

Propositional Logic is concerned with statements to which the truth values, “true” and “false”, can be assigned. The purpose is to analyze these statements either individually or in a composite manner.

Propositional Logic – Definition

A proposition is a collection of declarative statements that has either a truth value "true" or a truth value "false". A proposition consists of propositional variables and connectives. We denote the propositional variables by capital letters (P, Q, etc.).

Some examples of Propositions are given below –

"Man is Mortal", it returns truth value “TRUE”

" $10 + 18 = 12 - 5$ ", it returns truth value “FALSE”

The following is not a Proposition –

"P is less than 12". It is because unless we give a specific value of A, we cannot determine whether the statement is true or false.

Connectives

In propositional logic generally we use five connectives which are –

Disjunction/OR (\vee)

Conjunction/AND (\wedge)

Negation/ NOT (\neg)

Implication /if-then (\rightarrow)

Bidirectional Implication / If and only if (\Leftrightarrow).

OR (\vee)

The OR operation of two propositions A and B (written as $A \vee B$) is true if at least any of the propositional variable A or B is true.

The truth table is as follows –

A	B	$A \vee B$
True	True	True
True	False	True
False	True	True
False	False	False

AND (\wedge)

The AND operation of two propositions A and B (written as $A \wedge B$) is true if both the propositional variable A and B is true.

The truth table is as follows –

A	B	$A \wedge B$
True	True	True
True	False	False
False	True	False
False	False	False

Negation (\neg)

The negation of a proposition A (written as $\neg A$) is false when A is true and is true when A is false.

The truth table is as follows –

A	$\neg A$
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True	False
False	True

Implication / if-then (\rightarrow)

An implication $A \rightarrow B$ is the proposition “if A, then B”. It is false if A is true and B is false. Rest of the cases are true.

The truth table is as follows –

A	B	$A \rightarrow B$
True	True	True
True	False	False
False	True	True
False	False	True

If and only if (\Leftrightarrow)

$A \Leftrightarrow B$ is bi-conditional logical connective which is true when p and q are same, i.e. both are false or both are true.

The truth table is as follows –

A	B	$A \Leftrightarrow B$
True	True	True
True	False	False
False	True	False

False	False	True
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Tautologies

A Tautology is a formula which is always true for every value of its propositional variables. E.g. $P \vee \neg P$.

Contradictions

A Contradiction is a formula which is always false for every value of its propositional variables. E.g. $Q \wedge \neg Q$.

Logical Equivalence

Two propositions p and q are logically equivalent if their truth tables are the same.