7. Employ taylor series method to find y at $x = 0.1$ correct to 4 decimal places, given $\frac{dy}{dx} = 2y + 3e^x$; $y(0) = 0$.

Sol:- Given, $\frac{dy}{dx} = 2y + 3e^x = y'(x) \longrightarrow (1)$

and $y(0) = 0 \Rightarrow x_0 = 0,\ y_0 = 0$

$\Rightarrow y'(x_0) = 2y_0 + 3e^{x_0} = 2(0) + 3e^0 = 3$

$\therefore y''(x) = 2y' + 3e^x$

$\Rightarrow y''(x_0) = 2y_0' + 3e^{x_0} = 2(3) + 3 = 9$

$\therefore y'''(x) = 2y'' + 3e^x$

$\Rightarrow y'''(x_0) = 2y_0'' + 3e^{x_0} = 2(9) + 3(1) = 21$

$\therefore y^{IV}(x) = 2y''' + 3e^x$

$\Rightarrow y^{IV}(x_0) = 2y_0''' + 3e^{x_0} = 2(21) + 3 = 42 + 3 = 45$

WKT,

$$y(x) = y(x_0) + \frac{(x-x_0)^1}{1!}y'(x_0) + \frac{(x-x_0)^2}{2!}y''(x_0) + \frac{(x-x_0)^3}{3!}y'''(x_0) + \cdots$$

$$\Rightarrow y(x) = 0 + (x-0)(3) + \frac{(x-0)^2}{2}(9) + \frac{(x-0)^3}{6}(21) + \frac{(x-0)^4}{24}(45)$$

$$\Rightarrow y(x) = 3x + \frac{9x^2}{2} + \frac{21x^3}{6} + \frac{45x^4}{24}$$

$$\Rightarrow y(x) = 3x + \frac{9}{2}x^2 + \frac{7}{2}x^3 + \frac{15}{8}x^4$$

$$\Rightarrow y(0.1) = 3(0.1) + \frac{9}{2}(0.1)^2 + \frac{7}{2}(0.1)^3 + \frac{15}{8}(0.1)^4$$

$$\Rightarrow y(0.1) = 0.3 + 0.045 + 0.0035 + 0.0002$$

$$\Rightarrow y(0.1) = 0.3487.$$

4. Solve $\frac{dy}{dx} = x + \sin y$, $y(0) = 1$. Employ $y(0.4)$ by taking $h = 0.2$ using modified euler's method.

Sol. $\frac{dy}{dx} = x + \sin y = y'$

& $y(0) = 1 \Rightarrow x_0 = 0,\ y_0 = 1$

To find $y(x_1) = y_1$:-

$$\therefore y(x_0 + h) = y_1 = y_0 + hf(x_0, y_0)$$
$$= 1 + 0.2 f(0.1)$$
$$= 1 + 0.2(0.8415)$$
$$\Rightarrow y_1^{(1)} = 1.1683$$

$$\therefore y_1^{(2)} = y_0 + \frac{h}{2}\left[f(0,1) + f(x_1, y_1^{(1)})\right]$$
$$= 1 + \frac{0.2}{2}\left[(0.8415) + f(0.2, 1.1683)\right]$$
$$= 1 + 0.1\left[0.8415 + 1.1201\right]$$
$$\Rightarrow y_1^{(2)} = 1.1962$$

$$\therefore y_1^{(3)} = y_0 + \frac{h}{2}\left[f(0,1) + f(x_1, y_1^{(2)})\right]$$
$$= 1 + \frac{0.2}{2}\left[(0.8415) + f(0.2, 1.1962)\right]$$
$$= 1 + 0.1\left[0.8415 + 1.1307\right]$$
$$\Rightarrow y_1^{(3)} = 1.1972.$$

$$\therefore y_1^{(4)} = y_0 + \frac{h}{2}\left[f(0,1) + f(x_1, y_1^{(3)})\right]$$
$$= 1 + 0.1\left[(0.8415) + f(0.2, 1.1972)\right]$$
$$= 1 + 0.1973$$
$$\Rightarrow y_1^{(4)} = 1.1973$$

