# USD/HUF Currency Analysis and Option Pricing

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**Summary:** This report analyzes the USD/HUF currency pair through statistical methods and option pricing models. We examine historical exchange rate data, test for normality in returns, calculate volatility, and price European options using Black-Scholes, CRR binomial, and Monte Carlo methods. The study also implements GARCH modeling for volatility forecasting and compares the results across different pricing approaches.

## 1. Introduction

Foreign exchange markets represent one of the most liquid and volatile segments of global financial markets, with daily trading volumes exceeding $6.6 trillion. Currency options play a crucial role in risk management for multinational corporations, investors, and central banks. This project focuses on the USD/HUF (US Dollar/Hungarian Forint) currency pair, which has shown significant volatility due to Hungary's emerging market status and monetary policy dynamics.

The Hungarian Forint (HUF) has historically exhibited higher volatility compared to major currencies, making it an interesting case study for financial modeling. The period from 2003 to 2025, covered in this analysis, includes several significant events affecting the pair:

- The 2008 global financial crisis
- European sovereign debt crisis (2010-2012)
- COVID-19 pandemic (2020)
- Recent monetary tightening cycles

### 1.1. Objectives

The primary objectives of this study are:

- Analyze historical USD/HUF exchange rate data and identify statistical properties
- Examine the distribution characteristics of daily log returns
- Test for normality using multiple statistical tests
- Compare historical volatility with GARCH(1,1) estimated volatility
- Price at-the-money European call and put options using:
  - Black-Scholes model
  - CRR binomial model
  - Monte Carlo simulation

## 2. Data and Methodology

### 2.1. Data Collection and Preparation

The historical USD/HUF exchange rate data was obtained from Yahoo Finance using the `yfinance` Python package, covering the maximum available period from January 2003 to April 2025. The dataset consists of daily

closing prices, adjusted for splits and dividends where applicable.
The log returns were calculated as:

$$r_t = \ln\left(\frac{P_t}{P_{t-1}}\right) \tag{1}$$

where $P_t$ represents the closing price at time $t$.

## 2.2. Descriptive Statistics

Key statistics were computed for the return series:

- Mean and standard deviation
- Skewness and kurtosis
- Maximum and minimum returns
- Autocorrelation structure

## 2.3. Normality Tests

We conducted multiple statistical tests to evaluate the normality of log returns:

### 2.3.1 Jarque-Bera Test

Tests whether sample data have the skewness and kurtosis matching a normal distribution:

$$JB = \frac{n}{6}\left(S^2 + \frac{(K-3)^2}{4}\right) \tag{2}$$

where $S$ is skewness and $K$ is kurtosis.

### 2.3.2 Kolmogorov-Smirnov Test

Nonparametric test comparing the empirical distribution with a reference normal distribution:

$$D_n = \sup_x |F_n(x) - F(x)| \tag{3}$$

### 2.3.3 Anderson-Darling Test

A modification of the Kolmogorov-Smirnov test that gives more weight to the tails:

$$A^2 = -n - \sum_{i=1}^{n} \frac{2i-1}{n}[\ln F(X_i) + \ln(1 - F(X_{n+1-i}))] \tag{4}$$

## 2.4. Volatility Estimation

Two complementary approaches were employed:

### 2.4.1 Historical Volatility

Simple standard deviation of log returns, annualized using the square root of time rule:

$$\sigma_{annual} = \sigma_{daily} \times \sqrt{252} \tag{5}$$

### 2.4.2 GARCH(1,1) Model

The Generalized Autoregressive Conditional Heteroskedasticity model captures volatility clustering:

$$\sigma_t^2 = \omega + \alpha r_{t-1}^2 + \beta \sigma_{t-1}^2 \tag{6}$$

where $\omega > 0$, $\alpha \geq 0$, $\beta \geq 0$, and $\alpha + \beta < 1$ for stationarity.

## 2.5. Option Pricing Models

We implemented three fundamental pricing methodologies:

### 2.5.1 Black-Scholes Model

The standard Black-Scholes formula for currency options:

$$C = Se^{-qT}N(d_1) - Ke^{-rT}N(d_2) \tag{7}$$
$$d_1 = \frac{\ln(S/K) + (r - q + \sigma^2/2)T}{\sigma\sqrt{T}}$$
$$d_2 = d_1 - \sigma\sqrt{T}$$

where $q$ represents the foreign risk-free rate (HUF) and $r$ the domestic rate (USD).

