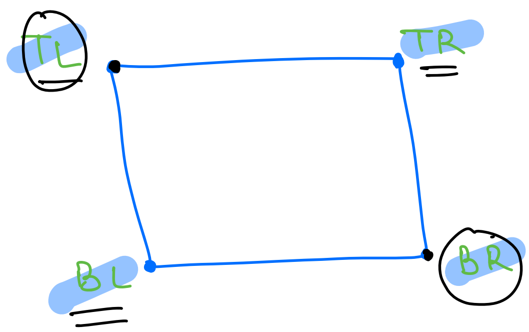
Q.1 Given a matrix of size N×M, for each query Q. find the sum of given submatrix.

Amazon

Contigous part of the matrix.

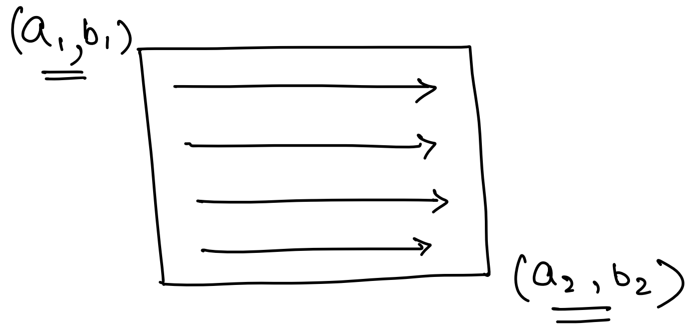


(a_1, b_1) (a_2, b_2)

⇒ TL & BR indexes can be used to uniquely identify a matrix

	0	1	2	3	4	5
0	7	1	-6	3	12	-2
1	10	5	-2	0	9	4
2	6	4	-3	8	11	3
3	13	-8	-5	12	4	6
4	3	2	1	9	3	9
5	4	3	-2	6	8	8

	TL	BR	
Q1:	(1,2)	(4,3)	⇒ 20
Q2:	(1,1)	(3,4)	⇒ 35



rows ⇒ a_1 to a_2

cols ⇒ b_1 to b_2

```
for ( k=1; k<=Q; k++ ){
        sum = 0
        for( i=a1 ; i<=a2 ; i++){
            for( j=b1; j<=b2; j++){
                sum += mat[i][j]
            }
        }
        print(sum);
}
```

N·M

1 Query ⇒ N·M

Q queries ⇒ $O(N \cdot M \cdot Q)$

SC : $O(1)$

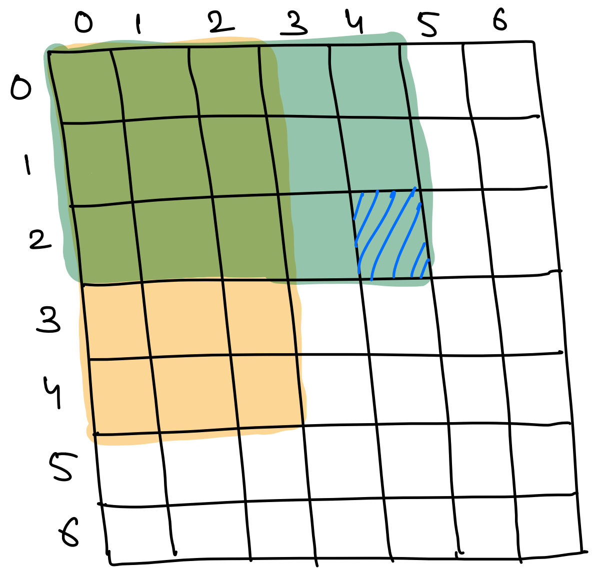
Optimization

1D Array :-

PS[i] ⇒ sum of elements from 0 to i.

2D Array

PS[i][j] ⇒ Sum of elements from [0, 0] to [i, j]

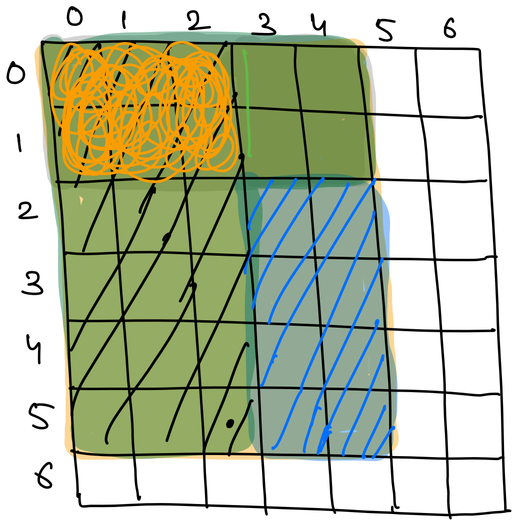


PS[2] [4] : sum from 0,0 to (2,4)

