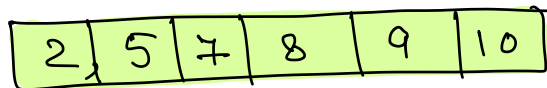
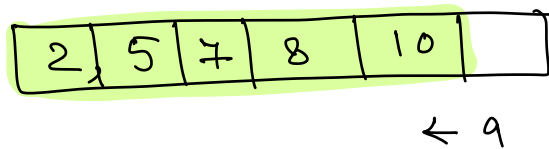
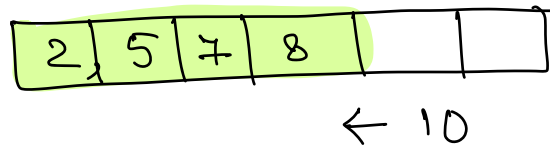
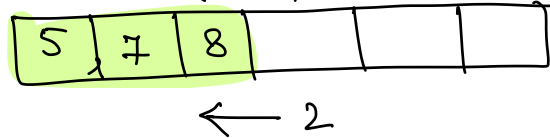
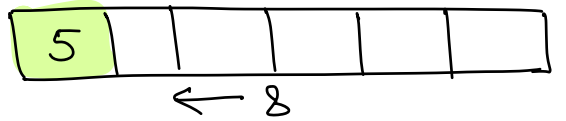


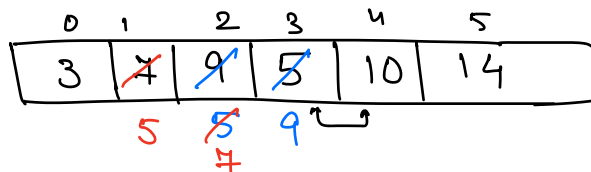
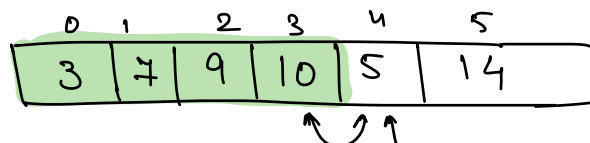
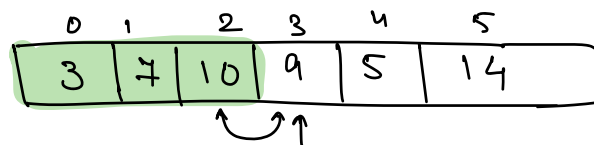
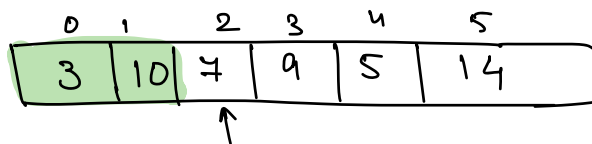
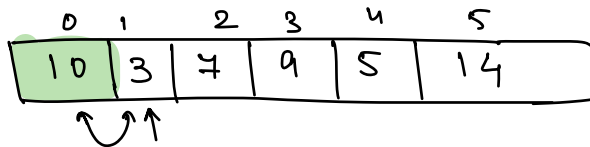
Stream of integers.



Insert the element
at its correct pos

Insertion Sort

Ex



| | | | | | | |
|---|---|---|---|----|----|--|
| 3 | 5 | 7 | 9 | 10 | 14 | |
|---|---|---|---|----|----|--|

↑

| | | | | | |
|---|---|---|---|----|----|
| 3 | 5 | 7 | 9 | 10 | 14 |
|---|---|---|---|----|----|


Sorted

```

void insertionSort (int A[], int N) {
    for (i = 1 ; i < N ; i++) {
        j = i - 1 ;
        index = i ;
        while (j >= 0) {
            if (A[index] < A[j]) {
                swap(A[j], A[index]);
                index = j ;
                j-- ;
            }
            else {
                break ;
            }
        }
    }
}

```

TC: $O(N^2)$:

Stable ? 

Inplace? ✓ SC: $O(1)$

No. of swaps (w.c) ?

1 2 3 4 7 8 $N = \underline{\underline{6}}$

$$1 + 2 + 3 + 4 + 5$$

No. of swaps = $1 + 2 + 3 + 4 + \dots + N-1$
 $= \frac{N(N-1)}{2}$

$$= \frac{N(N-1)}{2}$$

Best case

T.C

1, 2, 3, 4, 5 $\Rightarrow O(N)$ No swaps

TC: $O(N)$

If the Array is almost sorted then we can use Insertion sort.

1, 2, 3, 5, 4

1 maps.

Q: Given an Array of size N , rearrange the array s.t

① All elements $\leq A[0] \Rightarrow$ Go to left

② All elements $> A[0] \Rightarrow$ Go to right

{ 10, 3, 8, 15, 6, 12, 2, 18, 7, 15, 14 }

Partition

{ 3, 8, 6, 2, 7, 10, 15, 12, 18, 15, 14 }

≤ 10 > 10

TC: $O(N)$

SC: $O(1)$

{ 10, 3, 8, 15, 6, 12, 2, 18, 7, 15, 14 }

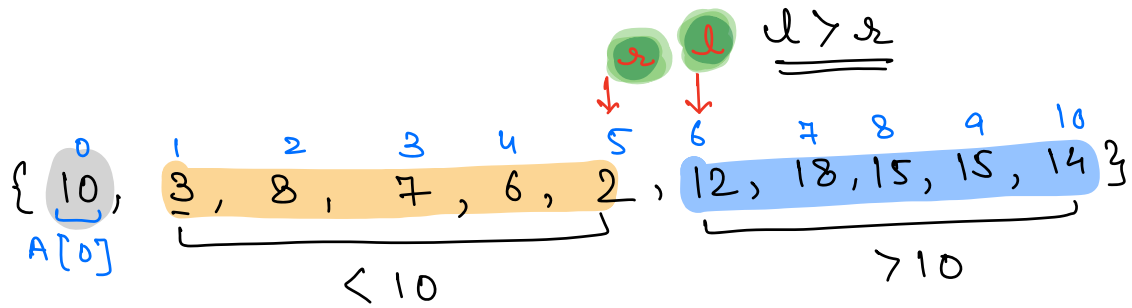
$A[0]$

\Downarrow

{ 10, 3, 8, 7, 6, 12, 2, 18, 15, 15, 14 }

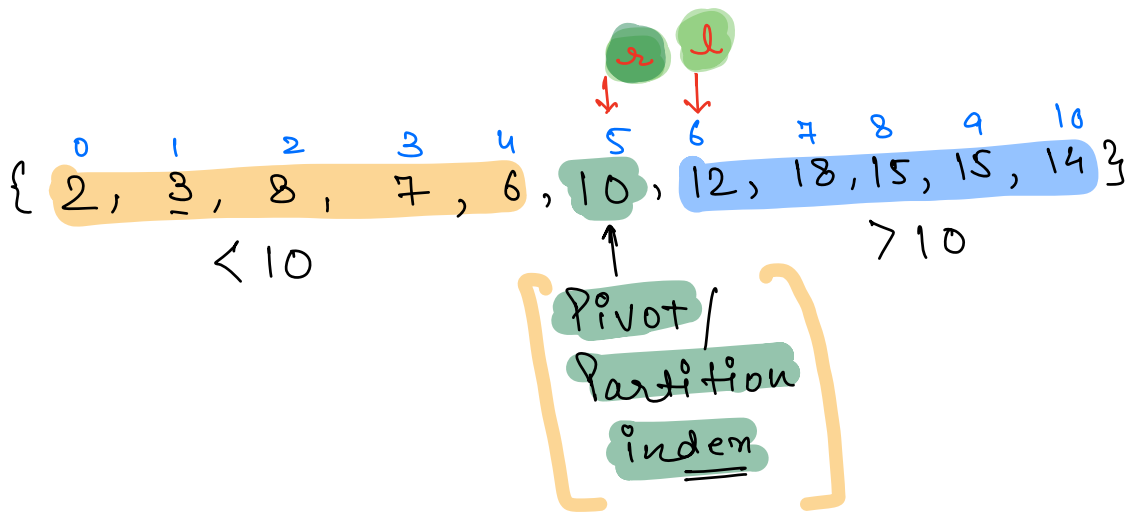
$A[0]$

\Downarrow



$\text{swap}(A[0], A[5]) \times$

$\text{swap}(A[0], A[5])$
 $\text{swap}(A[0], A[5-1])$ } ✓



```

void partition ( int A[], int N) {
    l = 1
    r = N-1;
    while ( l <= r ) {
        if ( A[l] <= A[0] )
            l++;
        else if ( A[r] > A[0] )
            r--;
        else {
            swap( A[l], A[r] );
            l++;
            r--;
        }
    }
    swap ( A[0], A[r] );
}

