

Today's Content

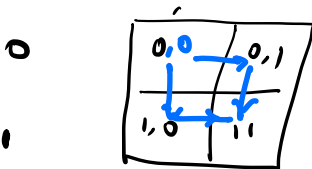
→ 2D matrices questions

- ✓ → No. of ways from $(0,0) \rightarrow$ BR \rightarrow Blocked cells
- ✓ → min cost to reach BR
- ✓ → Dungeons & Dragon.

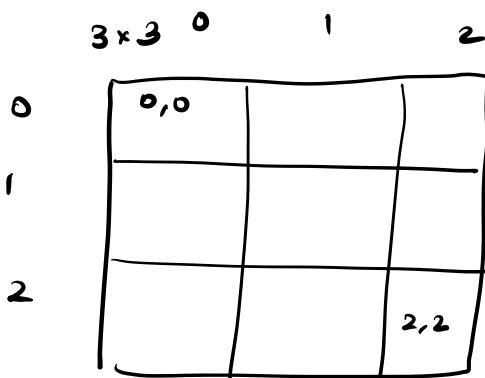
Q. No. of ways to go from $(0,0) \rightarrow$ BR cell $(n-1, m-1)$
 n, m .

Cell \rightarrow right, bottom movement
 is only allowed.

$n=2$ $m=2$



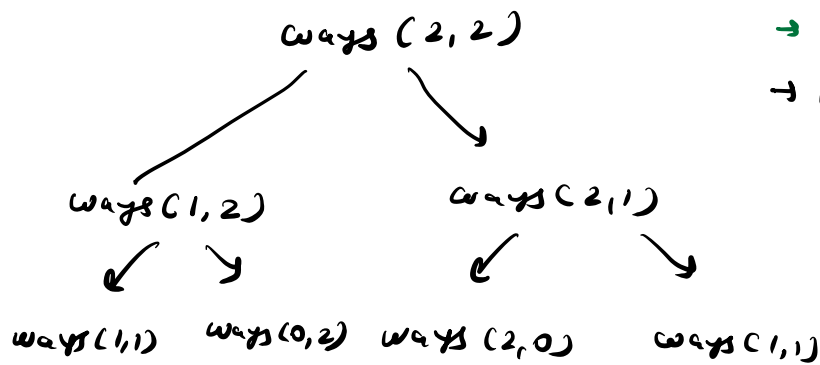
ans = 2



ans = 6

Path

- $(0,0) \rightarrow (0,1) \rightarrow (0,2) \rightarrow (1,2) \rightarrow (2,2)$
- $(0,0) \rightarrow (0,1) \rightarrow (1,1) \rightarrow (1,2) \rightarrow (2,2)$
- $(0,0) \rightarrow (0,1) \rightarrow (1,1) \rightarrow (2,1) \rightarrow (2,2)$
- $(0,0) \rightarrow (1,0) \rightarrow (1,1) \rightarrow (1,2) \rightarrow (2,2)$
- $(0,0) \rightarrow (1,0) \rightarrow (1,1) \rightarrow (2,1) \rightarrow (2,2)$
- $(0,0) \rightarrow (1,0) \rightarrow (2,0) \rightarrow (2,1) \rightarrow (2,2)$