$\therefore y(0.2) = 1.1973$, $x_1 = 0.2$, $y_1 = 1.1973$

To find $y(x_2) = y_2$:-

$$\therefore y(x_2) = y_2^{(1)} = y_1 + hf(x_1, y_1)$$
$$= 1.1973 + (0.2) f(0.2, 1.1973)$$
$$= 1.1973 + 0.2(1.1311)$$
$$\Rightarrow y_2^{(1)} = 1.4235$$

$$\therefore y_2^{(2)} = y_1 + \frac{h}{2}\left[f(x_1, y_1) + f(x_2, y_2^{(1)})\right]$$
$$= 1.1973 + \frac{0.2}{2}\left[f(0.2, 1.1973) + f(0.4, 1.42\ldots\right.$$

$[1.1311 + 1.3893]$

$$\therefore y_1^{(4)} = y_0 + \frac{h}{2}\left[f(0,1) + f(x_1, y_1^{(3)})\right]$$

$$= 1 + 0.1\left[(0.8415) + f(0.2, 1.1972)\right]$$

$$= 1 + 0.1973$$

$$\Rightarrow y_1^{(4)} = 1.1973$$

$$\therefore y(0.2) = 1.1973, \quad x_1 = 0.2, \quad y_1 = 1.1973$$

<u>To find $y(x_2) = y_2$</u> :-

$$\therefore y(x_2) = y_2^{(1)} = y_1 + hf(x_1, y_1)$$

$$= 1.1973 + (0.2) f(0.2, 1.1973)$$

$$= 1.1973 + 0.2(1.1311)$$

$$\Rightarrow y_2^{(1)} = 1.4235$$

$$\therefore y_2^{(2)} = y_1 + \frac{h}{2}\left[f(x_1, y_1) + f(x_2, y_2^{(1)})\right]$$

$$= 1.1973 + \frac{0.2}{2}\left[f(0.2, 1.1973) + f(0.4, 1.4235)\right]$$

$$= 1.1973 + 0.1\left[1.1311 + 1.3892\right]$$

$$\Rightarrow y_2^{(2)} = 1.4493$$

$$\therefore y_2^{(3)} = y_1 + \frac{h}{2}\left[f(x_1, y_1) + f(x_2, y_2^{(2)})\right]$$

$$= 1.1973 + \frac{0.2}{2}\left[f(0.2, 1.1973) + f(0.4, 1.4493)\right]$$

$$\Rightarrow y_2^{(3)} = 1.4497.$$

$$\therefore y_2^{(4)} = y_1 + \frac{h}{2}\left[f(x_1, y_1) + f(x_2, y_2^{(3)})\right]$$

$$= 1.1973 + 0.1\left[f(0.2, 1.1973) + f(0.4, 1.4497)\right]$$

$$= 1.1973 + 0.1\left[1.1311 + 1.3927\right]$$

$$\Rightarrow y_2^{(4)} = 1.4497.$$

$$\frac{dy}{dx} = 2y + 3e^{x}, \quad y(0) = 0, \quad x = 0.1, \quad h = 0.1$$

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$$x_0 = 0, \quad y_0 = 0$$

$$k_1 = hf(x_0, y_0) = 0.1\left(2(0) + 3e^{0}\right) = 0.3$$

$$k_2 = hf\left(\underbrace{x_0 + \frac{h}{2}}_{0.05}, \underbrace{y_0 + \frac{k_1}{2}}_{0.15}\right) = 0.1(3.458) = 0.3458$$

$$k_3 = hf\left(\underbrace{x_0 + \frac{h}{2}}_{0.05}, \underbrace{y_0 + \frac{k_2}{2}}_{0.1729}\right) = 0.1\left(\cancel{6.6118}\ 3.4996\right) = \cancel{0.66118}\ 0.34996$$

