Introduction to Matched Filtering, False Alarm Statistics

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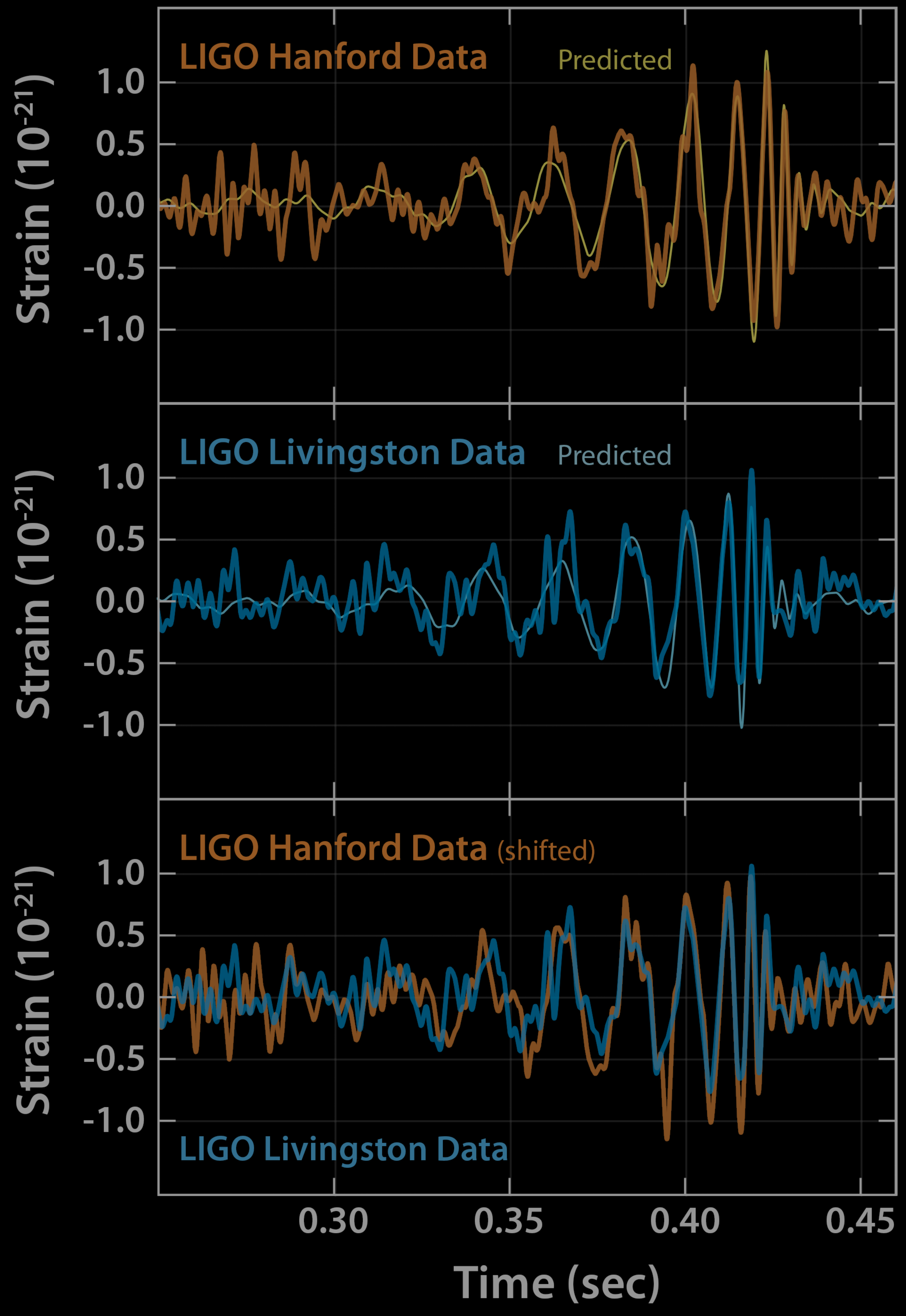


Credits : LIGO

Outline

- The detection problem
- Fourier Transform - the simplest matched filter
- General Matched Filter - ‘Noise weighted inner product’
- Different SNRs
- CBC searches :
  - Matched Filtering using GW templates
  - The template bank, marginalisation
  - Consistency checks

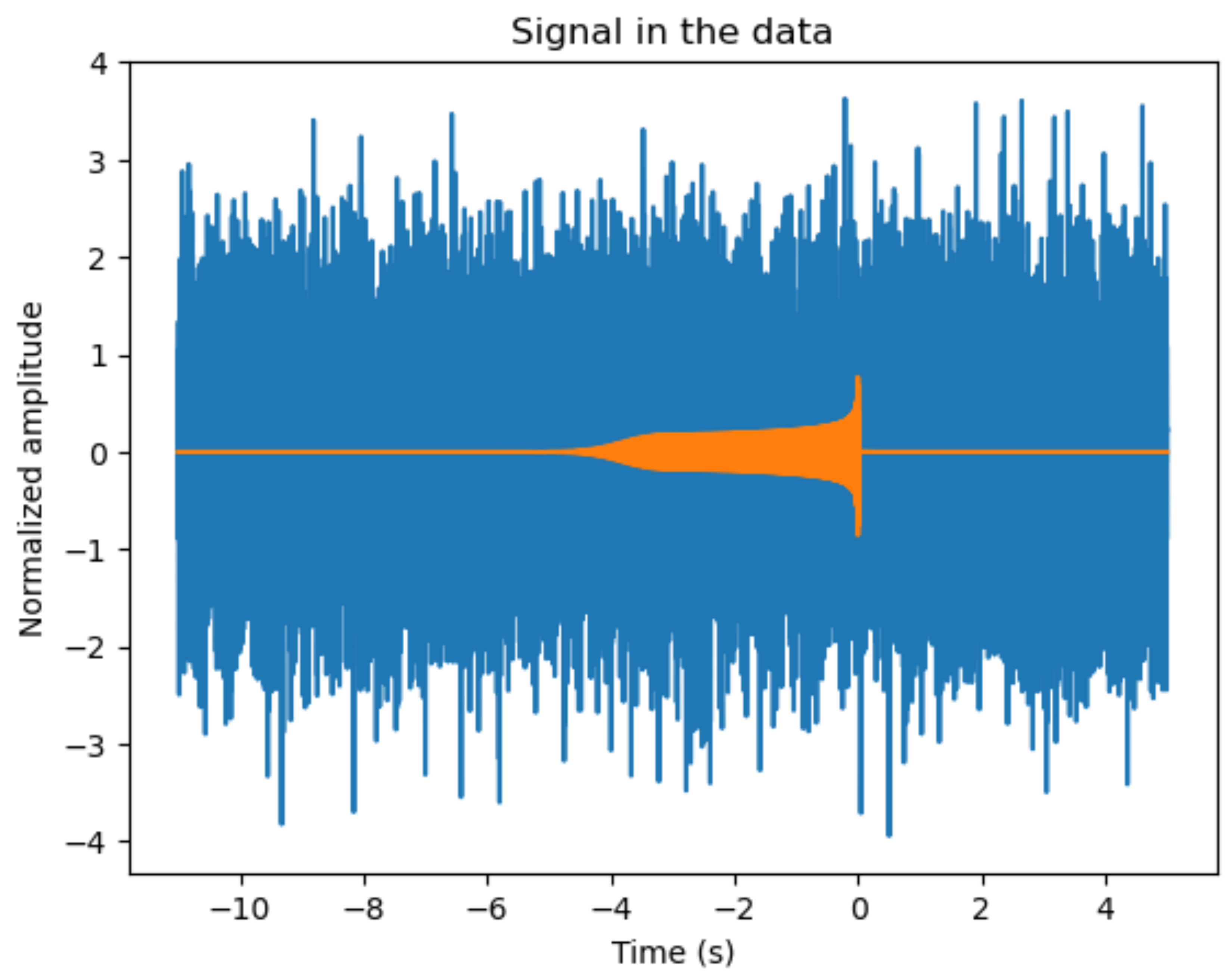
- Statistics of the detection problem : χ^2-consistency check, significance, 2 extremes

The Detection *problem*

In general, given a detector output, we want to know if there is any signal or not..

Not very straightforward when there is noise...

More severe, if the signal is buried deep inside the noise



Really becomes a 'problem' for GW, needs to be addressed systematically, when noises are *few orders of magnitude higher* than the signal.

The Detection *problem*

Null hypothesis $\mathscr{H}_0$: $s(t) = n(t)$

Alternate hypothesis $\mathscr{H}_1$: $s(t) = n(t) + h(t)$

We define the odds ratio, $\mathscr{O} = \dfrac{P(\mathscr{H}_1 | s(t))}{P(\mathscr{H}_0 | s(t))}$

$$\mathscr{O} = \boxed{\frac{P(\mathscr{H}_1)}{P(\mathscr{H}_0)}} \boxed{\frac{P(s(t) | \mathscr{H}_1)}{P(s(t) | \mathscr{H}_0)}}$$

Prior odds

Likelihood Ratio (Bayes factor)

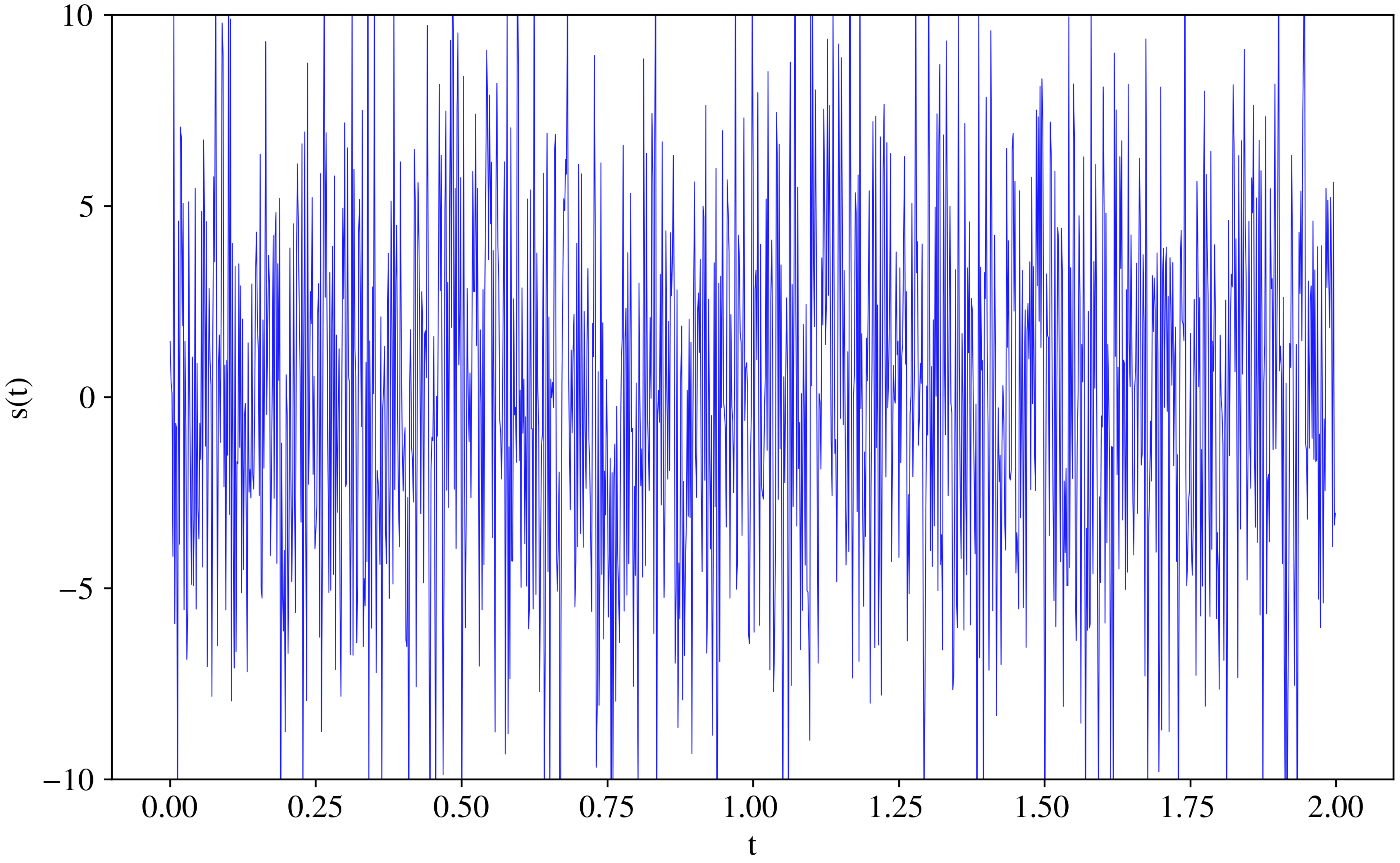
*low latency, need to point telescopes for EM counterpart

All we can say is this: *the data is ___ times more likely to contain a GW signal than to consist of pure noise.*

Monochromatic search

For a start, assume a ‘monochromatic’ signal buried inside a data:

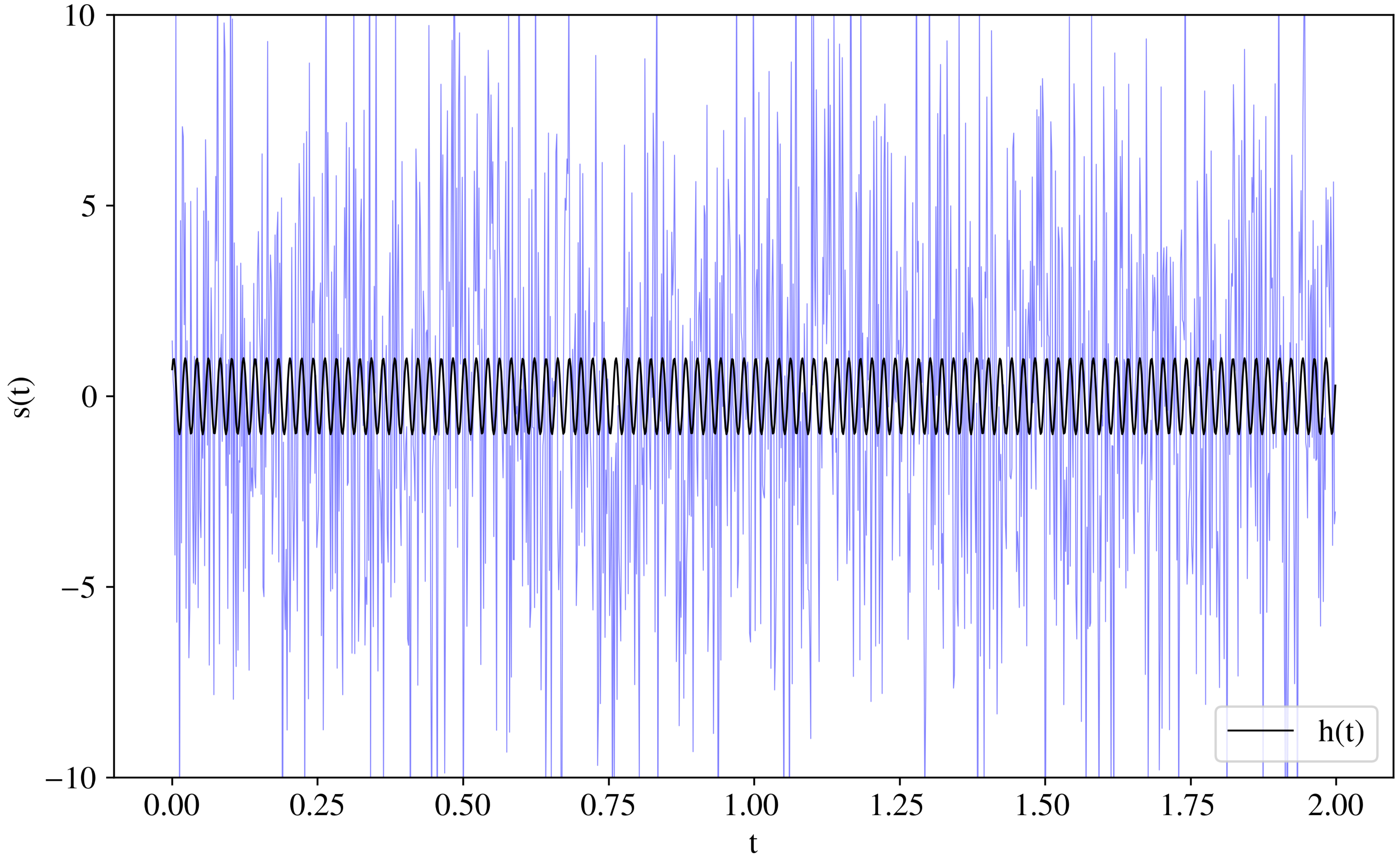
Discrete time-series, (finite interval)
well-sampled



Monochromatic search

For a start, assume a ‘monochromatic’ signal buried inside a data:

We need a Fourier transform!!



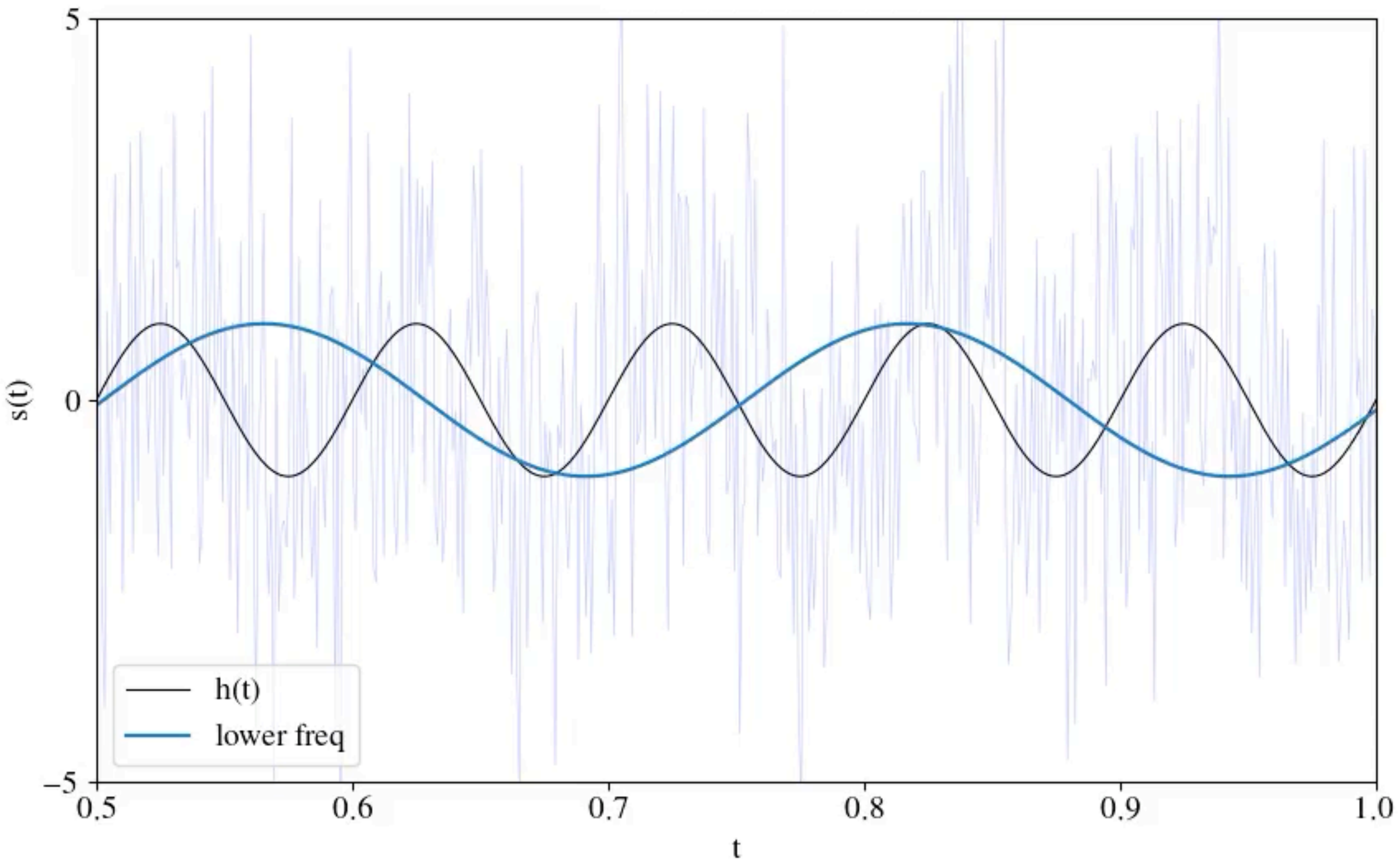
*needs windowing to prevent spectral leakage, unable to get info above f_{Nyquist}

What happens when we take the Fourier transform ?

In time domain,

1. we take monochromatic waves at every frequency
2. slide and multiply it with the data
3. calculate the integral

Formally, it is known as cross-correlation



Frequencies lower than what is present

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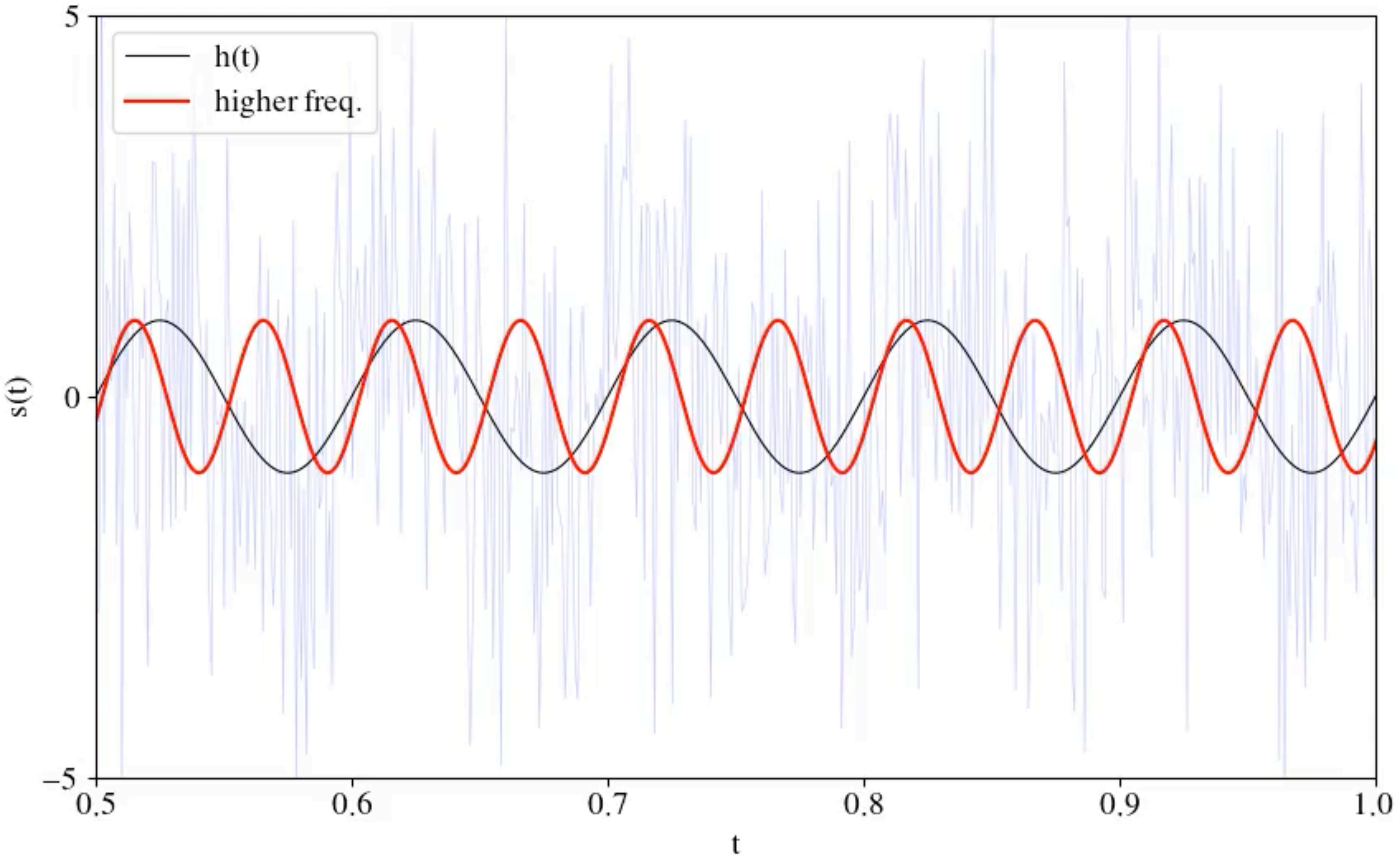

