

# 12\_MLP\_Architectures\_MNIST

April 11, 2019

```
In [1]: # import all required packages
import pandas as pd
import numpy as np
from datetime import datetime

# package for MLP
import tensorflow as tf

# Plot related packages
import matplotlib.pyplot as plt
import seaborn as sns
sns.set()

# math package
import math

# dataset related packages
from tensorflow.examples.tutorials.mnist import input_data
```

```
/home/amd_3/anaconda3/lib/python3.6/site-packages/h5py/__init__.py:36: FutureWarning: Conversion
from ._conv import register_converters as _register_converters
```

## 0.1 Load the data

```
In [2]: mnist = input_data.read_data_sets("MNIST_data/", one_hot=True)
```

```
Extracting MNIST_data/train-images-idx3-ubyte.gz
Extracting MNIST_data/train-labels-idx1-ubyte.gz
Extracting MNIST_data/t10k-images-idx3-ubyte.gz
Extracting MNIST_data/t10k-labels-idx1-ubyte.gz
```

The data is already standardized to fall in the range 0.0 to 1.0

## 1 UTIL functions

```
In [3]: def plot_loss_curve(train_metric_list, val_metric_list):
        """
```

```

This function plots the loss curve for train and validation
"""

# get x-label list
epoch_list = range(1, len(train_metric_list) + 1 )

# get train accuracy data
train_acc_list = [ item[1] for item in train_metric_list]

# get validation accuracy data
val_acc_list = [ item[1] for item in val_metric_list]

# plot both train, validation curve
plt.plot(epoch_list, train_acc_list, label='Train Loss', color='r')
plt.plot(epoch_list, val_acc_list, label='Validation Loss', color='b')
plt.xlabel('Training Epoch')
plt.ylabel('Cross Entropy Error')
plt.title('Training Loss Vs Validation Loss')
plt.legend()
plt.show()

In [4]: def plot_accuracy_curve(train_metric_list, val_metric_list):
"""
This function plots the accuracy curve for train and validation
"""

# get x-label list
epoch_list = range(1, len(train_metric_list) + 1 )

# get train accuracy data
train_acc_list = [ item[2] for item in train_metric_list]

# get validation accuracy data
val_acc_list = [ item[2] for item in val_metric_list]

# plot both train, validation curve
plt.plot(epoch_list, train_acc_list, label='Train Accuracy', color='r')
plt.plot(epoch_list, val_acc_list, label='Validation Accuracy', color='b')
plt.xlabel('Training Epoch')
plt.ylabel('Accuracy')
plt.title('Training Accuracy Vs Validation Accuracy')
plt.legend()
plt.show()

```

## 2 MODEL

```

In [5]: # declare placeholders for input and output
X = tf.placeholder(tf.float32, [None, 784])

```

```

y = tf.placeholder(tf.float32, [None, 10])
keep_probability = tf.placeholder(tf.float32) # This is for dropout in a layer

In [6]: def train_and_evaluate_model(custom_model, keep_prob_val, num_epochs = 2):
        """
        This function train and evaluate the model
        """

        batch_size = 100
        total_batches = int(mnist.train.num_examples / batch_size)

        # define the loss function
        cee = tf.reduce_mean(-tf.reduce_sum(y * tf.log(custom_model), reduction_indices=[1])
        # defien the train step
        train_step = tf.train.AdamOptimizer(1e-03).minimize(cee)
        # define the accuracy
        accuracy = tf.reduce_mean(tf.cast(tf.equal(tf.argmax(y,1), tf.argmax(custom_model,1))

        # declare two list for holding loss, accuracy for both train, validation
        train_metric_list = list()
        val_metric_list = list()

        # create a session and execute the code
        with tf.Session() as sess:

            # initialize variables
            tf.global_variables_initializer().run()

            # run multiple epochs
            for epoch in range(1, num_epochs + 1):

                # run batch by batch
                for batch_id in range(total_batches):

                    # get the train data into features and labels
                    X_train, y_train = mnist.train.next_batch(batch_size)

                    # run the training
                    _ = sess.run([train_step], feed_dict={X:X_train, y:y_train, keep_probabi

                    # find predicted value, loss and accuracy for both train & test data sets
                    tr_pred, tr_cee, tr_acc = sess.run([custom_model, cee, accuracy], feed_dict=
                                                                y:mnist.t
                                                                keep_prob

                train_metric_list.append((tr_pred, tr_cee, tr_acc,))

```

```

        val_pred, val_cek, val_acc = sess.run([custom_model, cek, accuracy], feed_dict={X : mnist.train.images, y:mnist.train.labels, keep_prob:keep_prob})

        val_metric_list.append((val_pred, val_cek, val_acc,))

    # Test the model
    ts_pred, ts_cek, ts_acc = sess.run([custom_model, cek, accuracy], feed_dict={X : mnist.train.images, y : mnist.train.labels, keep_prob:keep_prob})

    test_loss = ts_cek.mean()

    print('Test accuracy of model :%f, Test loss:%f'%(ts_acc, test_loss,))

    return (test_loss, ts_acc, train_metric_list, val_metric_list,)

```

## 2.1 a) MLP with 2 hidden layer, relu

```

In [7]: def build_model_a(X):
    """
    This functions builds the architecture for the MLP.
    """

    # model architecture
    num_input_neurons = 784
    num_hidden_neurons_1 = 512
    num_hidden_neurons_2 = 128
    num_output_neurons = 10

    # for relu activation function we need to use He normal initialization of weights as
    # If we sample weights from a normal distribution N(0,) we satisfy this condition with
    # weights => = (2/(fan_in+1)) => N(0,)

    # relu activation specific standard deviations
    l1_stddev = math.sqrt(2 / (num_input_neurons + 1))
    l2_stddev = math.sqrt(2 / (num_hidden_neurons_1 + 1))
    out_stddev = math.sqrt(2 / (num_hidden_neurons_2 + 1))

    # ===== hidden layer 1 =====
    l1_weights = tf.Variable(tf.random_normal(shape=[num_input_neurons, num_hidden_neurons_1],
                                                mean=0.0, stddev=l1_stddev))
    l1_biases = tf.Variable(tf.random_normal(shape=[num_hidden_neurons_1], mean=0.0, stddev=l1_stddev))

    l1_net_input = tf.matmul(X, l1_weights) + l1_biases
    l1_output = tf.nn.relu(l1_net_input) # activation function
    #=====

```

```

# ===== hidden layer 2 =====
l2_weights = tf.Variable(tf.random_normal(shape=[num_hidden_neurons_1, num_hidden_neurons_2],
                                           mean=0.0, stddev=l2_stddev))
l2_biases = tf.Variable(tf.random_normal(shape=[num_hidden_neurons_2], mean=0.0, stddev=l2_stddev))

l2_net_input = tf.matmul(l1_output, l2_weights) + l2_biases
l2_output = tf.nn.relu(l2_net_input) # activation function
#=====

# ===== output layer =====
l3_weights = tf.Variable(tf.random_normal(shape=[num_hidden_neurons_2, num_output_neurons],
                                           mean=0.0, stddev=out_stddev))
l3_biases = tf.Variable(tf.random_normal(shape=[num_output_neurons], mean=0.0, stddev=out_stddev))

l3_net_input = tf.matmul(l2_output, l3_weights) + l3_biases

# =====

# return output using the softmax layer
output = tf.nn.softmax(l3_net_input)

return output

