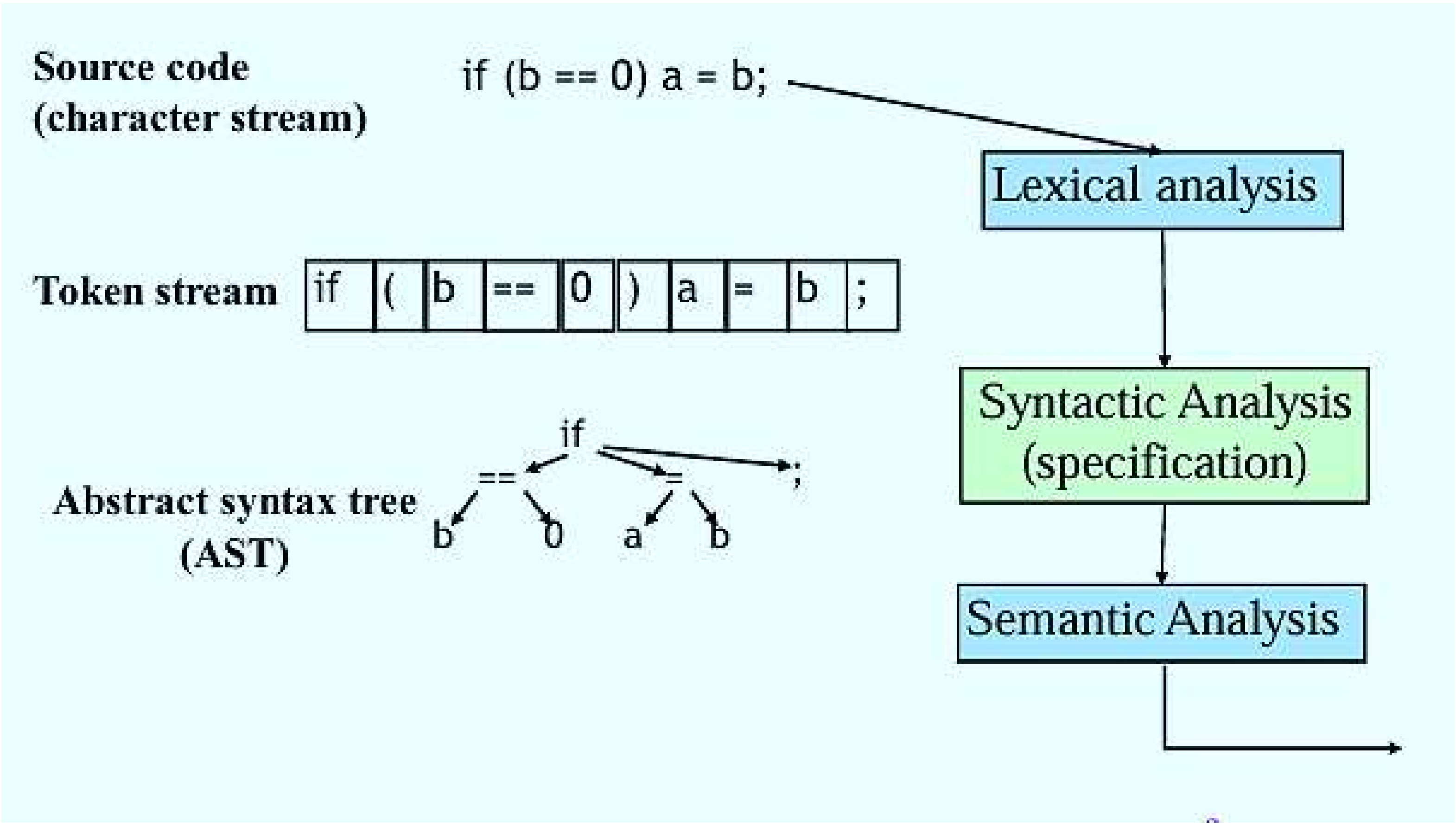
IT 302 Compiler Design

iNTRODUCTION TO pARSING

aMBIGUITY AND sYNTAX eRRORS

Where we are


Source code
(character stream)
if (b == 0) a = b;
Lexical analysis
Token stream
if (b == 0) a = b ;
Syntactic Analysis
(specification)
Abstract syntax tree
(AST)
if
==
b
0
=
a
b
;
Semantic Analysis

What is Syntactic Analysis?

