Q.1 Given an unsorted Array of integers, find the first missing natural number.

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.....

↳ 1, 2, 3, ...

arr[5] = { 3, -2, 1, 2, 7 }

1 → ✓
2 → ✓
3 → ✓
4 → ✗

4

Quiz

arr[8] = { 2, 4, -1, -6, 3, 7, 8, 4, -3 }

1 → ? ✗ ✓

Quiz

arr[6] = { 1, 0, -5, -6, 4, 2 }

1 → ✓
2 → ✓
3 → ✗

Quiz

arr[6] = { 1, 2, 5, 6, 4, 3 }

1 → ✓
2 → ✓
3 → ✓
4 → ✓
5 → ✓
6 → ✓
7 → ✗

N = 6

7

Brute force

1 → $O(N)$

2 → $O(N)$

3 → $O(N)$

4 → $O(N)$

⋮

ans → $O(N)$

\# of iterations = ans × N.

ans ∈ [1, N+1]

$N: \{\underbrace{1, 2, 3, 4, \text{------------} N}_{ans = (N+1)}\}$

Worst case : TC : $O(N^2)$

```
for (i = 1; i <= N; i++) {  =>  N
    Check if i is present
    in the Array
    If not present :
        return i;
}
return N+1;
```

N

HashSet | HashMap

HashSet< int > set;

(I) // Insert all array elements to set. $\Rightarrow O(N)$

(II) for (i = 1; i <= N; i++) { $\Rightarrow O(N)$

$O(1)$ { if (i is NOT present in the set)

return i;

}

return N+1;

TC: $O(N)$

SC: $O(N)$

↳ for set.

Sort

arr[6]: { 1, 0, -5, -6, 4, 2 }

↓ SORT $\Rightarrow N\log N$

: { -6, -5, 0, 1, 2, 4 } $\Rightarrow O(N)$

return 3

TC: $O(N\log N)$

SC: Depends on sorting Algo.

Approach with SC : O(1)

We can use the Array itself to search ?

$1 \rightarrow 0$
$2 \rightarrow 1$
$3 \rightarrow 2$
$4 \rightarrow 3$
$\vdots \qquad \vdots$
$N \rightarrow N-1$

⇒ We can use indices of the Array to mark the presence of Natural no's.

Assuming Array is containing +ve elements.

Ex Arr : { -7 -6 -1 $\boxed{2}$ 3 }   N = 5

indices: 0 1 2 3 4

$1 \rightarrow 0$
$2 \rightarrow 1$
$3 \rightarrow 2$
$\vdots$
$N \rightarrow N-1$

If arr[i] < 0 ⇒ (i+1) is present in the Array

⇒ return 4

Ex

1 → 0
2 → 1
3 → 2
4 → 3
⋮
N → N-1

Arr :- $\{-8, -1, -4, -2, 6, -3\}$ (indices 0, 1, 2, 3, 4, 5) N = 6

X, abs(-2), abs(-3)

⇓

⇒ $\{-8, -1, -4, -2, 6, -3\}$ (indices 0, 1, 2, 3, 4, 5); 4 → 4+1

return 5

Ex :-

Arr : $\{5, 9, 10, 11, 13\}$ (indices 0, 1, 2, 3, 4) N = 5

X X X X

⇒ $\{5, 9, 10, 11, -13\}$ (indices $\boxed{0}$, 1, 2, 3, 4)

At index = 0, a[i] > 0 ⇒ (i+1) is missing

Ex :-

⟹ $\{-1, -4, -3, -2, -5\}$ (indices 0, 1, 2, 3, 4) N = 5

abs(-2)

All the indices from 0 to N-1 are having -ve values ⇒ $\boxed{N+1}$ **Ans.**

Edge cases

(1) Array has -ve numbers.

Arr : { -8 , 1 , 5 , -2 , -6 , 3 } $N = 6$

indices: 0, 1, 2, 3, 4, 5

X abs(-2)

⇒ ans range : 1 to N+1.

Replace -ve numbers with N+2

$\{ -\cancel{1}, -\cancel{2}, 1, 3, 4 \}$ $N = 5$

↓ 1st step

{ -7 , 7 , -1 , -3 , 4 }

X X

arr[1] > 0 ⇒ 2 is missing

Can we replace -ve no's with ZERO?

$\{ -\cancel{1}, -\cancel{2}, 1, 3, 4 \}$

0 0

$\{ \boxed{-0}, 0, 1, 3, 4 \}$

-0 ? X

Ex :- $\{-\cancel{4}, 1, 2, -\cancel{8}, 5, 6\}$ $N=6$

$-4 \to N+1$, $-8 \to N+1$

$N=6 \Downarrow \geq N+1$

$\downarrow$

0	1	2	3	4	5
-7	-1	2	7	-5	-6
×			×		

$N=6$

$\longrightarrow$ return 3

Ex :-

0	1	2	3	4
4	1	$\cancel{0}$	9	$-\cancel{6}$
		6		6

$N=5$

0	1	2	3	4
-4	$\boxed{1}$	6	-9	6
		×	abs(-9) ×	×

$N=5$

$\longrightarrow$ return 2

Duplicates :-

Arr :

0	1	2	3	4	5
3	6	$-\cancel{8}$	3	$-\cancel{4}$	1
		7		7	

$N=6$

0	1	2	3	4	5
-3	$\boxed{6}$	$\boxed{-7}$	$\boxed{3}$	7	-1
		×		×	abs(-1)

At i=2, we have -ve already. Skip this.

