Q.1 Given an Array of size N of non negative integers. Calculate the sum of XOR of all possible pairs.

Arr : { 3 , 2 , 8 , 5 , 6 }

Possible pairs

(3,3)	(3,2)	(3,8)	(3,5)	(3,6)
(2,3)	(2,2)	(2,8)	(2,5)	(2,6)
(8,3)	(8,2)	(8,8)	(8,5)	(8,6)
(5,3)	(5,2)	(5,8)	(5,5)	(5,6)
(6,3)	(6,2)	(6,8)	(6,5)	(6,6)

Brute force :-

```
sum = 0
for( i=0; i< N; i++ ){
    for( j=0; j< N; j++ ){
        sum = sum + (A[i]^A[j]);
    }
}
return sum;
```

$$\boxed{\begin{array}{l} TC : O(N^2) \\ SC : O(1) \end{array}}$$

(3,3)	(3,2)	(3,8)	(3,5)	(3,6)
(2,3)	(2,2)	(2,8)	(2,5)	(2,6)
(8,3)	(8,2)	(8,8)	(8,5)	(8,6)
(5,3)	(5,2)	(5,8)	(5,5)	(5,6)
(6,3)	(6,2)	(6,8)	(6,5)	(6,6)

→ x

XOR = 0

$$\boxed{a \wedge a = 0}$$

find the sum of XOR of pairs in upper triangular matrix | lower triangular matrix

return 2 × sum;

```
sum = 0
for( i=0; i< N; i++ ){
    for( j=i+1; j< N; j++ ){
        sum = sum + (A[i] ^ A[j]);
    }
}
return 2*sum;
```

TC : $O(N^2)$

SC : $O(1)$

Arr : { 3, 2, 8, 5, 6 }

		2^3	2^2	2^1	2^0	
3^2	= 1	0	0	0	1	⇒ 2^0
3^8	= 11	1	0	1	1	= $2^0 + 2^1 + 2^3$
3^5	= 6	0	1	1	0	= $2^1 + 2^2$
3^6	= 5	0	1	0	1	= $2^0 + 2^2$
2^8	= 10	1	0	1	0	= $2^3 + 2^1$
2^5	= 7	0	1	1	1	= $2^2 + 2^1 + 2^0$
2^6	= 4	0	1	0	0	= 2^2
8^5	= 13	1	1	0	1	= $2^3 + 2^2 + 2^0$
8^6	= 14	1	1	1	0	= $2^3 + 2^2 + 2^1$
5^6	= 3	0	0	1	1	= $2^1 + 2^0$

$$X = 6 \times 2^0 + 6 \times 2^1 + 6 \times 2^2 + 4 \times 2^3$$

Idea :- At i^{th} bit position, count the no. of Set Bits $= S_i$

$$\text{Contribution} = S_i * 2^i$$

$$X = \sum_{i=0}^{31} S_i * 2^i$$

$$\boxed{ans = 2 * X}$$

TC : $O(N^2 * \log_2(Max))$

SC : $O(1)$

Can we get count of set bits at each i^{th} position without creating the XOR pairs.

$a \wedge b$ → 0 ⇒ a, b are same

$a \wedge b$ → 1 ⇒ a, b are different

Arr : {3, 2, 8, 5, 6}

3 ⇒ 0011   2 ⇒ 0010   8 ⇒ 1000   5 ⇒ 0101

6 ⇒ 0110

In given array, how many elements have 0^{th} Bit as set | Unset :-

0^{th} bit set ⇒ 3, 5 → 2

0^{th} bit unset ⇒ 2, 8, 6 → 3

(3, 2)	(5, 2)
(3, 8)	(5, 8)
(3, 6)	(5, 6)

} $2 \times 3 = 6$

# of pairs in which 0^{th} Bit will be set in XOR.

1st Bit

1st Bit unset ⇒ 8, 5

1st Bit set ⇒ 3, 2, 6

$$\left.\begin{matrix}(8,3) & (5,3)\\ (8,2) & (5,2)\\ (8,6) & (5,6)\end{matrix}\right\}$$ 1st Bit set in XOR ⇒ 6

