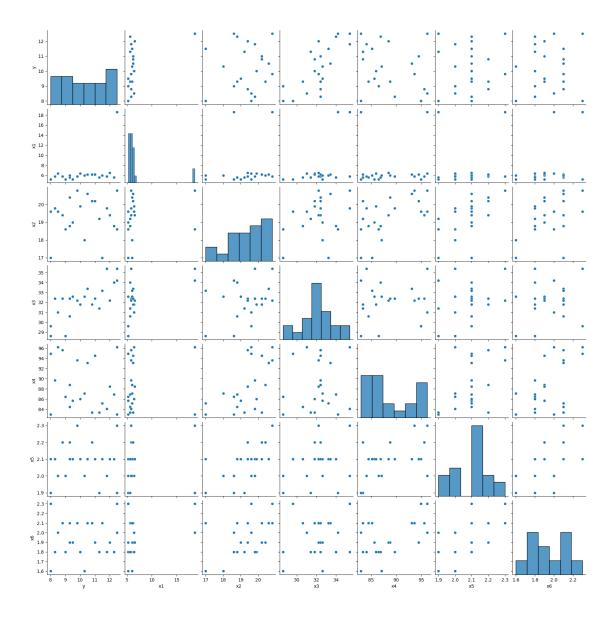
# Bhandari\_Nischal\_in\_Class\_assignment

# September 26, 2023

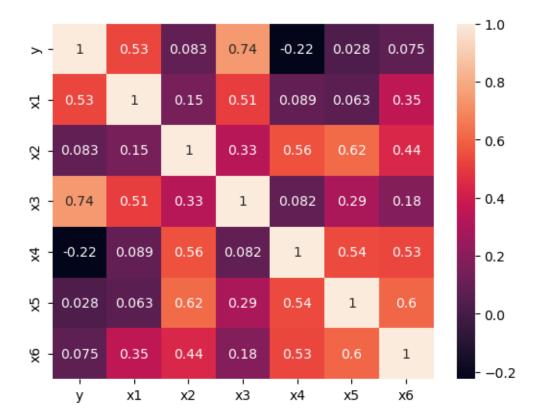
```
[1]: import pandas as pd
     wire_df = pd.read_excel('wire_bond.xlsx', index_col=0, header=0)
     wire_df.head()
[1]:
                                         ×1
                                                      x2
                                                                    x3 \
                              У
     observation
    NaN
                  pull strength
                                 die height post height
                                                          loop height
     1.0
                                        5.2
                              8
                                                       17
                                                                  28.6
    2.0
                                        5.2
                                                                  29.6
                              8
                                                     19.6
     3.0
                            8.3
                                        5.8
                                                     19.8
                                                                  32.4
     4.0
                            8.5
                                        6.4
                                                     19.6
                                                                    31
                            x4
                                                   x5
                                                                            ×6
     observation
    NaN
                  wire lenghth
                                bond width on the die
                                                       bond width on the post
     1.0
                            83
                                                   1.9
                                                                           1.6
     2.0
                          94.9
                                                  2.1
                                                                           2.3
     3.0
                          89.7
                                                  2.1
                                                                           1.8
     4.0
                          96.2
                                                    2
                                                                             2
[2]: # removing the first row
     wire_df = wire_df.iloc[1:]
     # renaming the columns
     wire_df.columns = ['y', 'x1', 'x2', 'x3', 'x4', 'x5', 'x6']
     # remove first column and set index default
     wire_df = wire_df.iloc[:, 0:].reset_index(drop=True)
     wire_df.head()
[2]:
                    x2
                          x3
                                x4
                                     x5
                                          x6
          У
              x1
     0
          8
            5.2
                    17
                        28.6
                                83
                                   1.9
                                         1.6
          8 5.2 19.6
     1
                       29.6
                            94.9
                                    2.1
                                         2.3
     2 8.3 5.8 19.8
                       32.4
                            89.7
                                    2.1
                                        1.8
     3 8.5 6.4
                  19.6
                              96.2
                                      2
                                           2
                          31
     4 8.8 5.8 19.4 32.4
                              95.6 2.2 2.1
```

```
[3]: wire_df.describe()
[3]:
                          x2
                                xЗ
                                      x4
                                            x5
                                                  x6
                    x1
               У
    count
            22.0 22.0 22.0 22.0 22.0
                                          22.0
                                                22.0
    unique 19.0
                   8.0 14.0 14.0 19.0
                                           5.0
                                                 6.0
    top
             8.0
                   5.2 17.0 32.4 83.0
                                           2.1
                                                 1.8
             2.0
                   4.0
                         2.0
                                     2.0 10.0
                                                 6.0
    freq
                               4.0
[4]: # checking for missing values
    wire_df.isna().sum()
[4]: y
          0
    x1
          0
    x2
          0
    xЗ
          0
    x4
    x5
          0
    x6
          0
    dtype: int64
[5]: # data analysis
    import seaborn as sns
    import matplotlib.pyplot as plt
    sns.pairplot(wire_df)
    plt.show()
```



