24 April

Thursday

Week - 17th

Days - 114-251

m is prime

$^nC_r \% m$ in $O(n)$ / Using Fermat Theorem

$(a * b) \% m = (a \% m * b \% m) \% m$

$(a / b) \% m \neq (a \% m / b \% m) \% m$

$(a / b) = a * b^{-1}$

$(a / b) \% m = (a * b^{-1}) \% m$

$= (a \% m * b^{-1} \% m) \% m$

Format's theorem :-

It states that if "m" is prime no. then for any integer "a" the number $a^m - a$ is an integral multiple of m

$$a^m - a = x\,m$$

for example $a=2$ & $m=3$

$2^3-2 = 8-2 = 6 = X*3 \quad \checkmark X=2$

$\Rightarrow (a^m - a = X*m) \;\%\; m$

$\Rightarrow (a^m - a) \;\%\; m = (X*m) \;\%\; m$

$\Rightarrow (a^m \;\%\; m - a \;\%\; m) \;\%\; m = (X \;\%\; m * \underbrace{m \;\%\; m}_{0}) \;\%\; m$

$\Rightarrow (a^m \;\%\; m - a \;\%\; m) \;\%\; m = 0$

$\Rightarrow a^m \;\%\; m = a \;\%\; m$

multiplying a^{-1} both side

$\Rightarrow (a^m \;\%\; m)\, a^{-1} = (a \;\%\; m)\, a^{-1}$

$\Rightarrow a^{m-1} \;\%\; m = a^{-1} \;\%\; m$

$\Rightarrow \boxed{a^{-1} \;\%\; m = a^{m-2} \;\%\; m}$ //

So, $(a/b) \;\%\; m = (a * b^{-1}) \;\%\; m = (a \;\%\; m * b^{-1} \;\%\; m) \;\%\; m$

$\boxed{(a/b) \;\%\; m = (a \;\%\; m * b^{m-2} \;\%\; m) \;\%\; m}$

APRIL '14

M	T	W	T	F	S	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30				

$$^{n}C_{r} \,\%\, m = \left(\frac{\lfloor n}{\lfloor n-r \,\lfloor r}\right) \%\, m$$

$$= \left(\lfloor n \,\%\, m * \lfloor (n-r)^{-1} \,\%\, m * \lfloor r^{-1} \,\%\, m\right) \%\, m$$

$$= \left(\lfloor n \,\%\, m * \lfloor r^{m-2} \,\%\, m * \lfloor (n-r)^{m-2} \,\%\, m\right) \%\, m$$

so,

$$\left[{}^{n}C_{r} \,\%\, m = \left(\lfloor n \,\%\, m * \lfloor r^{m-2} \,\%\, m * \lfloor (n-r)^{m-2} \,\%\, m\right) \%\, m\right]$$

Complexity $O(m+n)$

it will good if $m < 10^7$

SUNDAY 27