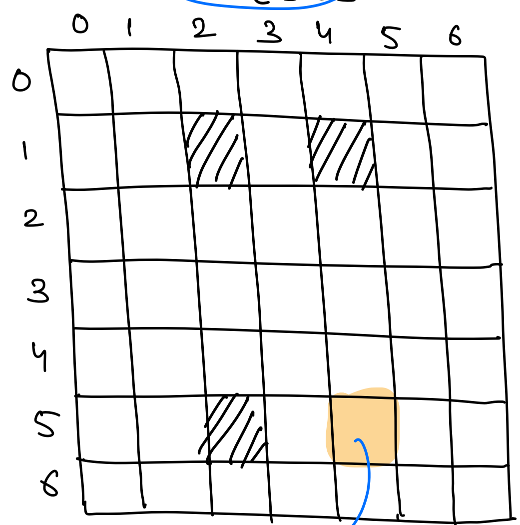
PS[4][2] : sum from 0,0 to (4,2)

Assuming we have PS matrix

mat[][]



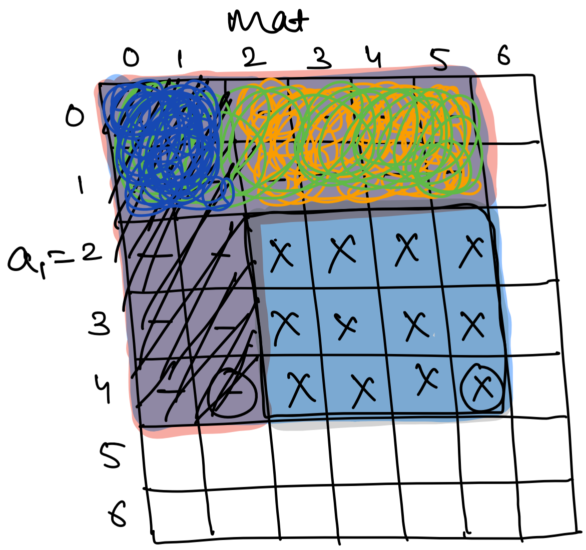
PS[][]

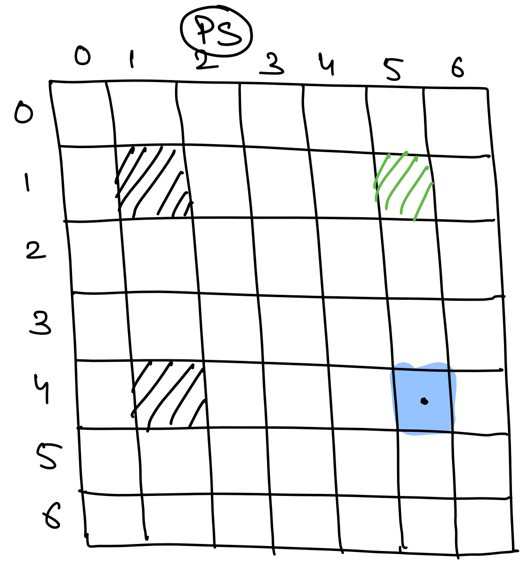


Sum from (0,0) to (5,4)

TL: (2,3)
BR: (5,4)

$$\text{Sum} = PS[5][4] - PS[5][2] - PS[1][4] + PS[1][2]$$

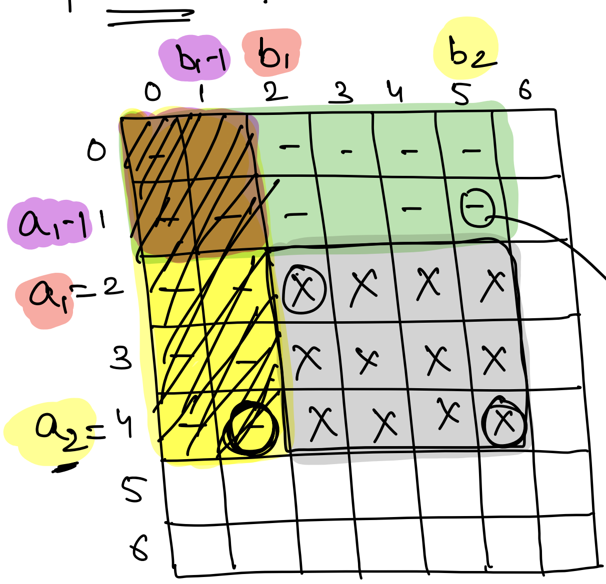




TL $(2, 2)$ to BR $(4, 5)$

$$\text{Sum} = PS[4][5] - PS[4][1] - PS[1][5] + PS[1][1]$$

General :-



TL : a_1, b_1

BR : a_2, b_2

$(a_1 - 1, b_2)$

$$\text{Sum} = PS[a_2][b_2] - PS[a_2][b_1 - 1] - PS[a_1 - 1][b_2] + PS[a_1 - 1][b_1 - 1]$$