pull strength seems to be positively correlated with die height (x1) and bond width on the post (x6)

```
[21]: # CORRELATION MATRIX
import seaborn as sns
corrMatrix = wire_df.corr()
sns.heatmap(corrMatrix, annot=True)
plt.show()
```



yeah, i was right above. only to the extent though. x1 has strong r coefficient than x6. x3 has the strongest correlation of all. let's see how the model places weights in these features.

## 0.0.1 Trying Linear Regression First

```
# Make predictions using the testing set
y_pred = regressor.predict(X_test)

# Evaluate the model
print('Mean Absolute Error:', metrics.mean_absolute_error(y_test, y_pred))
print('Mean Squared Error:', metrics.mean_squared_error(y_test, y_pred))
print('Root Mean Squared Error:', np.sqrt(metrics.mean_squared_error(y_test, u_sy_pred)))
```

Mean Absolute Error: 0.7758713710719516 Mean Squared Error: 1.1796743010856483 Root Mean Squared Error: 1.086128123697038

```
[9]: from sklearn.metrics import r2_score
    r2_score = r2_score(y_true=y_test,y_pred=y_pred)
    print(r2_score)
```

#### 0.455467918627378

The amount of variation explained is 45 %. So only 45% of the variation in the change in strength is explained by the other variables if linear regression method is used.

## 0.0.2 Next: Ordinary Least Square

### OLS Regression Results

Dep. Variable:	у	R-squared:	0.677
Model:	OLS	Adj. R-squared:	0.548
Method:	Least Squares	F-statistic:	5.244
Date:	Tue, 26 Sep 2023	Prob (F-statistic):	0.00430
Time:	15:29:46	Log-Likelihood:	-27.051
No. Observations:	22	AIC:	68.10
Df Residuals:	15	BIC:	75.74
Df Model:	6		

	coef	std err	t	P> t	[0.025	0.975]
Intercept	0.6737	5.891	0.114	0.910	-11.883	13.231
x1	0.0723	0.076	0.956	0.354	-0.089	0.233
x2	0.0226	0.286	0.079	0.938	-0.587	0.633

x3	0.5602	0.156	3.601	0.003	0.229	0.892	
x4	-0.0978	0.061	-1.592	0.132	-0.229	0.033	
x5	-1.0217	3.021	-0.338	0.740	-7.461	5.417	
x6	0.7078	1.664	0.425	0.677	-2.839	4.255	
=======		.=======			========		
Omnibus:		0.	.490 Durbi	in-Watson:		1.438	
Prob(Omnik	ous):	0.	.783 Jarqu	ie-Bera (JB):		0.288	
Skew:		0.	.265 Prob	(JB):		0.866	
Kurtosis:		2.	.814 Cond.	Cond. No.		2.67e+03	

#### Notes:

- [1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
- [2] The condition number is large, 2.67e+03. This might indicate that there are strong multicollinearity or other numerical problems.

In general, multiple linear regression would be applicable for this data since the data types in each columns is numerical. If we choose different models, OLS performs better because it accounts to  $\sim 68\%$  in the variability of pull strength based on all the features used compared to the linear regression model which only accounted for  $\sim 45\%$ .

The p-values for all features are greater than alpha value of 0.05 except x3 where alpha value where it is 0.03. So we reject the null hypothesis in case of x3.

In the hindsight, the correlation matrix also show a strong, positive correlation between y and x3.

The R-squared value for OLS model is 0.677. So it explains ~68\% of variability.

```
[11]: # 95% Confidence interval for each of the j's in your model
    confidence_interval = lm_mul.conf_int(alpha=0.05)
    print(' 95 % Confidence Interval:\n', confidence_interval)
```

## 95 % Confidence Interval:

```
0
                                 1
Intercept -11.883244 13.230595
           -0.088842
                        0.233431
x1
x2
           -0.587462
                        0.632592
xЗ
             0.228599
                        0.891846
x4
           -0.228725
                        0.033129
           -7.460726
                        5.417317
x5
           -2.839084
                        4.254750
x6
```

Interpretation of Confidence Interval of Model Coefficients  $\beta_0$  and  $\beta_1$  The 95% confidence interval for  $\beta_0$  is [-11.883244, 13.230595] and the 95% confidence interval for other features varies.