-1 * abs(-7)

$\Rightarrow$ $\boxed{-1 * abs(arr[i])}$

return 2

$\Rightarrow$ Replace -ve numbers or zero in the Array with a value >= N+1

TC : O(N)

SC : O(1)

Q.2 Given a row wise & col-wise sorted matrix, find if (k) is present in the matrix. $N \times M$

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	0	1	2	4
0	5	10	15	20
1	6	12	18	24
2	7	14	21	28
3	8	16	24	32

$K = 22 \Rightarrow$ false

$K = 14 \Rightarrow$ true.

Brute force $\Rightarrow O(N \cdot M)$

Binary Search $\Rightarrow O(N \log M)$

$14 < 20$

	0	1	2	4
0	5	10	15	$\boxed{20}$
1	6	12	18	24
2	7	14	21	28
3	8	16	24	32

$K = 14$

- Start with 20

$K < mat[i][j]$

$\Rightarrow 14 < 20$

$\Rightarrow 14 < 15$

$\Rightarrow 14 > 10$

$\Rightarrow 14 > 12$

$\Rightarrow 14 = 14$ $(2,1)$

TC: O(N+M)

SC: O(1)

```
i = 0, j = m-1
while ( // tho ) {
    if( k == mat[i][j])
        return true;
    if( k < mat[i][j])
        j--;
    else
        i++;
```

3

	0	1	2	4
0	5	10	15	20
1	6	12	18	24
2	7	13	21	28
3	14	16	24	32

14 < 20

14 < 15

14 > 10

14 > 12

14 > 13

14 < 16

Q.3 MAXIMUM ABSOLUTE DIFFERENCE

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Given an integer array, find the max value of

$$\boxed{f(i,j) = |A[i] - A[j]| + |i-j|}$$

$|x| \geq 0$

$|3| = 3$

$|-3| = 3$

Arr : $\{ \overset{0}{1} \quad \overset{1}{3} \quad \overset{2}{-1} \}$

i	j	$f(i,j)$
0	0	$\|1-1\| + \|0-0\| = 0$
0	1	$\|1-3\| + \|0-1\| = 3$
0	2	$\|1-(-1)\| + \|0-2\| = 4$
1	0	$\|3-1\| + \|1-0\| = 3$
1	1	$\|3-3\| + \|1-1\| = 0$
1	2	$\|3-(-1)\| + \|1-2\| = 5$
2	0	$\|-1-1\| + \|2-0\| = 4$
2	1	$\|-1-3\| + \|2-1\| = 5$
2	2	$\|-1-(-1)\| + \|2-2\| = 0$

$\Rightarrow 5$

Brute force

↳ Check for every possible pair.

```
for(i = 0; i < N; i++){
    for(j = 0; j < N; j++){
        // f(i, j) & maintain the MAX.
    }
}
```

TC : $O(N^2)$
SC : $O(1)$

Observations

$$\boxed{f(i,j) = |A[i] - A[j]| + |i-j|}$$

① $f(i,i) = 0$

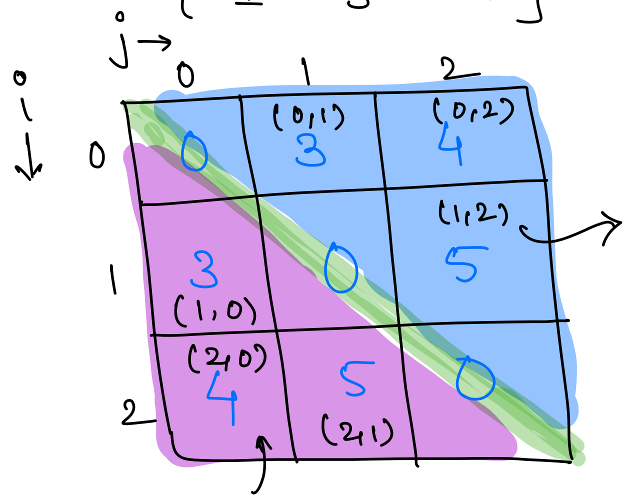
$\Rightarrow |A[i] - A[i]| + |i - i|$ → 0 , 0

② $f(i,j) = f(j,i)$

$$f(i,j) = \underbrace{|A[i] - A[j]|}_{\Rightarrow |A[j] - A[i]|} + \underbrace{|i-j|}_{|j-i|}$$

$N = 3$

$$\text{Arr} : \begin{array}{ccc} 0 & 1 & 2 \\ \{\ 1 & 3 & -1\ \} \end{array}$$



Upper triangular matrix
$(i < j)$

lower triangular matrix
$(i > j)$

$$|x| = +ve = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

$|7| = 7$ | $|-7| = 7$

↳ $-1 * -7 = 7$

$|-10| = -(-10)$

$= \underline{\underline{10}}$

$$|x|$$

x $(x \geq 0)$ $\qquad$ $-x$ $(x < 0)$

$$\boxed{f(i,j) = |A[i] - A[j]| + |i-j|}$$

$i-j>0$ $\qquad$ $i-j<0$

\# Upper triangular matrix $\Rightarrow (i<j)$

$\Rightarrow$

$$\Rightarrow \quad i-j<0$$

$$\boxed{|i-j| = -(i-j) = -i+j}$$

$$\boxed{|A[i] - A[j]|}$$

i) $A[i]-A[j]$ $\quad$ if $(A[i]-A[j])>0$

ii) $-(A[i]-A[j])$ $\quad$ if $(A[i]-A[j])<0$

i) $A[i]-A[j] \geq 0$

$$f(i,j) = A[i]-A[j] + j - i$$

$$= (A[i]-i) + j - A[j]$$

$$f(i,j) = (A[i]-i) - (A[j]-j)$$

$\hookrightarrow$ (1)

ii) $A[i] - A[j] < 0$

$$f(i,j) = -(A[i] - A[j]) + \cancel{j - i}$$

$$= -A[i] + A[j] + j - i$$

$$f(i,j) = (A[j] + j) - (A[i] + i)$$

$\hookrightarrow$ 2

<u>Ex</u> :-

$|A| - B = ?$

$A - B$ $(A \geq 0)$ | $-A - B$ $(A < 0)$

$A = 6$ $B = 2$ $\lvert A\rvert - B = 6 - 2$ $= 4$ ✓ i) $6 - 2 = 4$ ii) $-6 - 2 = -8$ $\max(4, -8)$ $\Rightarrow 4$ ✓	$A = -7$ $B = 4$ $\lvert A\rvert - B =$ $\lvert -7\rvert - 4 = 3$ i) $-7 - 4$ $= -11$ ii) $-(-7) - 4$ $7 - 4 = 3$ $\max(-11, 3)$ ✓	$A = 5$ $B = -1$ $\lvert A\rvert - B$ $= 5 - (-1)$ $= 6$ i) $5 - (-1)$ $= 6$ ii) $-5 - (-1)$ -4 $\max(6, -4)$	$A = -7$ $B = -5$ $\lvert A\rvert - B$ $= \lvert -7\rvert - (-5)$ $= 12$ i) $-7 - (-5)$ -2 ii) $-(-7) - (-5)$ $7 + 5 = 12$ $\max(-2, 12)$

<u>HW</u> Try to Derive this for <u>lower triangular matrix</u>. <u>$(i > j)$</u>

$$f(i,j) = \max\left(\underbrace{(A[i]-i)}_{\max} - \underbrace{(A[j]-j)}_{\min},\ \underbrace{(A[j]+j)}_{\max} - \underbrace{(A[i]+i)}_{\min}\right)$$

$$\max(\underbrace{x}_{\text{Max}} - \underbrace{y}_{\text{Min}})$$

$A[i]-i \Rightarrow$ max & min

$A[i]+i \Rightarrow$ max & min

A :

0	1	2
1	3	-1

$\downarrow A[i]-i$

B :

1	2	-3

max & min from Array B

min = -3

max = 2

$\Rightarrow 2-(-3) = 5$

A :

0	1	2
1	3	-1

$\downarrow A[i]+i$

C :

1	4	1

max & min from Array C

max = 4

min = 1

max - min = 3

$\max(5, 3) \Rightarrow 5$

TC : O(N)
SC : O(N)

↳ We can solve this in O(1) SC.

	0	1	2
A :	1	3	-1

$\downarrow$ A[i] - i

max = $-\infty$ ~~1~~ 2
min = ∞ ~~1~~ -3
i=0, A[0] - 0 = 1 - 0 = 1
i=1 A[1] - 1 = 3 - 1 = 2
i=2 A[2] - 2 = -1 - 2 = -3

	0	1	2
A :	1	3	-1

$\downarrow$ A[i] + i

max = $-\infty$ ~~1~~ 4
min = ∞ 1
i=0 ⇒ 1+0 = 1
i=1 ⇒ 3+1 = 4
i=2 ⇒ -1+2 = 1

⇒ $\boxed{SC \Rightarrow O(1)}$

⇒ find MAX (x) & MIN (y) of A[i] - i ∀ i

⇒ find MAX (a) & MIN (b) of A[i] + i ∀ i

⇒ return max(x-y, a-b)