for every query ⇒ (a_1, b_1) & (a_2, b_2)
TL BR

```
sum = PS[a2][b2]
if(b1 > 0)
    sum -= PS[a2][b1-1]
if(a1 > 0)
    sum -= PS[a1-1][b2]
if (a1 > 0 && b1 > 0)
    sum += PS[a1-1][b1-1]
```

$O(1)$

Create PS matrix

	0	1	2
0	a_0	a_1	a_2
1	b_0	b_1	b_2
2	c_0	c_1	c_2

① Prefix Sum row wise. ⇒ NM iterations.

	0	1	2
0	a_0	a_0+a_1	$a_0+a_1+a_2$
1	b_0	b_0+b_1	$b_0+b_1+b_2$
2	c_0	c_0+c_1	$c_0+c_1+c_2$

② Prefix Sum Column wise ⇒ NM iterations.

	0	1	2
0	a_0	a_0+a_1	$a_0+a_1+a_2$
1	a_0+b_0	$a_0+a_1+b_0+b_1$	$a_0+a_1+a_2+b_0+b_1+b_2$
2	$a_0+b_0+c_0$	$a_0+a_1+b_0+b_1+c_0+c_1$	$a_0+a_1+a_2+b_0+b_1+b_2+c_0+c_1+c_2$

Steps :-

1) find PS row wise $\Rightarrow O(N \times M)$

2) find PS column wise $\Rightarrow O(N \times M)$

TC : $O(\underbrace{N \times M}_{\text{creating PS matrix}} + Q)$ ↳ Q queries

SC : $O(\underbrace{N \times M}_{})$ ↳ PS matrix

Q.2 Given a matrix of size $N \times M$. Calculate the sum of all submatrix sum.

Sum of all subarray sums =

$$\left(\begin{array}{l}\text{No. of subarrays in} \\ \text{which } a[i] \text{ is present}\end{array}\right) * A[i]$$

$$= \sum_{i=0}^{N-1} a[i] * x$$

x ↓ No. of subarrays in which a[i] is present

Contribution Technique

Sum of all submatrix sum =

(No. of submatrices in which mat[i][j] is present) $*$ mat[i][j]

$$\begin{array}{c|c} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{bmatrix} 4 & 9 & 6 \\ 5 & -1 & 2 \end{bmatrix} \end{array}$$

\# mat[2][3]

$$[4] \to 4 \qquad [5] \to 5 \qquad [9] \to 9 \qquad [-1] \to -1 \qquad [6] \to 6 \qquad [2] \to 2$$

$$[4\ 9] \to 13 \qquad [9\ 6] \to 15 \qquad [5\ -1] \to 4 \qquad [-1\ 2] \to 1 \qquad [4\ 9\ 6] \to 19 \qquad [5\ -1\ 2] \to 6$$

$$\begin{bmatrix} 4 \\ 5 \end{bmatrix} \to 9 \qquad \begin{bmatrix} 9 \\ -1 \end{bmatrix} \to 8 \qquad \begin{bmatrix} 6 \\ 2 \end{bmatrix} \to 8$$

$$\begin{bmatrix} 4 & 9 \\ 5 & -1 \end{bmatrix} \to 17 \qquad \begin{bmatrix} 9 & 6 \\ -1 & 2 \end{bmatrix} \to 16 \qquad \begin{bmatrix} 4 & 9 & 6 \\ 5 & -1 & 2 \end{bmatrix} \to \underline{\underline{25}}$$