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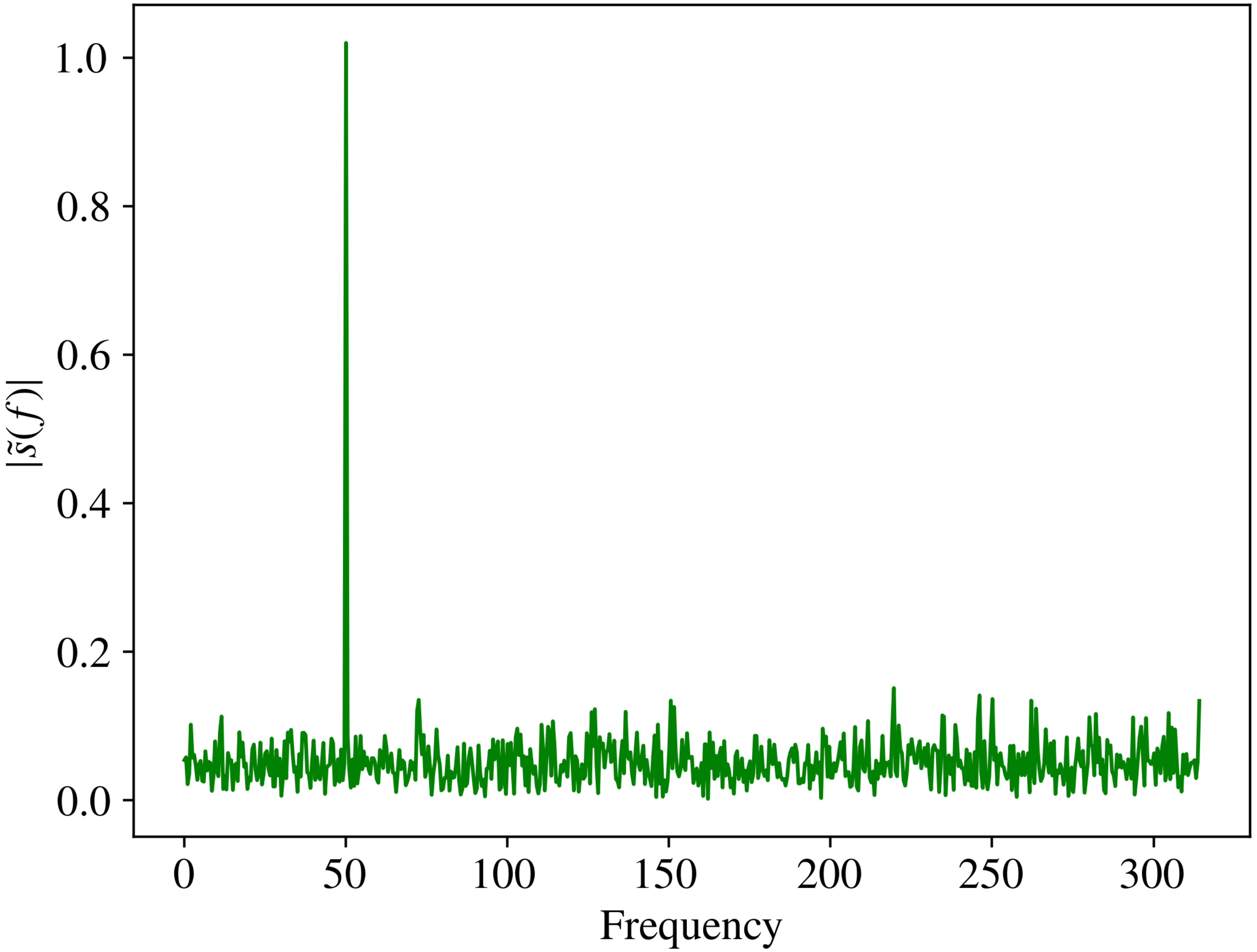
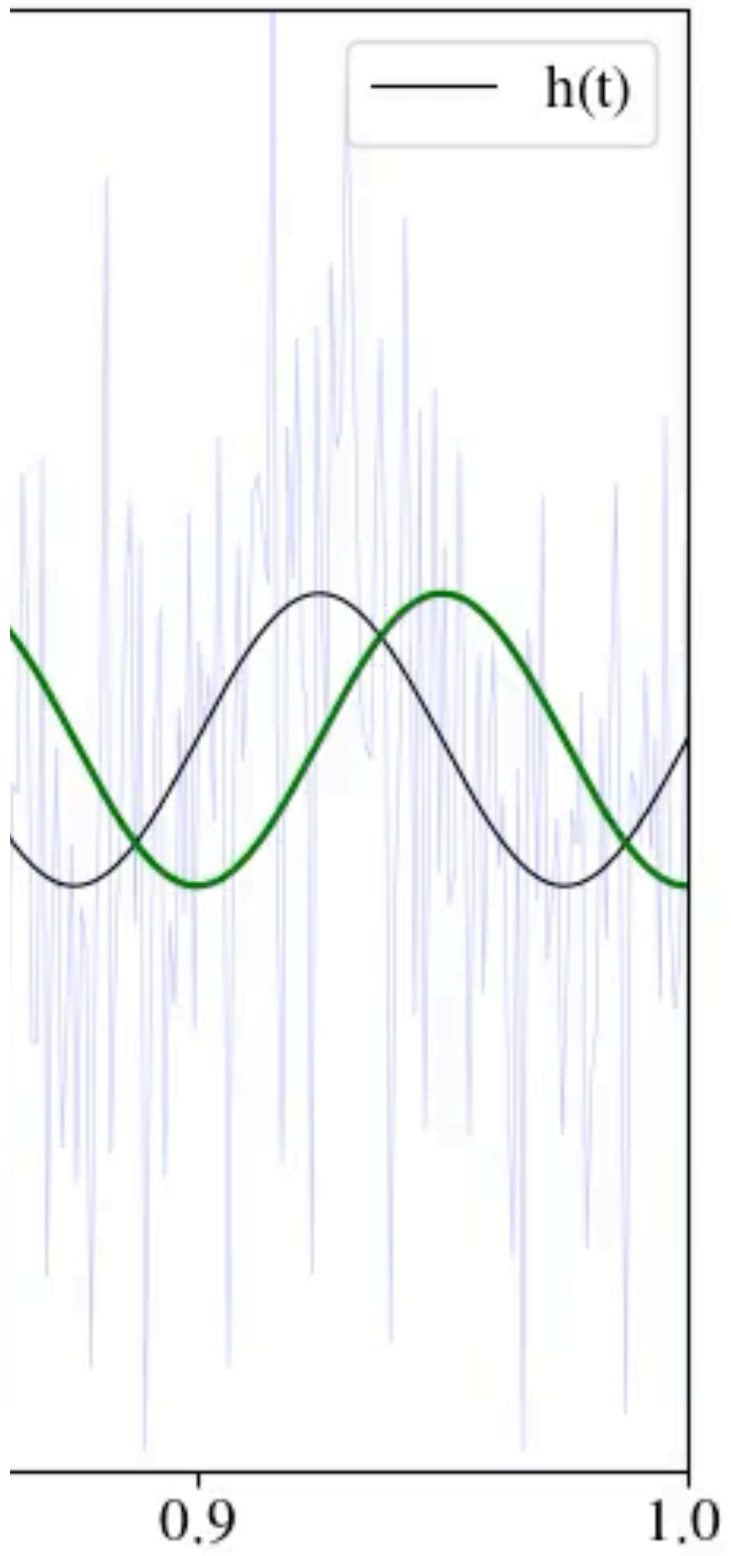




In other words, whe exactly matches, we get the maximum contribution !!

Maximum contribution when the plane wave frequency matches the signal frequency

Therefore, in Fourier transform, all we do is match the data with the simplest filter (the sines and cosines)

It is the simplest Matched Filter !!

Do the same to find any general pattern...

In time domain, multiply the data sliding the template each time, perform the integral, effectively a convolution time-series.

$$s * h\ (\tau) = \int_0^T s(t)h(t-\tau)dt$$

In frequency space, convolution becomes product:

$$\mathcal{F}(s * h(\tau)) = \tilde{s}(f)\ \tilde{h}(f)$$

Finally, integrate over all frequencies, detection statistic is: $\boxed{(s,h) = \int_0^\infty df \tilde{s}(f)\ \tilde{h}(f)}$

Naive matched filter

Almost correct...

Example with Sine-Gaussian

Assume signal of the form, $h(t) = Ae^{-\left(\frac{t-t_0}{\alpha}\right)^2} \sin(2\pi f_0(t-t_0)+\phi_0)$

Injected parameters:

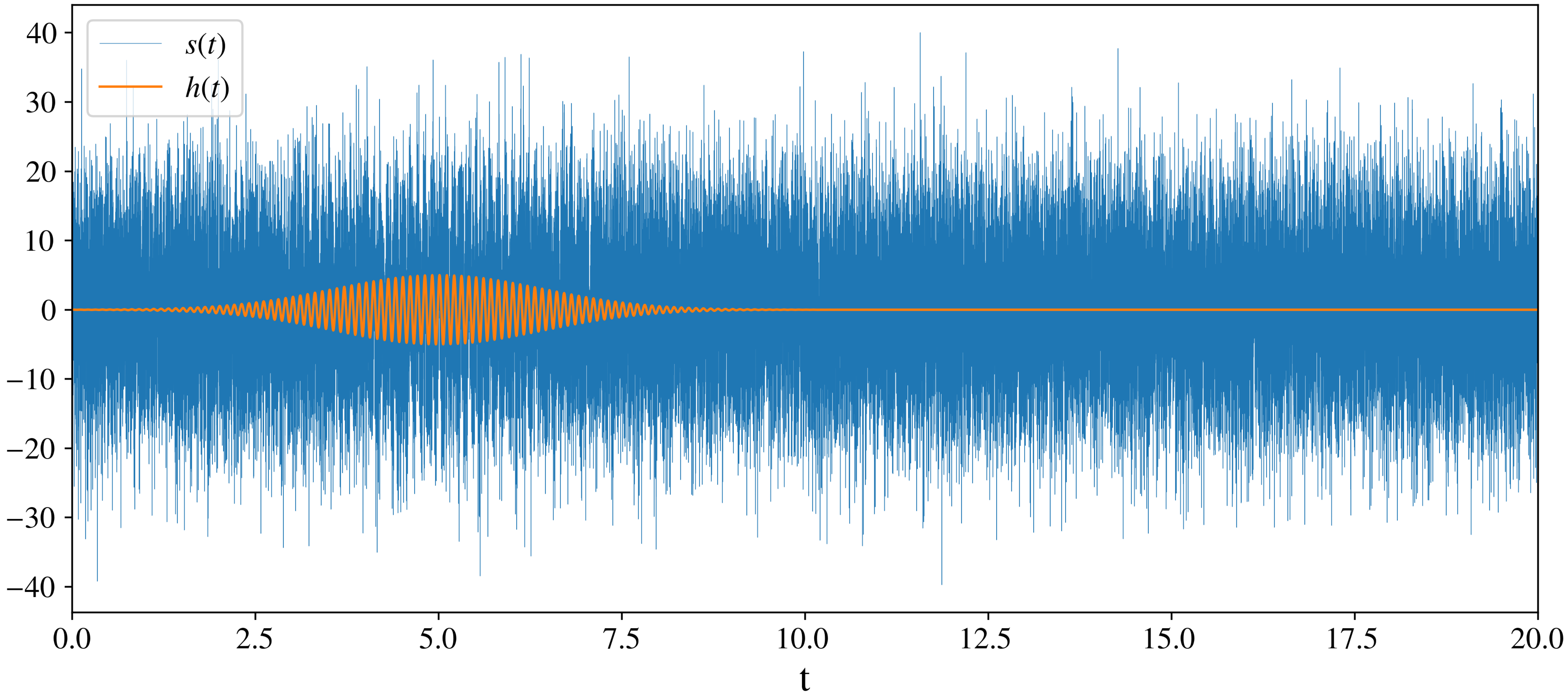
$A = 5$

$f_0 = 10\,\text{Hz}$

$\phi_0 = \pi/4$

$t_0 = 5$

$\alpha = 2$



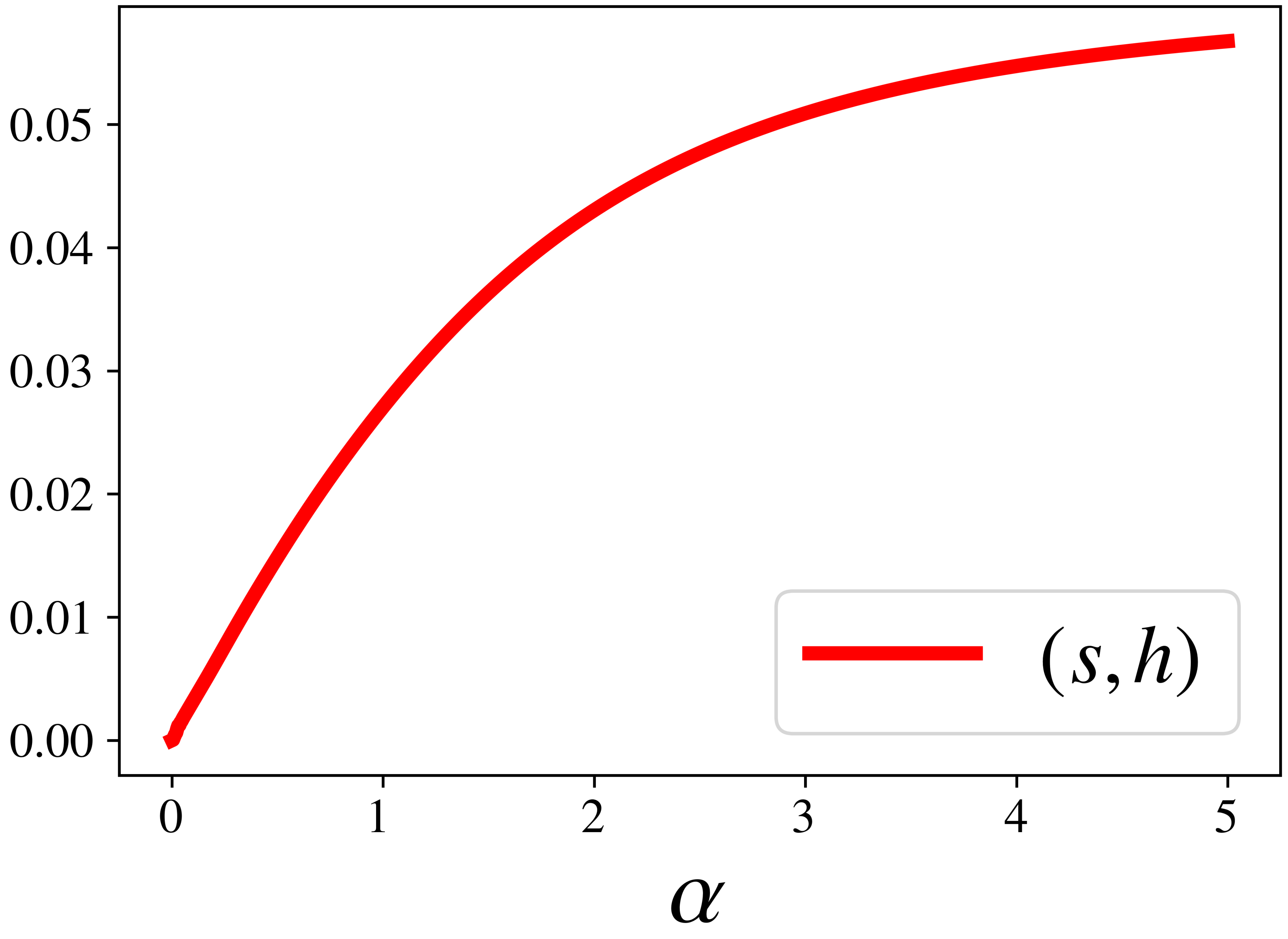
Example with Sine-Gaussian

$$h(t) = Ae^{-\left(\frac{t-t_0}{\alpha}\right)^2} \sin(2\pi f_0(t-t_0)+\phi_0)$$

Fixing other parameters, we aim to recover α,

Our naive matched-filter monotonically increases !!