We can conclude that in the absence of any other affects, pull strength of a wire, on average, fall somewhere between -11.883244 and 13.230595 units.

Other features have their own magnitude of effects. For example, one with the most positive effect

on the pull strength of a wire, pull strength increases from 22 to 89 units in general when loop height is increased by 100 units.

```
[12]: # just looking at the slope of x4 in multiple linear regression
slope_unit4 = lm_mul.params['x4']
print('Slope of wire length 4:', slope_unit4)
```

Slope of wire length 4: -0.09779833603224988

```
[13]: # printing all weights
lm_mul.params
```

```
[13]: Intercept 0.673675

x1 0.072295

x2 0.022565

x3 0.560223

x4 -0.097798

x5 -1.021704

x6 0.707833
```

dtype: float64

holding all features fixed, a unit change in x4 decresease the average value of y by 0.098 iin multiple linear regression.

d) Holding all else fixed, how does a unit change in x4 change the average value of y? ### Simple Linear Regression As SLS ignores the effect of other five variables and focus on mostly x4, let's try it.

```
[14]: import statsmodels.formula.api as smf

# 3. Train the model
lm = smf.ols(formula='y ~ x4', data=wire_df).fit()

# coefficients of the trained model
lm.params
```

[14]: Intercept 16.465740 x4 -0.070386 dtype: float64

Interpretation of Model Coefficients  $\beta_0$  and  $\beta_1$ 

$$y = \beta_0 + \beta_1 x = 16.465740 - 0.070386 \times x4$$

 $\beta_1=0.070386$ : An additional 1000 on wire length is associated with decreasing of pulling strength by approximately 704 of the wire. Note: excluding other features increase the effect of wirelength (from -0.098 to -0.070)

e. For a specimen with x1 = 5.5, x2 = 19.3, x3 = 30.2, x4 = 90, x5 = 2, and x6 = 1.85 find the predicted value of y.

## 0.0.3 Fitting a Multiple Linear Regression model

```
Y = \beta_0 + \beta_1 X_1 + \dots + \beta_n X_n
```

```
[15]: lm_mul = smf.ols(formula='y \sim x1 + x2 + x3 + x4 + x5 + x6', data=wire_df).fit()
```

The predicted value of y for the given specimen is: 8.889750471014537

# 0.0.4 Checking Model Assumptions

Predicted value of y for the randomly selected data is: 6 9.722465

```
dtype: float64
       9.3
У
       5.6
x1
      18.8
x2
xЗ
      30.6
      84.5
x4
       2.1
x5
x6
       2.1
Name: 6, dtype: float64
```

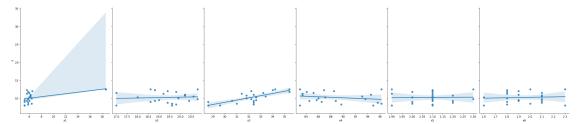
The predicted value is 9.722465 where as the actual value is 9.3, so the model is not super reliable but not extremely worse either.

## 0.0.5 Checking model assumptions

```
[18]: # linearity
import seaborn as sns
import matplotlib.pyplot as plt
```

```
sns.pairplot(data = wire_df, x_vars=['x1', 'x2', 'x3', 'x4', 'x5', 'x6'],u

y_vars=['y'],height=6, aspect=0.8, kind='reg')
plt.show()
```



```
[19]: # Mean of residuals for linear regression
residuals = y.values.reshape(-1,1)-y_pred
mean_residuals = np.mean(residuals)
print("Mean of Residuals for linear regression {}".format(mean_residuals))
```

Mean of Residuals for linear regression -0.25139825017226475

```
[20]: # Checking mean of residuals for OLS
mean_lm_mul_resid = np.mean(lm_mul.resid)
print("Mean of Residuals for OLS method {}".format(mean_lm_mul_resid))
```

Mean of Residuals for OLS method 8.881784197001252e-16

the assumptions of linear regression that the mean of the residuals should be zero is not validated by our models. But OLS performs much better and the mean difference is close to zero.

[]: