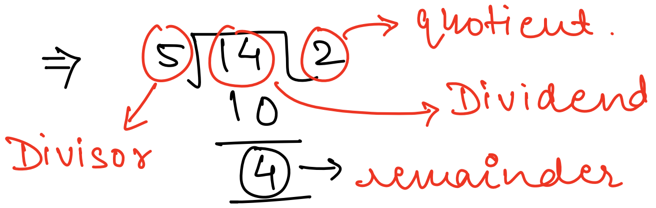
A % B ⇒ Remainder when Ⓐ is divided by B.

14 % 5 ⇒



⇒ Dividend = Divisor * quotient + remainder

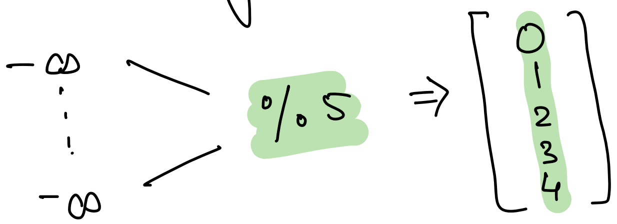
Division is a repeated subtraction :-

i) 14 % 5 ⇒ 14 − 5 = 9 − 5 = ④

ii) 40 % 6 ⇒ 40 − 6 = 34 − 6 = 28 − 6 = 22 − 6
= 16 − 6 = 10 − 6 = ④ = 40 − 36

⇒ Remainder = Dividend − (largest multiple of Divisor <= Dividend)

Importance of modulo operator :-



Quiz

a % m ∈ [0, M−1]

Modulo Arithmetic

1) $(a+b)\%m = (a\%m + b\%m)\%m$

$a=4,\ b=5,\ m=6$

$(4+5)\%6$

$9\%6 = 3$

$(4\%6 + 5\%6)\%6$

$(4+5)\%6$

$\Rightarrow 9\%6 = ③$

2) $(a*b)\%m = (a\%m * b\%m)\%m$

3) $a\%M = (a+\underline{M})\%M$

$= (a\%M + \underbrace{M\%M}_{0})\%M$

$= (a\%M)\%M$

$= \underline{\underline{a\%M}}$

4) $(a-b)\%M = (a\%M - b\%M + M)\%M$

$\underline{\underline{Ex}}$ $a=8,\ b=4,\ M=5$

$(8-4)\%5$

$4\%5 = \underline{\underline{④}}$

$(\underline{8\%5} - 4\%5)\%5$

$(3-4)\%5$

$$\begin{tikzcd} & -1\%5 \arrow[dl, "C/C++/Java"'] \arrow[dr, "Python"] & \\ \boxed{-1} \arrow[rr, "+5"] & & \boxed{4} \end{tikzcd}$$

Q.1 Given A, B

$A > B$, find M such that $A \% M = B \% M$.

$M > 1$

Ex :- $A = 16$, $B = 4$

$$16 \% M = 4 \% M$$

$M = 2, 3, 4,$ ~~5~~$, 6,$ - - - - - - - -

$$A \% M = B \% M$$

$$A \% M - B \% M = 0$$

$$\downarrow$$

$$\boxed{(A-B) \% M = 0}$$

$\Rightarrow$ M needs to be a factor of $(A-B)$.

$\Rightarrow$ $(A-B)$ is a factor of $(A-B)$.

$\Rightarrow$ return $(A-B)$. $O(1) \Rightarrow TC$

Q.2 Given N array elements, calculate the no. of pairs (i, j) such that

$$(a[i] + a[j]) \% M = 0 \quad , \quad i \,!\!= j$$

Note: pair (i, j) is same as pair (j, i)

$$A[6] : \{ \overset{0}{4}, \overset{1}{7}, \overset{2}{6}, \overset{3}{5}, \overset{4}{5}, \overset{5}{3} \} \qquad M = 3$$

i	j	A[i]	A[j]	
0	3	4	5	$\Rightarrow (4+5) \% 3 = 0$
0	4	4	5	$\Rightarrow (4+5) \% 3 = 0$
1	3	7	5	$\Rightarrow (7+5) \% 3 = 0$
1	4	7	5	$\Rightarrow (7+5) \% 3 = 0$
2	5	6	3	$\Rightarrow (6+3) \% 3 = 0$

$\Rightarrow$ ⑤

N = 4

$$A[4] \Rightarrow \{ \overset{0}{1} \quad \overset{1}{2} \quad \overset{2}{3} \quad \overset{3}{4} \}$$

$(i, j) \equiv (j, i)$

(0, 0)	(0, 1)	(0, 2)	(0, 3)
(1, 0)	(1, 1)	(1, 2)	(1, 3)
(2, 0)	(2, 1)	(2, 2)	(2, 3)
(3, 0)	(3, 1)	(3, 2)	(3, 3)

Brute force

```
ans = 0
for ( i= 0; i< N; i++) {
    for (j= i+1; j< N; j++){
        if((A[i] + A[j])%M == 0)
            ans++;
    }
}
return ans;
```

TC : $O(N^2)$

SC : $O(1)$

$$(a[i] + a[j]) \% M = 0$$

Ex

$A = 35$, $B = ?$, $M = 10$

$$(35 + B) \% 10 = 0$$

$$(35 \% 10 + B \% 10) \% 10 = 0$$

$$(5 + \underbrace{B \% 10}_{[0-9]}) \% 10 = 0$$

$$\Downarrow$$

⑤ $B \% 10 = 5$

$$(5 + 5) \% 10 = \underline{\underline{0}}$$

Ex $a = 8,\ b = ?\ ,\ M = 3$

$$(8 + b)\ \%\ 3 = 0$$

$$(8\%3 + b\%3)\ \%\ 3 = 0$$

b = ?

$$(2 + \underbrace{b\%3}_{\Downarrow})\ \%\ 3 = 0$$

$$0, 1, 2$$

$$(2+1)\ \%\ 3 = 0$$

$$b\%3 \Rightarrow 1 \Rightarrow \boxed{b=1}$$

Ex :-

$$A : \{\overset{0}{13}, \overset{1}{14}, \overset{2}{22}, \overset{3}{3}, \overset{4}{32}, \overset{5}{19}, 16\} \quad M=4$$

$$A[i]\ \%\ 4 \ \Big\downarrow\ \%4$$

$$\{\overset{0}{1}, \overset{1}{2}, \overset{2}{2}, \overset{3}{3}, \overset{4}{0}, \overset{5}{3}, \overset{6}{0}\}$$

M=4

(1,2) (0,3) (4,6)

(0,5)

$$\Rightarrow \textcircled{4}$$

Ex

M=10

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18

A:{29, 11, 21, 17, 2, 5, 4, 6, 23, 13, 26, 14, 18, 15, 30, 35, 50, 20, 40}

↑

$A[i] \% 10 \Rightarrow [0-9]$ M=10

i j

	0	1	2	③	4	5	6	7	8	9
Count:	4	2	1	2	2	3	2	1	1	1
	30 50 20 40	11 21	2	23 13	4 14	5 15 35	6 26	17	18	29

(2,18)
(23, 17)
(13,17)

(11,29)
(21,29)

(4,6)
(4,26)
(14,6)
(14,26)

ⓐ=3 $\begin{cases} 5 \\ 15 \\ 35 \end{cases}$

5 → 15, 35 ⇒ (5,15), (5,35)

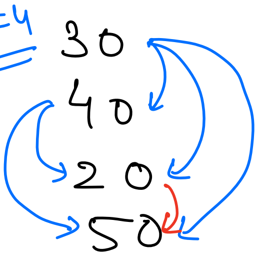
15 → 5, 35 ⇒ (15,5) (15,35)

35 → 5, 15 ⇒ (35,5), (35,15)