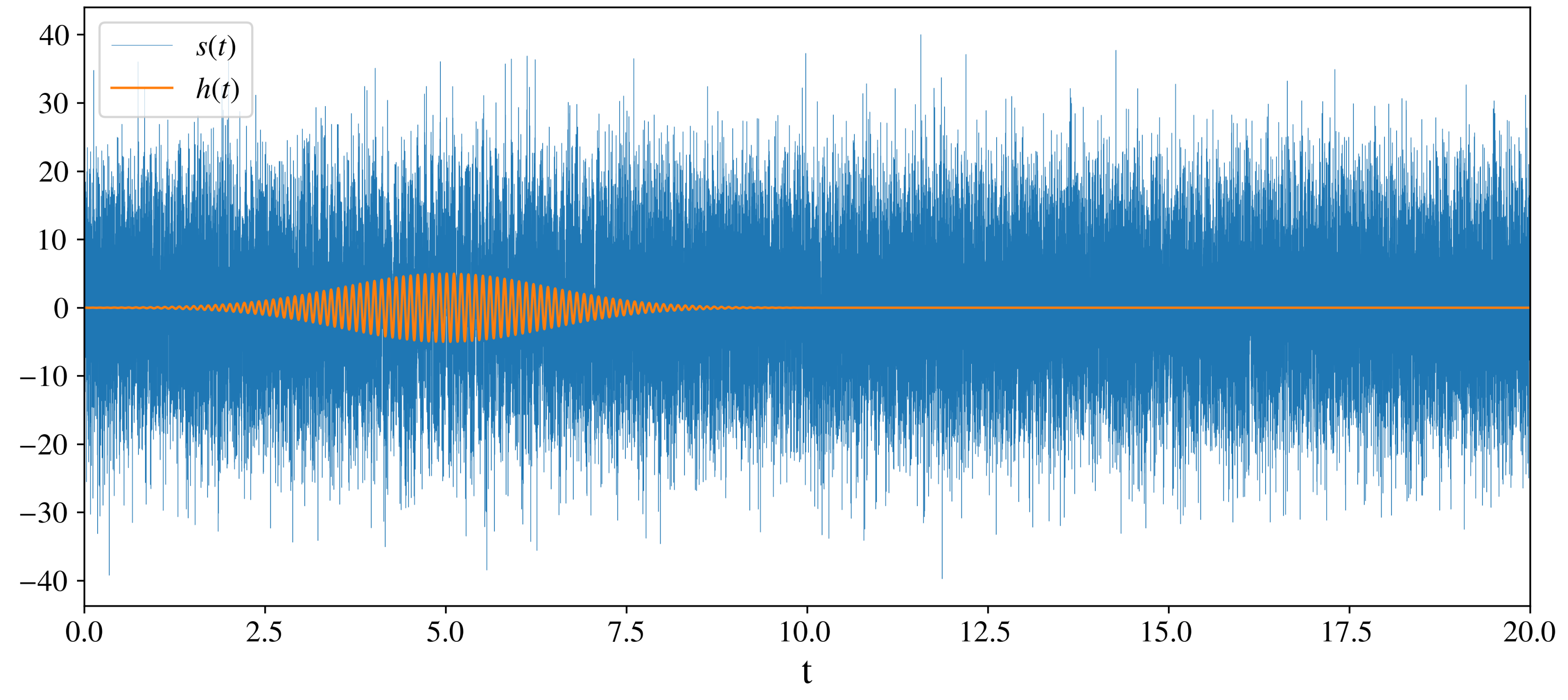
$$(s,h) = \int_0^\infty df \tilde{s}(f)\, \tilde{h}(f)$$

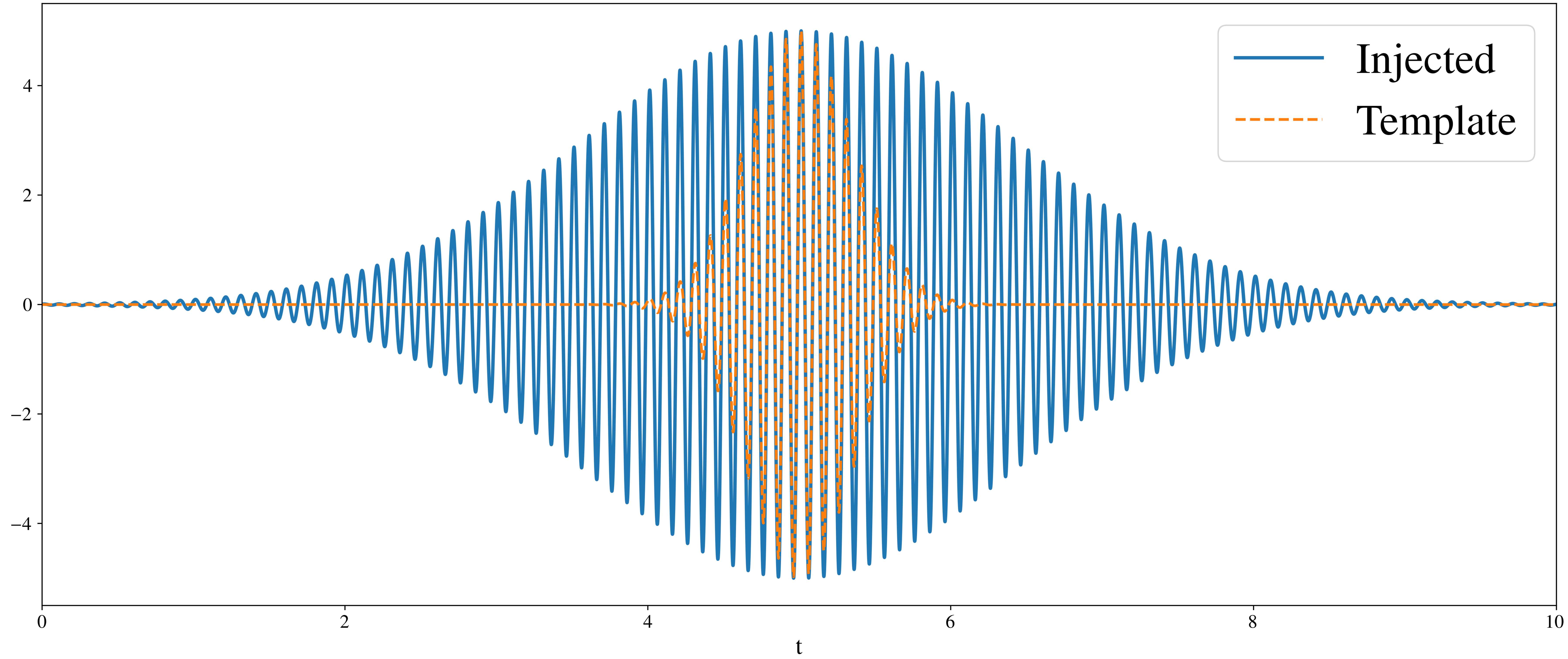


Example with Sine-Gaussian

$$h(t) = Ae^{-\left(\frac{t-t_0}{\alpha}\right)^2} \sin(2\pi f_0(t-t_0)+\phi_0)$$

$$\alpha = 0.5$$

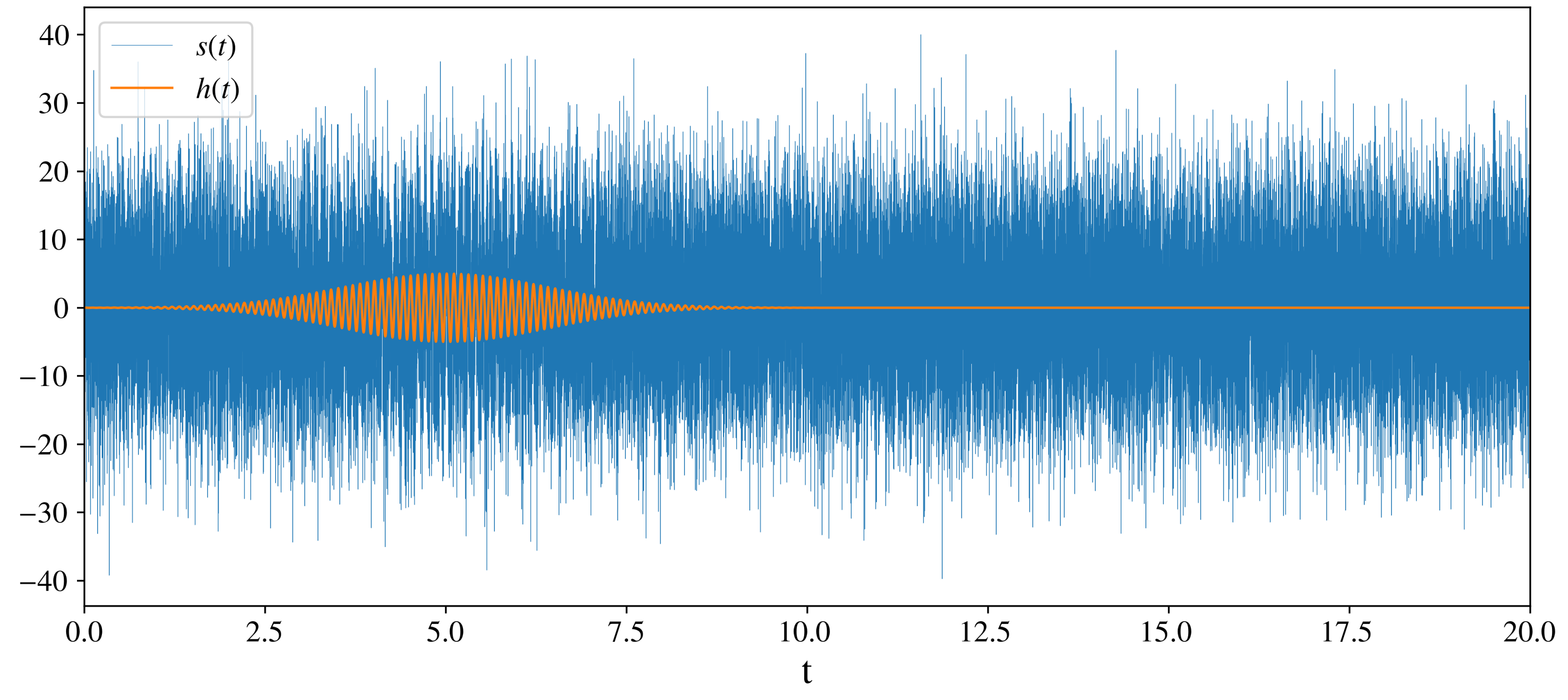


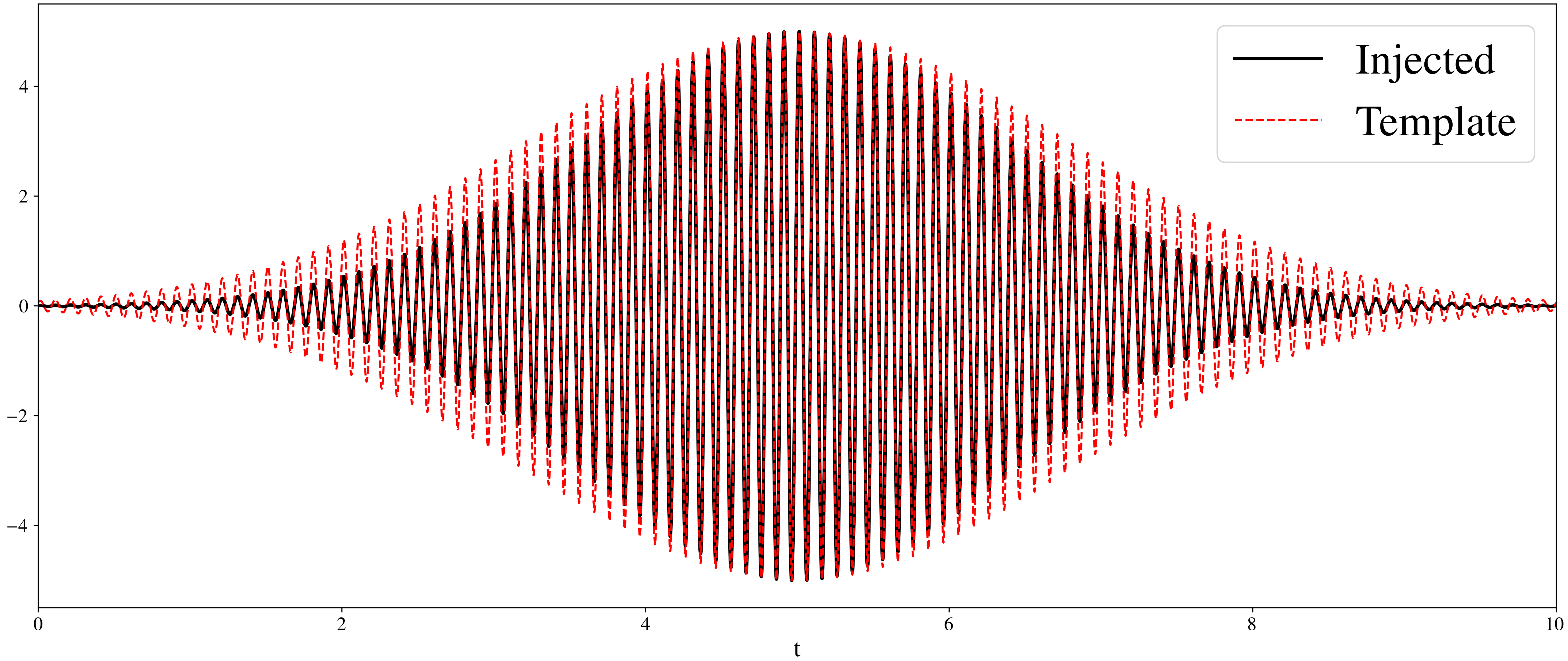


Example with Sine-Gaussian

$$h(t) = Ae^{-\left(\frac{t-t_0}{\alpha}\right)^2} \sin(2\pi f_0(t-t_0)+\phi_0)$$