$$k_4 = hf\left(\underbrace{x_0 + h}_{0.1}, \underbrace{y_0 + k_3}_{0.66118}\right) = 0.1\left(\cancel{4.639}\ 4.0154\right) = \cancel{0.4639}\ 0.40154$$

$$y(0.1) = y_0 + \frac{1}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right)$$

$$= 0 + \frac{1}{6}\left(0.3 + 0.6916 + 0.6999 + 0.40154\right)$$

$$y(0.1) = \frac{1}{6}(2.0930) = \underline{\underline{0.3488}}$$

4a) $\frac{dy}{dx} = 2e^x + y$ $\qquad y(0.4) = ?$

x	0	0.1	0.2	0.3	0.4
y	2.4	2.473	3.129	4.059	4.7083

$f = 2e^x + y$

$f_0 = 4.40$

$f_1 = 4.6833$

$f_2 = 5.5718$

$f_3 = 6.7587$

$f_4 = 7.6919$

$y_4^{(P)} = y_0 + \frac{4h}{3}(2f_1 - f_2 + 2f_3)$

$\Rightarrow 2.4 + \frac{4\times0.1}{3}(2\times 4.6833 - 5.5718 + 2\times 6.7587)$

$= 2.4 + \frac{0.4}{3}(9.3666 - 5.5718 + 13.5174)$

$y_4^{(P)} = \underline{4.7083}$

1st Iteration:

$y_4^{(C)} = y_2 + \frac{h}{3}(f_2 + 4f_3 + f_4^{(P)})$

$\Rightarrow 3.129 + \frac{0.1}{3}(5.5718 + 4\times 6.7587 + 7.6919)$

$y_4^{(C)} = \underline{4.4723}$ $\qquad y_4 = 4.4723$

$f_4 = 7.4559$

2nd Iteration:

$\Rightarrow 3.129 + \frac{0.1}{3}(5.5718 + 4\times 6.7587 + 7.4559)$

$\Rightarrow \underline{4.4644}$

$y_4 = 4.4644$

$f_4 = 7.4480$

3rd Iteration:

$\Rightarrow 3.129 + \frac{0.1}{3}(5.5718 + 4\times 6.7587 + 7.4480)$

$\Rightarrow \underline{4.4642}$

$$\Rightarrow \int_{-1}^{1} 4z^3\,dz = \left[\frac{\cancel{4}z^4}{\cancel{4}}\right]_{-1}^{1} \Rightarrow 1-1 = 0$$

3. $$I = \int_0^{\log 2}\int_0^{x}\int_0^{x+\log y} e^{x+y+z}\,dz\,dx\,dy$$

$$I = \int_0^{\log 2}\int_0^{x}\int_0^{x+\log y} e^x.e^y.e^z\,dz\,dx\,dy$$

$$= \int_0^{\log 2}\int_0^{x} e^x.e^y.\left[e^z\right]_0^{x+\log y} dx.dy$$

$$= \int_0^{\log 2}\int_0^{x} e^x.e^y\left[e^{x+\log y} - e^0 1\right]dx\,dy$$

$$= \int_0^{\log 2}\int_0^{x} e^x.e^y\left[e^x.e^{\log y} - 1\right]dx\,dy$$

$$= \int_0^{\log 2}\int_0^{x} e^x.e^y\left[e^x.y - 1\right]dx\,dy$$

$$= \int_0^{\log 2}\int_0^{x} e^{2x}e^y.y - e^x.e^y\,dx\,dy$$

$$= \int_0^{\log 2}\int_0^{x} e^{2x}e^y.y - e^x.e^y\,dy\,dx.$$

$$= \int_0^{\log 2} e^{2x}\left[\left[ye^y - e^y\right] - e^x.e^y\right]_0^x dx.$$

