FLOW RESISTANCE.

An overview of formulae for alluvial channel flow estimation.

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1 INTRODUCTION

There is a need to predict flow in ungauged rivers and to estimate water depths and velocities. However, estimation methods may produce large errors, especially where water depth is small relative to grain size.

The traditional use of these methods has been prediction of flood levels in large rivers, or the flow in hydro-power or irrigation canals. Today, flow estimates for smaller and shallower rivers are needed for ecological assessment and stream restoration. Many of the traditional methods were not developed for use in such conditions. Sophisticated 2d computational hydraulic models are being used which not only contain advanced numerical methods but also rely on a flow resistance equation. Although these models are used to predict the spread of floodwater on flood plains, ways to quantify the actual roughness of floodplains are poorly researched and understood.

2 HOW GOOD ARE FLOW RESISTANCE METHODS?

Where channel beds and banks are uniform, channels are wide and straight, there are no bed forms, no sediment movement or bank-side vegetation, the flow resistance can be confidently predicted. For canals and larger rivers without over-bank flows, good estimates can usually be made. There is less confidence if there are significant over-bank flows. Flow resistance estimation for flood plains is very difficult because of the variation in the “bed” e.g short grass, tall crops ploughed fields, fences, hedges, plantations, isolated buildings, dense dwellings, and so on. There is low confidence in predicting velocities and depths for streams with low water depth to grain size ratios. For these conditions there is much discussion as to the datum from which water depth should be measured.

3 FACTORS AFFECTING FLOW RESISTANCE

- Grain size,
- Bed forms such as ripples, dunes, bars, pools and riffles (bars have been shown to increase a resistance coefficient by 50-75%),

- River sinuosity,
- Water depth (H) to grain size (d) ratio,
- Sediment transport (sediment normally increases flow resistance, but when fine sediment covers a normally rough bed it may reduce the resistance coefficient by 50-70%),
- Cross-section shape, the channel aspect ratio (width:depth) and ratio of bank roughness to bed roughness have a strong influence on overall flow resistance,
- Vegetation (encroaching vegetation increases flow resistance, especially if the section is less than 50 metres wide).

4 TYPES OF FORMULAE INCORPORATING FLOW RESISTANCE

(a) Regime equations,

(b) Resistance formulae with a coefficient (e.g. Manning's n)

(c) Resistance formulae using roughness dimension(s). The roughness dimensions (e.g. median grain size) are used directly or used to calculate the coefficient in (b) above (e.g. Strickler's equation).

Several coefficients for roughness are used. The most common ones are related as follows:

$$\boldsymbol{f} = 8(\boldsymbol{V}*/\overline{\boldsymbol{V}})^2 = 8\,g\,/\,\boldsymbol{C}^2 = 8\,g\,\boldsymbol{n}^2\,/\,H^{1/3} \qquad (\,n \text{ in S.I. units}) \qquad \text{Eq.1}$$

where $\boldsymbol{f}$ is the friction factor, shear velocity $\boldsymbol{V^*} = (gHS)^{0.5}$, g is gravitational force per unit mass, H is the flow depth (use ratio of area to wetted perimeter for narrower channels), S is the energy grade or slope, $\bar{\boldsymbol{V}}$ is the depth averaged velocity in the channel, Chezy's $\boldsymbol{C} = \bar{V} / (HS)^{0.5}$ and $\boldsymbol{n}$ is Manning's coefficient.

Where a coefficient is used it may be determined by:

- Matching the river reach where an estimate is required with pictures of reaches with known resistance coefficient,
- Calibration from measurements made in the channel under consideration, or
- Descriptive methods where the likely resistance contribution of several factors (e.g. grain size, cross-section shape, H/d, bed forms, etc) is estimated and the roughness coefficient is obtained by adding the contributions.

5 CONDITIONS FOR EQUATION DEVELOPMENT

By understanding the conditions under which the various equations were developed and comparing them to the conditions of the channel for which a value is required it may become apparent that some methods may be more appropriate than others. Many of the equations have been developed under the following conditions:

- uniform flow
- Straight rectangular or trapezoidal channels
- Fully developed turbulent flow
- Negligible sediment transport
- Low bank roughness
- Large H/d, which implies deep water and/or small grain size
- Shallow slopes

6 REGIME EQUATIONS

Regime equations calculate velocity or flow from channel dimensions and slope without an explicit resistance parameter. They are usually not dimensionless, i.e. the coefficients depend on the measurement system used. The following regime equations are given in SI units and predict flow (Q) directly using channel slope (S), hydraulic radius (R) and cross-section area (A).

