Grid Problem

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1 Consider a 101x101 grid, if you were to throw 20 darts at this grid, on an average, how close are you to the Diagonal (assume repetitions are allowed)? Write a python code with 101 x 101 generalized to nxn (where n is odd) and 20 generalized to k darts.

We calculate the distance for any point (i, j) in the grid, the distance to the diagonal $y = x$ can be calculated using the formula for the distance of a point to a line. In this specific case, the formula simplifies to $\frac{|i-j|}{\sqrt{2}}$, because the diagonal line has a slope of 1 and an intercept of 0, and the grid is square and uniformly spaced. we find an expression for the n^2 points and this become general for the k points.

Average Distance Calculation

The average distance to the diagonal for points uniformly distributed over the grid is the expected value of the distance for a single randomly chosen point. Since each point is equally likely, this is the sum of the distances of all points from the diagonal divided by the total number of points (n^2).

Mathematical Expression

Considering a uniform distribution of points across the grid, the calculation of the average distance $\bar{D}$ involves summing up the distances of each point from the diagonal and then dividing by the total number of points:

$$\bar{D} = \frac{1}{n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{|i-j|}{\sqrt{2}}$$

This formula, however, does not easily simplify due to the absolute difference $|i-j|$, representing the distance to the diagonal. Now we can rewrite it into the

$$\bar{D} = 2\sum_{d=0}^{n-1} \frac{d(n-d)}{n^2\sqrt{2}}$$

And we can also see that the maximum distance from the diagonal to a point is $\frac{n}{\sqrt{2}}$, and there are only two points with that distance. The next maximum distance is $\frac{n-1}{\sqrt{2}}$, with four points at this distance, and so on. After generalizing this observation, we will derive the above expression.

d: Represents the distance of points from the diagonal in terms of grid units.

n-d: The number of points that are d units away from the diagonal. After expanding this expression the value of D is come is:

$$\bar{D} = \frac{n}{3\sqrt{2}}$$

and it will depend only on n, with change of k the value of average distance will not change. for n=101 the value come is approximately equal to 23.8.