- int a;
- a = "hello"; *// Error: assigning string to int*

- **Syntax analysis**: Says “grammar is fine”.
- **Semantic analysis**: Says “type mismatch: int variable cannot hold a string”.

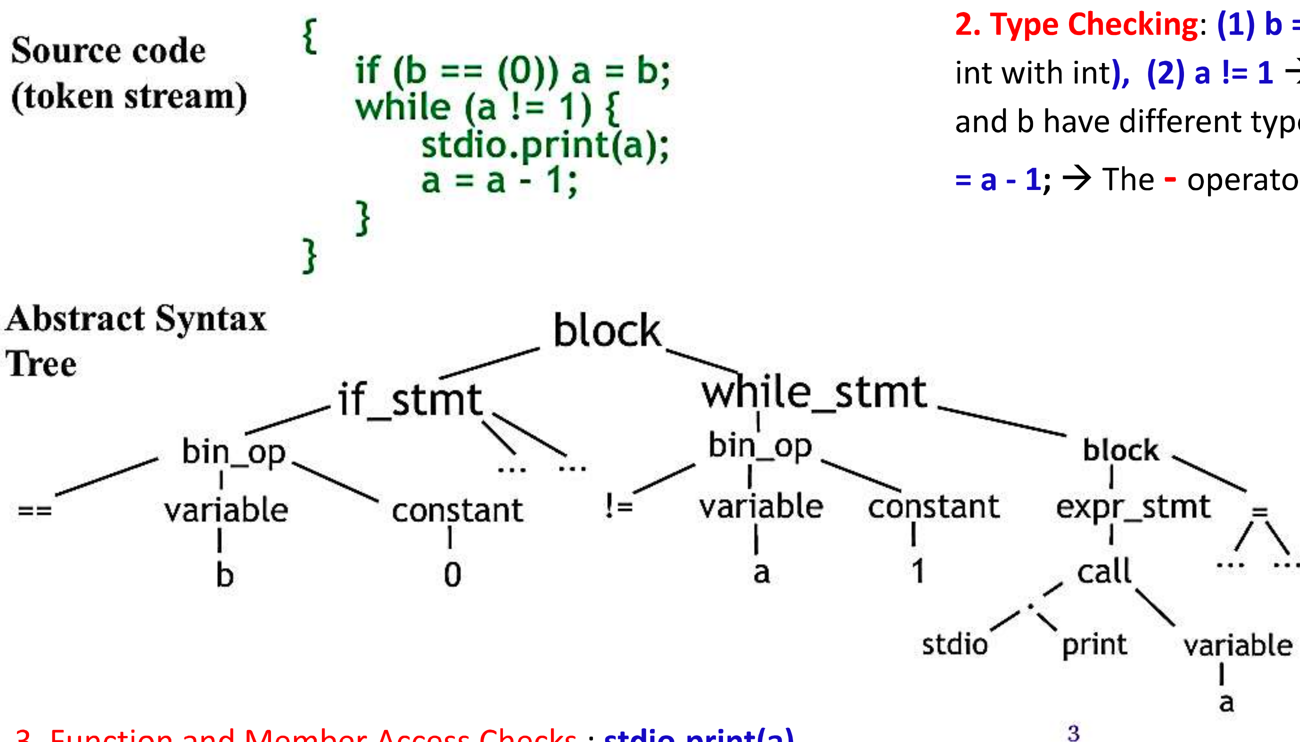
It verifies things such as:

- **Type checking →** Example: You can’t add an integer to a string unless the language defines how to do that.
- **Variable declarations →** Every variable must be declared before it is used.
- **Scope rules →** A variable can only be used in the scope it’s visible in.
- **Function usage →** The number and types of arguments must match the function definition.
- **Control flow rules →** Statements like break must be inside loops, return must be inside functions.

What is Syntactic Analysis?

1. Variable declaration check: (1) Is **b** declared before the if **(b == 0)?**, (2) Is **a** declared before use?

2. Type Checking: **(1) b == 0** → Both sides should be comparable (ex: int with int**), (2) a != 1** → must be comparable types, (3) **a = b;** → If a and b have different types (int vs. string) → **type mismatch error**. **(4) a = a - 1;** → The **-** operator must be valid for the type (numeric).