# of pairs in which 1st Bit will be set in XOR.

2nd Bit

2nd Bit set ⇒ 5, 6 ⇒ $x = 2$

2nd Bit unset ⇒ $N - x = 5 - 2 = 3$

# of pairs in which 2nd Bit will be set in XOR. ⇒ $2 * 3 = 6$

3rd Bit

3rd bit set ⇒ 8 ⇒ 1

3rd bit unset ⇒ 3, 2, 5, 6 ⇒ 4

# of pairs in which 3rd Bit will be set in XOR. ⇒ $1 \times 4 = 4$

Steps :-

for every bit position (i), find the no. of elements having 1 at i^{th} Bit position $= x$ and no. of elements having 0 at i^{th} Bit position $= N - x$.

$$\text{sum} = \text{sum} + x * (N-x) * \underbrace{2^i}_{(1<<i)}$$

return 2 * sum;

```
sum = 0
for ( i=0; i< 32 ; i++ ){
        x = 0
        for ( j = 0; j< N; j++ ){
              if ( checkBit( A[j], i ))
                   x++;
        }
        Sum = sum + x* (N-x) * (1<< i)
}
return 2*sum;
```

$\underbrace{(1<<i)}_{2^i}$ BoV of i^{th} Bit

TC : $O(\log_2(Max) * N)$

SC : $O(1)$

Q.2 Given an Array of non -ve integers.
Google Return the max & value of any pair. Return max(A[i] & A[j]), i != j

(& = AND)

Arr : { 27 , 18 , 20 }

27 → 11011, 18 → 10010, 20 → 10100

27 & 18

```
  11011
& 10010
-------
  10010
```

→ 18 max

27 & 20

```
  11011
& 10100
-------
  10000
```

→ 16

18 & 20

```
  10010
  10100
-------
  10000
```

→ 16

Observations :-

& → 0 { 0 & 1, 1 & 0, 0 & 0 }

& → 1 , both bits are 1
1 & 1 = 1

1) When both the bits are set, output of & will be 1

$a = 1\ 1\ 0\ 1\ 1$
$b = 1\ 0\ 0\ 0\ 0$
&
$\underline{\underline{x}} = 1\ 0\ 0\ 0\ 0$

2^4

$>$

$a = 1\ 1\ 0\ 1\ 1$
$c = 0\ 1\ 0\ 0\ 0$
&
$y = 0\ 1\ 0\ 0\ 0$

2^3

$\underline{\underline{x > y}}$

$a = 1\ 1\ 0\ 1\ 1$
$p = 1\ 0\ 0\ 0\ 0$
&
$s = 1\ 0\ 0\ 0\ 0$

$\underline{\underline{2^4}}$

$s = 16$

$>$

$a = 1\ 1\ 0\ 1\ 1$
$q = 0\ 1\ 0\ 1\ 1$
&
$r = 0\ 1\ 0\ 1\ 1$

$r = 2^3 + 2^1 + 2^0$
$= 8 + 2 + 1 = 11$

$\underline{\underline{s > r}}$

2) Ans will be maximum if set bit is more towards the MSB (left)

$1000 > 0111$

$8 \qquad 7$

Ex :-

{26, 13, 23, 28, 27, 7, 25}

26 : 1 1 0 1 0 ✓

13 : 0 1 1 0 1 ✗

23 : 1 0 1 1 1 ✗

28 : 1 1 1 0 0 ✗

27 : 1 1 0 1 1 ✓

7 : 0 0 1 1 1 ✗

25 : 1 1 0 0 1 ✗

1 1 0 1 _

Discarding 13, 7.

Discarding 23.

Discarding 25, 28.

Note :-

Try to set the MSB first.

$\boxed{11010}$ → Ans.