$$= \int_0^{\log 2} \cancel{e^{2x}\left[xe^x - e^x - e^x.e^x\right] dx}$$

$$e^{2x}[(xe^x - e^x) - (0-1)) - e^x(e^x-1)\,dx.$$

$$= \int_0^{\log 2} \cancel{e^{2x}\left[xe^x - e^x - e^{2x}\right] dx}$$

$$= \int_0^{\log 2} \cancel{x.e^{3x} - e^{3x} - e^{4x}}\,dx$$

$$= \int_0^{\log 2} e^{2x}\left[\left[xe^x - e^x + 1\right] - e^{2x} + e^x\right]dx.$$

$$= \int_0^{\log 2} xe^{3x} - e^{3x} + e^{2x} - e^{4x} + e^{3x}\,dx$$

$$= \int_0^{\log 2} xe^{3x} - e^{3x} + e^x\,dx$$

$$\Rightarrow \left[\frac{xe^{3x}}{3} - \frac{e^{3x}}{9} - \frac{e^{3x}}{3} + e^x\right]_0^{\log 2}$$

$$= \left[\frac{3xe^{3x} - e^{3x} - 3e^{3x} + 9e^x}{9}\right]_0^{\log 2}$$

$$\Rightarrow \left[\frac{3\log 2\, e^{3\log 2} - e^{3\log 2} - 3e^{3\log 2} + 9e^{\log 2}}{9}\right] -$$

$$\left[\frac{0 - 1 - 3 + 9}{9}\right]$$

$= \frac{3\log 2\, e^{3\log 2} - 4e^{3\log 2} + 9e^{\log 2}}{9} - \frac{5}{9}$

$\Rightarrow \frac{3\log 2\, e^{\log(2)^3} - 4e^{\log(2)^3} + 9e^{\log 2}}{9} - \frac{5}{9}$

$\Rightarrow \frac{8\times 3\times \log 2}{9} - \frac{4\times 8}{9} + \frac{9\times 2}{9} - \frac{5}{9}$

$\Rightarrow \frac{8\log 2}{3} - \frac{19}{9}$

→ Double integration:

4. $\int_0^1 \int_x^{\sqrt{x}} xy\, dy\, dx$

$I = \int_0^1 \int_x^{\sqrt{x}} xy\, dy\, dx$

$= \int_0^1 \left[\frac{xy^2}{2}\right]_x^{\sqrt{x}} dx$

$= \int_0^1 x\left[\frac{x}{2} - \frac{x^2}{2}\right] dx = \int_0^1 \frac{1}{2}\left[x^2 - x^3\right] dx$

$= \frac{1}{2}\left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1 \Rightarrow \frac{1}{2}\left[\frac{1}{3} - \frac{1}{4}\right]$

$\Rightarrow \frac{1}{2}\left[\frac{1}{12}\right] = \frac{1}{24}$

5. $\int_0^1 \int_x^{\sqrt{x}} (x^2+y^2)\, dy\, dx$

$I = \int_0^1 \int_x^{\sqrt{x}} x^2+y^2\, dy\, dx$

$= \int_0^1 \left[x^2y + \frac{y^3}{3}\right]_x^{\sqrt{x}} dx$

$= \int_0^1 \left[x^2\sqrt{x} + \frac{x\sqrt{x}}{3}\right] - \left[x^3 + \frac{x^3}{3}\right] dx$

$= \int_0^1 \frac{3x^2\sqrt{x} + x\sqrt{x}}{3} - \frac{4x^3}{3}\, dx$

$= \frac{1}{3}\int_0^1 3x^2\sqrt{x} + x\sqrt{x} - 4x^3\, dx$

$\frac{1}{2} + 2$

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4. $I = \int_0^1 \int_0^{\sqrt{1-y^2}} x^3 y \, dx \, dy$

$= \int_0^1 \left[y\frac{x^4}{4} \right]_0^{\sqrt{1-y^2}} dy = \frac{1}{4}\int_0^1 y(\sqrt{1-y^2})^4 dy$