Lacey's (1946) equations were derived for the channel forming discharge of regime channels in relatively fine sediment. They should be most appropriate for relatively high in-bank flows.

$$Q = 10.8\ A\ R^{2/3}\ S^{1/3} \qquad \text{(SI units)} \qquad \text{Eq.2}$$

For bank-full Canadian gravel bed rivers Bray (1982) found

$$Q = 8.0\ A\ R^{0.6}\ S^{0.29} \qquad \text{(SI units)} \qquad \text{Eq.3}$$

Jarrett's (1984) equation is recommended for steep streams with low H/d ratios.

$$Q = 3.17\ AR^{0.83}\ S^{0.12} \qquad \text{(SI units)} \qquad \text{Eq.4}$$

Riggs (1976) equation is:

$$Q = 1.55\ A^{1.33}\ S^{0.05 - 0.056 \log S} \qquad \text{(SI units)} \qquad \text{Eq.5}$$

This relationship was developed from a relatively small sample (62 data points) and not thoroughly validated. According to Dingman and Sharma (1997) the equation gives unreliable predictions when both discharges and Froude numbers (F) are small ($Q < 3\ m^3s^{-1}$ and $F < 0.2$).

Dingman and Sharma (1997) developed an equation using 520 data points and validated it with a further 100 points. It is recommended for use where flows are $> 3\ m^3s^{-1}$ and $F > 0.2$.

$$Q = 1.564\ A^{1.173}\ R^{0.400}\ S^{-0.0543 \log S} \qquad \text{(SI units).} \qquad \text{Eq.6}$$

7 RESISTANCE COEFICIENTS

The most popular resistance coefficient in the English speaking world is Manning's n. It is not always constant for a reach and often gets smaller as flow increases. There are many exceptions to this rule (See Hicks and Mason 1991).

$$V = R^{2/3} \sqrt{S} / n \qquad \text{S.I. units} \qquad \text{Eq.7}$$

Strickler (1923) and Keulegan (1938) presented techniques for estimating flow resistance based on bed particle-size:

$$n = 0.039 d_{50}^{\ 1/6} \qquad \text{Strickler (1923),} \qquad \text{S.I. units} \qquad \text{Eq.8}$$

$$n = 0.035 d_{90}^{\ 1/6} \qquad \text{Keulegan (1938),} \qquad \text{S.I. units} \qquad \text{Eq.9}$$

These techniques provide reasonable estimates of roughness in relatively low gradient streams with sediment of gravel size or smaller and where bed material is the primary source of resistance to flow. Equations such as this do not account for the resistance of bedforms if they exist in the channel. In steeper mountain streams with high relative roughness, these equations are often inaccurate and under-estimate Manning's *n* (Jarrett 1990).

Limerinos (1970), using 50 measurements of discharge and appropriate field surveys at 11 sites in California, related Manning's *n* to hydraulic radius and particle size as follows:

$$n = (0.113\, R^{0.16})/(1.16 + 2.0 \log (R/d_{84})) \qquad \text{(SI Units)} \qquad \text{Eq.10}$$

Jarrett's (1984, 1990) equation is recommended for steep streams with slopes up to 0.09 and for those with low h/d. His equation for *n* is:

$$n = 0.32\, \mathrm{S}^{0.38}\, \mathrm{R}^{-0.16} \qquad \text{SI units} \qquad \text{Eq.11}$$

Sauer's (1990) equation applies to channels with slopes between 0.0003 and 0.018 with *R* up to 5.8 m. This equation is limited to specific applications, such as estimating *n* values on narrow channels with dense stream bank vegetation (Coon 1998).

$$n = 0.11\, S^{0.18}\, R^{0.08} \qquad R \text{ in feet.} \qquad \text{Eq.12}$$

8 FRICTION FACTOR EQUATIONS

These equations are based on evaluating the Darcy-Weisbach friction factor *f* which is then used to calculate velocity.