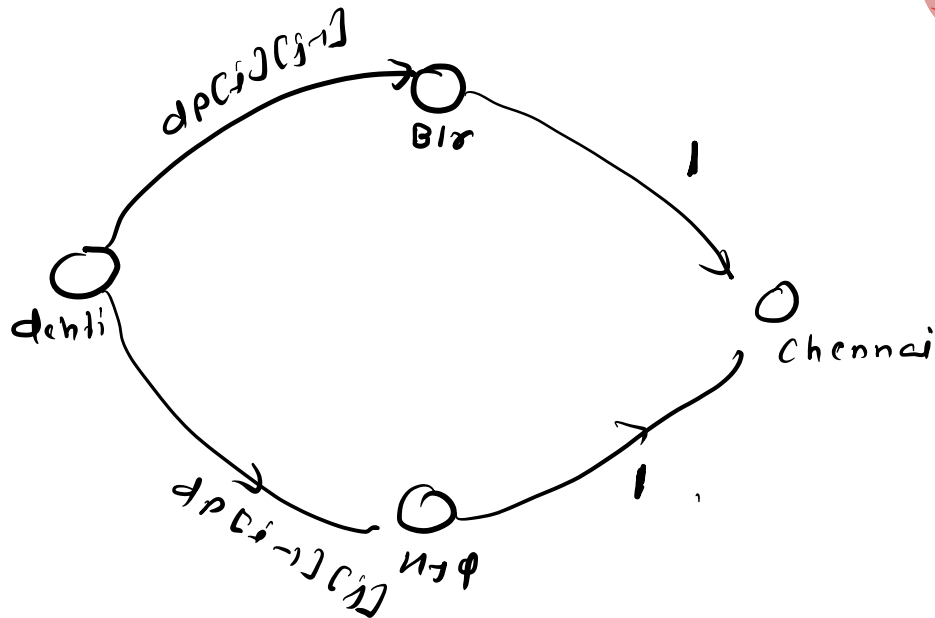


→ optimal substructure.

→ overlapping subproblems

$dp[i][j] = \# \text{ No. of ways to reach } (0,0) \rightarrow (i,j)$

$$dp[i][j] = dp[i][j-1] + dp[i-1][j] + \cancel{1} + \cancel{2}$$



$$dp[i][j] = dp[i-1][j] + dp[i][j-1]$$

Base condition

↳ if $(i == 0 \text{ || } j == 0)$ $dp[i][j] = 1$

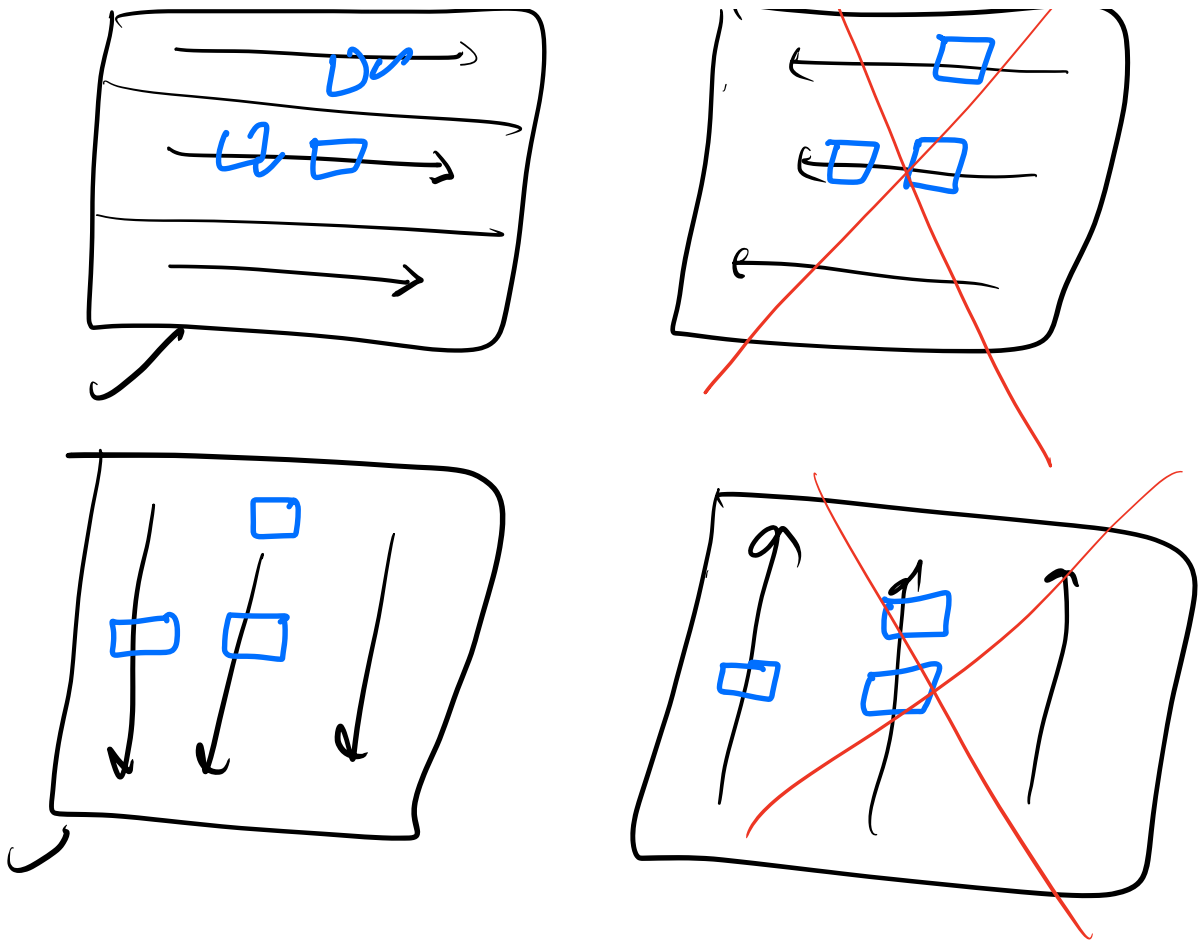
```
int ways (int N, int M)
{
    int dp[N][M]

    for (i = 0; i < n; i++)
        for (j = 0; j < m; j++)
        {
            if (i == 0 || j == 0) dp[i][j] = 1
            else
                dp[i][j] = dp[i-1][j] + dp[i][j-1]
        }

    return dp[n-1][m-1]
}
```

