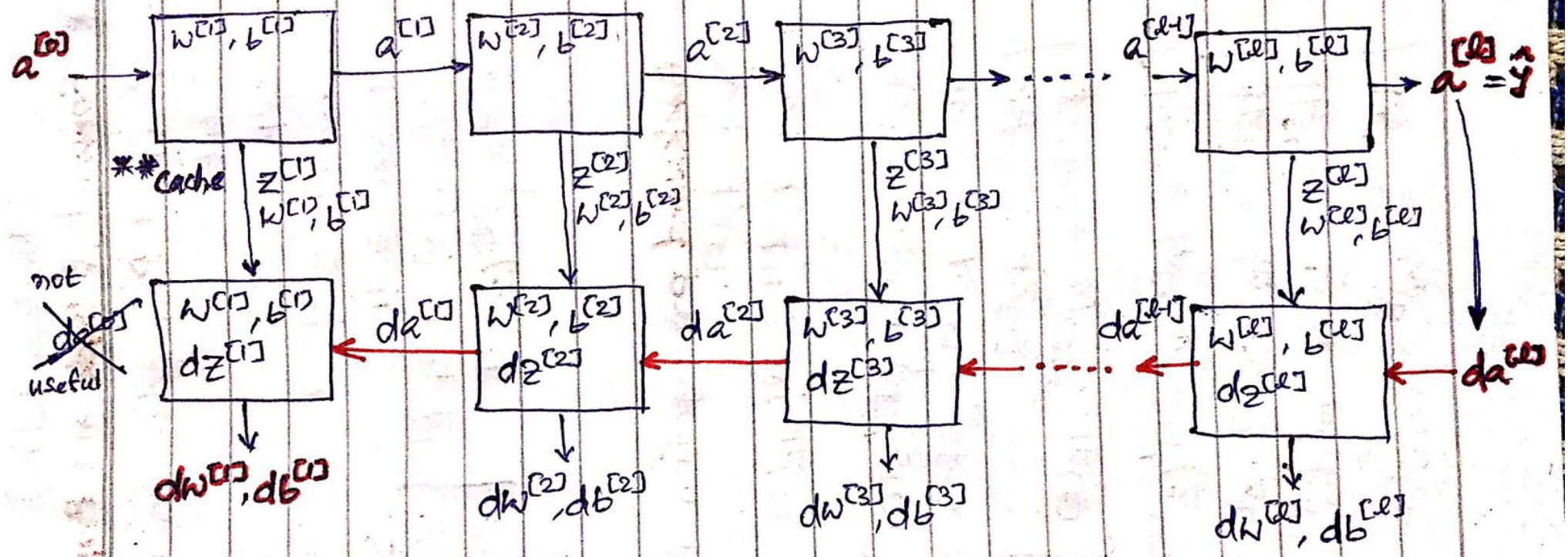
→ for an 'l' layer Deep Neural Network.



$$\boxed{\begin{aligned} W^{[l]} &= W^{[l]} - \alpha dW^{[l]} \\ b^{[l]} &= b^{[l]} - \alpha db^{[l]} \end{aligned}}$$

** for an implementation (programming) point of view cache $W^{[l]}, b^{[l]}$ too along with $Z^{[l]}$ so that it can be used in backward propagation step(s).

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(Implementation)

* Forward & Backward Propagation.

→ **Forward Propagation for Layer l**

Input: $a^{[l-1]}$

O/P : $a^{[l]}$

cache: $z^{[l]}, w^{[l]}, b^{[l]}$ — For implementation point of view cache it too

Non Vectorized:

$$z^{[l]} = w^{[l]} \cdot a^{[l-1]} + b^{[l]}$$
$$a^{[l]} = g^{[l]}(z^{[l]})$$

Vectorized:

$$Z^{[l]} = W^{[l]} \cdot A^{[l-1]} + b^{[l]}$$
$$A^{[l]} = g^{[l]}(Z^{[l]})$$

→ **Backward propagation for Layer l**:

I/P : $da^{[l]}$

O/P : $da^{[l-1]}, dw^{[l]}, db^{[l]}$

$$\left.\begin{cases} dz^{[l]} = da^{[l]} * g^{[l]\prime}(z^{[l]}) \\ dw^{[l]} = dz^{[l]} \cdot a^{[l-1]} \\ db^{[l]} = dz^{[l]} \\ da^{[l-1]} = w^{[l]T} \cdot dz^{[l]} \end{cases}\right\} \text{Non-Vectorized}$$

$$\text{Vectorized} \begin{cases} dZ^{[l]} = dA^{[l]} * g^{[l]\prime}(Z^{[l]}) \\ dW^{[l]} = \frac{1}{m} dZ^{[l]} \cdot A^{[l-1]T} \\ db^{[l]} = \frac{1}{m} np.sum(dZ^{[l]}, axis=1, keepdims=True) \\ dA^{[l-1]} = W^{[l]T} \cdot dZ^{[l]} \end{cases}$$

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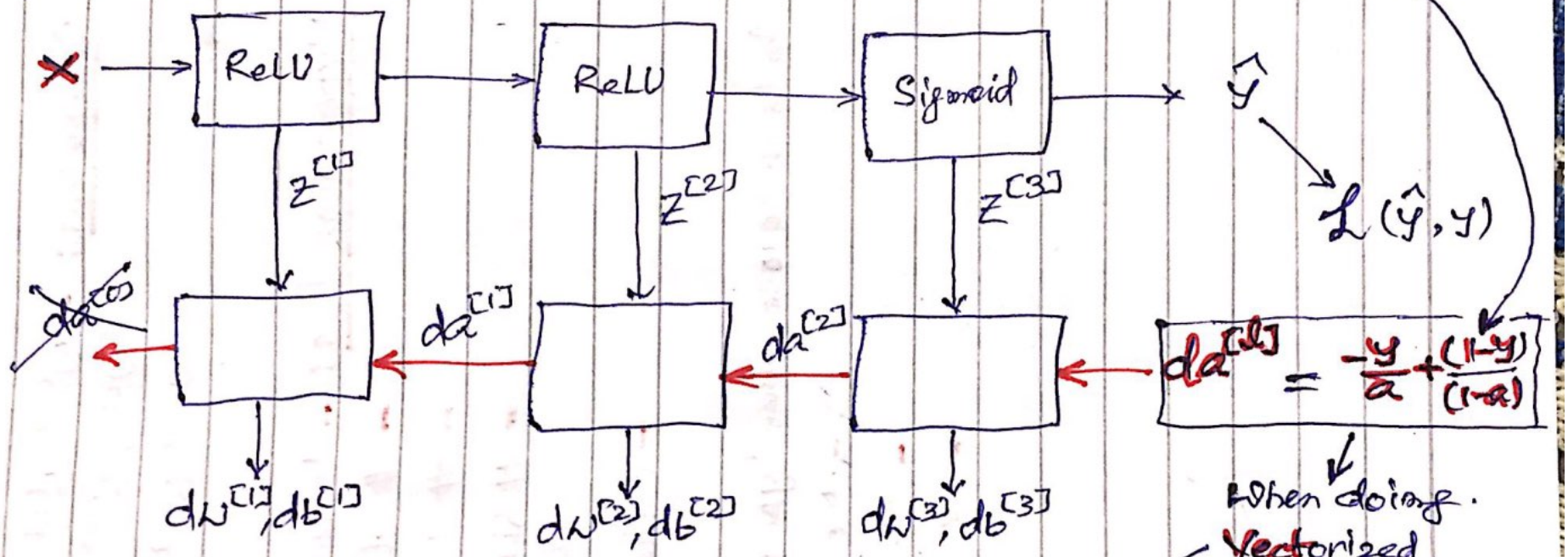
Summary:

When doing Logistic Reg.
(for Binary Classification)



When doing Vectorized Implementation

$$dA^{[L]} = -\frac{y^{(1)}}{a^{(1)}} + \frac{(1-y^{(1)})}{(1-a^{(1)})} \cdots\cdots - \frac{y^{(m)}}{a^{(m)}} + \frac{(1-y^{(m)})}{(1-a^{(m)})}$$

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* Parameters Vs. Hyperparameters.

Parameters: - $W^{[1]}, b^{[1]}$
$W^{[2]}, b^{[2]}$
$W^{[3]}, b^{[3]}$ .....

Controls.

Hyperparameters:
- Learning Rate α
- # iterations
- # hidden Layers L
- # hidden Units
  $n^{[1]}, n^{[2]}, ----$
- Choice of Activation fn

In later course, other hyperparameters:
- Momentum
- Minibatch Size
- Regularizations

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