5. Scope Checking: Here, **a** is used in both the if and while — so a must be in the outer scope.

3. Function and Member Access Checks : **stdio.print(a)**

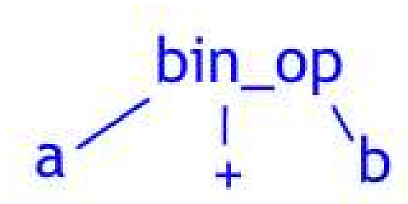
1. stdio must be a valid identifier in scope (library/module).
2. print must be a valid member/function of stdio.
3. Arguments passed (a) must match the expected type of print.

4. Control Flow Rules: (1) if_stmt → Condition must be a Boolean expression (result of b == 0 must be Boolean). (2) **while_stmt** → Condition must also be Boolean (a != 1).

Syntactic Analysis .. Contd.,

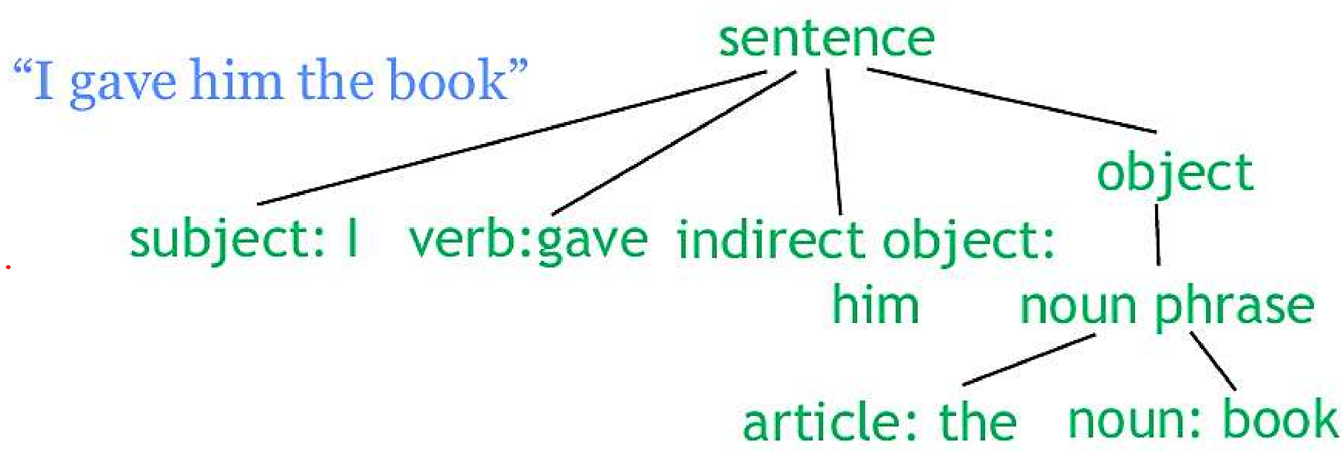
- Input: stream of tokens
- Output: abstract syntax tree
  - Abstract syntax tree removes extra syntax

$$a + b \approx (a) + (b) \approx ((a)+((b)))$$



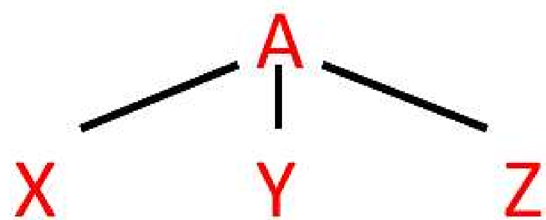
Parsing

- **Parsing:** recognizing whether a program (or sentence) is grammatically well formed & identifying the function of each component.



Parsing Trees

- A parse tree pictorially shows: How the start symbol of a grammar derives strings in language.



1. The root is labeled by start symbol
2. Each leaf is labeled by a terminal (T) or by ϵ
3. Each interior node is labeled by a non-terminals (NT)
4. If NT, X_1, X_2,… , X_n are labeled children of A from left to right then there must be production A, X_1, X_2, ,X_n, where X_1, X_2, ..X_n are eithe NT or T,
5. If A ϵ, then A may have single child ϵ

Parsing Trees: Properties

- A tree consists of one or more nodes
- Exactly one root node in a Tree
  - Root have no-parent, it is top node
  - Other node have exactly one parent
- **Leaf**: node with no children
  - N is parent of M,
  - M is child of N,
  - Children of one node is Siblings,
  - Ordered from left to right
  - Descendent (self, child*), Ancestor (self, parent*)

The Functionality of the Parser

- **Input:** sequence of tokens from lexer
- **Output:** parse tree of the program