```

```

In [8]: # build the model a
mlp_a = build_model_a(X)

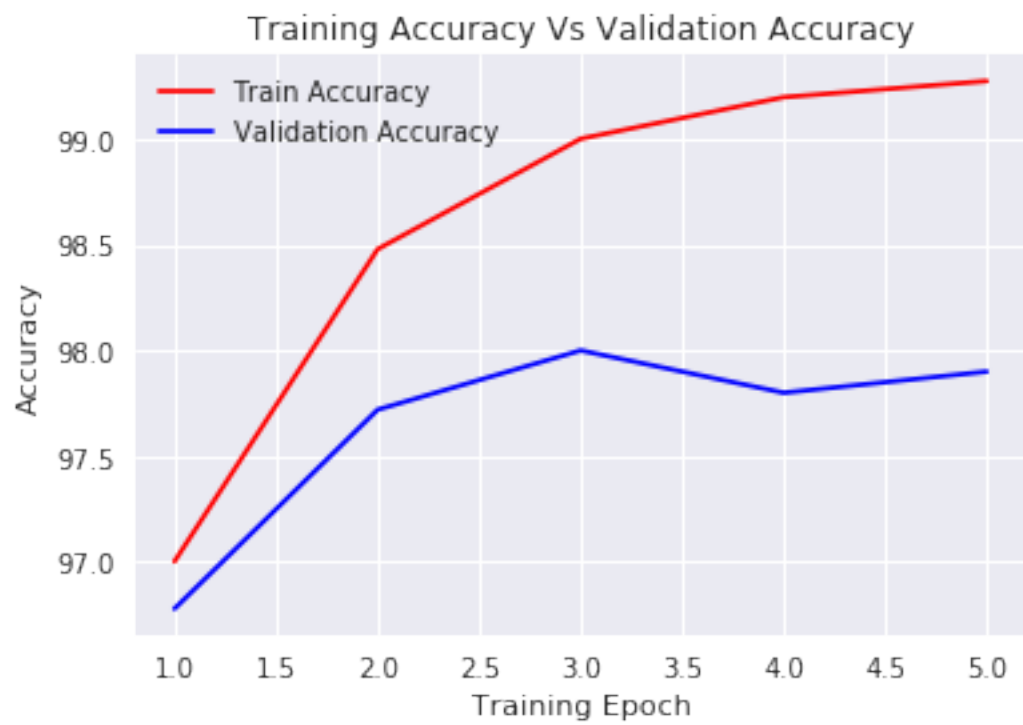
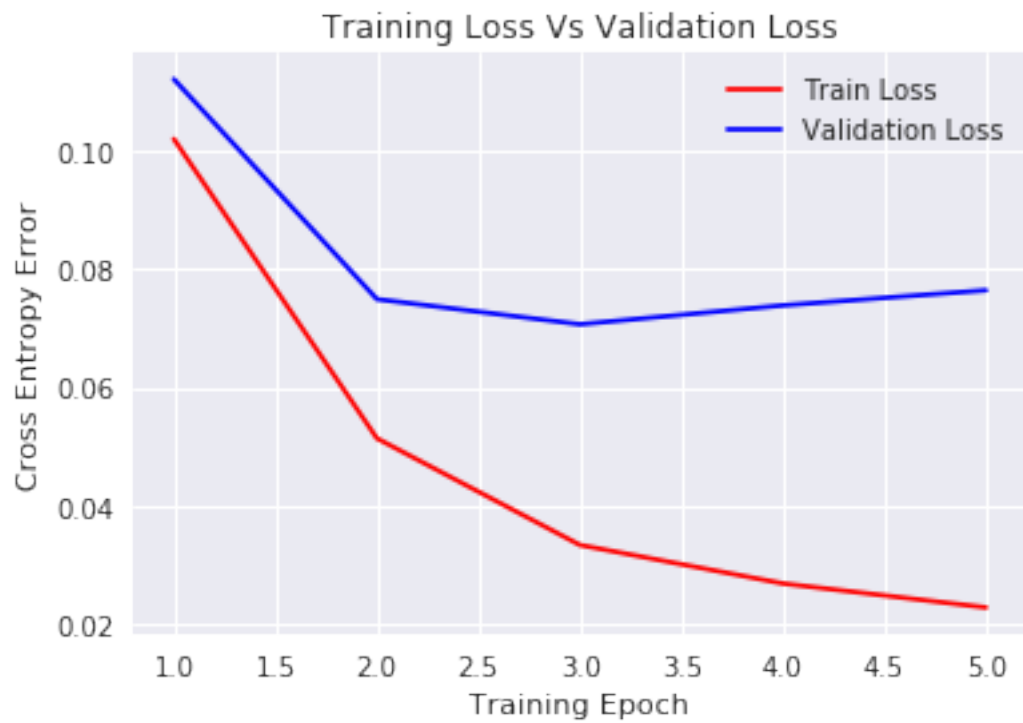
# train and evaluate the model
keep_prob_val = 1.0
num_epochs = 5

ts_loss_mlp_a, ts_acc_mlp_a, train_metric_list, val_metric_list = train_and_evaluate_model(mlp_a, X, y, num_epochs, keep_prob_val)

# plot the loss curve
plot_loss_curve(train_metric_list, val_metric_list)
# plot the accuracy curve
plot_accuracy_curve(train_metric_list, val_metric_list)

```

Test accuracy of model :98.000000, Test loss:0.074359



### 2.1.1 Observation

As the number of epochs pass 3, validation loss starts increasing slightly

## 2.2 b) MLP with 3 Hidden layers, dropout, batch normalization

```
In [9]: def build_model_b(X):
        """
        This functions builds the architecture for the MLP.
        """

        # model architecture
        num_input_neurons = 784
        num_hidden_neurons_1 = 432
        num_hidden_neurons_2 = 234
        num_hidden_neurons_3 = 110
        num_output_neurons = 10

        # for relu activation function we need to use He normal initialization of weights as
        # If we sample weights from a normal distribution  $N(0,)$  we satisfy this condition wi
        # weights =>  $(2/(fan\_in+1)) \Rightarrow N(0,)$ 

        # relu activation specific standard deviations
        l1_stddev = math.sqrt(2 / (num_input_neurons + 1))
        l2_stddev = math.sqrt(2 / (num_hidden_neurons_1 + 1))
        l3_stddev = math.sqrt(2 / (num_hidden_neurons_2 + 1))
        out_stddev = math.sqrt(2 / (num_hidden_neurons_3 + 1))

        # ===== hidden layer 1 =====
        l1_weights = tf.Variable(tf.random_normal(shape=[num_input_neurons, num_hidden_neurons_1],
                                                    mean=0.0, stddev=l1_stddev))
        l1_biases = tf.Variable(tf.random_normal(shape=[num_hidden_neurons_1], mean=0.0, stddev=l1_stddev))

        l1_net_input = tf.matmul(X, l1_weights) + l1_biases
        l1_output = tf.nn.relu(l1_net_input) # activation function
        #=====

        # ===== hidden layer 2 =====
        l2_weights = tf.Variable(tf.random_normal(shape=[num_hidden_neurons_1, num_hidden_neurons_2],
                                                    mean=0.0, stddev=l2_stddev))
        l2_biases = tf.Variable(tf.random_normal(shape=[num_hidden_neurons_2], mean=0.0, stddev=l2_stddev))

        l2_net_input = tf.matmul(l1_output, l2_weights) + l2_biases

        # apply batch normalization
        # Calculate the mean and variance of x.
        batch_mean_l2, batch_var_l2 = tf.nn.moments(l2_net_input, [0])
```

```

alpha_l2 = tf.Variable(tf.ones([num_hidden_neurons_2]))
beta_l2 = tf.Variable(tf.zeros([num_hidden_neurons_2]))

# do batch normalization on net input
epsilon = 1e-03
l2_net_input = tf.nn.batch_normalization(l2_net_input, batch_mean_l2, batch_var_l2,
                                         beta_l2, alpha_l2, epsilon)

l2_output = tf.nn.relu(l2_net_input) # activation function
l2_output = tf.nn.dropout(l2_output, keep_prob=keep_probability)
#=====

# ===== hidden layer 3 =====
l3_weights = tf.Variable(tf.random_normal(shape=[num_hidden_neurons_2, num_hidden_neurons_3],
                                           mean=0.0, stddev=l3_stddev))
l3_biases = tf.Variable(tf.random_normal(shape=[num_hidden_neurons_3], mean=0.0, stddev=l3_stddev))

l3_net_input = tf.matmul(l2_output, l3_weights) + l3_biases

# apply batch normalization
# Calculate the mean and variance of x.
batch_mean_l3, batch_var_l3 = tf.nn.moments(l3_net_input, [0])

alpha_l3 = tf.Variable(tf.ones([num_hidden_neurons_3]))
beta_l3 = tf.Variable(tf.zeros([num_hidden_neurons_3]))

# do batch normalization on net input
epsilon = 1e-03
l3_net_input = tf.nn.batch_normalization(l3_net_input, batch_mean_l3, batch_var_l3,
                                         beta_l3, alpha_l3, epsilon)

l3_output = tf.nn.relu(l3_net_input) # activation function
l3_output = tf.nn.dropout(l3_output, keep_prob=keep_probability)
#=====

# ===== output layer =====
l4_weights = tf.Variable(tf.random_normal(shape=[num_hidden_neurons_3, num_output_neurons],
                                           mean=0.0, stddev=out_stddev))
l4_biases = tf.Variable(tf.random_normal(shape=[num_output_neurons], mean=0.0, stddev=out_stddev))

l4_net_input = tf.matmul(l3_output, l4_weights) + l4_biases