TC : $O(N \times M)$

SC : $O(NM)$

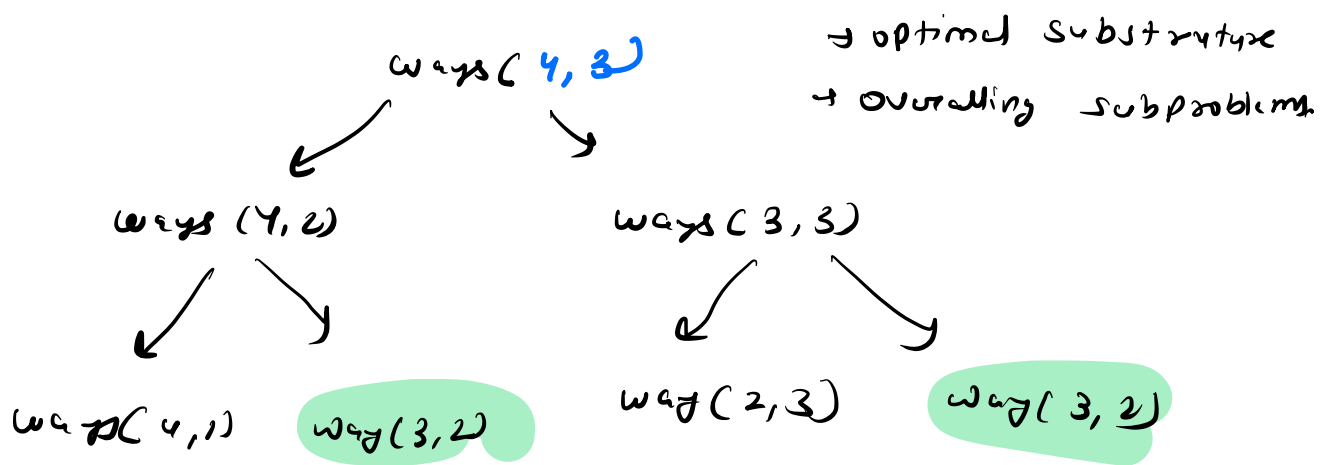


Q. no of ways to go from 0,0 to BR cell

cell \rightarrow right / bottom.

	0	1	2	3
0	1	1	1	1
1	1	2	2	3
2	2	3	4	6
3	3	6	10	20

ans = 3.



$$dp[i][j] = \begin{cases} \text{if } (mat[i][j] == 0) & dp[i][j] = 0 \\ \text{else} & \{ dp[i-1][j] + dp[i][j-1] \} \end{cases}$$

Base conditions,

$$i == 0 \quad || \quad j == 0 \quad \text{code fails,}$$

$$j = 0$$

	0	1	2	3
0	1	1	0	1
1	1	0	1	0

$$dp[0][3] = ?$$

$$= 0$$

2	0	1	1	1
3	1	0	1	1
4	1	1	1	1

$dp[0][2] = 0$
 $dp[0][1] = 1$
 $dp[0][0] = 1$

$dp[N][M] = \{0\}$

for ($i = 0$; $i < m$; $i++$)

