Time Complexity Analysis of Operations

The time complexity for each operation:

1. **Add Product (addProduct):**
   - Operation: Adding a product to the inventory.
   - Implementation: Uses the put method of HashMap.
   - Time Complexity: O(1) on average, O(n) in the worst case (when a rehash occurs due to high load factor).

2. **Update Product (updateProduct):**
   - Operation: Updating a product's details in the inventory.
   - Implementation: Also uses the put method of HashMap.
   - Time Complexity: O(1) on average, O(n) in the worst case (similar to the put method).

3. **Delete Product (deleteProduct):**
   - Operation: Removing a product from the inventory.
   - Implementation: Uses the remove method of HashMap.
   - Time Complexity: O(1) on average, O(n) in the worst case (when a rehash is needed).

Discussion on Optimizing Operations

1. Concurrency Management: Use `ConcurrentHashMap` to safely handle changes from multiple users at the same time.

2. Indexing and Search Optimization: Create additional ways to quickly find data by different attributes, not just the primary key.

3. Batch Operations: Group similar tasks together, like adding or updating products, to improve efficiency and reduce repetitive work.