6

$$\boxed{\frac{a*(a-1)}{2}} \Rightarrow$$

a=4



$$3+2+1 = \frac{3(4)}{2} = \frac{a(a-1)}{2}$$

```
Hashmap < int, int > map;
for ( i = 0; i < N; i++ ) {
O(N)    x = A[i] % M
        if ( x is present in map)                map[x]++;
            map[x] ++;
        else
            map.insert( x, 1 );
}
ans = 0
a = map[0]
ans += $\frac{a(a-1)}{2}$ ;
i = 1 , j = M-1
while ( i < j ) {
M       ans += map[i] * map[j];
        i++ ;
        j--;
}
if ( M % 2 == 0 ) {      || i == j.
        a = map[M/2]
        ans += $\frac{a*(a-1)}{2}$
}
return ans;
```

$$\boxed{\begin{array}{l} TC : O(N+M) \\ SC : O(M) \end{array}}$$

Q.3 **FB** Given an Array of all distinct integers where $0 \le A[i] \le N-1$, N is the size of the Array.

Replace $\boxed{A[i] \Rightarrow A[A[i]]}$

Ex :-

	0	1	2	3	4
A :	{ 3 ,	2 ,	4 ,	1 ,	0 }

$N=5$.

$0 \le A[i] \le 4$

$A[0] = A[A[0]] = A[3] = 1$

$A[1] = A[A[1]] = A[2] = 4$

$A[2] = A[A[2]] = A[4] = 0$

$A[3] = A[A[3]] = A[1] = 2$

$A[4] = A[A[4]] = A[0] = 3$

A : {1, 4, 0, 2, 3}

Quiz

	0	1	2	3	4	↓ 5	6
A :	{ ③ ,	1 ,	4 ,	6 ,	5 ,	0 ,	2 }
	↓	↓	↓	↓	↓	↓	↓
	6	1	5	2	0	3	4

$A[i] = A[A[i]]$

$A[4] = A[A[4]]$
$\neq A[5]$

$A[6] = A[A[6]]$
$= A[2]$

①
```
int B[N]
for( i = 0; i < N; i++ ){
        B[i] = A[A[i]]
}
```

TC : O(N)
SC : O(N)

* SC : O(1) {Inplace}

Big Bang Theory

00:00

Day → 0
Hour → 0

	Day	Hour
23 hrs	0	23
46 hrs	1	22
100 hrs	4	4
125 hrs	5	5
x hrs	$x/24$ Quotient	$x\%24$ Remainder

\# x hrs → $x/24$ ⇒ Day
→ $x\%24$ ⇒ Hour.

$$\begin{tikzcd}
 & \underline{\underline{\text{Old}}} \arrow[r, Rightarrow] & A[i] \\
A[i] \arrow[ur, "/N"] \arrow[dr, "\%N"'] & & \\
 & \underline{\underline{\text{New}}} \arrow[r, Rightarrow] & A[A[i]]
\end{tikzcd}$$

$$\boxed{x = A[i] * N + A[A[i]]} \Leftarrow$$

$$x / N \Rightarrow A[i]$$

$$x \% N \Rightarrow A[A[i]]$$

①

$$\boxed{\begin{tikzcd}
0 \le A[i] \le N-1 \arrow[d, "\%N"] \\
A[i]
\end{tikzcd}}$$

②

$$\boxed{\begin{tikzcd}
0 \le A[i] \le N-1 \arrow[d, "/N"] \\
0
\end{tikzcd}}$$

$$\boxed{A[i] = A[i] * N + A[A[i]]}$$

$$\frac{A[i]}{N} = \frac{A[i] * \cancel{N}}{\cancel{N}} + \cancelto{0}{\frac{A[A[i]]}{N}}$$

$$= A[i] \ (\text{Older})$$

$$A[i] \% N = \Big(\underbrace{(A[i] * N) \% N}_{0} + A[A[i]] \% N\Big) \% N$$

$$= A[A[i]] \% N$$

$$= A[A[i]]$$

Ex

$$A : \{ \overset{0}{\underline{3}} \quad \overset{1}{①} \quad \overset{2}{4} \quad \overset{3}{\underline{6}} \quad \overset{4}{5} \quad \overset{5}{0} \quad \overset{6}{2} \}$$