{
 if ($mat[0][i] == 1$) $dp[0][i] = 1$
 else break
 }

$d^r = 0$

	0	1	2	3
0	1	1	0	1
1	1	0	1	0
2	0	1	1	1
3	1	0	1	1
4	1	1	1	1

`dp[N][M] = 0;`

`for (i = 0; i < n; i++)`

`{`
 `if (mat[i][0] == 1) dp[i][0] = 1`
 `else break`
`}`

`int ways (int mat[][], int n, int m)`

`int dp[N][M] = {0}`

`// row 0 is taken care`

`for (i = 0; i < m; i++)`

`{`
 `if (mat[0][i] == 1) dp[0][i] = 1`
 `else break`
`}`

`// col 0 is taken care`

`for (i = 0; i < n; i++)`

`{`
 `if (mat[i][0] == 1) dp[i][0] = 1`
`}`

```

    else break
    for (j=1 ; j < n ; j++)
        for (j=1 ; j < m ; j++)
            if (mat[i][j] == 0) dp[i][j] = 0
            else
                dp[i][j] = dp[i-1][j] + dp[i][j-1]
    return dp[N-1][m-1].

```

T.C : $O(N \times M)$

S.C : $O(N \times M)$


```
int ways (int mat[][], N, m)
```

```
{  
    int dp[N][M] = {-1}
```

```
    return func (mat, dp, N-1, m-1)
```

```
int func (int mat[][], int dp[][], i, j)
```

```
{  
    if (i < 0 || j < 0) return 0
```

```
    if (mat[i][j] == 0) return 0
```

```
    if (i + j == 0) return 1
```

```
    // same  
    if (i == 0 || j == 0)
```

```
    if (dp[i][j] == -1)
```

```
{  
    dp[i][j] = func (mat, dp, i-1, j)
```

```
        +  
        func (mat, dp, i, j-1)
```

```
    return dp[i][j]
```

In recursive code . writing base cases

is generally relatively easier.

TC: $O(N \times M)$

SC: $O(NM)$.

Break

10:23

10:32

TODO.

Q. Min cost to reach 0,0 to BR cell

cell \rightarrow right / bottom.

3	2	8
6	7	5
4	3	2
1	8	10

Q. Given $mat[N][M]$, where $mat[i][j]$ indicate health gain at this cell.

Find min health you should start with

at (0,0) so that you can reach $N-1, M-1$

\rightarrow cell \rightarrow bottom / right

\rightarrow If your health reach zero, game over

→ (0,0) → n-1, n-1

-3	-5
-2	1

~~H=4~~

~~H=5~~

H=6

10
5

-3	2		
-6			

min H (0,2)



min (1,0)

Expression.

Case I

x

-5

$$x - 5 = 1$$

$$x = 6$$

Case II

x

-2
-5

$$x - 2 = 6$$

$$x = 8$$

Case III

$$x \quad \begin{array}{|c|c|} \hline -4 & -5 \\ \hline \end{array} \quad \boxed{6}$$

$$x - 4 = 6$$

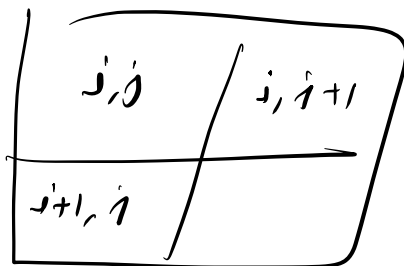
$$x = 10$$

Case IV

$$x \quad \begin{array}{|c|c|} \hline -3 & -2 \\ \hline -4 & -5 \\ \hline \end{array} \quad \begin{array}{c} \boxed{8} \\ \boxed{10} \end{array}$$