```



```

# =====

# return output using the softmax layer
output = tf.nn.softmax(l4_net_input)

return output

In [10]: # build the model a
mlp_b = build_model_b(X)

#keep_prob_val = 0.92
keep_prob_val_list_b = [0.80, 0.84, 0.88, 0.92]
num_epochs = 5

# decalre a list for all the dropout value metrics
metric_list_b = list()

# try with different dropout rate
for keep_prob_val in keep_prob_val_list_b:

    print('='*100)

    print('With keep probability: %f'%keep_prob_val)

    # train and evaluate the model
    ts_loss_mlp_b, ts_acc_mlp_b, train_metric_list, val_metric_list = train_and_evaluate(
                                                                    keep_prob_val,

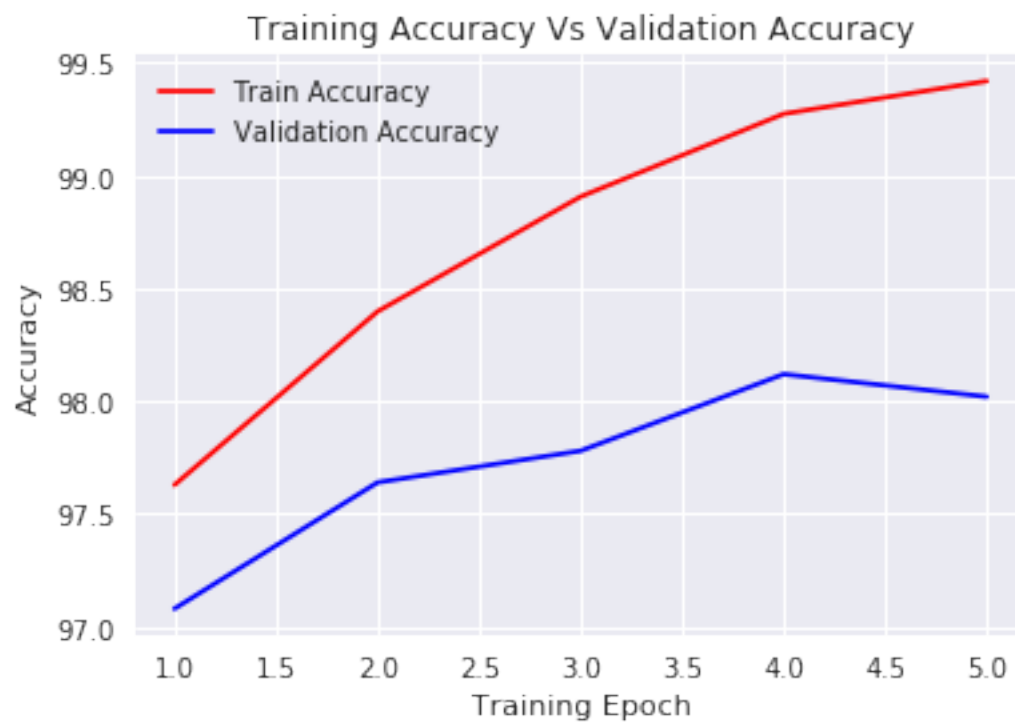
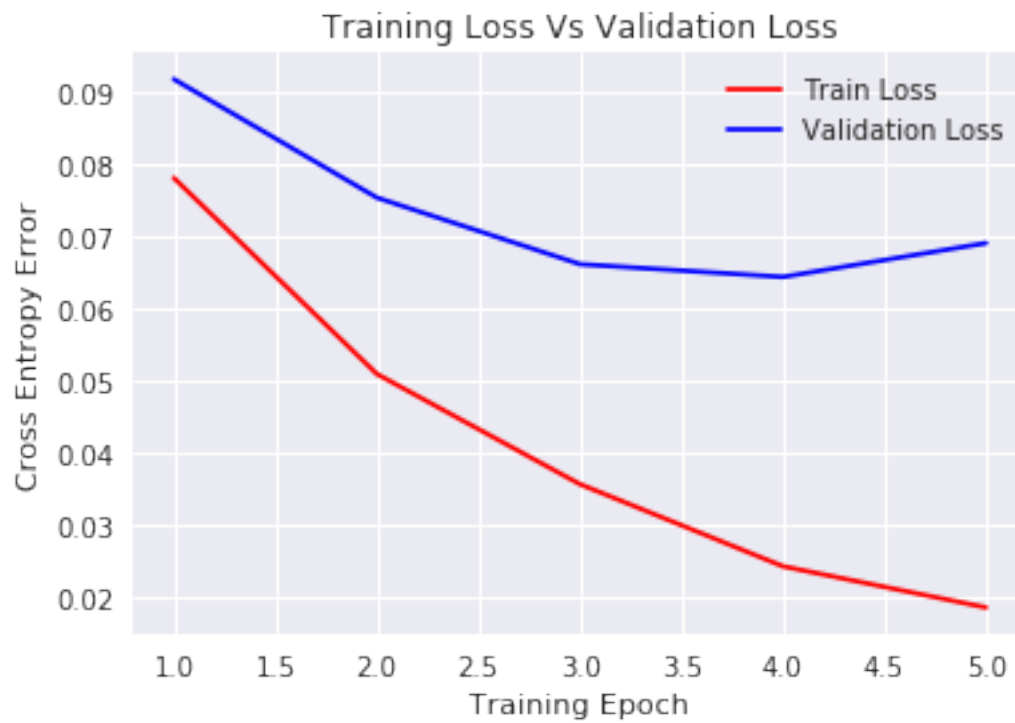
    # append to list
    metric_list_b.append(( ts_loss_mlp_b, ts_acc_mlp_b,))

    # plot the loss curve
    plot_loss_curve(train_metric_list, val_metric_list)
    # plot the accuracy curve
    plot_accuracy_curve(train_metric_list, val_metric_list)

    print('='*100)

=====
With keep probability: 0.800000
Test accuracy of model :98.009995, Test loss:0.069162

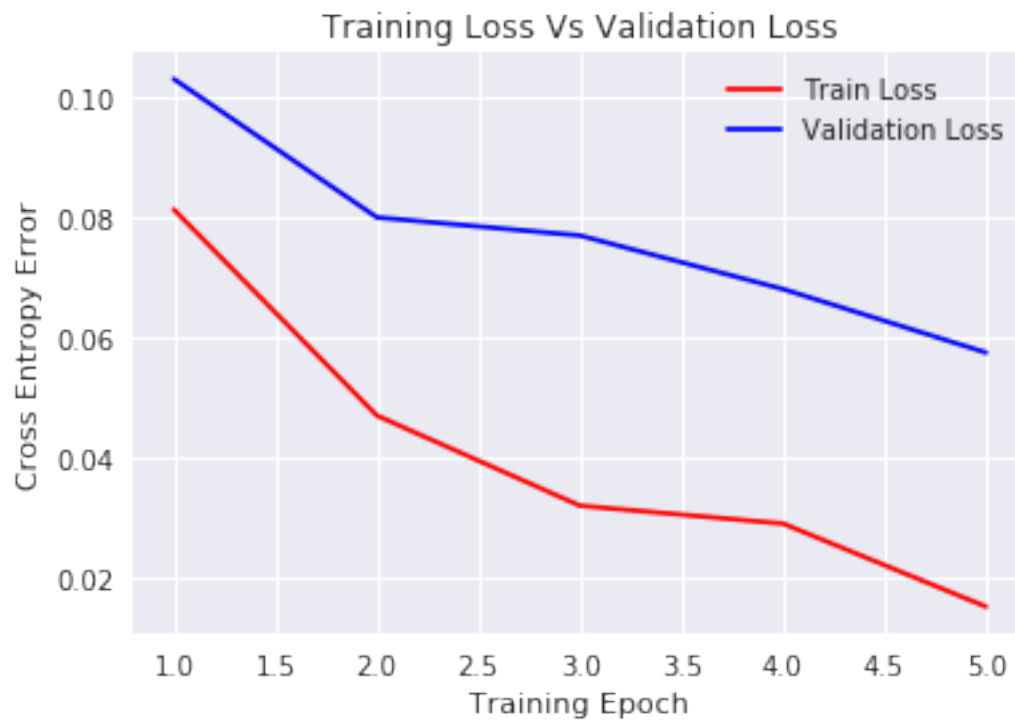
```

