RandomVariables
Discrete & Continuous

Random Variable

Random variable is a real valued function whose domain is the sample space associated with a random experiment and range is the real line.

Discrete Random variable

Discrete Random Variable. A random variable is said to be discrete if it takes only a finite number of values.

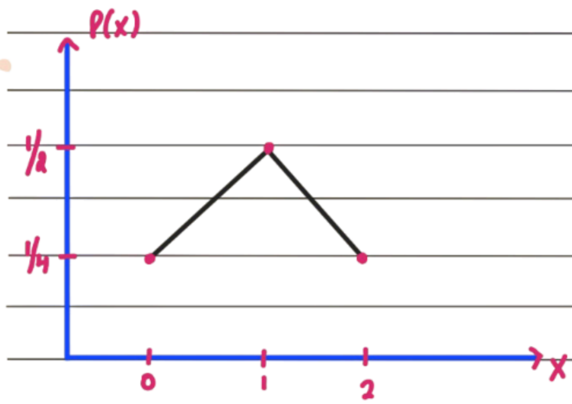
For example, in an experiment of tossing a pair of coins, if we define the random variable X as the number of heads obtained, then the values of random variable X are 0,1,1,2 corresponding to the outcomes TT, HT, TH and HH respectively.

Continuous Random variable

Continuous Random variable. A random variable is said to be continuous if it assumes any possible value between cases it takes any value in a certain limit. In such an interval.

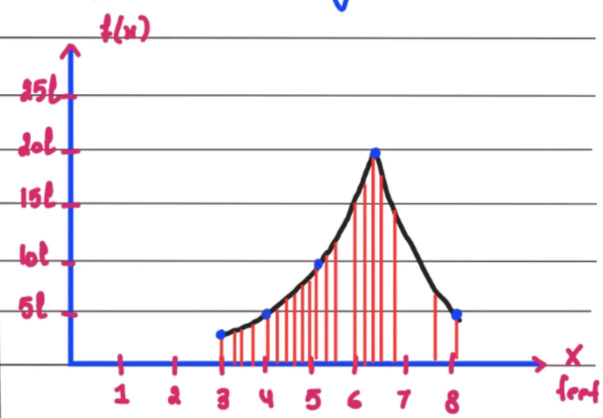
For example, the height of a person chosen at random from a population of 1000 persons lies between 140 cm and 160 cm. Similarly age, weight etc. are continuous variables.

Discrete Random Variable	Continuous Random Variable
• Finite No. (Countable)	• Infinite (Non Countable)
• For Mean $(E(x)) \rightarrow \sum x_i p_i$	• For Mean $(E(x)) \rightarrow \int_{-\infty}^{\infty} x f(x)\, dx$
• Small sample space. eg- 2 Coins are tossed	• Large sample space. eg- Weight of persons
• Integers ✓	• Decimal ✓
• Spread is small	Spread is large.



Two coins are tossed → x (Tail)

• $P(x) \geq 0 \rightarrow \sum P(x) = 1$



Weight of persons in Haryana

• $f(x) \geq 0 \quad \int_{-\infty}^{\infty} f(x) = 1$

Probability distribution of a Random Variable

If a random variable x assume value X1,X2 .....Xn with respective probabilities P1, P2,..........Pn such that :

a) 0 < pi ≤ 1 for i = 1, 2,…, n
b) P1 + P2 +.......+ Pn = 1

then the random variable X possesses the following probability distribution :

X :	x1	x2	x3………………..xn	
P(x) :	p1	p2	p3………………..pn	

Example 1 - A coin is biased so that the head is 3 times as likely to occur as a tail. If the coin is tossed twice, find the probability distribution for the number of tails.

Solution :

Let the random variable

$$X = \text{number of tails in two tosses}$$

Since **Head is 3 times as likely as Tail**, let:

$$P(T) = p, \quad P(H) = 3p \Rightarrow p + 3p = 1 \Rightarrow 4p = 1 \Rightarrow p = \frac{1}{4}$$

So,

$$P(T) = \frac{1}{4}, \quad P(H) = \frac{3}{4}$$

Possible values of X: $0, 1, 2$

Probability Distribution of X

1. $X = 0$ (No tails → HH)

$$P(X = 0) = P(HH) = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$$

2. $X = 1$ (One tail → HT or TH)

$$P(X = 1) = 2 \cdot \frac{3}{4} \cdot \frac{1}{4} = \frac{6}{16} = \frac{3}{8}$$

3. $X = 2$ (Two tails → TT)

$$P(X = 2) = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$$

X (Number of Tails)	0	1	2
$P(X)$	9/16	3/8	1/16

Example 2 - Two cards are drawn simultaneously from a pack of 52 cards without replacement. Find the probability distribution of the number of kings.

Solution :

Let the random variable

$$X = \text{number of kings in two cards drawn}$$

Total cards = 52
Kings = 4, Non-kings = 48
Total ways to draw 2 cards:

$$^{52}C_2 = 1326$$

Possible values of X: $0, 1, 2$

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1. Probability that $X = 0$ (No king)

Both cards are non-kings:

$$P(X = 0) = \frac{^{48}C_2}{^{52}C_2} = \frac{1128}{1326}$$

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2. Probability that $X = 1$ (Exactly one king)

One king and one non-king:

$$P(X = 1) = \frac{^{4}C_1 \cdot {}^{48}C_1}{^{52}C_2} = \frac{192}{1326}$$

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3. Probability that $X = 2$ (Two kings)

Both cards are kings:

$$P(X = 2) = \frac{^{4}C_2}{^{52}C_2} = \frac{6}{1326}$$

✅ **Final Probability Distribution of X**

X (No. of Kings)	0	1	2
$P(X)$	$1128/1326$	$192/1326$	$6/1326$

Example 3 - From a lot of 10 items containing 3 defectives, a sample of 4 items is drawn at random. Let the random variable X denote the number of defective items in the sample. If the sample is drawn without replacement, find

(i) the probability distribution of X.

