

Exercise 02

PAVA - I

c) Perform PAVA on data by hand to obtain Isotonic regression.

Concentrations	x_1	x_2	x_3	x_4	x_5	x_6	x_7
$\hat{\mu}(x_i)$	0.5	2.5	1.5	4.5	2	1.5	5
n_i	3	3	7	8	6	4	2

$$x_2 < x_3 < x_4 < x_5 < x_6 < x_7.$$

$\hat{\mu}(x_i)$ - MLE of responses

n_i - sample sizes, $i = 1, \dots, 7$

\Rightarrow We got the first violation at $(i=2)$.

Since, $\hat{\mu}(x_2) = 2.5 > \hat{\mu}(x_3) = 1.5$

\Rightarrow Hence, we pool the observations for 2 conflicting values:

$$\hat{\mu}(x_{i+1}, x_i) = \frac{n_i \hat{\mu}(x_i) + n_{i+1} \hat{\mu}(x_{i+1})}{n_i + n_{i+1}}$$

$$\hat{\mu}(x_2, x_3) = \frac{n_2 \hat{\mu}(x_2) + n_3 \hat{\mu}(x_3)}{n_2 + n_3} = \frac{3 \times 2.5 + 7 \times 1.5}{3 + 7} = \underline{\underline{1.5}}$$

\Rightarrow Now, $i=2, 3$ are pooled with $\hat{\mu}(x_2, x_3) = \underline{\underline{1.5}}$.

\Rightarrow At $i=4$, $\hat{\mu}(x_4) = 4.5$, we got violation,

Since $\hat{\mu}(x_4) > \hat{\mu}(x_5) = 2$

Hence, $\hat{\mu}(x_4, x_5) = \frac{n_4 \hat{\mu}(x_4) + n_5 \hat{\mu}(x_5)}{n_4 + n_5} = \frac{8 \times 4.5 + 6 \times 2}{8 + 6} = \frac{48}{14} \approx 3.4286 \approx \underline{\underline{3.43}}$

\Rightarrow At $i=6$, we got 2nd violation, since $\hat{\mu}(x_4, x_5) > \hat{\mu}(x_6) = 1.5$

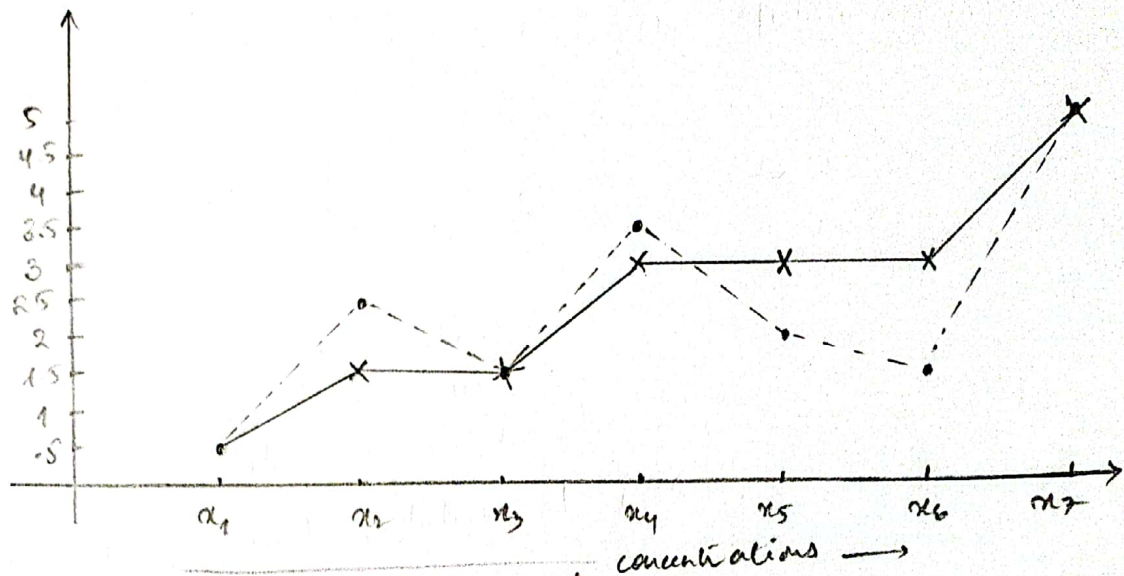
we pooled,
$$\hat{\mu}(x_4, x_5, x_6) = \frac{n_4 x_4 + n_5 x_5 + n_6 x_6}{n_4 + n_5 + n_6} = \frac{8 \times 4.5 + 6 \times 2 + 4 \times 1.5}{8 + 6 + 4} = \frac{54}{18} = \underline{\underline{3.00}}$$

\Rightarrow At $i=7$, $\hat{\mu}(x_4, x_5, x_6) < \hat{\mu}(x_7) = 5$
So, no violation further.
we will stop the algorithm.

Final result :-

$$(0.5, 1.5, 1.5, 3.00, 3.00, 3.00, 5)$$

MCE $\mu(m)$



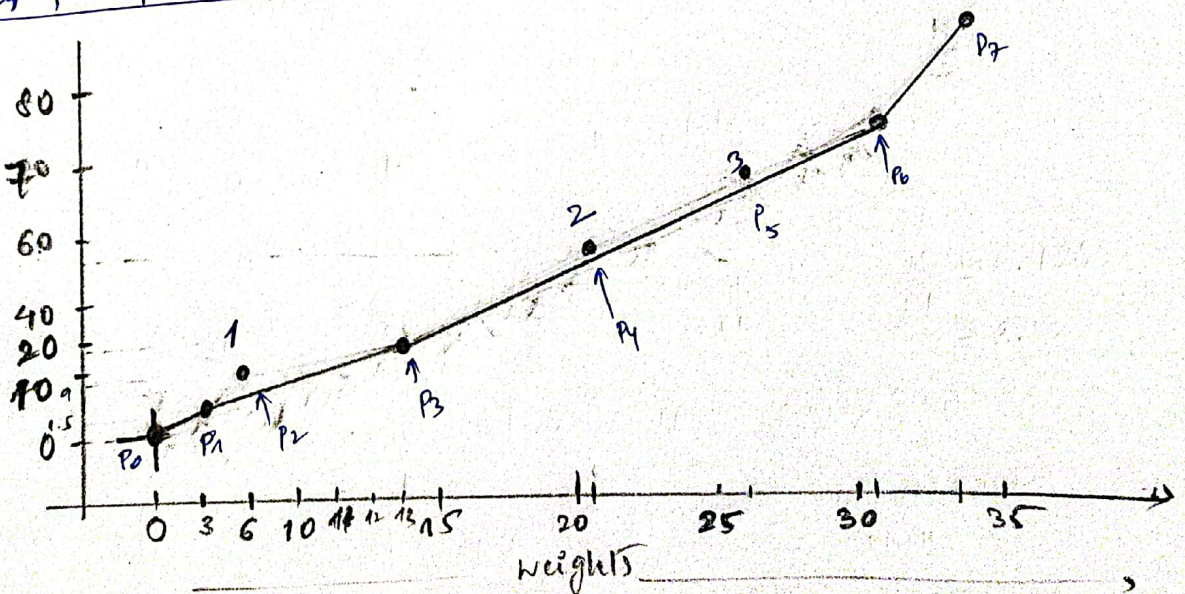
b) Calculate CSD and GCM by hand.

CSD - Cumulative Sum Diagram

GCM - Greatest Convex Minorant.

	w_i	$q(x_i)$
α_1	3	1.5
α_2	6	9
α_3	13	19.5
α_4	21	55.5
α_5	27	67.5
α_6	31	73.5
α_7	33	83.5

Height MCE



The points 1 & 2, 3 are above line of plot GCM, which need to plot in their own segments of line.
 GCM has 4 line segments from $(P_0 \text{ to } P_1, P_1 \text{ to } P_3, P_3 \text{ to } P_6, P_6 \text{ to } P_7)$
 PAVA yields 4 set of results from $P_1 = 0.5$
 4 $P_2, P_3 = 1.5$
 4 $P_4, P_5, P_6 = 3$
 4 $P_7 = 5$

Ans