$$\alpha = 2.5$$



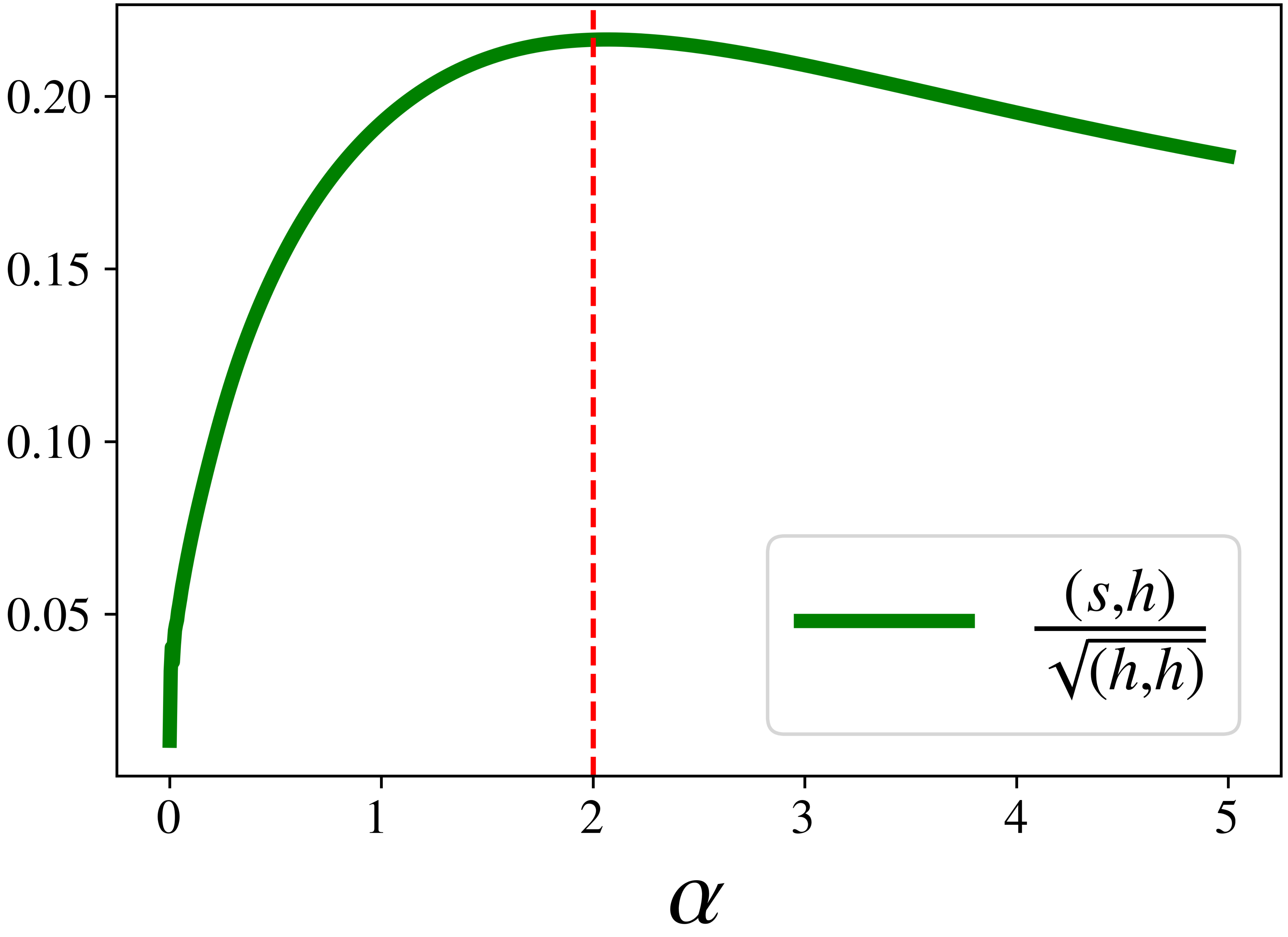


Example with Sine-gaussian

$$h(t) = Ae^{-\left(\frac{t-t_0}{\alpha}\right)^2} \sin(2\pi f_0(t-t_0)+\phi_0)$$

Normalising the naive matched-filter solves the problem!

$$\frac{(s,h)}{\sqrt{(h,h)}} = \frac{\int_0^\infty df \tilde{s}(f)\,\tilde{h}(f)}{\sqrt{\int_0^\infty df \tilde{h}(f)\,\tilde{h}(f)}}$$

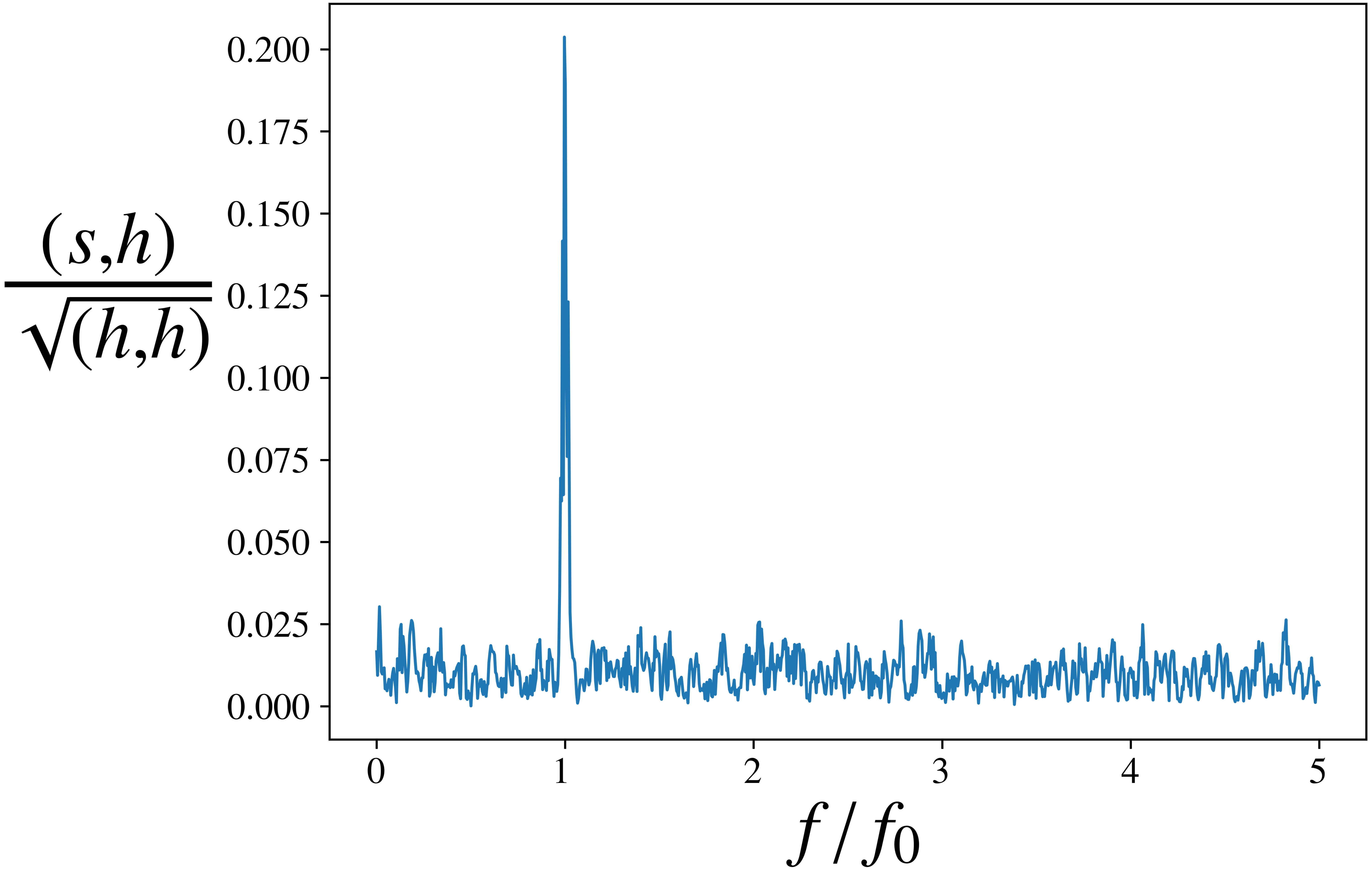


Example with Sine-Gaussian

$$h(t) = Ae^{-\left(\frac{t-t_0}{\alpha}\right)^2} \sin(2\pi f_0(t - t_0) + \phi_0)$$

Fixing other parameters,
we now aim to recover f_0,

Note the discriminating power of matched filtering between amplitude modulation and phase modulation!!



Noise weighted inner-product

Let's have a *formal* derivation:

$$p_x(\{x_j\}) = \left(\frac{1}{\sqrt{2\pi}\sigma}\right)^N \exp\left[-\frac{1}{2\sigma^2}\sum_{j=0}^{N-1} x_j^2\right]$$

Zero-mean Gaussian noise, with variance σ^2

With the definition of PSD, $S_x(f) = \lim_{\Delta t \to 0} 2\sigma^2 \Delta t$

We now show,

$$\lim_{\Delta t \to 0} \exp\left[-\frac{1}{2\sigma^2}\sum_{j=0}^{N-1} x_j^2\right] = \exp\left[-\frac{1}{2}4\int_0^\infty \frac{|\tilde{x}(f)|^2}{S_x} df\right]$$

Correct for white noise...

Noise weighted inner-product

Consider a linear process that produces the time-series of the non-white noise,

$$\gamma(t) = \int_{-\infty}^{\infty} K(t - t')\ x(t')\ dt'$$

$$\tilde{\gamma}(f) = \tilde{K}(f)\ \tilde{x}(f)$$

$$p_\gamma[\gamma(t)] \propto \exp\left[-\frac{1}{2}4\int_0^\infty \frac{|\tilde{\gamma}(f)|^2}{S_\gamma(f)}df\right]$$

$$(a, b) \equiv 4\mathscr{R}\int_0^\infty \frac{\tilde{a}(f)\tilde{b}^*(f)}{S(f)}df$$

With this definition, $\boxed{p_x(x(t)) \propto e^{-(x,\ x)/2}}$ Gaussian Noise probability

But how do we arrive at the inner product between the signal and the template ?

Matched filter SNR-the maximum detection statistics

Recall the likelihood ratio:

$$\Lambda(\vec{\lambda}, \mathscr{H}_1 \,|\, s(t)) = \frac{p(s(t)\,|\,\vec{\lambda}, \mathscr{H}_1)}{p(s(t)\,|\,\mathscr{H}_0)}$$

Now, for a fixed BBH parameter set $\vec{\lambda}$

$$p(s\,|\,\mathscr{H}_0) = p_n[s(t)] \propto e^{-(s,s)/2}$$

$$p(s\,|\,\vec{\lambda}, \mathscr{H}_1) = p_n[s(t) - h(t;\vec{\lambda})] \propto e^{-(s-h,\, s-h)/2}$$

$$\Lambda(\vec{\lambda}, \mathscr{H}_1 \,|\, s(t)) = \frac{e^{-(s-h,\, s-h)/2}}{e^{-(s,s)/2}} = e^{(s,\,h)}e^{-(h,\,h)/2}$$

Need to construct a statistic that will maximise the likelihood ratio, while taking care of the template overall normalisation

Matched filter SNR-the maximum detection statistics

$$\Lambda(\vec{\lambda}, \mathscr{H}_1 \,|\, s(t)) = \frac{e^{-(s-h,\ s-h)/2}}{e^{-(s,s)/2}} = e^{(s,\ h)} e^{-(h,\ h)/2}$$

$$h(\vec{\lambda}) = \rho(\vec{\lambda})\ \hat{h}(\lambda) \implies \Lambda(\vec{\lambda}, \mathscr{H}_1 \,|\, s(t)) = e^{\rho\,(s,\hat{h}) - \rho^2(\hat{h},\hat{h})/2}$$

$$(h(\vec{\lambda}), h(\vec{\lambda})) = \rho^2(\vec{\lambda})(\hat{h}(\vec{\lambda}, \hat{h}(\vec{\lambda})) = \rho^2(\vec{\lambda})$$

$$\frac{\partial \Lambda(\vec{\lambda})}{\partial \rho} = 0 \implies \rho_{\rm mf}(\vec{\lambda}) = (s, \hat{h}(\vec{\lambda})) = \frac{\boxed{(s, h(\vec{\lambda}))}}{\boxed{\sqrt{(h(\vec{\lambda})), h(\vec{\lambda}))}}}$$

Takes care of the phasing

Takes care of the overall amplitude

Easy to show that,

$$\langle \rho_{\rm mf}(\vec{\lambda}) \rangle = \rho(\vec{\lambda})$$

Matched filter SNR

Optimal SNR

Next job is to find the Bayes factor...

The parameters (a quick recap)

Intrinsic parameters $m_1, m_2, \vec{S}_1, \vec{S}_2$ Masses and spins

Extrinsic parameters
$$\begin{cases} RA, DEC, d_L & \text{Location} \\ \iota, \psi & \text{Orientation} \\ t_c, \phi_c & \text{Arrival time and phase} \end{cases}$$

Location and orientation: can be combined to form an effective amplitude A

*with more physics comes more parameters

$$\Lambda(\mathscr{H}_1 \,|\, s(t)) = \int d\vec{\lambda}\; p(\vec{\lambda})\; \Lambda(\vec{\lambda}, \mathscr{H}_1 \,|\, s(t))$$

When a strong signal is present, the likelihood ratio is strongly peaked at the true value.