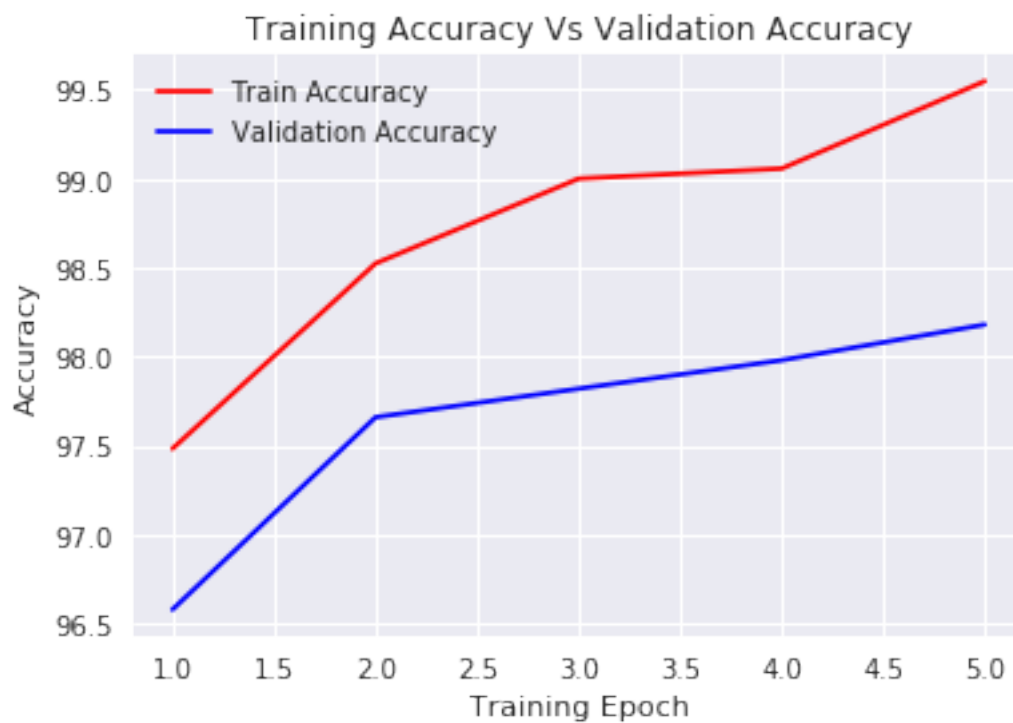


=====

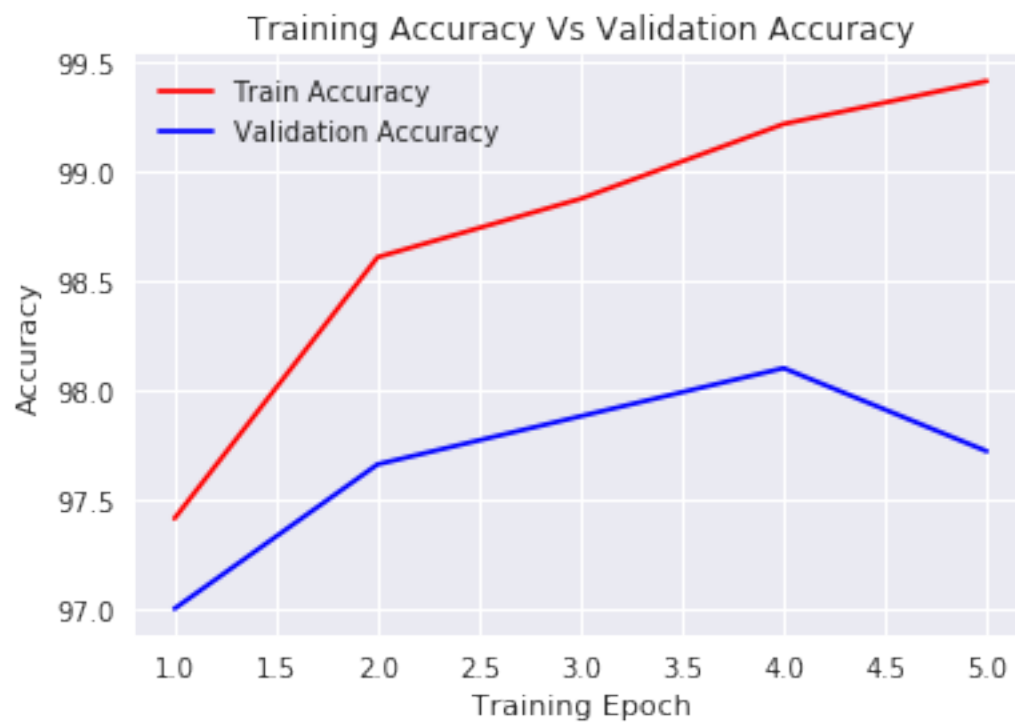
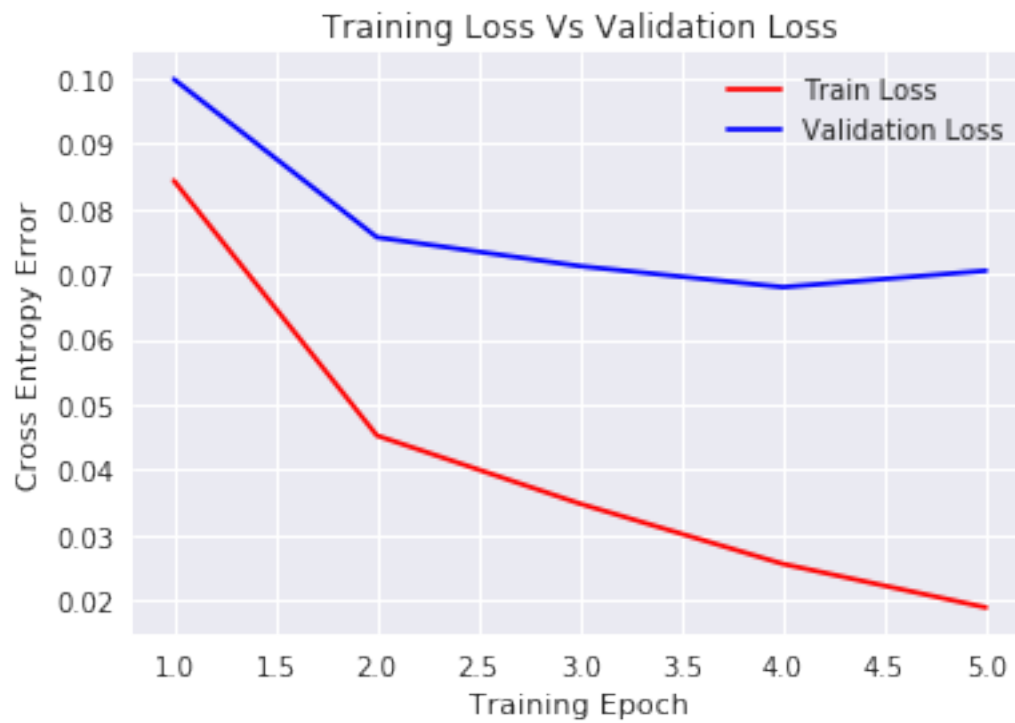
With keep probability: 0.840000