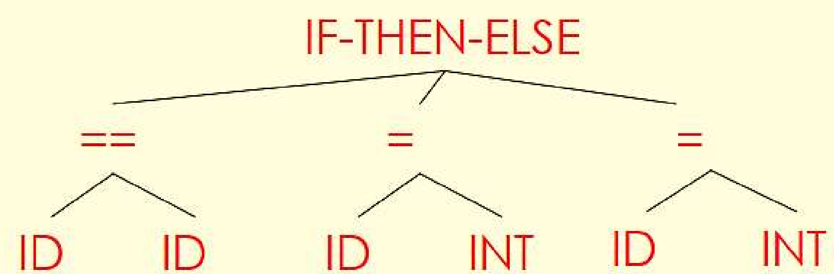
- If-then-else statement

  if (x == y) then z =1; else z = 2;

- Parser input

  IF (ID == ID) THEN ID = INT; ELSE ID = INT;

- Possible parser output



Phase	*Input*	*Output*
Lexer	Sequence of characters	Sequence of tokens
Parser	Sequence of tokens	Parse tree

What Parsing doesn't do

- Doesn't check many things: type agreement, variables declared, variables initialized, etc.
  - int x = true;
  - int y; z = f(y);
- Postponed until semantic analysis

Parsing ≠ Full Program Correctness

- The parser's job is **only** to check whether the program's structure matches the grammar rules — the "shape" of the code.
- It **does not**:
  - Check if types match (**type agreement**).
  - Verify if variables are declared before use.
  - Ensure variables are initialized before being read.

Recap: Languages and Automata

- Formal languages are very important in CS
  - Especially in programming languages
- Regular languages
  - The weakest formal languages widely used
  - Many applications

Specifying Language Syntax

- First problem: how to describe language syntax precisely and conveniently
- Last time: can describe tokens using regular expressions
- Regular expressions easy to implement, efficient (by converting to DFA)
- Why not use regular expressions (on tokens) to specify programming language syntax?

    - Programming languages need **nested, recursive structures**, but **regular expressions can't handle recursion**.

if (x) { y = 1; } else { z = 2; }

This involves:

- Matching **balanced parentheses {} or ()**.
- Handling **nested if-statements**.
- Matching **expressions** that can contain subexpressions.
- Regular languages **cannot** match:
    - **Balanced brackets**: { { } { { } } }

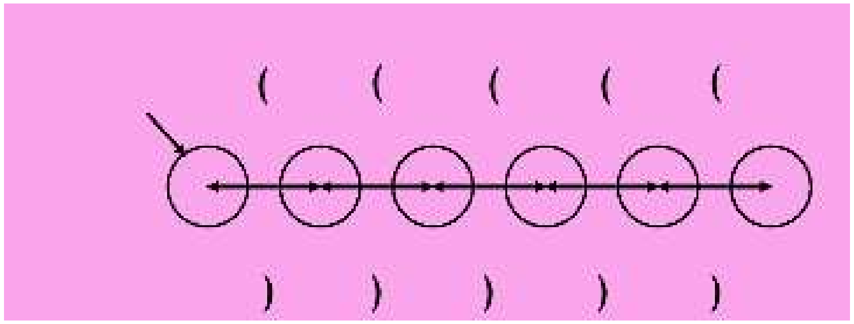
Limitations of Regular Languages

- A finite automaton that runs long enough must repeat states
  - A finite automaton cannot remember # of times it has visited a particular state
  - Because a finite automaton has finite memory
    - Only enough to store in which state it is
    - Cannot count, except up to a finite limit
  - Many languages are not regular
  - ***Ex:*** language of balanced parentheses is not regular: $\{ (^i\ )^i \mid i \geq 0\}$

Problem: need to keep track of number of parentheses seen so far: *unbounded counting*

Contd.,

- RE = DFA
- DFA has only finite number of states; cannot perform unbounded counting



maximum depth: 5 parenthesis

The Role of the Parser

- Not all sequences of tokens are programs . . .
- . . . Parser must distinguish between valid and invalid sequences of tokens

- We need
  - A language for describing valid sequences of tokens
  - A method for distinguishing valid from invalid sequences of tokens

Regexes

Compiler Writer

Scotty, we need more power!

Context-Free Grammars

- Many programming language constructs have a recursive structure

- A STMT is of the form

  if COND then STMT else STMT , or

  while COND do STMT , or

  …

- Context-free grammars are a natural notation for this recursive structure

CFGs (Cont.)

- A CFG consists of
  - A set of terminals **T**
  - A set of non-terminals **N**
  - A start symbol **S** (a non-terminal)
  - A set of productions

Assuming $X \in N$ the productions are of the form

$X \rightarrow \varepsilon$, or

$X \rightarrow Y_1 Y_2 \ldots Y_n$ where $Y_i \in N \cup T$

Contd., Let's begin by defining the parts of a CFG

- A **terminal** is a discrete symbol that can appear in the language, other wise known as a token.
  - Examples of terminals are keywords, operators, and identifiers.
  - We will use *lower-case letters to represent terminals*.

- A **non-terminal** represents a structure that can occur in a language, but is not a literal symbol.
  - Example of non-terminals are declarations, statements, and expressions.
  - We will use *upper-case letters to represent non-terminals*: P for program, S for statement, E for expression, etc.

- A **sentence** → a valid sequence of terminals in a language, while a sentential form → a valid sequence of terminals and non-terminals.
  - Greek symbols to represent sentential forms. Ex: **α, β, γ** and represent (possibly) mixed sequences of terminals and non-terminals.
  - We will use a sequence like Y1, Y2,… Yn to indicate the individual symbols in a sentential form: Yi may be either a terminal or a non-terminal.

Contd., Let’s begin by defining the parts of a CFG

- A **context-free grammar (CFG)** is a list of rules that formally describe the allowable sentences in a language.
  - The left-hand side of each rule is always a single non-terminal.
  - The right-hand side of a rule is a sentential form that describes an allowable form of that non-terminal.

    Ex: the rule A → xXy

    - indicates the non-terminal A represents a terminal x followed by a non-terminal X and a terminal y.
    - The right-hand side of a rule can be indicated **ϵ** that the rule produces nothing.
- The first rule is special: it is the top-level definition of a program and its non-terminal known as start symbol.

For example, here is a simple CFG that describes expressions involving addition, integers, and identifiers:

1. P → E
2. E → E+E
3. E → ident
4. E → int

- This grammar can be read as follows:
  (1) A complete program consists of one expression.
  (2) An expression can be any expression plus any expression.
  (3) An expression can be an identifier.
  (4) An expression can be an integer.

- *For shortness:* a common left-hand side by combining all of the right-hand sides with a logical-or symbol, like this:

E → E + E **|** ident **|** int

Notational Conventions

- In this Parsing
  - Non-terminals are written upper-case
  - Terminals are written lower-case
  - The start symbol is the left-hand side of the first production

Grammar for simple arithmetic expressions

Examples of CFGs

- A fragment of our example language (simplified)

$$
\begin{aligned}
\text{STMT} \rightarrow\ & \text{if COND then STMT else STMT} \\
\mid\ & \text{while COND do STMT} \\
\mid\ & \text{id = int}
\end{aligned}
$$

$$
\begin{aligned}
E \rightarrow\ & E * E \\
\mid\ & E + E \\
\mid\ & (E) \\
\mid\ & \text{id}
\end{aligned}
$$

The Language of a CFG

- Read productions as replacement rules

$X \rightarrow Y_1 \ldots Y_n$

Means X can be replaced by $Y_1 \ldots Y_n$

$X \rightarrow \varepsilon$

Means X can be erased (replaced with empty string)

Key Idea

(1) Begin with a string consisting of the start symbol "S"

(2) Replace any non-terminal X in the string by a right-hand side of some production

(3) Repeat (2) until there are no non-terminals in the string

The Language of a CFG (Cont.)

Describing **formal derivations** in **context-free grammars** in simpler terms:

More formally, we write

$$X_1 \cdots X_i \cdots X_n \to X_1 \cdots X_{i-1} Y_1 \cdots Y_m X_{i+1} \cdots X_n$$

if there is a production

$$X_i \to Y_1 \cdots Y_m$$

Write

$$X_1 \cdots X_n \overset{*}{\to} Y_1 \cdots Y_m$$

if

$$X_1 \cdots X_n \to \cdots \to \cdots \to Y_1 \cdots Y_m$$

in 0 or more steps

- **In simple words:**

  $\to$ means "one step of replacement"

  $\to$* means "any number of steps of replacement" (including zero)

The Language of a CFG

- Let **G** be a context-free grammar with start symbol **S.** Then the language of **G** is:

$$\left\{ a_1 \ldots a_n \mid S \xrightarrow{*} a_1 \ldots a_n \text{ and every } a_i \text{ is a terminal} \right\}$$

- This is the **language defined by the grammar**:

  L(G)={all terminal strings derivable from the start symbol} Where G is your CFG, S is its start symbol.

