

Set Theory

The set is represented by listing all the elements comprising it. The elements are enclosed within braces and separated by commas.

Example 1 – Set of vowels in English alphabet, $B = \{a, e, i, o, u\}$

Example 2 – Set of odd numbers less than 20, $A = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$

Set Builder Notation

The set is defined by specifying a property that elements of the set have in common. The set is described as $P = \{x: a(x)\}$

Example 1 – The set $\{a, e, i, o, u\}$

is written as –

$P = \{y : y \text{ is a vowel in English alphabet}\}$

Example 2 – The set $\{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$

is written as –

$A = \{y : 1 \leq y < 20 \text{ and } (y \% 2) \neq 0\}$

If an element y is a member of any set A , it is denoted by $y \in A$

and if an element x is not a member of set A , it is denoted by $x \notin A$.

Cardinality of a Set

Cardinality of a set A , denoted by $|A|$, is the number of elements of the set.

The number is also referred as the cardinal number. If a set has an infinite number of elements, its cardinality is ∞

Example – $|\{1,2,3,4\}|=4$, $|\{1,2,3,4,\dots\}|=\infty$.

Types of Sets

Sets can be classified into many types. Some of which are finite, infinite, subset, universal, proper set, etc.

Finite Set

A set which contains a definite number of elements is called a finite set.

Example – $A = \{y \mid y \in \mathbb{N} \text{ and } 50 > y > 40\}$.

Infinite Set

A set which contains infinite number of elements is called an infinite set.

Example – $A = \{y \mid y \in \mathbb{N} \text{ and } y > 20\}$

Subset

A set B is a subset of set C (Written as $B \subseteq C$) if every element of B is an element of set C .

Example 1 – Let, $C = \{1,2,3,4,5,6\}$

and $B = \{1,2\}$.

Here set B is a subset of set C as all the elements of set B are in set C .

Proper Subset

The term “proper subset” can be defined as “subset of but not equal to”. A Set B is a proper subset of set C (Written as $B \subset C$) if every element of B is an element of set C and $|B| < |C|$.

Universal Set

It is a collection of all elements in a particular context or application. All the sets in that context or application are essentially subsets of this universal set. Universal sets are represented as U .

Example – We may define U as the set of all sport events. In this case, set of all format of cricket is a subset of U

, set of all lawn tennis is a subset of U

, and so on.

Empty Set or Null Set

An empty set contains no elements. It is denoted by \emptyset .

As the number of elements in an empty set is finite, empty set is a finite set. The cardinality of empty set or null set is zero.

Example – $S = \{ x \mid x \in \mathbb{N}$

And $7 < x < 8 \} = \emptyset$.

Venn Diagrams

In the late 1800's, an English logician named John Venn developed a method to represent relationship between sets. He represented these relationships using diagrams, which are now known as Venn diagrams. A Venn diagram represents a set as the interior of a circle. Often two or more circles are enclosed in a rectangle where the rectangle represents the universal set.

Venn Diagram in case of two elements

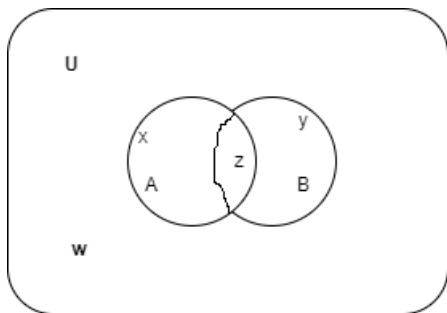
Where;

x = number of elements that belong to set A only

y = number of elements that belong to set B only

z = number of elements that belong to set A and B both ($A \cap B$)

w = number of elements that belong to none of the sets A or B is



From the above figure, it is clear that

$$n(A) = x + z ;$$

$$n(B) = y + z ;$$

$$n(A \cap B) = z;$$

$$n(A \cup B) = x + y + z.$$

$$\text{Total number of elements} = x + y + z + w.$$

Set Operations

Set Operations include Set Union, Set Intersection, Set Difference, Complement of Set, and Cartesian Product.

Set Union

The union of sets A and B (denoted by $A \cup B$) is the set of elements which are in A, in B, or in both A and B. Hence, $A \cup B = \{x | x \in A \text{ OR } x \in B\}$.

Example – If $A = \{10, 11, 12, 13\}$

and $B = \{13, 14, 15\}$

, then $A \cup B = \{10, 11, 12, 13, 14, 15\}$

Set Intersection

The intersection of sets A and B (denoted by $A \cap B$) is the set of elements which are in both A and B. Hence, $A \cap B = \{x | x \in A \text{ AND } x \in B\}$.

Set Difference/ Relative Complement

The set difference of sets A and B (denoted by $A-B$) is the set of elements which are only in A but not in B. Hence, $A-B = \{x | x \in A \text{ AND } x \notin B\}$

Example – If $A = \{10, 11, 12, 13\}$

and $B = \{13, 14, 15\}$,

then $(A-B) = \{10, 11, 12\}$

and $(B-A) = \{14, 15\}$.

Here, we can see $(A-B) \neq (B-A)$

Cartesian Product / Cross Product

The Cartesian product of n number of sets A_1, A_2, \dots, A_n

denoted as $A_1 \times A_2 \cdots \times A_n$

can be defined as all possible ordered pairs (x_1, x_2, \dots, x_n)

where $x_1 \in A_1, x_2 \in A_2, \dots, x_n \in A_n$

Example – If we take two sets $A = \{a, b\}$

and $B = \{1, 2\}$

The Cartesian product of A and B is written as – $A \times B = \{(a, 1), (a, 2), (b, 1), (b, 2)\}$

The Cartesian product of B and A is written as – $B \times A = \{(1, a), (1, b), (2, a), (2, b)\}$