```

Pivot index \Rightarrow r

TC : $O(N)$

SC : $O(1)$

Q: Given an Array of size N , rearrange the subarray from s to e s.t

① All elements $\leq A[s] \Rightarrow$ Go to left

② All elements $> A[s] \Rightarrow$ Go to right

A: { 4, 1, 5, 6, 7, 8, 2, 3, 1, 7 }

↓

A: { 4, 1, 2, 3, 5, 6, 7, 8, 1, 7 }


```

int partition ( int A[], int s, int e )
    l = s + 1
    r = e ;
    while ( l <= r ) {
        if ( A[l] <= A[s] )
            l++;
        else if ( A[r] > A[s] )
            r--;
        else {
            swap ( A[l], A[r] );
            l++;
            r--;
        }
    }
    swap ( A[s], A[r] );
    return (r) ;

```

3

 ↳ Pivot index

{ ⁰10, ¹3, ²8, ³15, ⁴6, ⁵12, ⁶2, ⁷18, ⁸7, ⁹15, ¹⁰14 }

Partition

{ ⁰2, ¹3, ²8, ³7, ⁴6, ⁵10, ⁶12, ⁷18, ⁸15, ⁹15, ¹⁰14 }

After partition, 10 has come to its right position.

{ ⁰2, ¹3, ²8, ³7, ⁴6, ⁵10, ⁶12, ⁷18, ⁸15, ⁹15, ¹⁰14 }

{ ⁰2, ¹3, ²8, ³7, ⁴6 }

{ 2, 3, 8, 7, 6 }

{ 3, 8, 7, 6 }

{ 8, 7, 6 }

{ 6, 7, 8 }

{ 6, 7 }

{ 6, 7 }

{ ⁶12, ⁷18, ⁸15, ⁹15, ¹⁰14 }

{ 12, 18, 15, 15, 14 }

{ 14, 15, 15, 18 }

{ 14, 15, 15 }

{ 15, 15 }

{ 15, 15 }

{ 15 }

X {7}

{2, 3, 6, 7, 8, 10, 12, 14, 15, 15, 18}

\Rightarrow sorted.

Quick Sort

* Assumption: quicksort(A[], s, e) will sort the subarray from s to e.

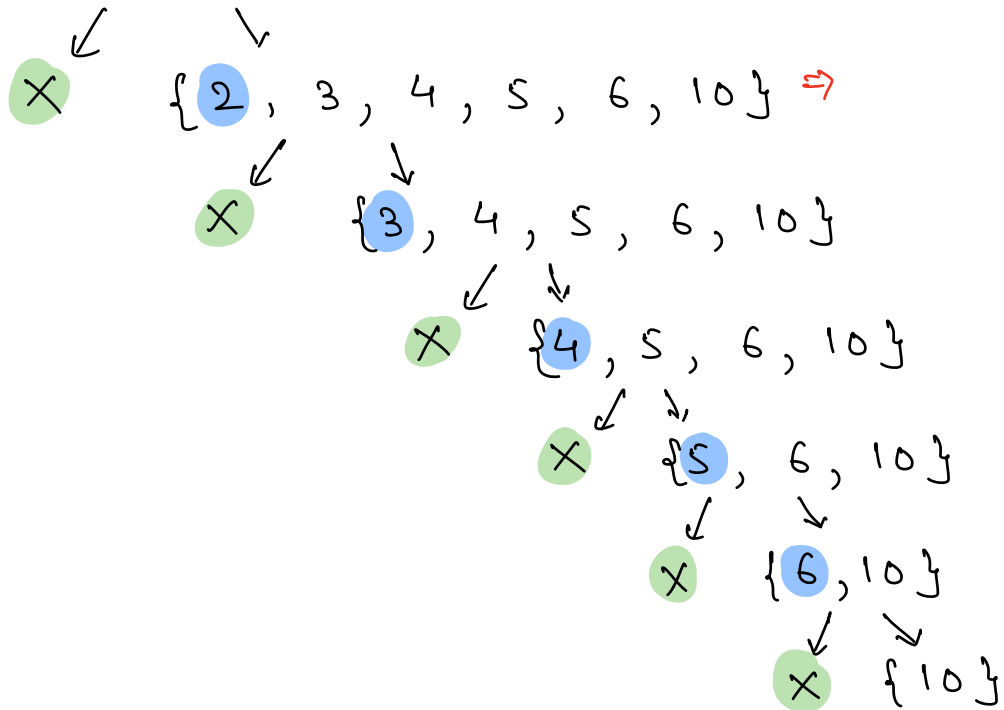
```
void quicksort ( A[] , s , e ) {  
    if ( s >= e )  
        return ;  
  
    int pivot = partition ( A , s , e );  
    quicksort ( A , s , pivot - 1 );  
    quicksort ( A , pivot + 1 , e );  
}
```