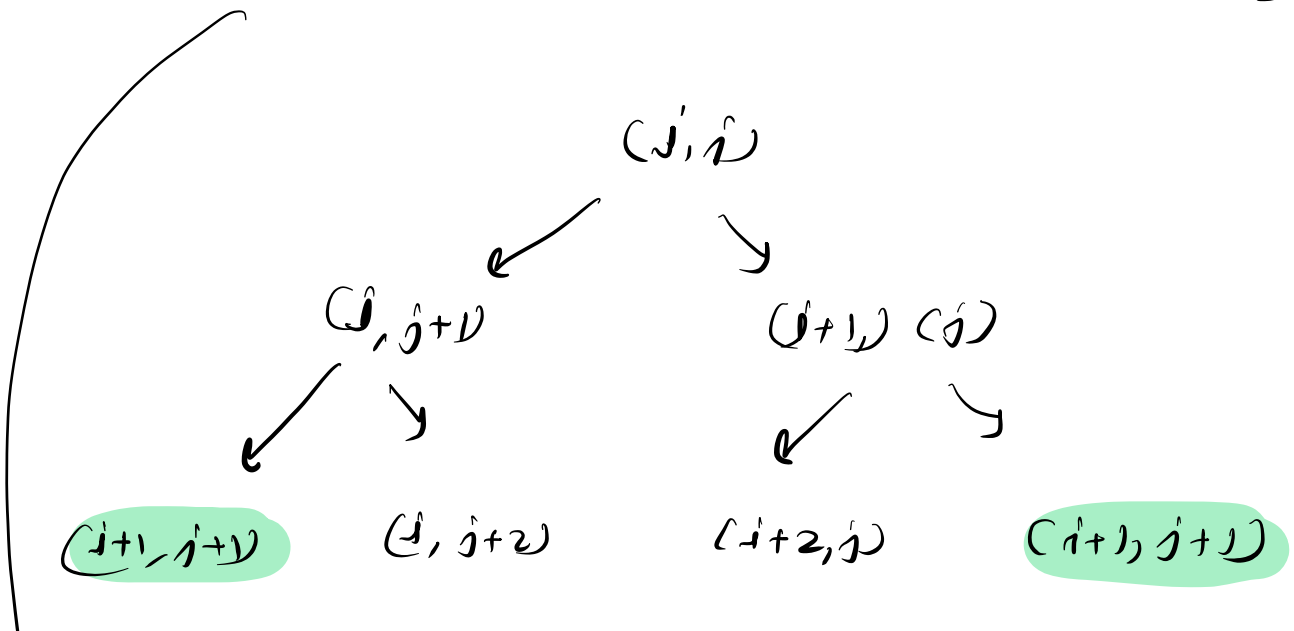
$$x - 3 = \min(8, 10)$$

$$x = 11$$



→ optimal substructure,
→ overlapping subproblems.

$$x + \max[i][j] = \min(dp[i+1][j], dp[i][j+1])$$



$$dp[i][j] = \min(dp[i+1][j], dp[i][j+1]) - \text{mat}[i][j]$$

$dp[N][N]$ = min health you will need start (j, j) and reach $(N-1, M-1)$

	8
20	-2
-4	-5
10	

$$x + 20 = 8$$

$$x = -12$$

$$dp[i][j] = \max(1, \min(dp[i+1][j], dp[i][j+1]) - \text{mat}[i][j])$$

Base cases

$j == N-1 \parallel j == M-1 \Rightarrow \text{code fail.}$

int minH (int mat[N][M], N, M)

int dp[N][M] = {-1}

dp[N-1][M-1] = max(1, 1 - mat[N-1][M-1])

```
return fun(mat, N, m, dp, 0, 0)
```

```
int fun(mat, n, m, dp, i, j)
```

```
if (i ≥ N || j ≥ m) return INT_Max
```

```
if (dp[i][j] == -1)
```

```
int x = fun(mat, n, m, dp, i+1, j)
```

```
int y = fun(mat, n, m, dp, i, j+1)
```

```
dp[i][j] = max(1, min(x, y) - mat[i][j])
```

```
return dp[i][j]
```

10

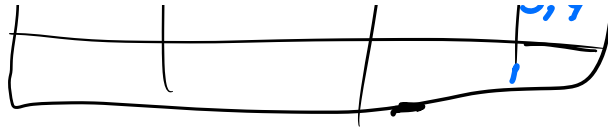
15

-3	2		
-6			

4,4

4,5

5,4



Passing by value

Passing by reference



sending array will

not create copy.