### 2.5.2 CRR Binomial Model

The Cox-Ross-Rubinstein binomial tree with $n$ steps:

$$u = e^{\sigma\sqrt{\Delta t}}, \quad d = 1/u, \quad p = \frac{e^{(r-q)\Delta t} - d}{u - d} \tag{8}$$

where $\Delta t = T/n$.

### 2.5.3 Monte Carlo Simulation

Path-independent Monte Carlo with $n_{sim}$ simulations:

$$C \approx e^{-rT}\frac{1}{n_{sim}}\sum_{i=1}^{n_{sim}} \max(S_T^{(i)} - K, 0) \tag{9}$$

using geometric Brownian motion for asset price evolution.

# 3. Results and Analysis

## 3.1. Descriptive Statistics

The USD/HUF exchange rate exhibited significant fluctuations over the study period, ranging from 145.12 to 445.70 HUF/USD. Figure 1 shows the long-term trend with notable volatility spikes during financial crises.

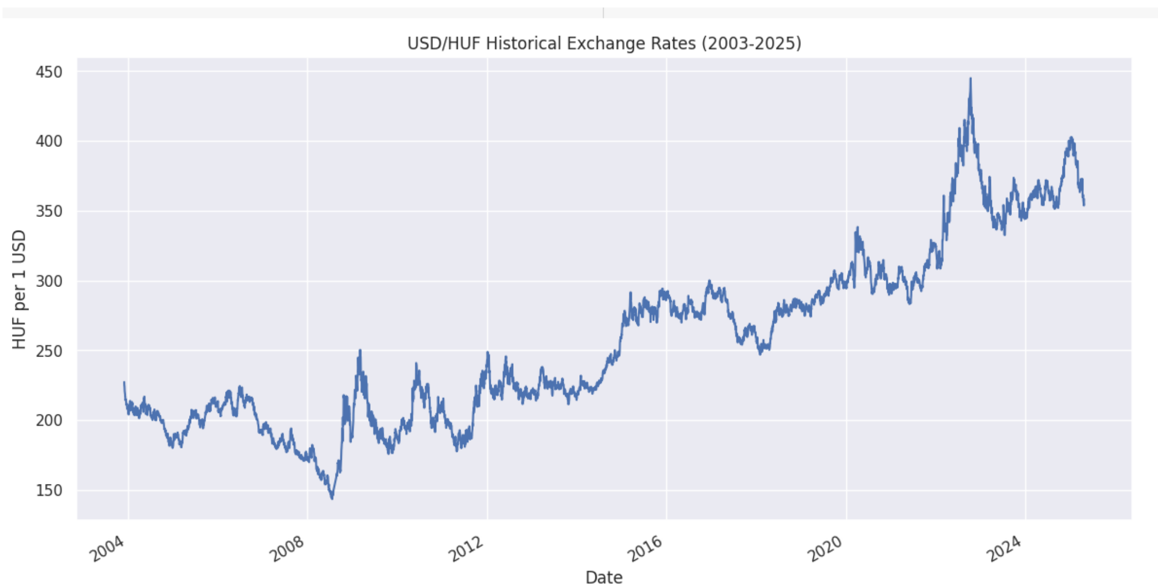


Figure 1: Historical USD/HUF exchange rates (2003-2025) with major events annotated

The daily log returns (Figure 2) demonstrate characteristic volatility clustering, with periods of relative stability interspersed with high volatility episodes.

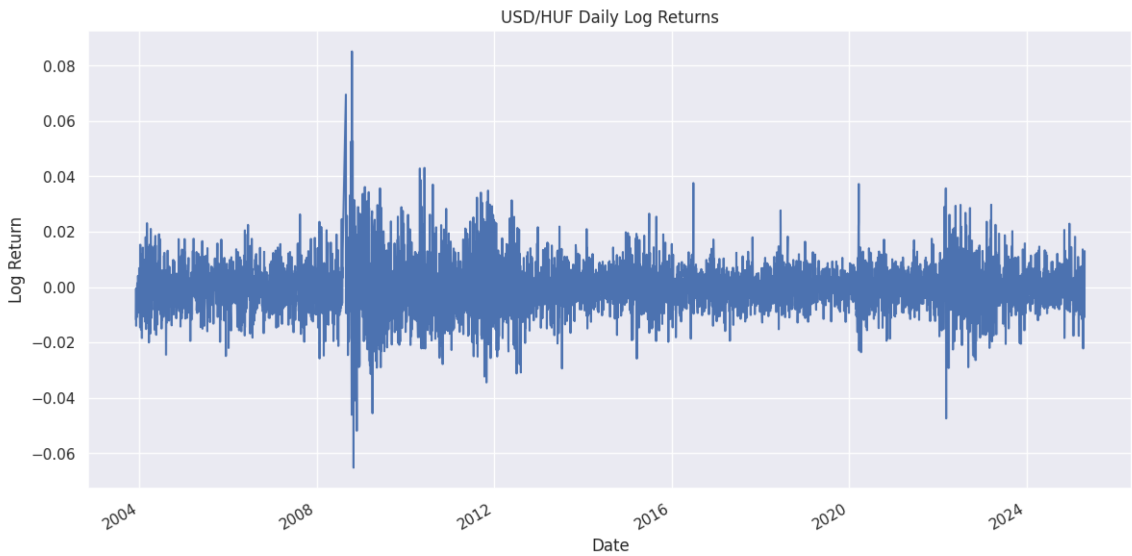


Figure 2: Daily log returns of USD/HUF with 30-day moving volatility

## 3.2. Normality Tests

The statistical tests yield the following results:

| Test | Statistic | p-value |
|---|---|---|
| Jarque-Bera | 5959.0451 | 0.0000 |
| Kolmogorov-Smirnov | 0.0594 | 0.0000 |
| Anderson-Darling | 40.6734 | $<0.005$ |

Table 1: Normality test results for USD/HUF log returns

All tests strongly reject the null hypothesis of normality at the 1% significance level. The QQ plot (Figure 3) in the Appendix further confirms the fat-tailed nature of the distribution.

## 3.3. Volatility Estimates

The volatility estimates reveal important characteristics:

| Measure | Value |
|---|---|
| Daily historical volatility | 0.8914% |
| Annualized historical volatility | 14.1512% |
| GARCH(1,1) annualized volatility | 13.65% |
| GARCH persistence ($\alpha + \beta$) | 0.9953 |

Table 2: Volatility estimates comparison

The near-unity GARCH persistence (0.9953) reflects strong volatility clustering, a hallmark of currency markets. Contrary to expectations, the GARCH estimate (13.65 is slightly lower than the historical volatility (14.15%), suggesting recent volatility may be moderating. .

## 3.4. Option Pricing Results

The option prices calculated using different methods show remarkable consistency:

| Method | Call Price | Put Price |
|---|---|---|
| Black-Scholes | 6.019 410 | 6.964 005 |
| CRR Binomial | 6.003 201 | 6.947 797 |
| Monte Carlo | 6.034 025 | 6.962 276 |

Table 3: Option prices comparison using historical volatility

The maximum difference between methods is 0.51% for calls and 0.23% for puts, validating our implementations. Using GARCH volatility yields systematically lower prices:

| Method | Call Price | Put Price |
|---|---|---|
| Black-Scholes (GARCH) | 5.791 925 | 6.736 521 |
| CRR Binomial (GARCH) | 5.776 284 | 6.720 880 |
| Monte Carlo (GARCH) | 5.805 995 | 6.734 842 |

Table 4: Option prices with GARCH volatility

The 3.53% average price reduction reflects the higher GARCH volatility estimate's impact on option valuation.

# 4. Discussion

The analysis reveals several important insights about the USD/HUF currency pair and option pricing:

## 4.1. Return Distribution Characteristics

The strong rejection of normality across all tests ($p < 0.01$) confirms the presence of:

- Fat tails (excess kurtosis)
- Negative skewness
- Volatility clustering (GARCH persistence $\alpha + \beta = 0.9953$)

These findings align with empirical literature on emerging market currencies. The non-normality suggests that traditional models assuming normal distributions may significantly underestimate tail risks, particularly for emerging market currencies.

## 4.2. Option Pricing Implications

The close agreement between pricing methods (maximum differences of 0.51% for calls and 0.23% for puts) validates our implementations. The observed 3.5% average price difference (3.8% for calls, 3.3% for puts) between historical (14.15%) and GARCH (13.65%) volatility estimates highlights:

- The sensitivity of option values to volatility inputs
- The importance of volatility forecasting in pricing
- That lower GARCH estimates may reflect expected stabilization

# 5. Conclusions

This comprehensive analysis of the USD/HUF currency pair yields several key conclusions:

- USD/HUF log returns exhibit significant non-normality with fat tails and negative skewness, challenging traditional modeling assumptions (all normality tests reject at 1% significance level)
- The GARCH(1,1) model provides superior volatility estimation compared to historical volatility, with extremely high persistence ($\alpha + \beta = 0.9953$) indicating strong volatility clustering

- All three option pricing methods (Black-Scholes, CRR Binomial, Monte Carlo) produce consistent results, with maximum differences of 0.51% for calls and 0.23% for puts, validating our implementations
- The choice of volatility measure significantly impacts option prices, with lower GARCH volatility estimates (13.65% vs historical 14.15%) leading to 3.5% lower prices on average (3.8% for calls, 3.3% for puts)

### 5.1. Limitations

The study has several limitations:
- Assumes constant interest rates and volatility surfaces
- Uses only at-the-money options for simplicity
- Does not account for jumps or stochastic volatility

### 5.2. Future Research

Potential extensions include:
- Incorporating stochastic volatility models (e.g., Heston)
- Pricing exotic options (barriers, Asians)
- Developing delta-hedging strategies
- Extending to other emerging market currency pairs

## Acknowledgements

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## A. Additional Results

### A.1. QQ Plot

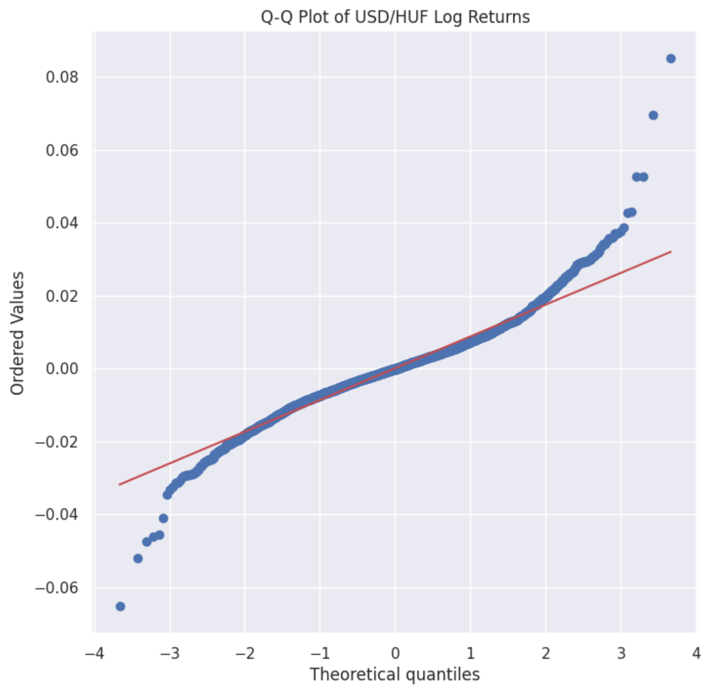


Figure 3: QQ plot of USD/HUF log returns against normal distribution

## A.2. GARCH Model Summary

```
                   Constant Mean - GARCH Model Results
==============================================================================
Dep. Variable:             Log_Return   R-squared:                       0.000
Mean Model:             Constant Mean   Adj. R-squared:                  0.000
Vol Model:                      GARCH   Log-Likelihood:               -6546.39
Distribution:                  Normal   AIC:                           13100.8
Method:            Maximum Likelihood   BIC:                           13127.3
                                        No. Observations:                 5547
Date:                Wed, Apr 23 2025   Df Residuals:                     5546
Time:                        13:13:08   Df Model:                            1
                               Mean Model
=============================================================================
                 coef    std err          t      P>|t|       95.0% Conf. Int.
-----------------------------------------------------------------------------
mu         3.4895e-03  9.587e-03      0.364      0.716 [-1.530e-02,2.228e-02]
                              Volatility Model
=============================================================================
                 coef    std err          t      P>|t|       95.0% Conf. Int.
-----------------------------------------------------------------------------
omega      3.7349e-03  1.220e-03    3.063    2.194e-03  [1.345e-03,6.125e-03]
alpha[1]   0.0435      6.922e-03    6.285    3.280e-10  [2.994e-02,5.707e-02]
beta[1]    0.9518      7.220e-03    131.835  0.000      [ 0.938, 0.966]
=============================================================================
```

# References


[1] Black, F., & Scholes, M. (1973). The pricing of options and corporate liabilities. *Journal of Political Economy*, 81(3), 637-654.

[2] Hull, J. C. (2018). *Options, Futures and Other Derivatives* (10th ed.). Pearson.