(ii) P(x<1)

(iii) P(0<x<2)

Solution :

Let

$$X = \text{number of defective items in the sample}$$

Total items = 10, Defective = 3, Good = 7

Sample size = 4 (without replacement)

Total possible samples:

$$^{10}C_4 = 210$$

Possible values of X: $0, 1, 2, 3$

(i) Probability Distribution of X

$X = 0$ **(No defective)**

$$P(X=0) = \frac{^3C_0\,^7C_4}{^{10}C_4} = \frac{35}{210} = \frac{1}{6}$$

$X = 1$ **(One defective)**

$$P(X=1) = \frac{^3C_1\,^7C_3}{^{10}C_4} = \frac{3 \cdot 35}{210} = \frac{105}{210} = \frac{1}{2}$$

$X = 2$ **(Two defectives)**

$$P(X=2) = \frac{^3C_2\,^7C_2}{^{10}C_4} = \frac{3 \cdot 21}{210} = \frac{63}{210} = \frac{3}{10}$$

$X = 3$ **(Three defectives)**

$$P(X=3) = \frac{^3C_3\,^7C_1}{^{10}C_4} = \frac{7}{210} = \frac{1}{30}$$

✅ **Final Probability Distribution**

X	0	1	2	3
$P(X)$	1/6	1/2	3/10	1/30

(ii) $P(X < 1)$

$$P(X < 1) = P(X = 0) = \boxed{\frac{1}{6}}$$

(iii) $P(0 < X < 2)$

$$P(0 < X < 2) = P(X = 1) = \boxed{\frac{1}{2}}$$

Example 4 - Assume that the pair of dice is thrown and the random variable X is the sum of numbers that appears on two dice. Find the mean or the expectation of the random variable X.

Solution :

$$X = \text{sum of the numbers on two dice}$$

Possible values of X:
$2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12$

Total outcomes when two dice are thrown:

$$6 \times 6 = 36$$

Probability Distribution of X

x	2	3	4	5	6	7	8	9
$P(X = x)$	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36

Mean (Expectation) of Random Variable X

$$E(X) = \sum x\, P(X = x)$$

$$E(X) = \frac{1}{36}[2(1) + 3(2) + 4(3) + 5(4) + 6(5) + 7(6) + 8(5) + 9(4) + 10(3) + 11(2) + 12(1)]$$

$$E(X) = \frac{1}{36}[2 + 6 + 12 + 20 + 30 + 42 + 40 + 36 + 30 + 22 + 12]$$

$$E(X) = \frac{252}{36} = 7$$

Question 5 -

Q4) If X denotes Random Variable then $P(X)$,

$$P(X=x) = \begin{cases} 0.1 \text{ if } x = 0 \\ kx \text{ if } x = 1 \text{ or } 2 \\ k(5-x) \text{ if } x = 3 \text{ or } 4 \\ 0 \text{ otherwise} \end{cases}$$

a) Find the value of k.

b) $P(X<3)$? c) $P(0<X<2)$?

Solution :

Given the random variable X with probability mass function:

$$P(X=x) = \begin{cases} 0.1, & x = 0 \\ kx, & x = 1, 2 \\ k(5-x), & x = 3, 4 \\ 0, & \text{otherwise} \end{cases}$$

(a) Find the value of k

Sum of all probabilities = 1

$$P(0) + P(1) + P(2) + P(3) + P(4) = 1$$

$$0.1 + k(1) + k(2) + k(5-3) + k(5-4) = 1$$

$$0.1 + k + 2k + 2k + k = 1$$

$$0.1 + 6k = 1$$

$$6k = 0.9 \Rightarrow k = 0.15$$

✅ $k = 0.15$

(b) Find $P(X < 3)$

$$P(X < 3) = P(0) + P(1) + P(2)$$

$$= 0.1 + k(1) + k(2)$$

$$= 0.1 + 0.15 + 0.30 = 0.55$$

✅ $P(X < 3) = 0.55$

(c) Find $P(0 < X < 2)$

Only $x = 1$ satisfies $0 < x < 2$

$$P(0 < X < 2) = P(1) = k(1) = 0.15$$

✅ $P(0 < X < 2) = 0.15$

Assignment

Question 1 :
Assume that the pair of dice is thrown and the random variable X is the sum of numbers that appears on two dice. Find the mean or the expectation of the random variable X

Question 2 :
Find the probability distribution for the number of doublets in the three throws of a pair of dice.

Question 3 :
Let X be a discrete random variable with the following pmf: P(X = 1) = 0.1, P(X = 2) = 0.3, P(X = 3) = 0.4, and P(X = 4) = 0.2. Find the expected value E(X).

Question 4 :
An urn contains 5 mangoes and 4 apples. Three fruits are taken at random. If the number of

apples taken is a random variable, then find the values of the random variable and number of points in its inverse images.

Question 5 :

In a pack of 52 playing cards, two cards are drawn at random simultaneously. If the number of black cards drawn is a random variable, find the values of the random variable and number of points in its inverse images.

Answer Sheet

Q1: $E(X) = 7$

Q2: $P(X = k) = C(3, k)(1/6)^k(5/6)^{(3-k)}$, $k = 0,1,2,3$

Q3: $E(X) = 2.7$

Q4: Values of X: 0, 1, 2, 3 Inverse image points: 10, 40, 30, 4

Q5: Values of X: 0, 1, 2 Inverse image points: 325, 676, 325