

Random Variables

Discrete & Continuous

Random Variable

Random variable is a real valued function whose domain is the sample space associated with a random experiment and range is the real line.

Discrete Random variable

Discrete Random Variable. A random variable is said to be discrete if it takes only a finite number of values.

For example, in an experiment of tossing a pair of coins, if we define the random variable X as the number of heads obtained, then the values of random variable X are 0,1,1,2 corresponding to the outcomes TT, HT, TH and HH respectively.

Continuous Random variable

Continuous Random variable. A random variable is said to be continuous if it assumes any possible value between cases it takes any value in a certain limit. In such an interval.

For example, the height of a person chosen at random from a population of 1000 persons lies between 140 cm and 160 cm. Similarly age, weight etc. are continuous variables.

Probability distribution of a Random Variable

If a random variable x assume value X_1, X_2, \dots, X_n with respective probabilities P_1, P_2, \dots, P_n such that :

- a) $0 < p_i \leq 1$ for $i = 1, 2, \dots, n$
b) $P_1 + P_2 + \dots + P_n = 1$

then the random variable X possesses the following probability distribution :

$$\begin{array}{llll} X : & x_1 & x_2 & x_3, \dots, x_n \\ P(x) : & p_1 & p_2 & p_3, \dots, p_n \end{array}$$

Example 1 - A coin is biased so that the head is 3 times as likely to occur as a tail. If the coin is tossed twice, find the probability distribution for the number of tails.

Solution :

Let the random variable

X = number of tails in two tosses

Since Head is 3 times as likely as Tail, let:

$$P(T) = p, \quad P(H) = 3p \Rightarrow p + 3p = 1 \Rightarrow 4p = 1 \Rightarrow p = \frac{1}{4}$$

So,

$$P(T) = \frac{1}{4}, \quad P(H) = \frac{3}{4}$$

Possible values of X : 0, 1, 2

Probability Distribution of X

1. $X = 0$ (No tails \rightarrow HH)

$$P(X = 0) = P(HH) = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$$

2. $X = 1$ (One tail \rightarrow HT or TH)

$$P(X = 1) = 2 \cdot \frac{3}{4} \cdot \frac{1}{4} = \frac{6}{16} = \frac{3}{8}$$

3. $X = 2$ (Two tails \rightarrow TT)

$$P(X = 2) = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$$

X (Number of Tails)	0	1	2
$P(X)$	9/16	3/8	1/16

Example 2 - Two cards are drawn simultaneously from a pack of 52 cards without replacement. Find the probability distribution of the number of kings.

Solution :

Let the random variable

$$X = \text{number of kings in two cards drawn}$$

Total cards = 52

Kings = 4, Non-kings = 48

Total ways to draw 2 cards:

$${}^{52}C_2 = 1326$$

Possible values of X : 0, 1, 2

1. Probability that $X = 0$ (No king)

Both cards are non-kings:

$$P(X = 0) = \frac{{}^{48}C_2}{{}^{52}C_2} = \frac{1128}{1326}$$

2. Probability that $X = 1$ (Exactly one king)

One king and one non-king:

$$P(X = 1) = \frac{{}^4C_1 \cdot {}^{48}C_1}{{}^{52}C_2} = \frac{192}{1326}$$

3. Probability that $X = 2$ (Two kings)

Both cards are kings:

$$P(X = 2) = \frac{{}^4C_2}{{}^{52}C_2} = \frac{6}{1326}$$

Final Probability Distribution of X

X (No. of Kings)	0	1	2
$P(X)$	1128/1326	192/1326	6/1326

Example 3 - From a lot of 10 items containing 3 defectives, a sample of 4 items is drawn at random. Let the random variable X denote the number of defective items in the sample. If the sample is drawn without replacement, find

(i) the probability distribution of X .

(ii) $P(X < 1)$

(iii) $P(0 < X < 2)$

Solution :

Let

X = number of defective items in the sample

Total items = 10, Defective = 3, Good = 7

Sample size = 4 (without replacement)

Total possible samples:

$${}^{10}C_4 = 210$$

Possible values of X : 0, 1, 2, 3

(i) Probability Distribution of X

$X = 0$ (No defective)

$$P(X = 0) = \frac{{}^3C_0 {}^7C_4}{{}^{10}C_4} = \frac{35}{210} = \frac{1}{6}$$

$X = 1$ (One defective)

$$P(X = 1) = \frac{{}^3C_1 {}^7C_3}{{}^{10}C_4} = \frac{3 \cdot 35}{210} = \frac{105}{210} = \frac{1}{2}$$

$X = 2$ (Two defectives)

$$P(X = 2) = \frac{{}^3C_2 {}^7C_2}{{}^{10}C_4} = \frac{3 \cdot 21}{210} = \frac{63}{210} = \frac{3}{10}$$

$X = 3$ (Three defectives)

$$P(X = 3) = \frac{{}^3C_3 {}^7C_1}{{}^{10}C_4} = \frac{7}{210} = \frac{1}{30}$$

Final Probability Distribution

X	0	1	2	3
$P(X)$	1/6	1/2	3/10	1/30

$$(ii) P(X < 1)$$

$$P(X < 1) = P(X = 0) = \boxed{\frac{1}{6}}$$

$$(iii) P(0 < X < 2)$$

$$P(0 < X < 2) = P(X = 1) = \boxed{\frac{1}{2}}$$

Example 4 - Assume that the pair of dice is thrown and the random variable X is the sum of numbers that appears on two dice. Find the mean or the expectation of the random variable X .

Solution :

X = sum of the numbers on two dice

Possible values of X :

2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12

Total outcomes when two dice are thrown:

$$6 \times 6 = 36$$

Probability Distribution of X

x	2	3	4	5	6	7	8	9
$P(X = x)$	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36

Mean (Expectation) of Random Variable X

$$E(X) = \sum x P(X = x)$$

$$E(X) = \frac{1}{36}[2(1) + 3(2) + 4(3) + 5(4) + 6(5) + 7(6) + 8(5) + 9(4) + 10(3) + 11(2) + 12(1)]$$

$$E(X) = \frac{1}{36}[2 + 6 + 12 + 20 + 30 + 42 + 40 + 36 + 30 + 22 + 12]$$

$$E(X) = \frac{252}{36} = 7$$

Question 5 -

(Q4) If X denotes Random Variable then $P(X)$,

$$P(X = x) = \begin{cases} 0.1 & \text{if } x=0 \\ kx & \text{if } x=1 \text{ or } 2 \\ k(5-x) & \text{if } x=3 \text{ or } 4 \\ 0 & \text{otherwise} \end{cases}$$

a) Find the value of k .

b) $P(X < 3)$? c) $P(0 < X < 2)$?

Solution :

Given the random variable X with probability mass function:

$$P(X = x) = \begin{cases} 0.1, & x = 0 \\ kx, & x = 1, 2 \\ k(5 - x), & x = 3, 4 \\ 0, & \text{otherwise} \end{cases}$$

(a) Find the value of k

Sum of all probabilities = 1

$$P(0) + P(1) + P(2) + P(3) + P(4) = 1$$

$$0.1 + k(1) + k(2) + k(5 - 3) + k(5 - 4) = 1$$

$$0.1 + k + 2k + 2k + k = 1$$

$$0.1 + 6k = 1$$

$$6k = 0.9 \Rightarrow k = 0.15$$

✓ $k = 0.15$

(b) Find $P(X < 3)$

$$\begin{aligned}P(X < 3) &= P(0) + P(1) + P(2) \\&= 0.1 + k(1) + k(2) \\&= 0.1 + 0.15 + 0.30 = 0.55\end{aligned}$$

$P(X < 3) = 0.55$

(c) Find $P(0 < X < 2)$

Only $x = 1$ satisfies $0 < x < 2$

$$P(0 < X < 2) = P(1) = k(1) = 0.15$$

$P(0 < X < 2) = 0.15$

Assignment

Question 1 :

Assume that the pair of dice is thrown and the random variable X is the sum of numbers that appears on two dice. Find the mean or the expectation of the random variable X

Question 2 :

Find the probability distribution for the number of doublets in the three throws of a pair of dice.

Question 3 :

Let X be a discrete random variable with the following pmf: $P(X = 1) = 0.1$, $P(X = 2) = 0.3$, $P(X = 3) = 0.4$, and $P(X = 4) = 0.2$. Find the expected value $E(X)$.

Question 4 :

An urn contains 5 mangoes and 4 apples. Three fruits are taken at random. If the number of

apples taken is a random variable, then find the values of the random variable and number of points in its inverse images.

Question 5 :

In a pack of 52 playing cards, two cards are drawn at random simultaneously. If the number of black cards drawn is a random variable, find the values of the random variable and number of points in its inverse images.

Answer Sheet

Q1: $E(X) = 7$

Q2: $P(X = k) = C(3, k)(1/6)^k(5/6)^{3-k}$, $k = 0, 1, 2, 3$

Q3: $E(X) = 2.7$

Q4: Values of X: 0, 1, 2, 3 Inverse image points: 10, 40, 30, 4

Q5: Values of X: 0, 1, 2 Inverse image points: 325, 676, 325