- This condition ensures that the derived string contains **no nonterminals** left — it's a complete sentence in the language.

Terminals

- Terminals are tokens and there are no rules for replacing them
- Once generated, terminals are permanent

Examples

L(G) is the language of the CFG G

→→→→→→→→

Strings of balanced parentheses $\{(^i)^i \mid i \geq 0\}$

Two grammars:

$$S \rightarrow (S)$$
$$S \rightarrow \varepsilon$$

OR

$$S \rightarrow (S) \mid \varepsilon$$

CFG for Arithmetic Example

Simple arithmetic expressions:

$$E \rightarrow E+E \mid E * E \mid (E) \mid id$$

Some elements of the language:

id	$id + id$
(id)	$id * id$
$(id) * id$	$id * (id)$

We have one non-terminal: **E** (stands for “expression”)

Production rules:

E → E + E // addition
E → E * E // multiplication
E → (E) // parentheses
E → id // identifier (variable or number)

context-free grammar for **simple arithmetic expressions** and some example strings it can generate.

Derivations and Parse Trees

- A derivation is a sequence of productions

$$S \rightarrow \rightarrow \ldots \rightarrow \rightarrow \ldots \rightarrow \rightarrow \rightarrow$$

- A derivation can be drawn as a tree
  - Start symbol is the tree's root

- For a production X $\rightarrow$ $Y_1 \ldots\ldots Y_n$ add children Y_1 ….. Y_n to node X

Derivation Example

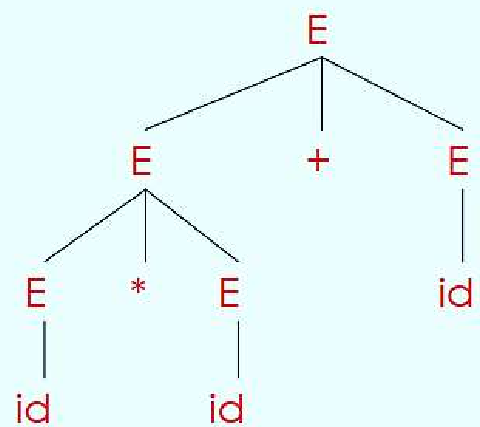
- Grammar

$$E \rightarrow E+E \mid E * E \mid (E) \mid id$$

- String

$$id * id + id$$

$$
\begin{aligned}
& E \\
\rightarrow\ & E+E \\
\rightarrow\ & E * E+E \\
\rightarrow\ & id * E + E \\
\rightarrow\ & id * id + E \\
\rightarrow\ & id * id + id
\end{aligned}
$$

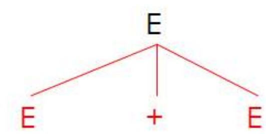


Derivation in Detail

E

E

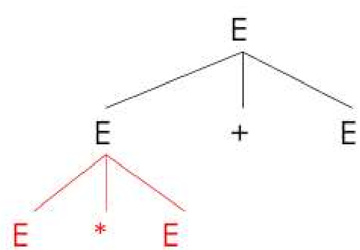
Step 1



$$\begin{aligned} & E \\ \rightarrow\ & E+E \end{aligned}$$

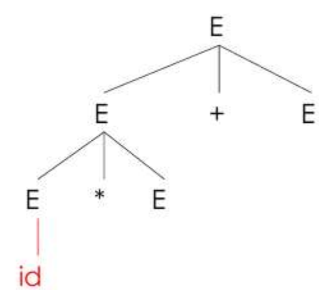
Step 2

$$\begin{aligned} & E \\ \rightarrow\ & E+E \\ \rightarrow\ & E*E+E \end{aligned}$$



Step 3

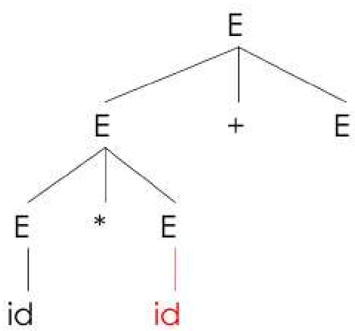
$$\begin{aligned} & E \\ \rightarrow\ & E+E \\ \rightarrow\ & E*E+E \\ \rightarrow\ & id*E+E \end{aligned}$$