$\boxed{26 \& 27 = 11010}$

26 : 1 1 0 1 0 $\qquad$ ans = 0

13 : 0 $\cancel{1}$ $\cancel{1}$ 0 $\cancel{1}$ 0 ✗

23 : 1 $\cancel{0}$ $\cancel{1}$ $\cancel{1}$ $\cancel{1}$

28 : 1 1 1 $\cancel{0}$ $\cancel{0}$

27 : 1 1 0 1 1

7 : 0 0 $\cancel{1}$ $\cancel{1}$ $\cancel{1}$ ✗

25 : 1 1 0 $\cancel{0}$ $\cancel{1}$

ans = 1 1 0 1 0 ⇒ max And value.

maximise

```
for( i = 31; i >= 0; i-- ){
        setBitCount = 0
        for( j = 0; j < N; j++ ){
              if( CheckBit( A[j], i ))
                    setBitCount ++
        }
        if ( setBitCount >= 2 ) {
              ans = ans | (1<<i) ;
              for( j = 0; j < N; j++ ){
                    if( ! CheckBit( A[j], i ))
                          A[j] = 0   // Discard
              }
        }
}
```

return ans;

$$TC \Rightarrow O(\log_2(Max) * N)$$

$$SC \Rightarrow O(1)$$

Q.3 Given an Array of non negative elements (≥ 0), find the pair with minimum XOR value, return any pair.

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A : { 0 , 2 , 7 , 5 }

0000   0010   0111   0101

0 ^ 2	7 ^ 5	2 ^ 7
0000	0111	0010
0010	0101	0111
^ 0010 → 2	^ 0010 → 2	0101 = 5

min XOR = 2

Brute force

$$\boxed{\begin{array}{l} TC : O(N^2) \\ SC : O(1) \end{array}}$$

Observations

^ → 0 : Bits are same
^ → 1 : Bits are different

Minimum XOR ⇒ find XOR of elements with as many simillar bits as possible.

$$\begin{array}{rl} a = & 1\,0\,1\,1\,0\,1 \\ b = & 0\,0\,1\,1\,0\,1 \\ \hline a\,\hat{}\,b = & 1\,0\,0\,0\,0\,0 \end{array} \quad > \quad \begin{array}{rl} a = & 1\,0\,1\,1\,0\,1 \\ c = & 1\,1\,1\,1\,0\,1 \\ \hline a\,\hat{}\,c = & 0\,1\,0\,0\,0\,0 \end{array}$$

a^b > a^c

$$\begin{array}{rl} x = & 1\,0\,1\,1\,0\,1 \\ y = & 0\,0\,1\,1\,0\,1 \\ \hline x\,\hat{}\,y = & 1\,0\,0\,0\,0\,0 \end{array} \quad > \quad \begin{array}{rl} x = & 1\,0\,1\,1\,0\,1 \\ z = & 1\,1\,0\,0\,1\,0 \\ \hline x\,\hat{}\,z = & 0\,1\,1\,1\,1\,1 \end{array}$$

x^y > x^z

$100 \Rightarrow 1100100$

$99 \Rightarrow 1100011$

$100 \Rightarrow 1100100$

$1 \Rightarrow 0000001$

$100 \wedge 99 < 100 \wedge 1$

$$64 = \overset{6}{1}\overset{5}{0}\overset{4}{0}\overset{3}{0}\overset{2}{0}\overset{1}{0}\overset{0}{0}$$
$$63 = 0111111$$
$$64 \wedge 63 = \underset{6}{1}\underset{5}{1}\underset{4}{1}\underset{3}{1}\underset{2}{1}\underset{1}{1}\underset{0}{1}$$
$$= \underline{\underline{127}}$$

$$63 = 0111111$$
$$32 = 0100000$$
$$63 \wedge 32 = 00\underbrace{11111}_{}$$
$$= \underline{\underline{31}}$$

$$64 = 1000000$$
$$32 = 0100000$$
$$\wedge \quad 1100000 = \underline{\underline{96}}$$

63, 64, 32

$$\downarrow$$

$$32 \quad 63 \quad 64$$

127

31

min(31, 127)

$= \underline{\underline{31}}$

$$64 : 01000000$$
$$32 : 00100000$$
$$\wedge \quad 01100000 \Rightarrow 64 + 32 = 96$$

$$64 \; 63 = \underbrace{\overset{6}{1}\overset{5}{1}\overset{4}{1}\overset{3}{1}\overset{2}{1}\overset{1}{1}\overset{0}{1}}_{} \Rightarrow$$

① Sort the Array
② find the XOR of every consecutive Pair.

TC: $O(N \log N)$
SC: depends on sorting algo.