Test accuracy of model :98.199997, Test loss:0.061258





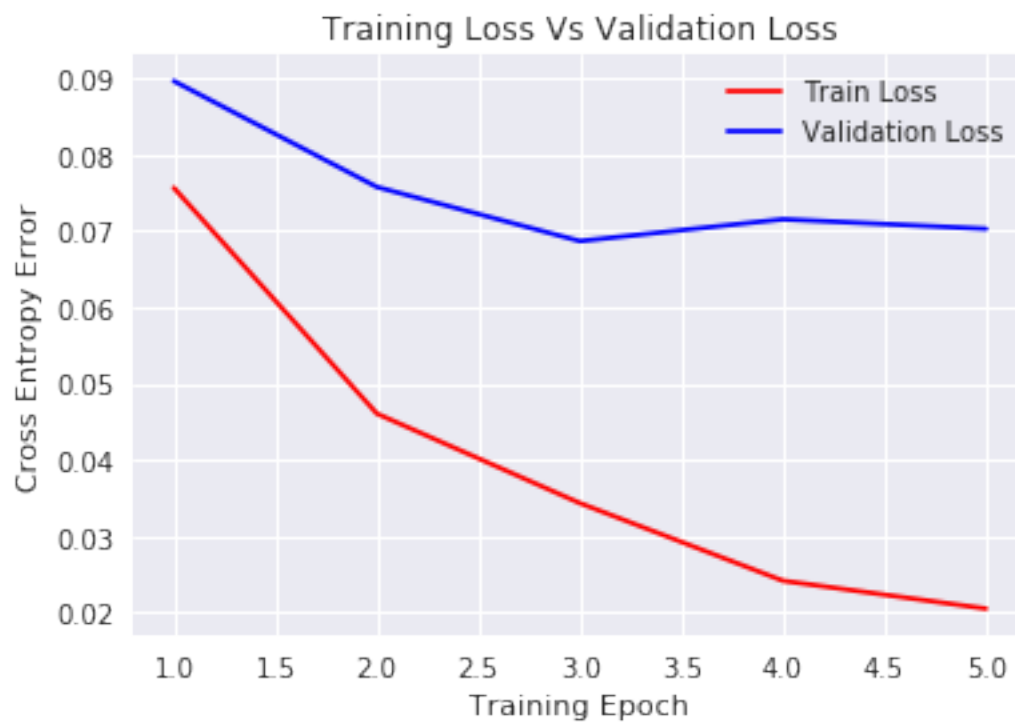
=====  
=====  
With keep probability: 0.880000  
Test accuracy of model :97.939995, Test loss:0.067430

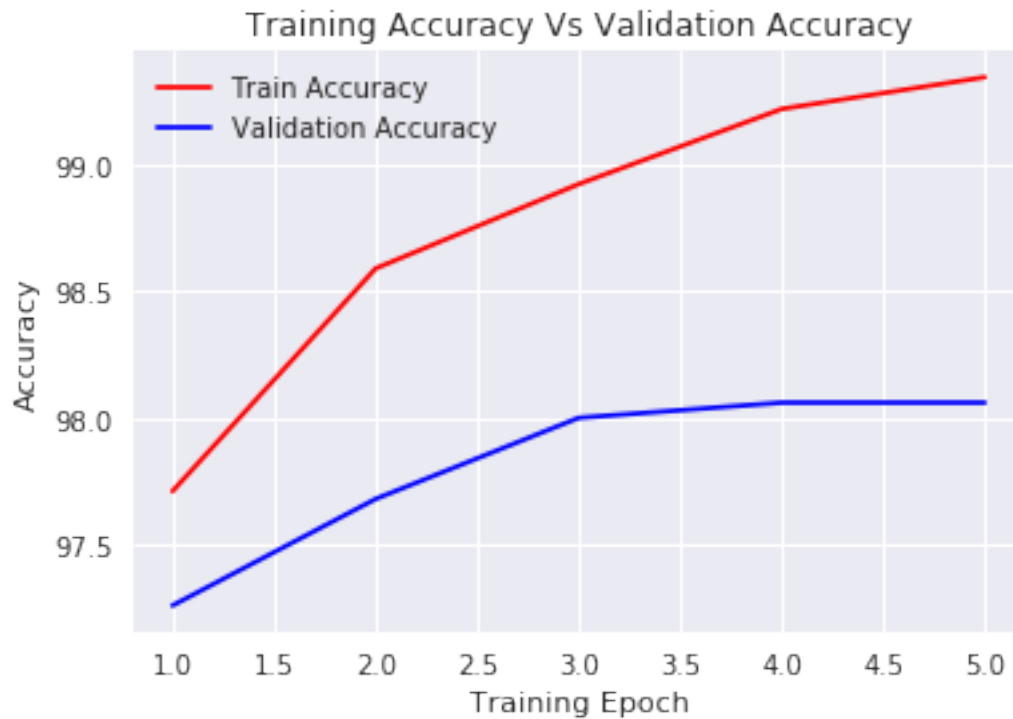


=====

With keep probability: 0.920000

Test accuracy of model :97.899994, Test loss:0.067173





### 2.3 Observation

0.84 is the best keep\_prob for which the model showed best performance

### 2.4 c) MLP with 5 Hidden layers, dropout, batch normalization

```
In [11]: def build_model_c(X):
         """
         This functions builds the architecture for the MLP.
         """

         # model architecture
         num_input_neurons = 784
         num_hidden_neurons_1 = 489
         num_hidden_neurons_2 = 265
         num_hidden_neurons_3 = 112
         num_hidden_neurons_4 = 40
         num_hidden_neurons_5 = 22
         num_output_neurons = 10

         # for relu activation function we need to use He normal initialization of weights a
```

```

# If we sample weights from a normal distribution  $N(0,)$  we satisfy this condition u
# weights =>  $\sigma^2 = (2/(fan\_in+1)) \Rightarrow N(0,)$ 

# relu activation specific standard deviations
l1_stddev = math.sqrt(2 / (num_input_neurons + 1))
l2_stddev = math.sqrt(2 / (num_hidden_neurons_1 + 1))
l3_stddev = math.sqrt(2 / (num_hidden_neurons_2 + 1))
l4_stddev = math.sqrt(2 / (num_hidden_neurons_3 + 1))
l5_stddev = math.sqrt(2 / (num_hidden_neurons_4 + 1))
out_stddev = math.sqrt(2 / (num_hidden_neurons_5 + 1))

# ===== hidden layer 1 =====
l1_weights = tf.Variable(tf.random_normal(shape=[num_input_neurons, num_hidden_neurons_1],
                                          mean=0.0, stddev=l1_stddev))
l1_biases = tf.Variable(tf.random_normal(shape=[num_hidden_neurons_1], mean=0.0, stddev=l1_stddev))

l1_net_input = tf.matmul(X, l1_weights) + l1_biases
l1_output = tf.nn.relu(l1_net_input) # activation function
#=====

# ===== hidden layer 2 =====
l2_weights = tf.Variable(tf.random_normal(shape=[num_hidden_neurons_1, num_hidden_neurons_2],
                                          mean=0.0, stddev=l2_stddev))
l2_biases = tf.Variable(tf.random_normal(shape=[num_hidden_neurons_2], mean=0.0, stddev=l2_stddev))

l2_net_input = tf.matmul(l1_output, l2_weights) + l2_biases

# apply batch normalization
# Calculate the mean and variance of x.
batch_mean_l2, batch_var_l2 = tf.nn.moments(l2_net_input, [0])

alpha_l2 = tf.Variable(tf.ones([num_hidden_neurons_2]))
beta_l2 = tf.Variable(tf.zeros([num_hidden_neurons_2]))

# do batch normalization on net input
epsilon = 1e-03
l2_net_input = tf.nn.batch_normalization(l2_net_input, batch_mean_l2, batch_var_l2,
                                         beta_l2, alpha_l2, epsilon)

l2_output = tf.nn.relu(l2_net_input) # activation function
#=====

# ===== hidden layer 3 =====
l3_weights = tf.Variable(tf.random_normal(shape=[num_hidden_neurons_2, num_hidden_neurons_3],
                                          mean=0.0, stddev=l3_stddev))