$= \frac{1}{4}\int_0^1 y(1-y^2)^2 dy$

$= \frac{1}{4}\int_0^1 y[1+y^4-2y^2]\,dy = \int_0^1 \frac{1}{4}[y+y^5-2y^3]\,dy$

$= \frac{1}{4}\left[\frac{y^2}{2}+\frac{y^6}{6}-\frac{2y^4}{4}\right]_0^1$

$= \frac{1}{4}\left[\frac{1}{2}+\frac{1}{6}-\frac{1}{2}\right] \Rightarrow \frac{1}{24}$

5. $I = \int_0^1 \int_0^{\sqrt{1+x^2}} \frac{1}{\sqrt{1+x^2+y^2}} \, dy \, dx$ $\qquad \int \frac{dx}{\sqrt{a^2+x^2}} = \log(x+$

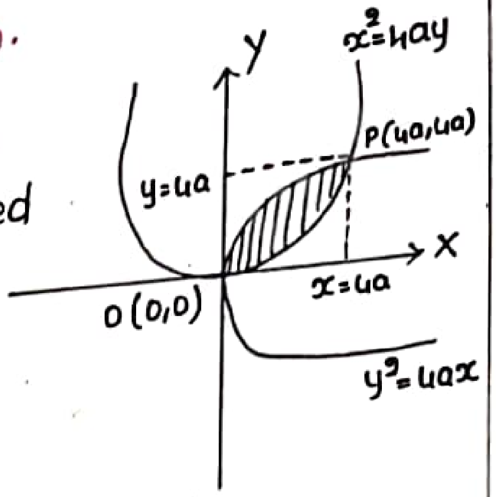
$= \int_0^1 \left[\log\left(y+\sqrt{1+x^2+y^2}\right)\right]_0^{\sqrt{1+x^2}} dx$ $\qquad$ Let $1+x^2 = a$

$= \int_0^1 \left[\log\left(y+\sqrt{a^2+y^2}\right)\right]_0^{a}$

$= \int_0^1 \left(\log(a+\sqrt{2}a) - \log a\right) dx$

$= \int_0^1 \log\left[(1+\sqrt{2}) \cancel{\times} \log(1+x^2)\right] - \log(1+x^2)$

1. Find the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ using double integration.



Sol:- Given, $y^2 = 4ax$ & $x^2 = 4ay$ be the parabolas then the area bounded between $y^2 = 4ax$ & $x^2 = 4ay$.

$$A = \iint dx\,dy$$

$$\Rightarrow A = \int_{x=0}^{4a} \int_{y=\frac{x^2}{4a}}^{2\sqrt{ax}} dy\,dx$$

$$A = \int_{x=0}^{4a} \left| y \right|_{\frac{x^2}{4a}}^{2\sqrt{ax}} dx$$

$$A = \int_{x=0}^{4a} \left| 2\sqrt{ax} - \frac{x^2}{4a} \right| dx$$

$$A = 2\sqrt{a} \int_{x=0}^{4a} x^{1/2}.\,dx - \frac{1}{4a} \int_{x=0}^{4a} x^2.\,dx$$

$$A = 2\sqrt{a} \left| \frac{x^{3/2}}{3/2} \right|_0^{4a} - \frac{1}{4a} \left| \frac{x^3}{3} \right|_0^{4a}$$

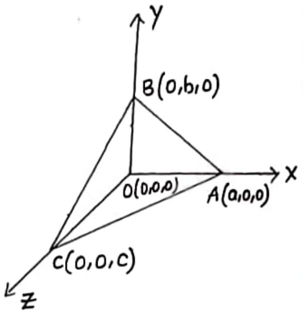
$$A = \frac{4\sqrt{a}}{3} \left[x^{3/2} \right]_0^{4a} - \frac{1}{12a} \left[x^3 \right]_0^{4a}$$

