Q2) Use Strassen's algorithm to compute the matrix product

$$\begin{pmatrix} 1 & 3 \\ 7 & 5 \end{pmatrix} \begin{pmatrix} 6 & 8 \\ 4 & 2 \end{pmatrix}$$

$a = \begin{bmatrix} 1 & 3 \\ 7 & 5 \end{bmatrix}$ $b = \begin{bmatrix} 6 & 8 \\ 4 & 2 \end{bmatrix}$

$S_1 = a_{11} + a_{22}$

$\quad = 1 + 5$

$\quad = 6$

$S_2 = b_{11} + b_{22}$

$\quad = 6 + 2$

$\quad = 8$

$S_3 = a_{21} + a_{22}$

$\quad = 7 + 5$

$\quad = 12$

$S_4 = b_{12} - b_{22}$

$\quad = 8 - 2$

$\quad = 6$

$S_5 = b_{21} - b_{11}$

$\quad = 4 - 6$

$\quad = -2$

$S_6 = a_{11} + a_{12}$

$\quad = 1 + 3$

$\quad = 4$

$S_7 = a_{21} - a_{11}$

$\quad = 7 - 1$

$\quad = 6$

$S_8 = b_{11} + b_{12}$

$\quad = 6 + 8$

$\quad = 14$

$S_9 = a_{12} - a_{22}$

$\quad = 3 - 5$

$\quad = -2$

$S_{10} = b_{21} + b_{22}$

$\quad = 4 + 2$

$\quad = 6$

Strassen's 7 formulas:

1) $D_1 = S_1 . S_2$
$= (a_{11} + a_{22}) . (b_{11} + b_{22})$
$= 6 \times 8$
$= 48$

2) $D_2 = S_3 . b_{11}$
$= (a_{21} + a_{22}) . b_{11}$
$= 12 \times 6$
$= 72$

3) $D_3 = S_4 . a_{11}$
$= (b_{212} - b_{22}) . a_{11}$
$= 6 \times 1$
$= 6$

4) $D_4 = S_5 . a_{22}$
$= (b_{21} - b_{11}) . a_{22}$
$= (-2) \times 5$
$= -10$

5) $D_5 = S_6 . b_{22}$
$= (a_{11} + a_{12}) . b_{22}$
$= 4 \times 2$
$= 8$

6) $D_6 = S_7 . S_8$
$= (a_{21} - a_{11}) . (b_{11} + b_{12})$
$= 6 \times 14$
$= 84$

7) $D_7 = S_9 . S_{10}$
$= (a_{12} - a_{22}) . (b_{21} + b_{22})$
$= (-2) . (6)$
$= -12$

Result Matrix.

$C_{11} = D_1 + D_4 - D_5 + D_7$

$\quad = 48 + (-10) - 8 + (-12)$

$\quad = 18$.

$C_{12} = D_3 + D_5$

$\quad = 6 + 8$

$\quad = 14$

$C_{21} = D_2 + D_4$

$\quad = 72 - 10$

$\quad = 62$

$C_{22} = D_1 + D_3 - D_2 - D_6$

$\quad = 48 + 6 - 72 - 84$

$\quad = 66$.

$$\therefore \begin{pmatrix} 1 & 3 \\ 7 & 5 \end{pmatrix} \begin{pmatrix} 6 & 8 \\ 4 & 2 \end{pmatrix} = \begin{pmatrix} 18 & 14 \\ 62 & 66 \end{pmatrix}$$

Time Complexity of Strassen's method:

Addition and Subtraction of two matrices takes $O(N^2)$ time. So time complexity can be written as,

$$T(N) = 7T(N/2) + O(N^2)$$

..... (Since there are 7 subproblems).

From Master's method time complexity of Strassen Method is,

$$O(N^{\log 7}) = O(N^{2.8074}).$$