Prior probabilities

Maximisation can be done analytically

$$\left.\frac{\partial \ln \Lambda(H_{\vec{\lambda}} \,|\, s(t))}{\partial \vec{\lambda}}\right|_{\vec{\lambda}=\vec{\lambda}_{\max}} = 0$$

$$\left.\left(s - h(\vec{\lambda}), \frac{\partial}{\partial \vec{\lambda}} h(\vec{\lambda})\right)\right|_{\vec{\lambda}=\vec{\lambda}_{\max}} = 0$$

Marginalisation over other extrinsic parameters

$$\left.\frac{\partial \ln \Lambda(H_{\vec{\lambda}} \,|\, s(t))}{\partial \vec{\lambda}}\right|_{\vec{\lambda}=\vec{\lambda}_{\max}} = 0$$

$$\left.\left(s - h(\vec{\lambda}, \frac{\partial}{\partial \vec{\lambda}} h(\vec{\lambda})\right)\right|_{\vec{\lambda}=\vec{\lambda}_{\max}} = 0$$

Arrival time marginalisation :

Assume a template $h(t, \overrightarrow{\mu}) = Ag(t - t_0, \overrightarrow{\mu})$

$$(s, h) = 2A \int_{-\infty}^{\infty} \frac{\tilde{s}(f)\tilde{g}^*(f)}{S_n(f)} e^{2\pi i f t_0} df$$

Marginalisation over extrinsic parameters

$$\left.\frac{\partial \ln \Lambda(H_{\vec{\lambda}} \,|\, s(t))}{\partial \vec{\lambda}}\right|_{\vec{\lambda}=\vec{\lambda}_{\max}} = 0$$

$$\left.\left(s - h(\vec{\lambda}, \frac{\partial}{\partial \vec{\lambda}} h(\vec{\lambda})\right)\right|_{\vec{\lambda}=\vec{\lambda}_{\max}} = 0$$

Phase marginalisation :

Assume that the data contains $\tilde{d}(f; \overrightarrow{\mu}, \phi) = \tilde{h}(f; \overrightarrow{\mu})e^{i\phi_c}$

Generate two templates, $\tilde{h}(\phi_c = 0)$ and $\tilde{h}(\phi_c = \pi/2)$

$$(s \,|\, h)_{\max\ \phi} = \sqrt{(s, h(0))^2 + (s, h(\pi/2))^2}$$

Notice, $(s, h(0)) = (h(0), h(0))\cos\phi_c$

$$(s, h(\pi/2)) = (h(0), h(0))\ \sin\phi_c$$

- For the intrinsic parameters, it is not possible to have an explicit form of maximum likelihood estimator
- Need to resort to a collection of pre-computed templates to search from...

The template bank

- Grid up the continuous (intrinsic) parameter space (3 or 4-dimensional)!
- Sufficiently finely that the maximum likelihood ratio can be approximately identified as the best match among the collection
- The available set $\{h(t, \vec{\lambda})\}$ so densely packed that the true signal will lie close enough to one of the templates

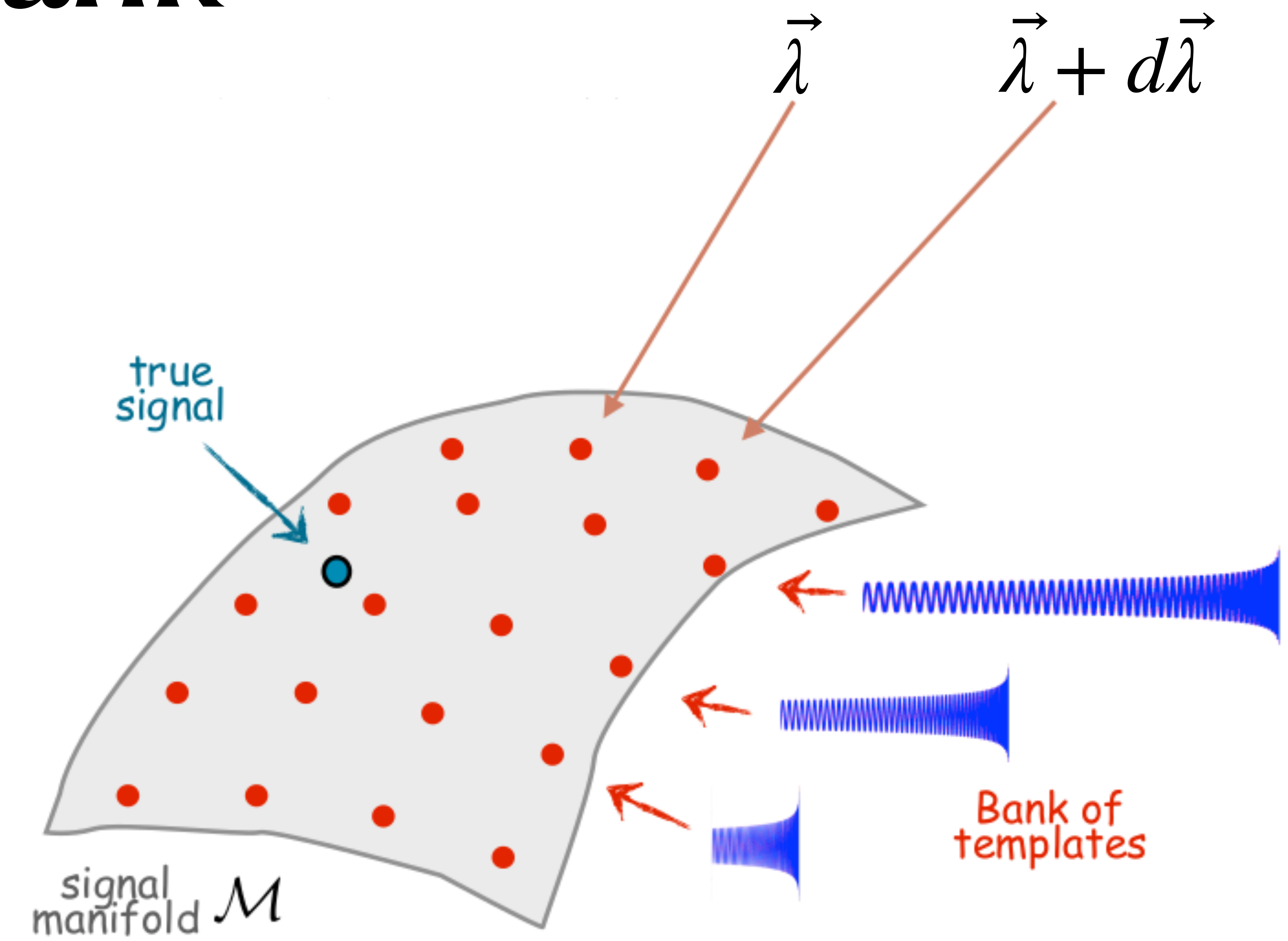


Illustration credit: P. Ajith

Accuracy required

Ben Owen, 1999

Assume true signal $h(t) = \rho u(t, \vec{\lambda}_t)$

$$\rho_{\text{opt}} = \rho$$

Nearest template in the bank, $\rho u(t, \vec{\lambda}_t + \Delta\vec{\lambda})$

$$\rho_{\text{opt}} = \rho' = \rho(u(t, \vec{\lambda}_t),\ u(t, \vec{\lambda}_t + \Delta\vec{\lambda}))$$

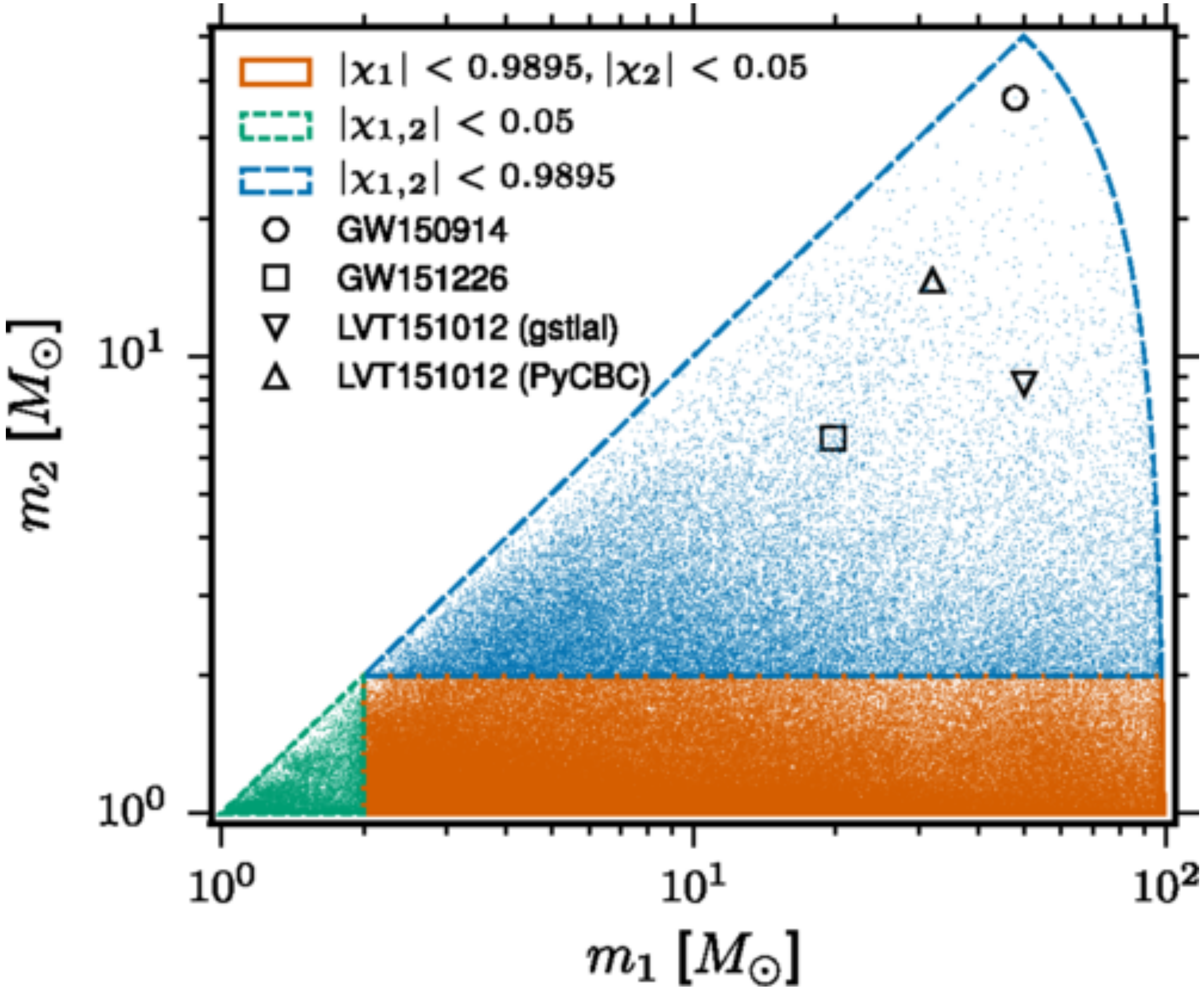
Fractional loss: $\dfrac{\rho - \rho'}{\rho} = 1 - (u(t, \vec{\lambda}_t),\ u(t, \vec{\lambda}_t + \Delta\vec{\lambda})) = 1 - \mathscr{A}$

$$\mathscr{A} = 1 - \left|-\frac{1}{2}\left(u(\vec{\lambda}), \frac{\partial^2 u}{\partial\lambda_i \partial\lambda_j}(\vec{\lambda}_t)\right)\right| d\lambda_i\, d\lambda_j = 1 - g_{ij}(\vec{\lambda}_t)\ d\lambda_i\, d\lambda_j$$

need much more finer resolution
whenever the waveform varies drastically

We demand for a fixed fractional loss, leading to fixing $d\lambda_i$ at different regimes

Actual template bank used



Phys. Rev. X 6, 041015

Summary so-far

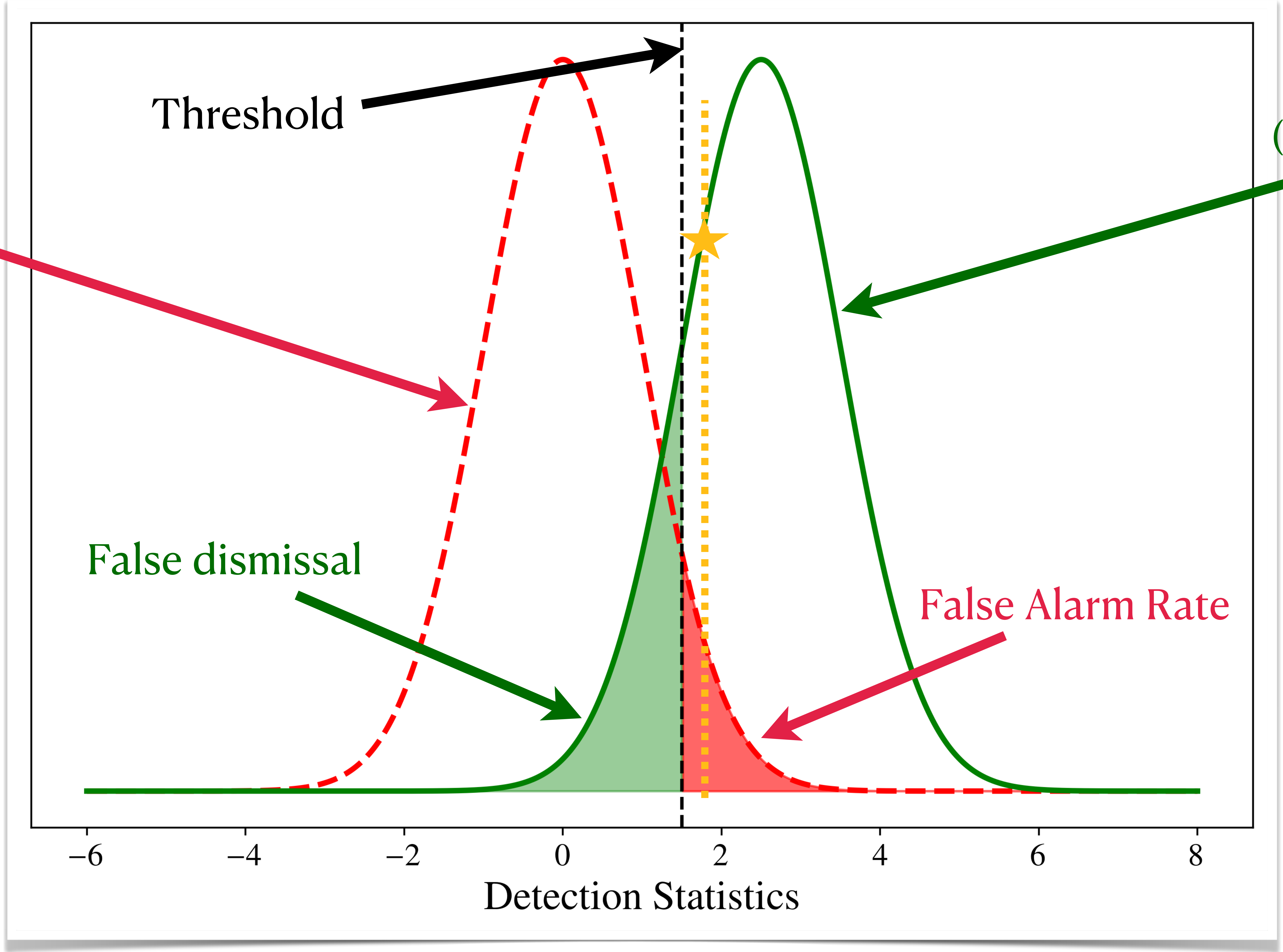
- Matched filtering is central to GW detection, but only optimal under Gaussian noise.
- Noise modeling and template construction are crucial for accuracy.
- Statistical methods (likelihood ratios, hypothesis testing) form the foundation for detection significance.