$$A = \frac{4\sqrt{a}}{3} (4a)^{3/2} - \frac{1}{12a} (4a)^3$$

Sol:- $\therefore V = \iiint dx\,dy\,dz$

$$V = \int_{x=0}^{a}\int_{y=0}^{\frac{b}{a}(a-x)}\int_{z=0}^{c\left[1-\frac{x}{a}-\frac{y}{b}\right]} 1.\,dz\,dy\,dx$$

$$V = \int_{x=0}^{a}\int_{y=0}^{\frac{b}{a}(a-x)} \left|z\right|_{0}^{c\left[1-\frac{x}{a}-\frac{y}{b}\right]} dy\,dx$$



$$V = c\int_{x=0}^{a}\int_{y=0}^{\frac{b}{a}(a-x)} \left[1-\frac{x}{a}-\frac{y}{b}\right] dy\,dx$$

$$V = c\int_{x=0}^{a}\left[y-\frac{xy}{a}-\frac{y^2}{2b}\right]_{0}^{\frac{b}{a}(a-x)} dx$$

$$V = c\int_{x=0}^{a}\left|\frac{b}{a}(a-x)-\frac{x}{a}\cdot\frac{b}{a}(a-x)-\frac{1}{2\cancel{b}}\cdot\frac{b^{\cancel{2}}}{a^2}(a-x)^2\right| dx$$

$$V = c\int_{x=0}^{a}\left\{\frac{b}{a}(a-x)\left[1-\frac{x}{a}\right]-\frac{b}{2a^2}(a-x)^2\right\} dx$$

$$V = c\int_{x=0}^{a}\left|\frac{b}{a}(a-x)\frac{(a-x)}{a}-\frac{b}{2a^2}(a-x)^2\right\} dx$$

$$V = c\int_{x=0}^{a}\frac{b}{a^2}(a-x)^2-\frac{b}{2a^2}(a-x)^2$$

$$V = \frac{c}{2}\int_{x=0}^{a}\frac{b}{a^2}(a-x)^2.\,dx$$

$$V = \frac{c}{2}\int_{x=0}^{a}\frac{b}{a^2}(x-a)^2\,dx$$

$$V = \frac{bc}{2a^2}\int_{x=0}^{a}(x-a)^2\,dx$$

$$V = \frac{bc}{2a^2}\left[\frac{(x-a)^3}{3}\right]_{0}^{a}$$

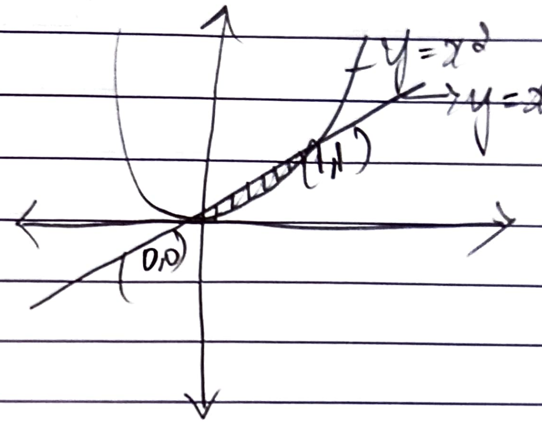
$$V = \frac{bc}{2a^2}\left|\frac{0-(-a)^3}{3}\right|$$

$$\Rightarrow V = \frac{bc}{2a^{\cancel{2}}}\times\frac{a^{\cancel{3}}}{3} \Rightarrow V = \frac{abc}{6}$$

$$\boxed{V = \frac{abc}{6}\text{ Cubic units}}$$

6. Find the volume of the solid bounded by the

10 a) $I = \int_0^1 \int_{x^2}^{x} 1 \, dy \, dx$



$\Rightarrow \int_0^1 [y]_{x^2}^{x} \, dx$

$\Rightarrow \int_0^1 (x - x^2) \, dx$

$\Rightarrow \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$

$\Rightarrow \frac{1}{2} - \frac{1}{3} \Rightarrow \underline{\underline{\frac{1}{6}}}$