Translational Mechanical Systems Transfer Functions - Q.26

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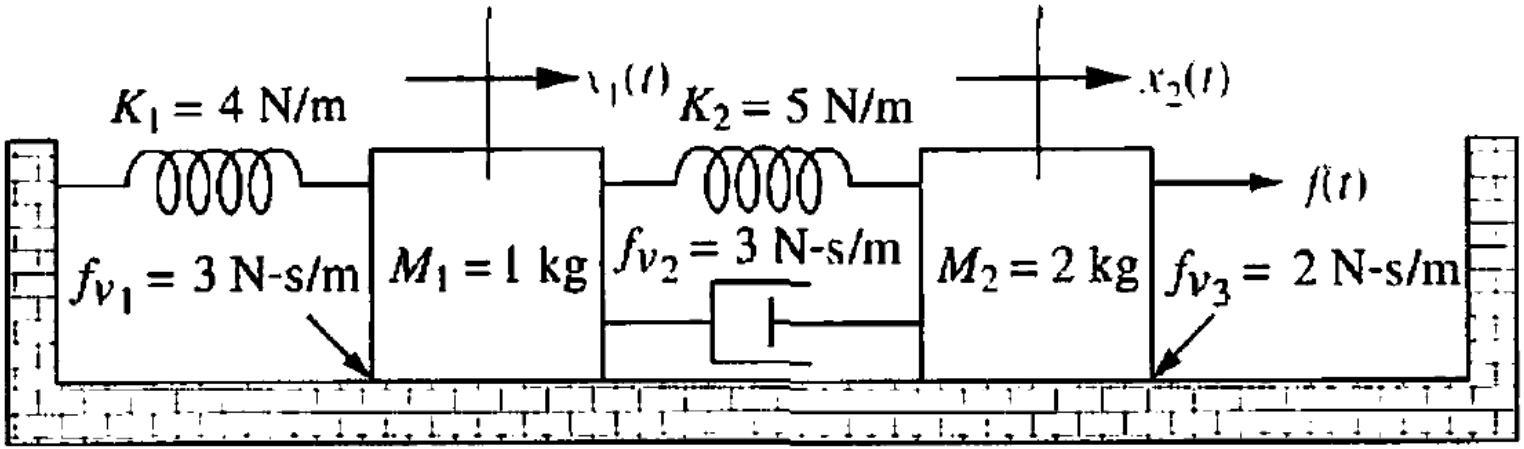
Contents

Given Question

Problem 26

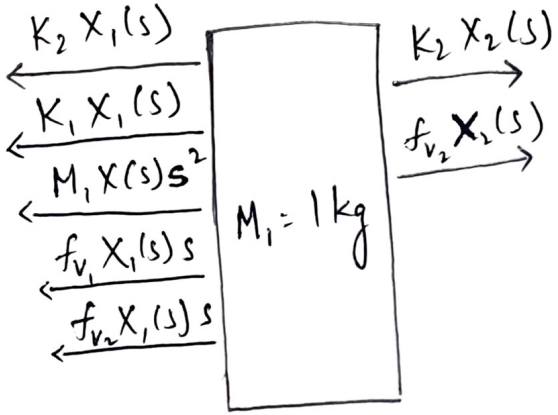
Q.26) For the system in the Figure below, find the transfer function

$G(s) = X_1(s)/F(s)$.

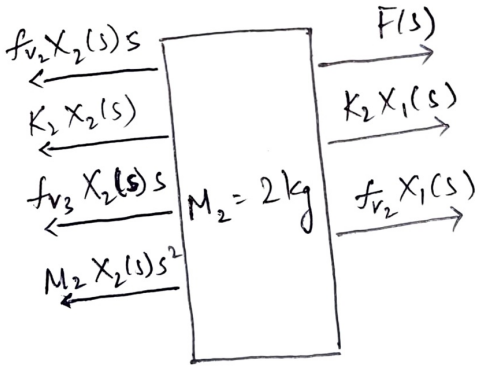


Solution

Free Body Diagrams



Forces acting on first block



Forces acting on second block

Solution

Equations

From the Free Body Diagrams, we get the equations as below,

Equation of the first body,
$\implies 0 = (\mathrm{K}_1 + K_2)X_1(s) + (f_{v_1} + f_{v_2})sX_1(s) + M_1 s^2 X_1(s) - K_2 X_2(s) - f_{v_2} s X_2(s)$

Equation of the second body,
$\implies \mathrm{F(s)} = \mathrm{K}_2 X_2(s) + (f_{v_2} + f_{v_3})sX_2(s) + M_2 s^2 X_2(s) - K_2 X_1(s) - f_{v_2} s X_1(s)$

Solution

Given Values

Substituting these values in the equations above,

$K_1 = 4N/m$

$K_2 = 5N/m$

$M_1 = 1Kg$

$M_2 = 2Kg$

$f_{v_1} = 3Ns/m$

$f_{v_2} = 3Ns/m$

$f_{v_3} = 2Ns/m$

Solution

Simplified Equations

$$\implies (s^2+6s+9)X_1(s) - (3s+5)X_2(s) = 0$$

$$\implies (2s^2+5s+5)X_2(s) - (3s+5)X_1(s) = F(s)$$

Solving for $X_1(s)$,

$$\implies \mathrm{X_1}(s) = \frac{\begin{vmatrix} 0 & -(3s+5) \\ F(s) & (2s^2+5s+5) \end{vmatrix}}{\begin{vmatrix} (s^2+6s+9) & -(3s+5) \\ -(3s+5) & (2s^2+5s+5) \end{vmatrix}}$$

$$\implies \mathrm{X_1}(s) = \frac{(3s+5)F(s)}{2s^4+17s^3+44s^2+45s+20}$$

Solution

Final Answer

$\therefore$ The Transfer Function is,

$$\implies \mathrm{G(s)} = \mathrm{X_1}(s)/F(s) = \frac{(3s+5)}{2s^4+17s^3+44s^2+45s+20}$$