

Design And Analysis of Algorithm

LA-2

PAGE NO.:

DATE 20/10/20

2018BTECS00061

Q2) Use Strassen's algorithm to compute the matrix product

$$\begin{pmatrix} 1 & 3 \\ 7 & 5 \end{pmatrix} \begin{pmatrix} 6 & 8 \\ 4 & 2 \end{pmatrix}$$

$$a = \begin{bmatrix} 1 & 3 \\ 7 & 5 \end{bmatrix}$$

$$b = \begin{bmatrix} 6 & 8 \\ 4 & 2 \end{bmatrix}$$

$$s_1 = a_{11} + a_{22}$$

$$= 1 + 5$$

$$= 6$$

$$s_9 = a_{12} - a_{22}$$

$$= 3 - 5$$

$$= -2$$

$$s_2 = b_{11} + b_{22}$$

$$= 6 + 2$$

$$= 8$$

$$s_{10} = b_{21} + b_{22}$$

$$= 4 + 2$$

$$= 6$$

$$s_3 = a_{21} + a_{22}$$

$$= 7 + 5$$

$$= 12$$

$$s_4 = b_{12} - b_{22}$$

$$= 8 - 2$$

$$= 6$$

$$s_5 = b_{21} - b_{11}$$

$$= 4 - 6$$

$$= -2$$

$$s_6 = a_{11} + a_{12}$$

$$= 1 + 3$$

$$= 4$$

$$s_7 = a_{21} - a_{11}$$

$$= 7 - 1$$

$$= 6$$

$$s_8 = b_{11} + b_{12}$$

$$= 6 + 8$$

$$= 14$$

Strassen's 7 formulas:

$$\begin{aligned} 1) D_1 &= S_1 \cdot S_2 \\ &= (a_{11} + a_{22}) \cdot (b_{11} + b_{22}) \\ &= 6 \times 8 \\ &= 48 \end{aligned}$$

$$\begin{aligned} 2) D_2 &= S_3 \cdot b_{11} \\ &= (a_{21} + a_{22}) \cdot b_{11} \\ &= 12 \times 6 \\ &= 72 \end{aligned}$$

$$\begin{aligned} 3) D_3 &= S_4 \cdot a_{11} \\ &= (b_{212} - b_{22}) \cdot a_{11} \\ &= 6 \times 1 \\ &= 6 \end{aligned}$$

$$\begin{aligned} 4) D_4 &= S_5 \cdot a_{22} \\ &= (b_{21} - b_{11}) \cdot a_{22} \\ &= (-2) \times 5 \\ &= -10 \end{aligned}$$

$$\begin{aligned} 5) D_5 &= S_6 \cdot b_{22} \\ &= (a_{11} + a_{12}) \cdot b_{22} \\ &= 4 \times 2 \\ &= 8 \end{aligned}$$

$$\begin{aligned} 6) D_6 &= S_7 \cdot S_8 \\ &= (a_{21} - a_{11}) \cdot (b_{11} + b_{12}) \\ &= 6 \times 14 \\ &= 84 \end{aligned}$$

$$\begin{aligned} 7) D_7 &= S_9 \cdot S_{10} \\ &= (a_{12} - a_{22}) \cdot (b_{21} + b_{22}) \\ &= (-2) \cdot (6) \\ &= -12 \end{aligned}$$

Result Matrix:

$$\begin{aligned} C_{11} &= D_1 + D_4 - D_5 + D_7 \\ &= 48 + (-10) - 8 + (-12) \\ &= 18 \end{aligned}$$

$$\begin{aligned} C_{12} &= D_3 + D_5 \\ &= 6 + 8 \\ &= 14 \end{aligned}$$

$$\begin{aligned} C_{21} &= D_2 + D_4 \\ &= 72 - 10 \\ &= 62 \end{aligned}$$

$$\begin{aligned} C_{22} &= D_1 + D_3 - D_2 - D_6 \\ &= 48 + 6 - 72 - 84 \\ &= 66 \end{aligned}$$

$$\therefore \begin{pmatrix} 1 & 3 \\ 7 & 5 \end{pmatrix} \begin{pmatrix} 6 & 8 \\ 4 & 2 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 18 & 14 \\ 62 & 66 \end{pmatrix}}}$$

Time Complexity of Strassen's method:

Addition and Subtraction of two matrices takes $O(N^2)$ time. So time complexity can be written as,

$$T(N) = 7T(N/2) + O(N^2)$$

..... (Since there are 7 subproblems).

From Master's method time complexity of Strassen Method is,

$$O(N^{\log 7}) = O(N^{2.8074})$$