3
Time Complexity

\Rightarrow Best case :-

Ex If the Array is already sorted.

A: { ^P1, 2, 3, 4, 5, 6, 10 }



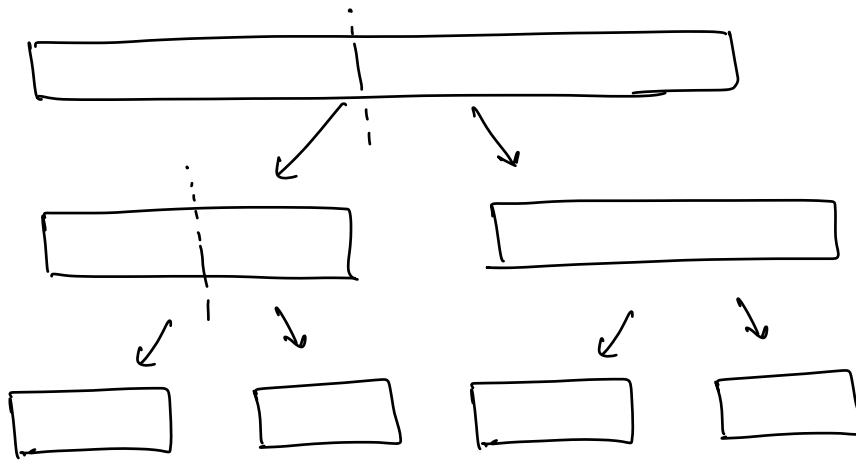
of iterations :-

$$= N + (N-1) + (N-2) + \dots + 1$$

$$= \frac{N(N+1)}{2}$$

TC: $O(N^2)$: Worst Case

Best Case : Pivot partitions the array into exactly half every time.



$$TC: O(N \log N)$$

$$T(N) = 2T(N/2) + O(N) \quad \text{Recurrence relation}$$

$$A: \{1, 2, 3, 4, 5, 6, 10\}$$

$$\frac{1}{N} \times \frac{1}{N-1} \times \frac{1}{N-2} \dots \Rightarrow 0 \quad \left. \vphantom{\frac{1}{N} \times \frac{1}{N-1} \times \frac{1}{N-2}} \right\} \text{Practically impossible.}$$

Probability of getting $O(N^2)$ case if we are picking the random index everytime is practically zero.

\Rightarrow Randomized Quick Sort

Code

```
randomIndex = rand(——)  $\Rightarrow$   $O(1)$   
Swap(A[0], A[randomIndex]);  
P = partition(A, s, e);  
QS(A, s, P-1);  
QS(A, P+1, e);
```

Avg case :- $O(N \log N)$

TC of Q.S

Best case : $O(N \log N)$
Worst case : $O(N^2)$
Avg case : $O(N \log N)$

* See the implementation of library sort method.

Is Q.S stable ? NO

S.C Inplace Sorting

Worst case : $O(N)$ } skewed Tree } Call Stack
Best case : $O(\log N)$

* Are we creating any extra space explicitly?
 \Rightarrow NO

Count Sort

a c b a d e

⇓

a a b c d e

If the input is range specific then we can apply the Count Sort.