```



```

l3_biases = tf.Variable(tf.random_normal(shape=[num_hidden_neurons_3], mean=0.0, s

l3_net_input = tf.matmul(l2_output, l3_weights) + l3_biases

# apply batch normalization
# Calculate the mean and variance of x.
batch_mean_l3, batch_var_l3 = tf.nn.moments(l3_net_input, [0])

alpha_l3 = tf.Variable(tf.ones([num_hidden_neurons_3]))
beta_l3 = tf.Variable(tf.zeros([num_hidden_neurons_3]))

# do batch normalization on net input
epsilon = 1e-03
l3_net_input = tf.nn.batch_normalization(l3_net_input, batch_mean_l3, batch_var_l3,
                                         beta_l3, alpha_l3, epsilon)

l3_output = tf.nn.relu(l3_net_input) # activation function
l3_output = tf.nn.dropout(l3_output, keep_prob=keep_probability)
#=====

# ===== hidden layer 4 =====
l4_weights = tf.Variable(tf.random_normal(shape=[num_hidden_neurons_3, num_hidden_n
                                         mean=0.0, stddev=l4_stddev))
l4_biases = tf.Variable(tf.random_normal(shape=[num_hidden_neurons_4], mean=0.0, s

l4_net_input = tf.matmul(l3_output, l4_weights) + l4_biases

# apply batch normalization
# Calculate the mean and variance of x.
batch_mean_l4, batch_var_l4 = tf.nn.moments(l4_net_input, [0])

alpha_l4 = tf.Variable(tf.ones([num_hidden_neurons_4]))
beta_l4 = tf.Variable(tf.zeros([num_hidden_neurons_4]))

# do batch normalization on net input
epsilon = 1e-03
l4_net_input = tf.nn.batch_normalization(l4_net_input, batch_mean_l4, batch_var_l4,
                                         beta_l4, alpha_l4, epsilon)

l4_output = tf.nn.relu(l4_net_input) # activation function
l4_output = tf.nn.dropout(l4_output, keep_prob=keep_probability)
#=====

# ===== hidden layer 5 =====
l5_weights = tf.Variable(tf.random_normal(shape=[num_hidden_neurons_4, num_hidden_n

```

```

                                mean=0.0, stddev=l5_stddev))
l5_biases = tf.Variable(tf.random_normal(shape=[num_hidden_neurons_5], mean=0.0, s

l5_net_input = tf.matmul(l4_output, l5_weights) + l5_biases

# apply batch normalization
# Calculate the mean and variance of x.
batch_mean_l5, batch_var_l5 = tf.nn.moments(l5_net_input, [0])

alpha_l5 = tf.Variable(tf.ones([num_hidden_neurons_5]))
beta_l5 = tf.Variable(tf.zeros([num_hidden_neurons_5]))

# do batch normalization on net input
epsilon = 1e-03
l5_net_input = tf.nn.batch_normalization(l5_net_input, batch_mean_l5, batch_var_l5,
                                         beta_l5, alpha_l5, epsilon)

l5_output = tf.nn.relu(l5_net_input) # activation function
l5_output = tf.nn.dropout(l5_output, keep_prob=keep_probability)
#=====

# ===== output layer =====
l6_weights = tf.Variable(tf.random_normal(shape=[num_hidden_neurons_5, num_output_n
                                mean=0.0, stddev=out_stddev))
l6_biases = tf.Variable(tf.random_normal(shape=[num_output_neurons], mean=0.0, std

l6_net_input = tf.matmul(l5_output, l6_weights) + l6_biases

# =====

# return output using the softmax layer
output = tf.nn.softmax(l6_net_input)

return output

```

```

In [12]: # buid the model a
mlp_c = build_model_c(X)

# train and evaluate the model
#keep_prob_val = 0.92
keep_prob_val_list_c = [0.82, 0.86, 0.90, 0.94]
num_epochs = 5

# decalre a list for all the dropout value metrics
metric_list_c = list()

# try with different dropout rate

```

```

for keep_prob_val in keep_prob_val_list_c:

    print('='*100)

    print('With keep probability: %f'%keep_prob_val)

    ts_loss_mlp_c, ts_acc_mlp_c, train_metric_list, val_metric_list = train_and_evaluate(
                                                                    keep_prob_val)

    # append results to list
    metric_list_c.append(( ts_loss_mlp_c, ts_acc_mlp_c,))

    # plot the loss curve
    plot_loss_curve(train_metric_list, val_metric_list)
    # plot the accuracy curve
    plot_accuracy_curve(train_metric_list, val_metric_list)

    print('='*100)

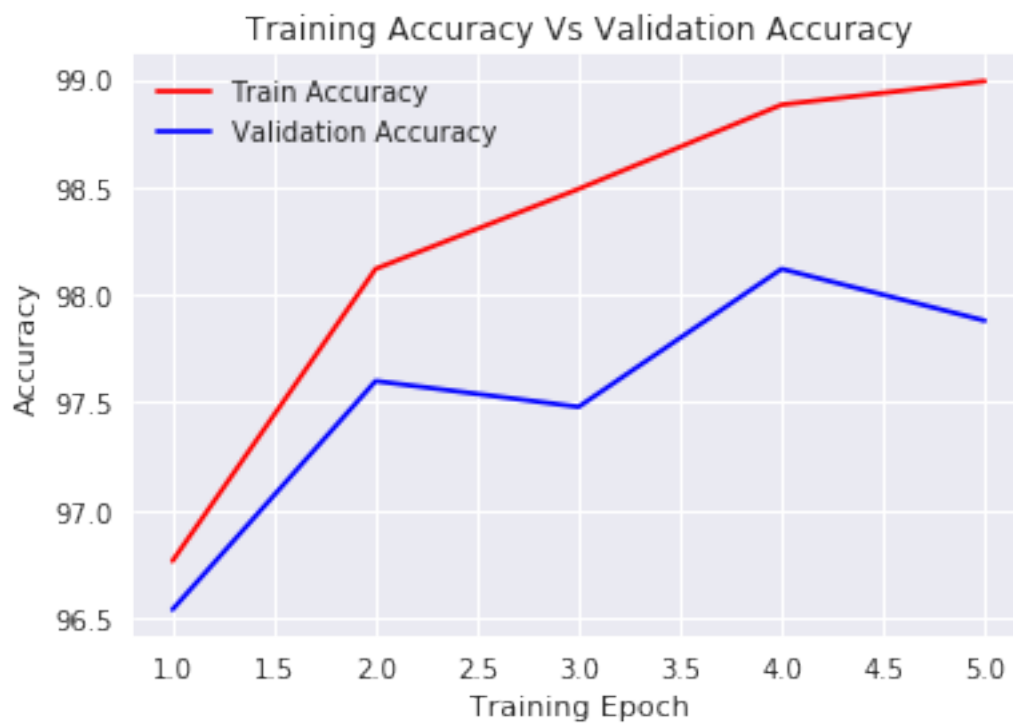
```

```

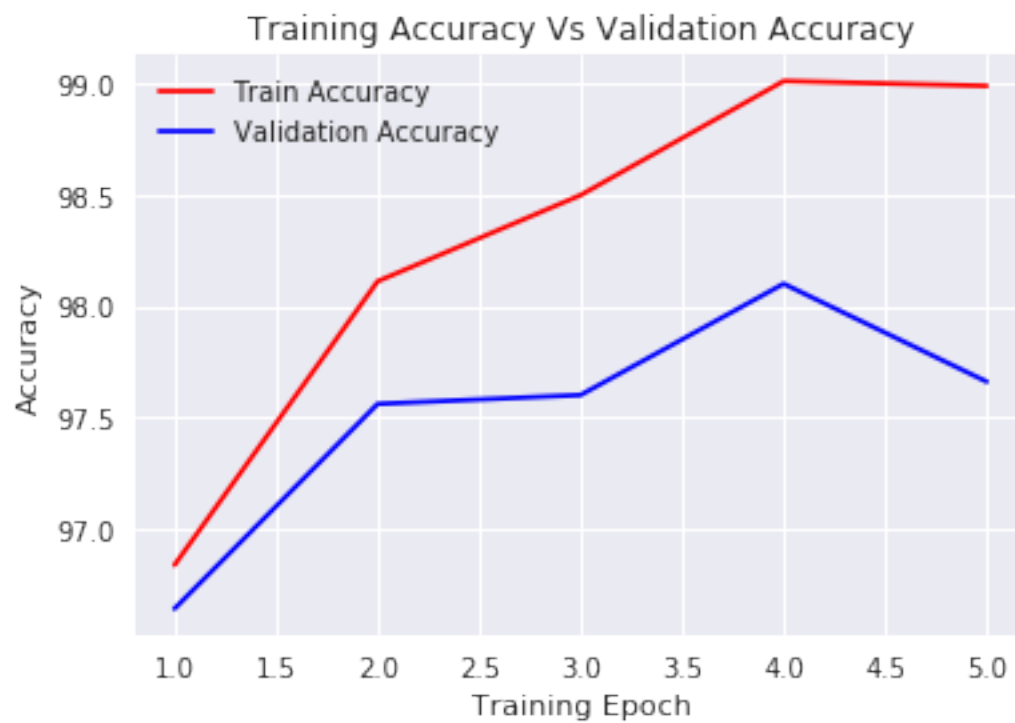
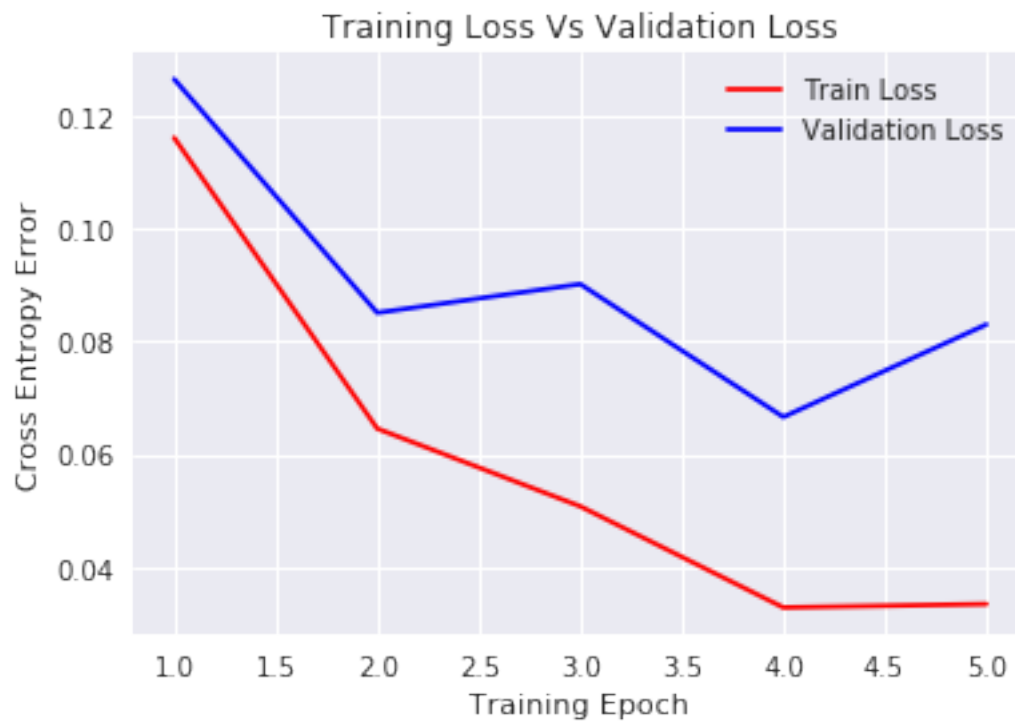
=====
With keep probability: 0.820000
Test accuracy of model :97.809998, Test loss:0.082611

```





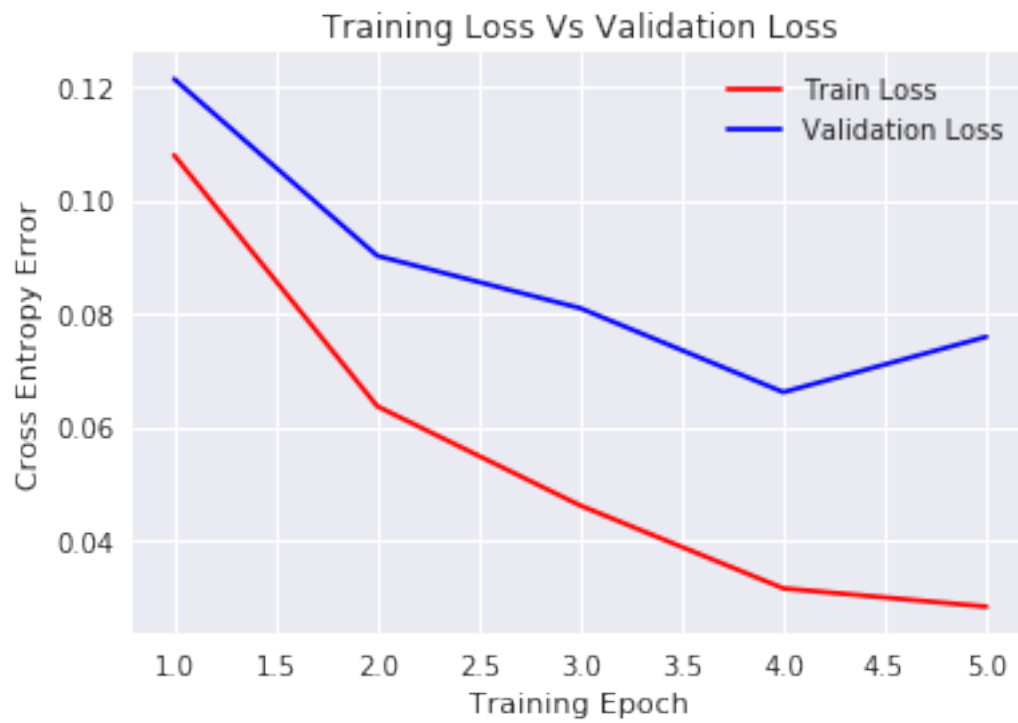
=====  
=====  
With keep probability: 0.860000  
Test accuracy of model :98.029999, Test loss:0.076954

