

Mean, Variance, Standard Deviation

Mean and Variance of a Random Variable

Let X be a discrete random variable which assumes values x_1, x_2, \dots, x_n with respective probabilities p_1, p_2, \dots, p_n such that $\sum_{i=1}^n p_i = 1$. Then

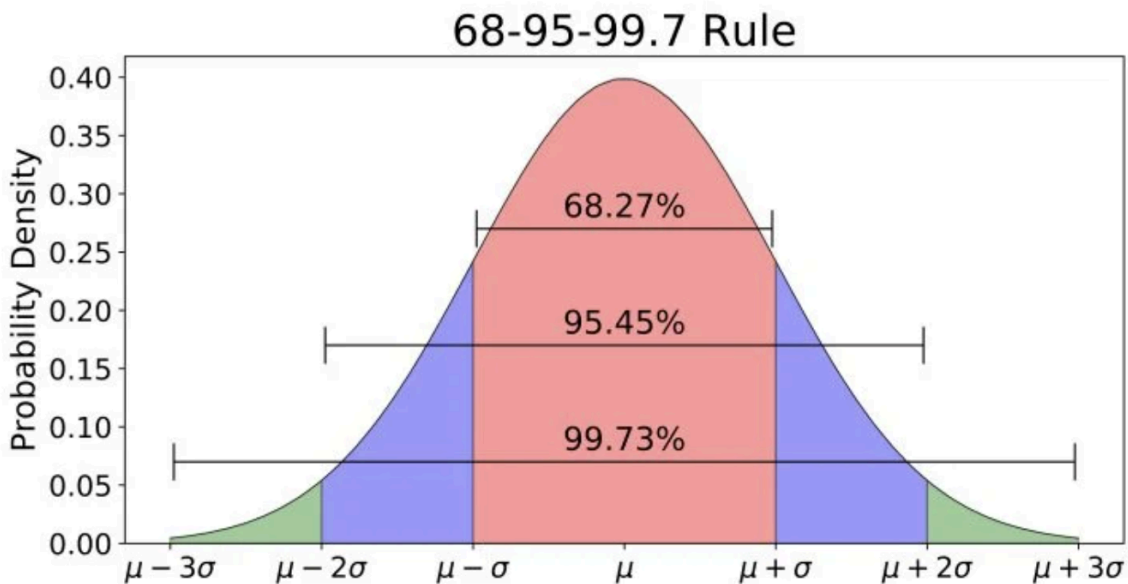
$$\text{Mean}(\mu) = \frac{\sum_{i=1}^n p_i x_i}{\sum_{i=1}^n p_i} = \sum_{i=1}^n p_i x_i \quad \left[\because \sum_{i=1}^n p_i = 1 \right]$$

$$\text{Variance}(\sigma) = \sum_{i=1}^n p_i x_i^2 - \left(\sum_{i=1}^n p_i x_i \right)^2$$

$$\text{Standard deviation} = \sqrt{\text{variance}} = \sqrt{\sum p_i x_i^2 - (\sum p_i x_i)^2}$$

❖ 68-95-99 Rule

The 68-95-99.7 rule defines the spread of data in a bell-shaped normal distribution graph, stating that approximately 68% of the data falls within one standard deviation of the mean, 95% within two standard deviations, and 99.7% within three standard deviations.



Example 1 : Find the mean and variance of the probability distribution of the random variable "number of tails" when four coins are tossed.

Solution : Let **X = number of tails** obtained when **4 fair coins** are tossed.

Each coin has

$P(\text{Tail}) = 1/2$, so this is a **binomial distribution** with

$n = 4$, $p = 1/2$.

1. Probability Distribution of X

$$P(X = x) = \binom{4}{x} \left(\frac{1}{2}\right)^4$$

x (No. of Tails)	P(X = x)
0	$\frac{1}{16}$
1	$\frac{4}{16}$
2	$\frac{6}{16}$
3	$\frac{4}{16}$
4	$\frac{1}{16}$

2. Mean of X

For a binomial distribution:

$$\begin{aligned}\text{Mean} &= np \\ &= 4 \times \frac{1}{2} = 2\end{aligned}$$

✓ Mean = 2

3. Variance of X

$$\text{Variance} = npq$$

where $q = 1 - p = \frac{1}{2}$

$$\begin{aligned}&= 4 \times \frac{1}{2} \times \frac{1}{2} \\ &= 1\end{aligned}$$

✓ Variance = 1

✓ Final Answer

Mean = 2, Variance = 1

Example 2 :

Out of a group of 30 honest people, 20 always speak the truth. Two persons are selected at Mandom from the group. Find the probability distribution of the number of selected persons who speak the truth. Also find the mean of the distribution.

Solution :

Let the random variable

X = number of truth-speakers among the 2 selected persons

From the group:

- Total people = 30
- Truth-speakers = 20
- Liars = 10

Two persons are selected **at random without replacement**.

1 Total Number of Possible Selections

$$\text{Total ways} = \binom{30}{2} = \frac{30 \times 29}{2} = 435$$

2 Possible Values of X

$$X = 0, 1, 2$$

3 Probability Distribution

✓ Case 1: $X = 0$ (No truth-speaker \rightarrow both are liars)

$$P(X = 0) = \frac{\binom{10}{2}}{\binom{30}{2}} = \frac{45}{435} = \frac{3}{29}$$

✓ Case 2: $X = 1$ (One truth-speaker, one liar)

$$P(X = 1) = \frac{\binom{20}{1}\binom{10}{1}}{\binom{30}{2}} = \frac{20 \times 10}{435} = \frac{200}{435} = \frac{40}{87}$$

✓ Case 3: $X = 2$ (Both are truth-speakers)

$$P(X = 2) = \frac{\binom{20}{2}}{\binom{30}{2}} = \frac{190}{435} = \frac{38}{87}$$

✓ Probability Distribution Table

X (No. of truth-speakers)	$P(X)$
0	$\frac{3}{29}$
1	$\frac{40}{87}$
2	$\frac{38}{87}$

4 Mean of the Distribution

$$E(X) = \sum xP(x)$$

$$E(X) = 0 \cdot \frac{3}{29} + 1 \cdot \frac{40}{87} + 2 \cdot \frac{38}{87}$$

$$E(X) = \frac{40}{87} + \frac{76}{87} = \frac{116}{87} = \frac{4}{3}$$

✓ Final Answer

- **Probability Distribution:** as given in the table above
- **Mean of the distribution:**

$$E(X) = \frac{4}{3}$$

Assignment

Question 1

Four coins are tossed at a time. Let the random variable (X) denote the number of heads obtained.

- Construct the probability distribution of (X).
- Find the mean and variance of (X).

Question 2

A die is thrown two times. Let (X) be the number of times 6 appears.

- Construct the probability distribution of (X).
- Find the mean and variance of (X).

Question 3

Out of a group of 25 people, 15 always speak the truth. Two persons are selected at random.

Let (X) be the number of truth-speakers selected.

- (i) Construct the probability distribution of (X) .
- (ii) Find the mean and variance of the distribution.

Question 4

Three coins are tossed simultaneously. Let (X) be the number of tails.

- (i) Construct the probability distribution of (X) .
- (ii) Find the mean and variance of (X) .

Question 5

A bag contains 6 red balls and 4 black balls. Two balls are drawn at random without replacement.

Let (X) be the number of red balls drawn.

- (i) Construct the probability distribution of (X) .
- (ii) Find the mean and variance of (X) .

Answer Sheet

Q1:

Mean = 2, Variance = 1

Q2:

Mean = $\frac{1}{3}$, Variance = $\frac{5}{18}$

Q3:

Mean = $\frac{6}{5}$, Variance = $\frac{23}{50}$

Q4:

Mean = $\frac{3}{2}$, Variance = $\frac{3}{4}$

Q5:

Mean = $\frac{6}{5}$, Variance = $\frac{32}{75}$