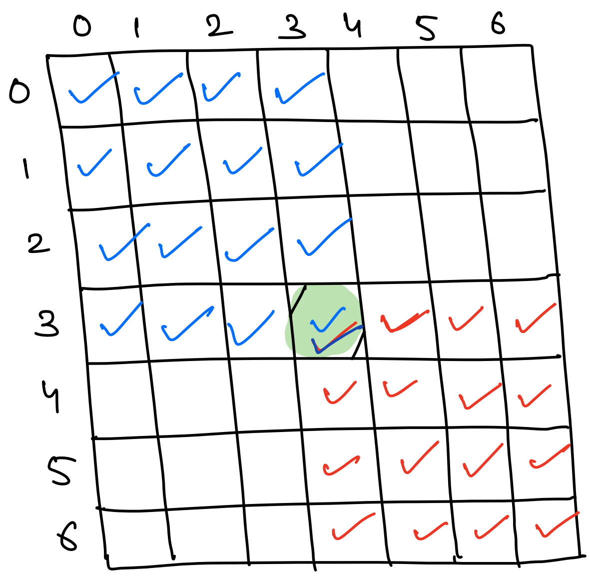
sum = $\underline{166}$

$$\text{Sum} = 4 * 6 + 5 \times 6 + 9 \times 8 + (-1) \times 8 + 6 \times 6 + 2 \times 6$$

$$= 24 + 30 + 72 - 8 + 36 + 12$$

$$= \underline{\underline{166}}$$

Ex :-



mat[3][3]

Submatrix : TL & BR
(a_1, b_1) (a_2, b_2)

TL
BR

Choices for TL ⇒ 16

Choices for BR ⇒ 16

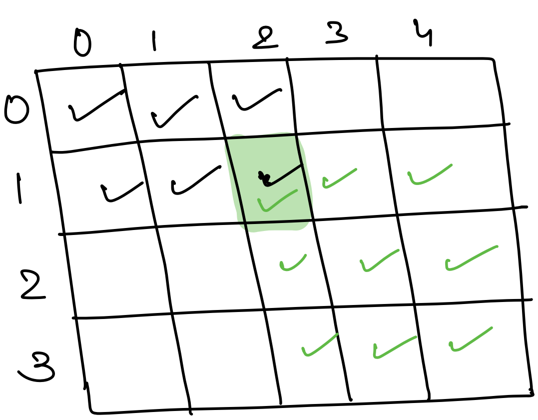
Total no. of submatrices in which mat[3][3] is present = 16 × 16

= 256

Quiz-1

4×5

mat[1][2]



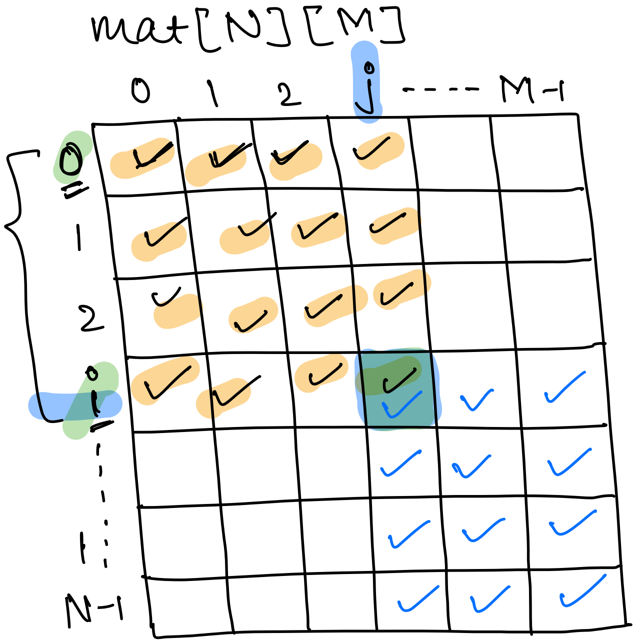
TL ⇒ 6

BR ⇒ 9

No. of matrices in which mat[i][j] will be present = 6×9

$= \underline{\underline{54}}$

⇒ Generalisation



mat[i][j]

row col

TL

BR

Quiz 2 :-

Choices for TL :

TL: rows ⇒ 0 to i : [0, i] ⇒ i+1

Cols ⇒ 0 to j : [0, j] ⇒ j+1

$$\boxed{\# \text{ of choices} = (i+1) \times (j+1)}$$

Quiz-3

Choices for BR :

rows : i to $N-1$: $[i, N-1] \Rightarrow N-i$

Cols : j to $M-1$: $[j, M-1] \Rightarrow M-j$

$$\boxed{\text{Choices for BR} : (N-i) * (M-j)}$$

Contribution of mat[i][j] = TL × BR

$= (i+1) \times (j+1) \times (N-i) \times (M-j)$

Code :-

```
for ( i= 0; i< N; i++ ) {
    for( j= 0; j< M; j++){
        TL => (i+1)x(j+1)
        BR => (N-i)x (M-j)

        sum += (TL x BR x mat[i][j]);
    }
}
return sum;
```

TC : $O(N \cdot M)$

SC : $O(1)$

Q Max submatrix sum.

↳ Saturday 12:00 pm