Step 4

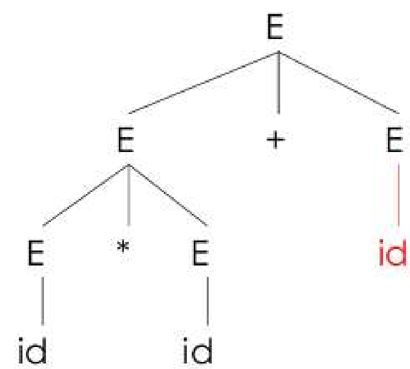
Derivation in Detail .. Contd.,

$$
\begin{aligned}
& E \\
\rightarrow\ & E+E \\
\rightarrow\ & E * E+E \\
\rightarrow\ & id * E + E \\
\rightarrow\ & id * id + E
\end{aligned}
$$



Step 5

$$
\begin{aligned}
& E \\
\rightarrow\ & E+E \\
\rightarrow\ & E * E+E \\
\rightarrow\ & id * E + E \\
\rightarrow\ & id * id + E \\
\rightarrow\ & id * id + id
\end{aligned}
$$



Step 6

Notes on Derivation

- A parse tree has
  - Terminals at the leaves
  - Non-terminals at the interior nodes

- An in-order traversal of the leaves is the original input

- The parse tree shows the association of operations; the input string does not

Left-most and Right-most Derivations

- The example is a *left-most derivation*
  - At each step, replace the left-most non-terminal
- There is an equivalent notion of a *right-most derivation*

$$
\begin{aligned}
& E \\
\rightarrow\ & E+E \\
\rightarrow\ & E+id \\
\rightarrow\ & E * E + id \\
\rightarrow\ & E * id + id \\
\rightarrow\ & id * id + id
\end{aligned}
$$

E → E+E

Right-most Derivation in Detail

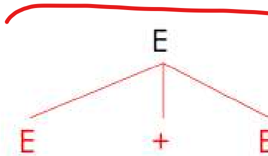
E

E

Step 1

E

→ E+E

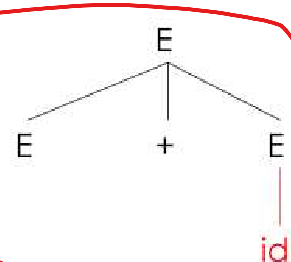


Step 2

E

→ E+E

→ E+id



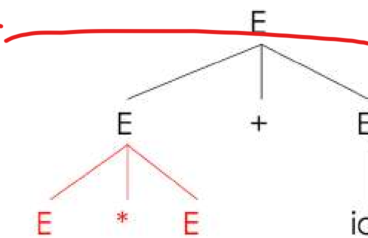
Step 3

E

→ E+E

→ E+id

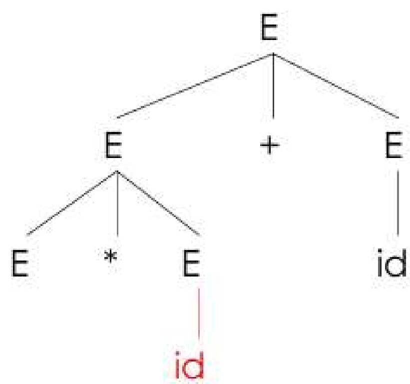
→ $E * E + id$



Step 4

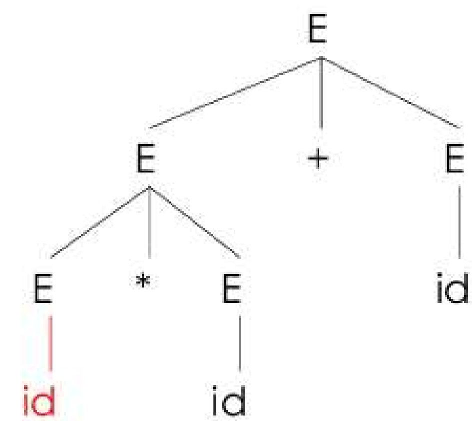
Contd.,

$$
\begin{aligned}
& E \\
\rightarrow\ & E{+}E \\
\rightarrow\ & E{+}id \\
\rightarrow\ & E * E + id \\
\rightarrow\ & E * id + id
\end{aligned}
$$



Step 5

$$
\begin{aligned}
& E \\
\rightarrow\ & E{+}E \\
\rightarrow\ & E{+}id \\
\rightarrow\ & E * E + id \\
\rightarrow\ & E * id + id \\
\rightarrow\ & id * id + id
\end{aligned}
$$



Step 6

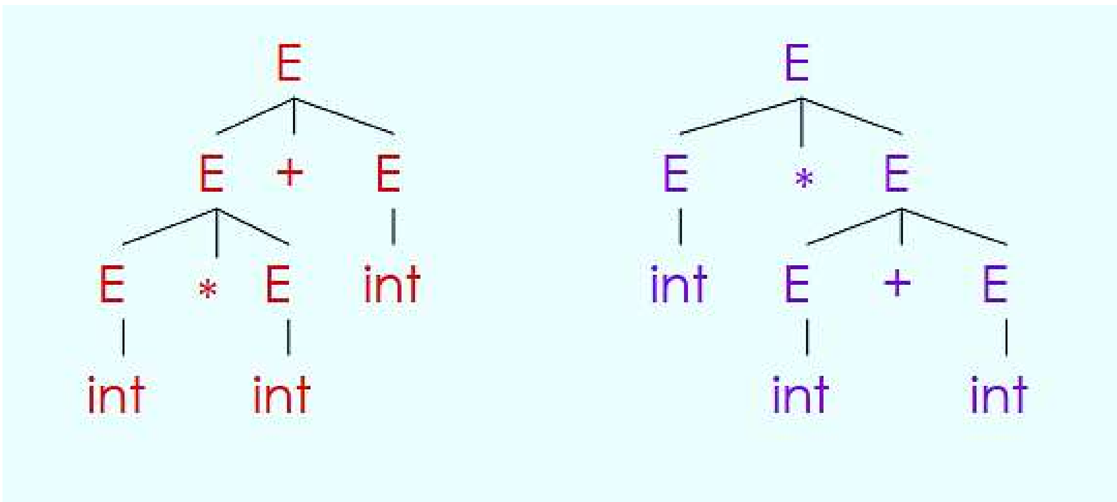
Derivations and Parse Trees

- Note that right-most and left-most derivations have the same parse tree
- The difference is just *in the order in which branches are added*
- We are not just interested in whether s ∈ L(G)
  - We need a parse tree for **s**
- A derivation defines a parse tree
  - But one parse tree may have many derivations
- Left-most and right-most derivations are important in parser implementation

Ambiguity

- Grammar: E → E + E | E * E | (E) | int
- String: int * int + int

This string has two parse trees



Ambiguity (Cont.)

- A grammar is ambiguous if it has more than one parse tree for some string
  - Equivalently, there is more than one right-most or left-most derivation for some string
- Ambiguity is bad
  - Leaves meaning of some programs ill-defined
- Ambiguity is common in programming languages
  - Arithmetic expressions
  - IF-THEN-ELSE

Dealing with Ambiguity

- There are several ways to handle ambiguity
- Most direct method is to rewrite grammar unambiguously

$$E \rightarrow T + E \mid T$$
$$T \rightarrow int * T \mid int \mid (E)$$

Enforces precedence of * over +

Ambiguity: The Dangling Else

- Consider the following grammar

$$
\begin{aligned}
S \rightarrow\ & \text{if } C \text{ then } S \\
\mid\ & \text{if } C \text{ then } S \text{ else } S \\
\mid\ & \text{OTHER}
\end{aligned}
$$

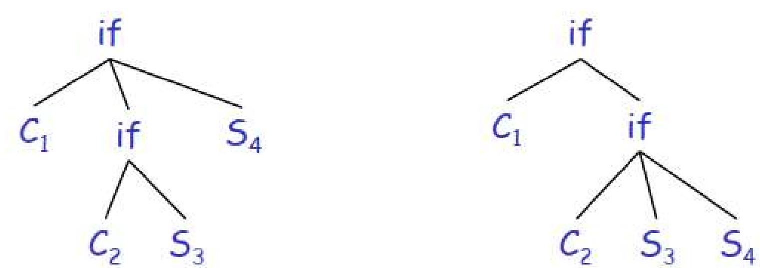
This grammar is also ambiguous

The Dangling Else: Example

- The expression

  if C_1 then if C_2 then S_3 else S_4

  has two parse trees



Typically, we want the second form