$N = 7$

$$\Big\downarrow \boxed{*7}$$

$A[i] = A[i] * N + A[A[i]]$

$$\{ 3*7,\ 1*7,\ 4*7,\ 6*7,\ 5*7,\ 0*7, 2*7 \}$$

$$\Big\downarrow + \underline{A[A[i]]}$$

Ⓐ

$$\{ \overset{0}{3*7+6},\ \overset{1}{1*7+1},\ \overset{2}{4*7+5},\ \overset{3}{6*7+2},\ \overset{4}{5*7+0},\ 0*7+3,\ 2*7+4 \}$$

$$\Big\downarrow \underline{\underline{\% 7}}$$

Code

```
for (i = 0; i< N; i++ ){
        A[i] *= N;
}
for ( i= 0; i<N ; i++ ){
        index = A[i] / N;
        newValue= A[index]/ N ;
        A[i] += newvalue;
}
for ( i = 0; i< N; i++){
        A[i] = A[i] % N
}
```

\# $(a|b)\ \%\ M \neq (a\ \%\ M)\ /\ (b\ \%\ M)$

$a = 2,\ b = 2,\ M = 2$

$(2|2)\ \%\ 2$

$1\ \%\ 2 = ①$

$(2\ \%\ 2)\ |\ (2\ \%\ 2)$

$\boxed{0|0}$ ×

$$\Rightarrow \quad (a/b)\ \%\ M = (a \times b^{-1})\ \%\ M$$

$$= \left((a\ \%\ M) * \underbrace{(b^{-1}\ \%\ M)}_{\text{Inverse Modulo.}}\right)\ \%\ M$$

Inverse Modulo

Given b, M

$b^{-1}\ \%\ M$ exists only if $\underbrace{gcd(b, M)}_{\text{greatest common divisor.}} = 1$

$$\Rightarrow \quad b^{-1} = \frac{1}{b}$$

$$b * \frac{1}{b} = 1$$

$$(b * b^{-1})\ \%\ M = 1$$

$$(b\%M * \underbrace{b^{-1}\%M}_{\downarrow\ \cancel{0}})\%M = 1$$

$$b^{-1}\%M \in [1, M-1]$$

$b^{-1}\%M$ can't be zero.

$$x = b^{-1}\%M \in [1, M-1]$$

```
for (x = 1; x <= M-1; x++) {
    if ((b % M * x) % M == 1) {
        return x;
    }
}
```

$$TC : O(M)$$

$b = 10$, $M = 7$ $b^{-1}\%M = x = ?$

$x = 1$ $(10\%7 * 1)\%7 = 3$

$x = 2$ $(10\%7 * 2)\%7 = 6$

⋮

$x = 5$ $(10\%7 \times 5)\%7 = (3 \times 5)\%7 =$ ①

$x = 6$

$$\boxed{x = b^{-1}\%M = 5}$$

Fermat Little Theorem :-

Given b, M ⇒ $b^{-1} \% M$ exists only if $\gcd(b, M) = 1$

If M is a prime no :-

$$\boxed{b^{M-1} \% M = 1}$$

$$\Big\downarrow \; b^{-1} \% M$$

$$(b^{M-1} \% M * b^{-1} \% M) \% M = b^{-1} \% M$$

$$(b^{M-1} * b^{-1}) \% M = b^{-1} \% M$$

$$(b^{M-2}) \% M = b^{-1} \% M.$$

$$b^{-1} \% M = \underline{b^{M-2} \% M}$$

$$b^{-1} \% M = \text{power}(b, M-2, M)$$

{M is a prime No}

$$\boxed{TC :- O(\log_2 M)}$$

* * *