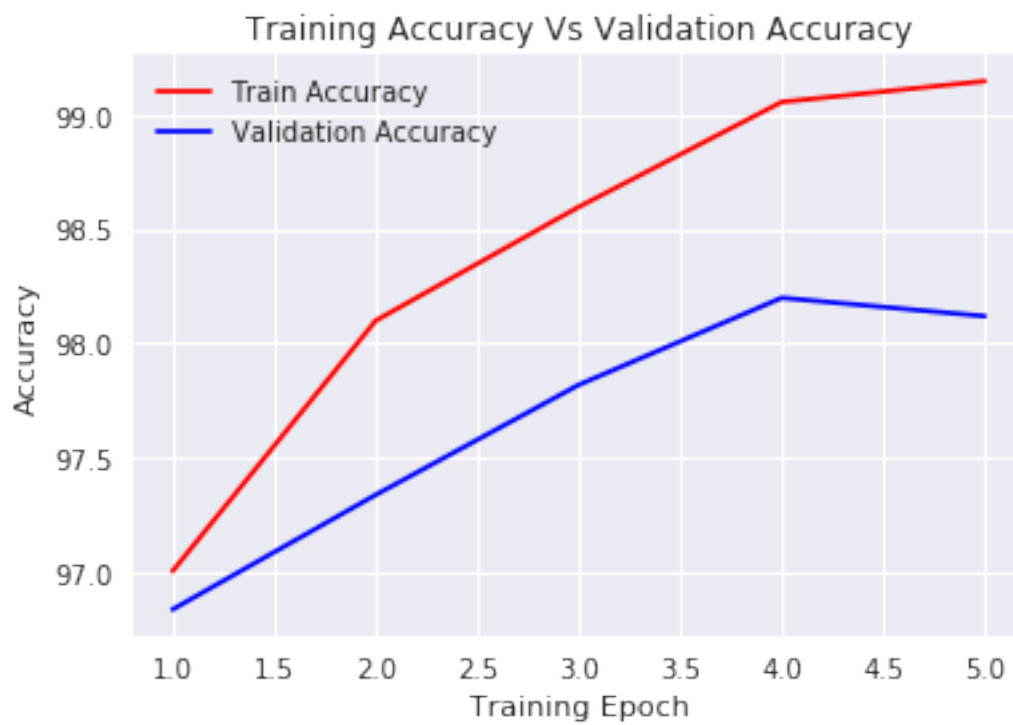


=====

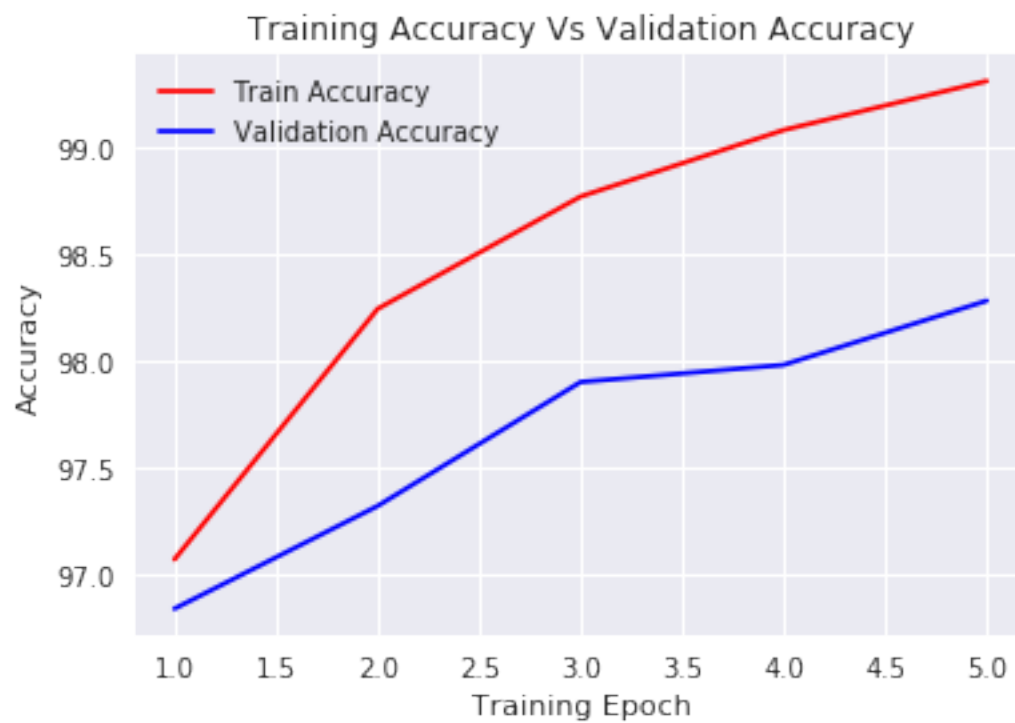
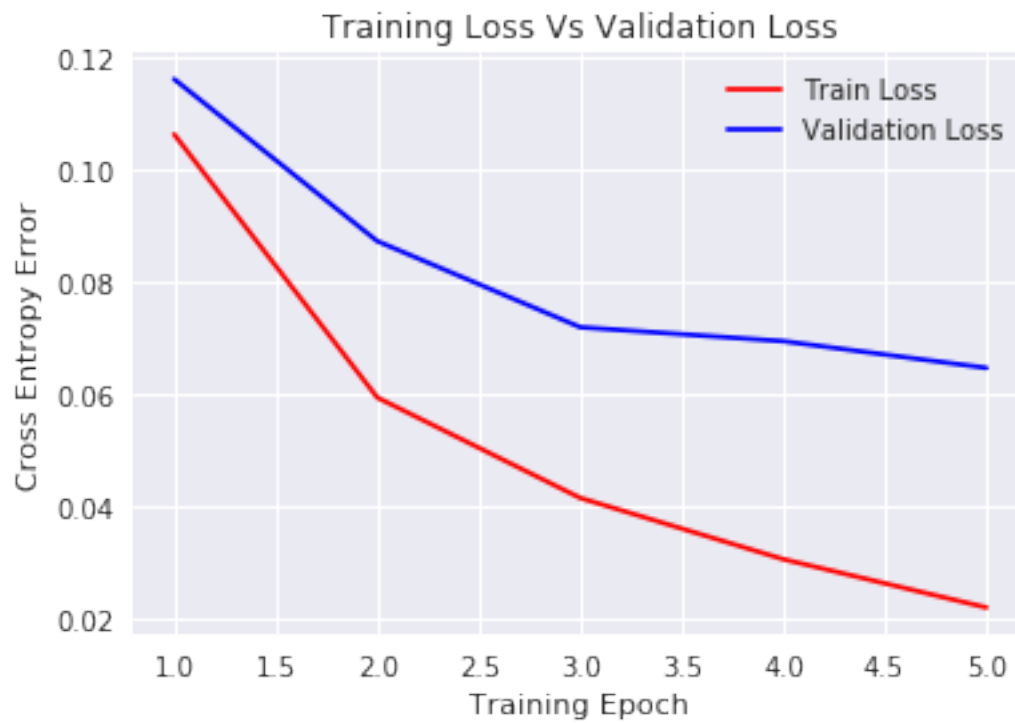
With keep probability: 0.900000

Test accuracy of model :97.860001, Test loss:0.080251





=====  
=====  
With keep probability: 0.940000  
Test accuracy of model :98.009995, Test loss:0.070729





## 2.5 Observation

Model showed best performance on test data set with 0.86 keep\_prob value

## 3 Procedure Summary

Design various MLP networks with different number of layers, neurons, batch normalization, dropout rate

Train and evaluate all the models to see its performance

Select the best model based on the loss value as well as accuracy on unseen data

## 4 Results Summary

```
In [13]: from prettytable import PrettyTable
```

```
In [14]: ptable = PrettyTable()
         ptable.title = 'Comparison of MLP Models'
         ptable.field_names = ['Model', 'Architecture', 'Keep_Prob', 'Loss', 'Accuracy']
```

```
In [15]: # append result of model a with no dropout
         ptable.add_row(['MLP-a', '2-HL', 1.0, ts_loss_mlp_a, ts_acc_mlp_a, ])
```

```
         # append results of model b
```

```
         for index, keep_prob_val in enumerate(keep_prob_val_list_b):
```

```
             ts_loss_mlp_b = metric_list_c[index][0]
```

```
             ts_acc_mlp_b = metric_list_c[index][1]
```

```
             ptable.add_row(['MLP-b', '3-HL, 2-Dropout, 2-BN', keep_prob_val, ts_loss_mlp_b, ts_a
```

```
         # append results of model c
```

```
         for index, keep_prob_val in enumerate(keep_prob_val_list_c):
```

```
             ts_loss_mlp_c = metric_list_c[index][0]
```

```
             ts_acc_mlp_c = metric_list_c[index][1]
```

```
             ptable.add_row(['MLP-c', '5-HL, 3-Dropout, 4-BN', keep_prob_val, ts_loss_mlp_c, ts_a
```

```
In [16]: print(ptable)
```

```
+-----+
|                                     Comparison of MLP Models                                     |
+-----+-----+-----+-----+-----+
| Model | Architecture | Keep_Prob | Loss | Accuracy |
+-----+-----+-----+-----+-----+
| MLP-a | 2-HL | 1.0 | 0.074358895 | 98.0 |
| MLP-b | 3-HL, 2-Dropout, 2-BN | 0.8 | 0.08261111 | 97.81 |
| MLP-b | 3-HL, 2-Dropout, 2-BN | 0.84 | 0.07695403 | 98.03 |
| MLP-b | 3-HL, 2-Dropout, 2-BN | 0.88 | 0.08025092 | 97.86 |
```

MLP-b	3-HL, 2-Dropout, 2-BN	0.92	0.070729315	98.009995	
MLP-c	5-HL, 3-Dropout, 4-BN	0.82	0.08261111	97.81	
MLP-c	5-HL, 3-Dropout, 4-BN	0.86	0.07695403	98.03	
MLP-c	5-HL, 3-Dropout, 4-BN	0.9	0.08025092	97.86	
MLP-c	5-HL, 3-Dropout, 4-BN	0.94	0.070729315	98.009995	
+-----+-----+-----+-----+-----+					

## 5 Conclusion

In this assignment, various architecture of MLPs have been tried and evaluated on MNIST data set

All models showed test accuracy above 97%

Batch Normalization version showed less deviation between train, validation loss

The network with more layers and complicated architectures (model\_b, model\_c) showed a tendency to overfit when compared with simple model\_a

For MNIST dataset, simple model (model\_a) can be used as it showed relatively small loss and good accuracy