$$V = \sqrt{(8gRS/f)} \qquad \text{Eq.13}$$

Note that *f* is related to the other resistance coefficients given in Equation 1.
Most equations relate the friction factor to channel dimension(s) and grain size. Where comparisons between measured and predicted friction factors have been made, many

investigations show reaches where the friction factor is more than 25% in error. These investigations would be more representative if measured velocities were compared with those predicted using a friction factor formula because the friction factor is not measured directly and spurious correlation can be introduced. The investigations showed that the following equation of Hey (1979) performs as well as any other equations tested for riffles and runs:

$$1/\sqrt{f} = 2.03 \log(aR/3.5d_{84}) \qquad \text{Eq.14}$$

The coefficient *a* varies between 11.1 and 13.46 and reflects the cross-sectional shape.
Karim and Kennedy (1990) recommend different equations depending on Shield's dimensionless shear stress $\theta = R\ S/((\gamma_s/\gamma\ -1)d)$ where γ_s and γ are the specific weight of the grains and water respectively. Their equations are:

$$\frac{f}{f_o} = 1.2 + 8.92\left[0.08 + 2.24\left(\frac{\theta}{3}\right) - 18.13\left(\frac{\theta}{3}\right)^2 + 70.9\left(\frac{\theta}{3}\right)^3 - 88.33\left(\frac{\theta}{3}\right)^4\right] \qquad \theta < 1.5$$

$$\frac{f}{f_o} = 1.2 \qquad \theta > 1.5 \qquad \text{Eq. 15}$$

where the grain roughness factor $f_o = 8/\left[6.25 + 2.5\ln(H/2.5d_{50})\right]^2$ and d_{50} is the median bed particle size.
Equations which use the Froude number, F, should be treated with caution because the definitions of *f* and F are closely related. When *f* and F are calculated from the same data close correlation can be found even though the predicted velocities may contain large errors.
An example containing F is that of Afzalimehr and Ancil (1998):

$$\sqrt{(1/f)} = 2.03 \log(\Psi h/d) + 2.96F - 0.18\ \tau_s/\tau_c - 0.83 \qquad \text{Eq.16}$$

where $\Psi = (p/w)^{1/2}$ is a cross-sectional form factor, *w* is channel width, *p* is the channel wetted perimeter and $\tau_c = 0.03$ in SI units.

Another equation, given by Colosimo (1988), may avoid the spurious correlation problem as it is designed to be solved iteratively.

$$1/\sqrt{f} = 2.03 \log(\alpha y_m/md_{84}) + (2.54\ F - 1.65) + (0.75 - 0.68Y/Y_c) \qquad \text{Eq. 17}$$

α is a cross-section shape factor, m is calculated from the grain size distribution, y_m is the mean depth, $Y = V^2_* / g d_\phi (\gamma_s / \gamma - 1)$ is a sediment mobility factor γ_s and γ are the specific weight of the grains and the liquid respectively. Y_c is the critical value of Y. V_* is the mean shear velocity and d_ϕ is the particle size for which $_\phi$ percent of the particles are finer.

All of the above equations for f use the bed material grain size, however statistical investigations show there is often a very low correlation between grain size dimension and the roughness parameter. An equation based on the concept of a coefficient of friction for permeable beds has been recently developed by Smart (1999) and uses a parameter M related to the bed dynamic friction angle.

$$1/\sqrt{f} = 2.03 \log(0.368\, M / S) \qquad \text{Eq. 18}$$

M should be calibrated from data at the reach under consideration or from geologically similar reaches (with similar channel slope). When tested on bankfull Canadian gravel bed rivers ($M = 0.33$) and New Zealand gravel bed rivers ($0.15 < M < 2$) this equation gave better predictions of velocity than could be obtained using grain size based equations.

9 DESCRIPTIVE METHODS

One descriptive method is to use references such as Barnes (1967) or Hicks and Mason (1991) for New Zealand rivers, where pictures of rivers with measured Manning's n values can be compared with the river for which a value is required. Tests have shown that only experience engineers are able to choose correct values. However, this method can be used as a rough check on other methods.

Cowan's (1956) component method takes Manning's n values for components such as sediment size, surface irregularity, variation in cross-section, obstruction, vegetation and degree of meandering to obtain a composite n value. Tests show that the success of this method is very dependent on individual assessments. Those trained in lowland streams under estimated n in mountain streams as they were uncomfortable with the large (probably realistic) values obtained using the method. Because of the subjectivity this method is not recommended.

10 CONCLUSIONS

- There is a need to understand the conditions for which flow resistance equations were developed so that they are applied appropriately.
- It is recommended that several appropriate methods be used to obtain a consensus.
- There may be large errors in estimates and conservative values should be used for design purposes.

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