Approximately, the data is $\Lambda(\vec{\lambda}_{\max}, \mathscr{H}_1 \mid s(t))\dfrac{P(\mathscr{H}_1)}{P(\mathscr{H}_0)}$ *times more likely to contain a GW signal than to consist of pure noise.*

Frequentist's model selection

- Since there is noise in the data, measurement results(detection statistics) have a distribution.
- Subjective to (a) problem at hand (b) the detection statistics considered

Model 1
(Null hypothesis)
There is no signal!



Model 2
(True hypothesis)
Signal!

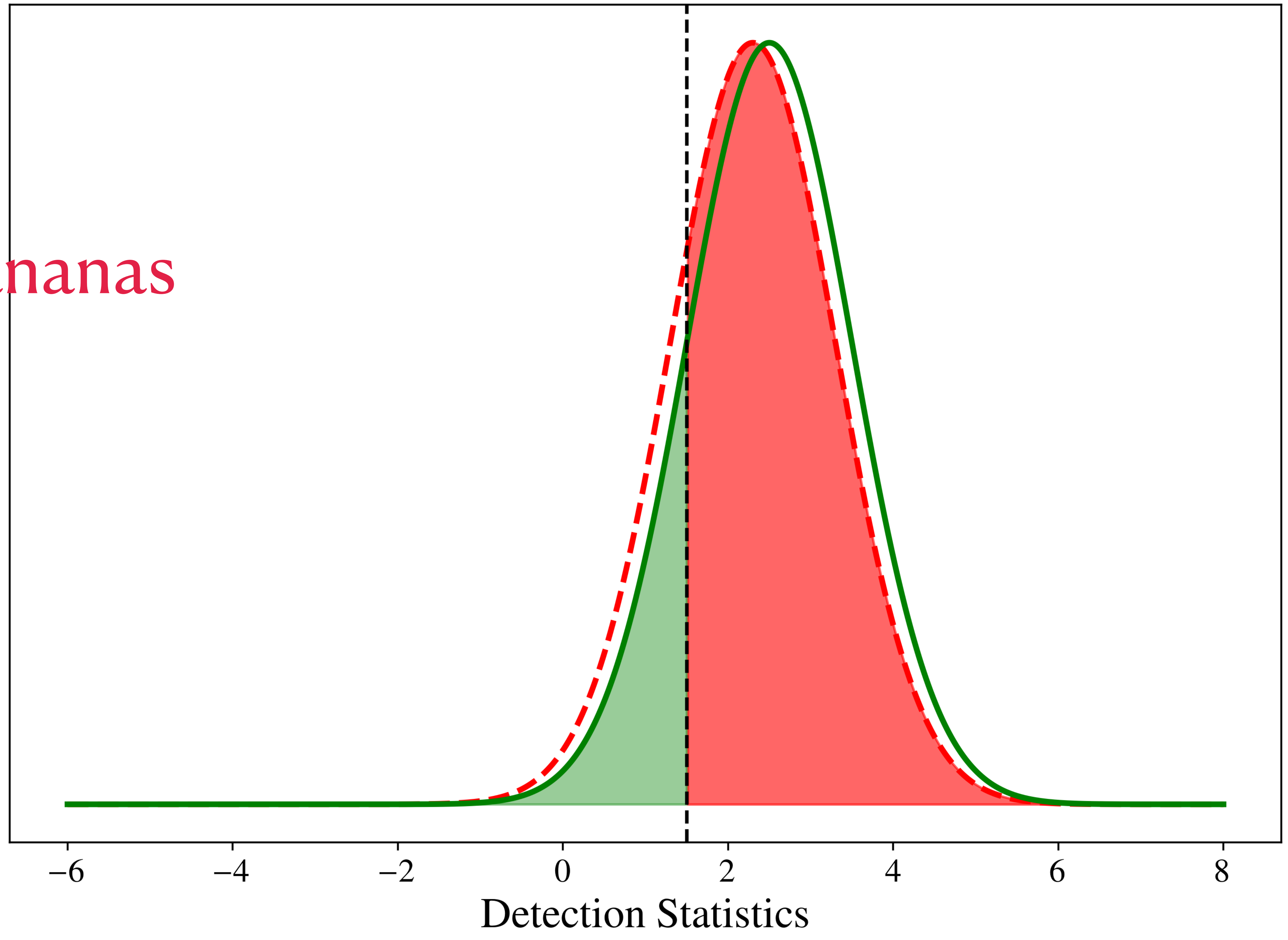
p-value
probability of obtaining results observed at least as extreme as the result actually observed

A very simple example

Let's take a simple example of distinguishing mangoes and bananas

Only choose colour as the detection statistics

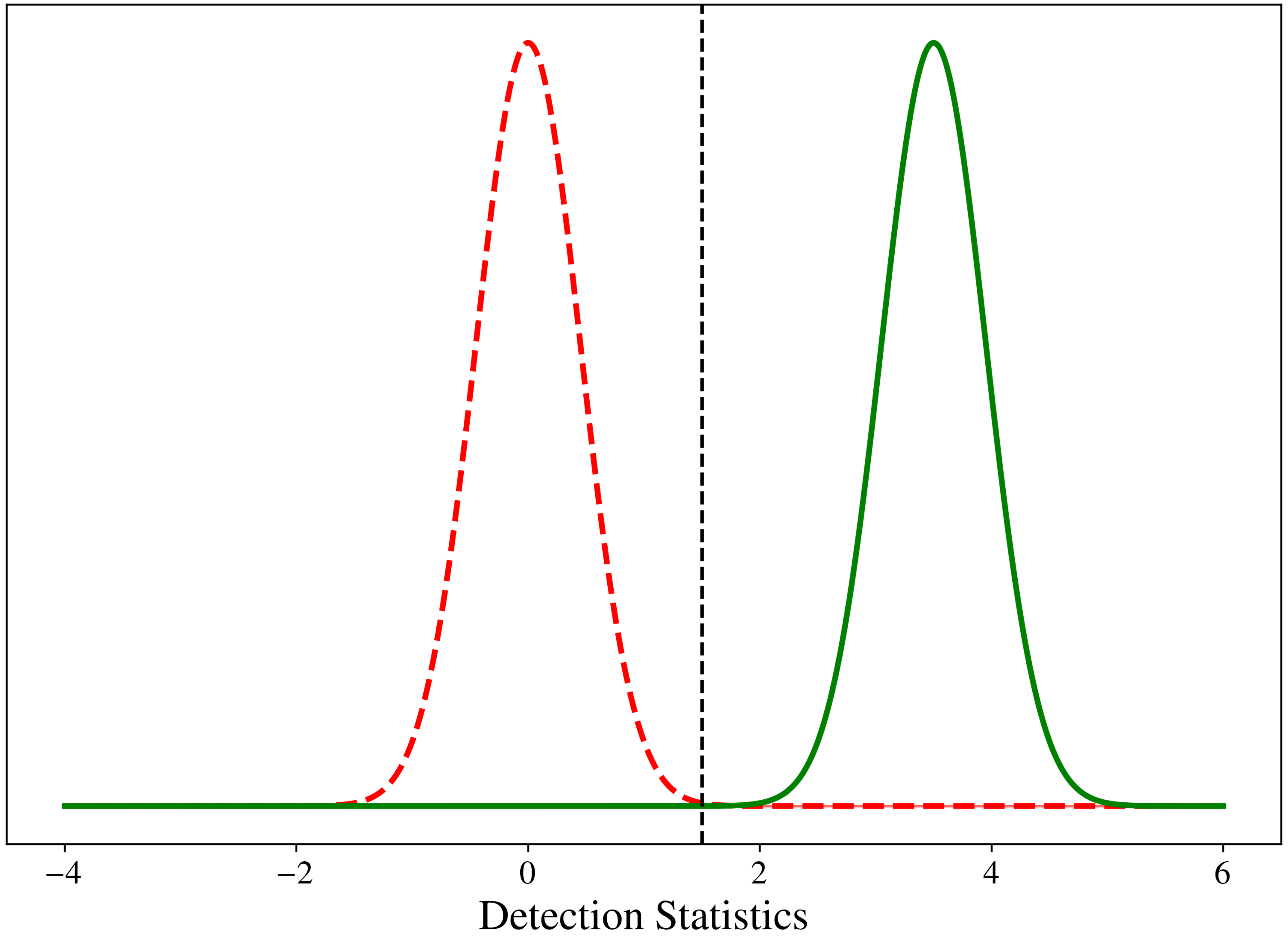
More false alarms, identifying bananas as mangoes !



A very simple example

Let's take a simple example of distinguishing mangoes and bananas

Combine colour, shape and the smell



Connecting FAR and p-value in GW

For the null hypothesis we have

$$p(\rho_{\rm mf}\,|\,\mathscr{H}_0) = \frac{1}{\sqrt{2\pi}} e^{-\rho_{\rm mf}^2/2}$$

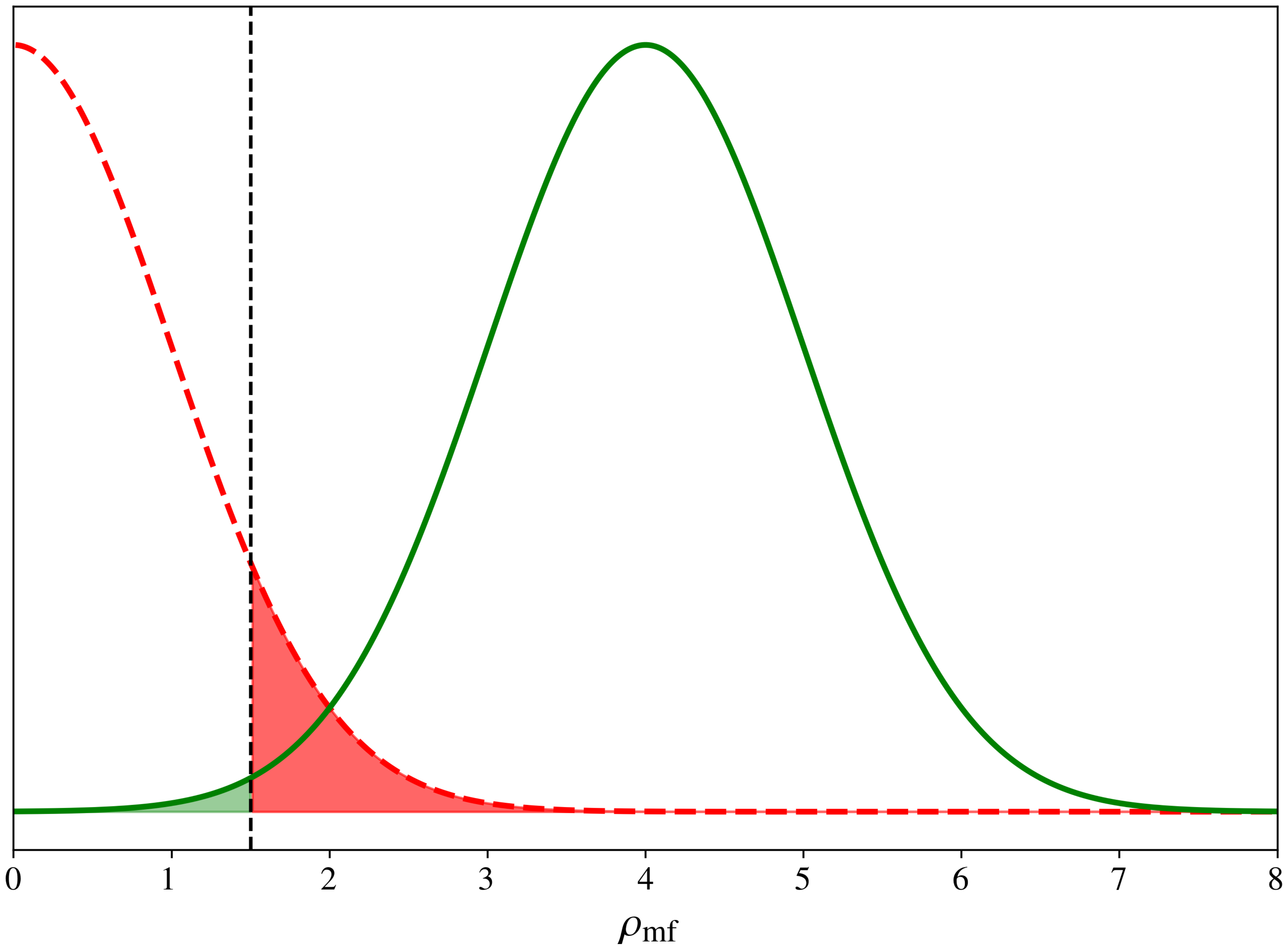
For the detection hypothesis we have

$$p_1(\rho_{\rm mf}\,|\,\mathscr{H}_1) = \frac{1}{\sqrt{2\pi}} e^{-(\rho_{\rm mf}^2-\rho^2)/2}$$

Setting a threshold of ρ_* gives the false alarm and false dismissal probabilities:

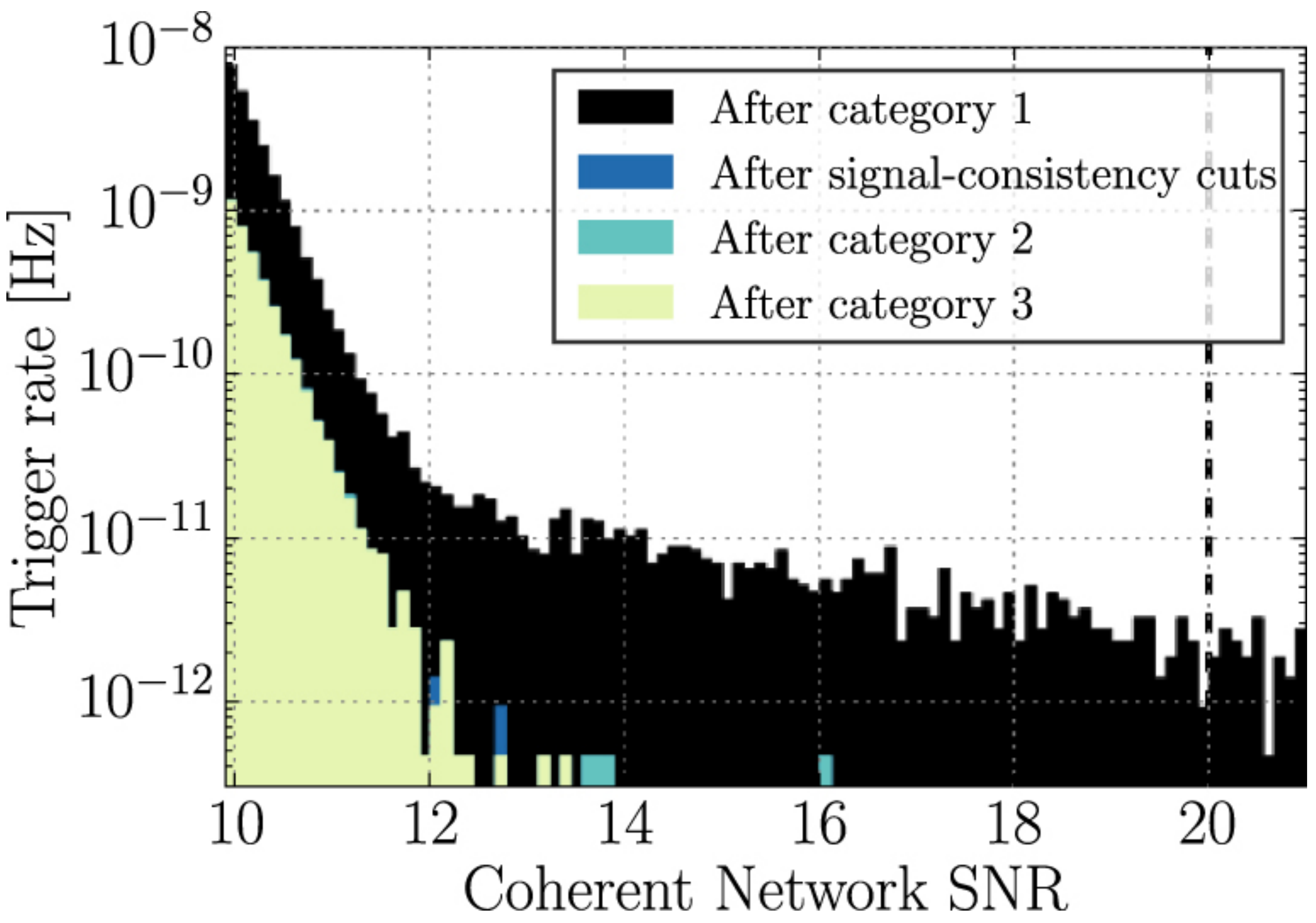
$$P_{\rm FA} = \frac{1}{2}\,\mathrm{erfc}\left(\frac{\rho_*}{\sqrt{2}}\right)$$

$$P_{\rm FD} = \frac{1}{2}\,\mathrm{erfc}\left(\frac{\rho_* - \rho}{\sqrt{2}}\right)$$



Signal detection in non-gaussian noise

- Sometimes glitches(noisy bursts) can accidentally mimic real signals, producing large values of SNR,
- SNR distribution of the background doesn't look like a gaussian anymore
- Matched-filtering is not always reliable.



Need extra checks!!

B P Abbott et al 2016 Class. Quantum Grav. 33 134001

Signal detection in non-gaussian noise

- Chi-square test : Check whether the frequency distribution of power is consistent with what is expected from a GW signal
- Veto out: Continuous monitoring of seismic, atmospheric, magnetic, and interferometric signals helps veto events mimicked by local disturbances.
- Multi-detector coincidence : The triggers from multiple detectors be coincident in time and consistent in template parameters
- Null-stream test : Construct null-combinations of the data streams, any signal present in the Individual detectors will cancel by construction, glitches or other transients will most likely show up

Exploits the fact that, if the detectors are not co-located, or co-aligned, non-gaussian noises are local, whereas astrophysical GW transient should be there in all detectors

Signal detection in non-gaussian noise

- Chi-square test : Break the detector bandwidth into several smaller bands and check if the response in each band is consistent with what would be expected from a signal

Actual SNR contribution from each bin

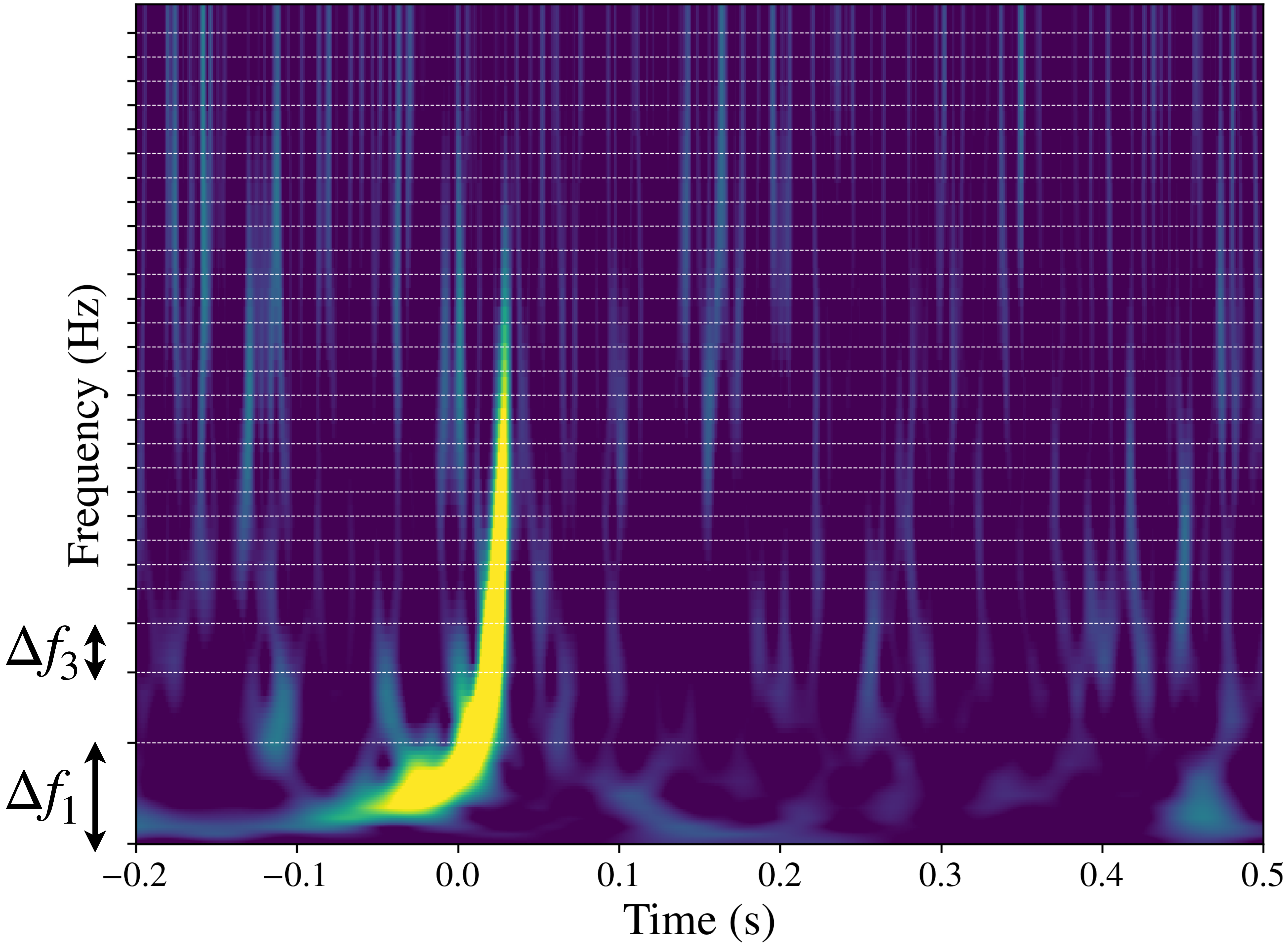
Total SNR

$$\Delta\rho_j \equiv \rho_j - \frac{\rho}{p}$$

Number of bins

$$\chi^2 \equiv \chi^2(\rho_1, \rho_2, \ldots \rho_p) = p \sum_{j=i}^{p} (\Delta\rho_j)^2$$

$$\langle \chi^2 \rangle = p - 1$$

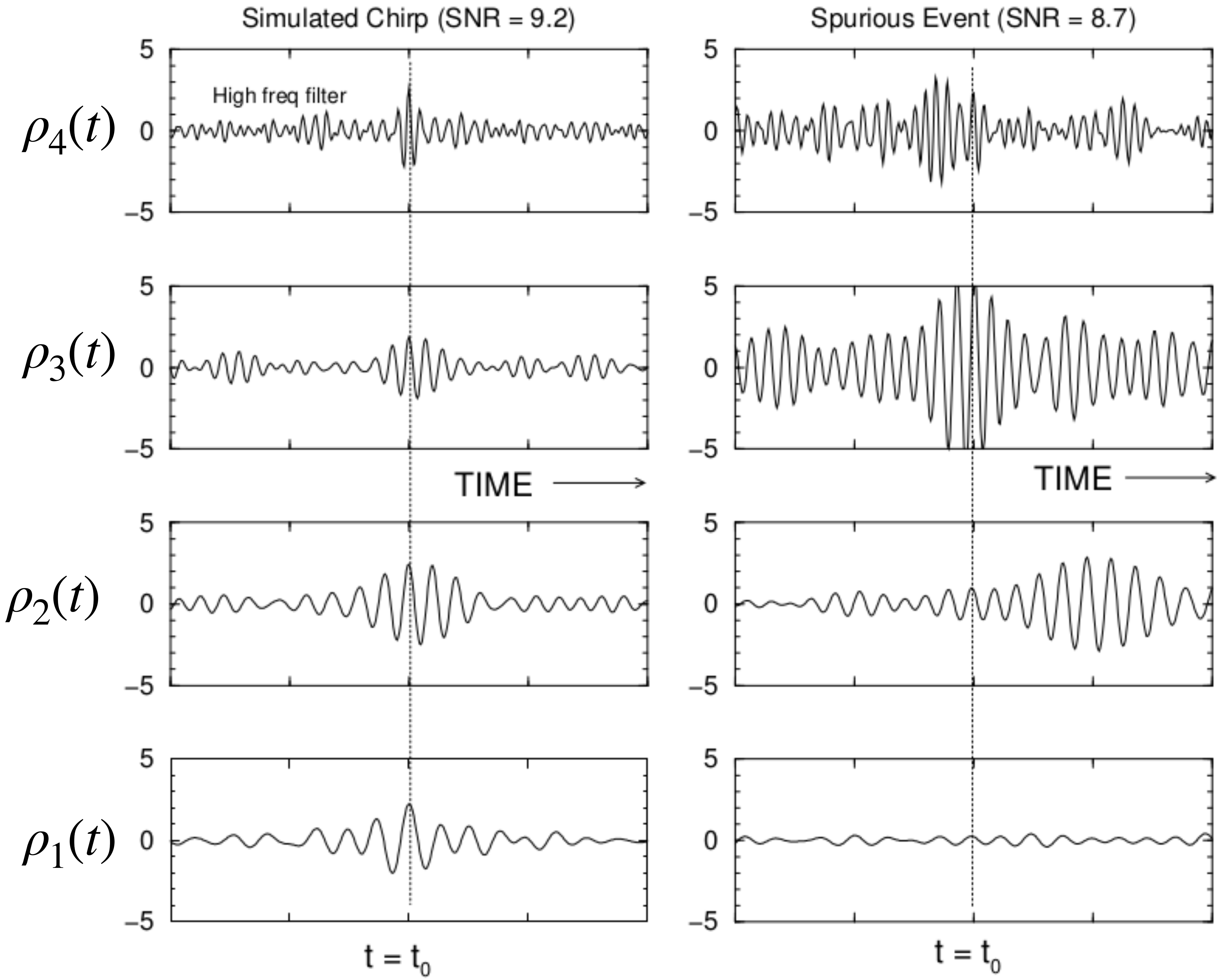


A toy-example

$$\rho_1 = 2.25,\ \rho_2 = 2.44,$$
$$\rho_3 = 1.87,\ \rho_4 = 2.64$$

$$\chi^2 = 4\sum_{i=1}^{4}(\rho_i - \frac{9.2}{4})^2 = 1.296$$

$$P_{\chi^2 \geq 1.296} = 0.73$$



$$\rho_1 = 0.23,\ \rho_2 = 0.84,$$
$$\rho_3 = 5.57,\ \rho_4 = 2.33$$

$$\chi^2 = 4\sum_{i=1}^{4}(\rho_i - \frac{8.7}{4})^2 = 68.4$$

$$P_{\chi^2 \geq 68.4} = 9.4 \times 10^{-15}$$

B. Allen Phys. Rev. D 71, 062001

Time-consistency test

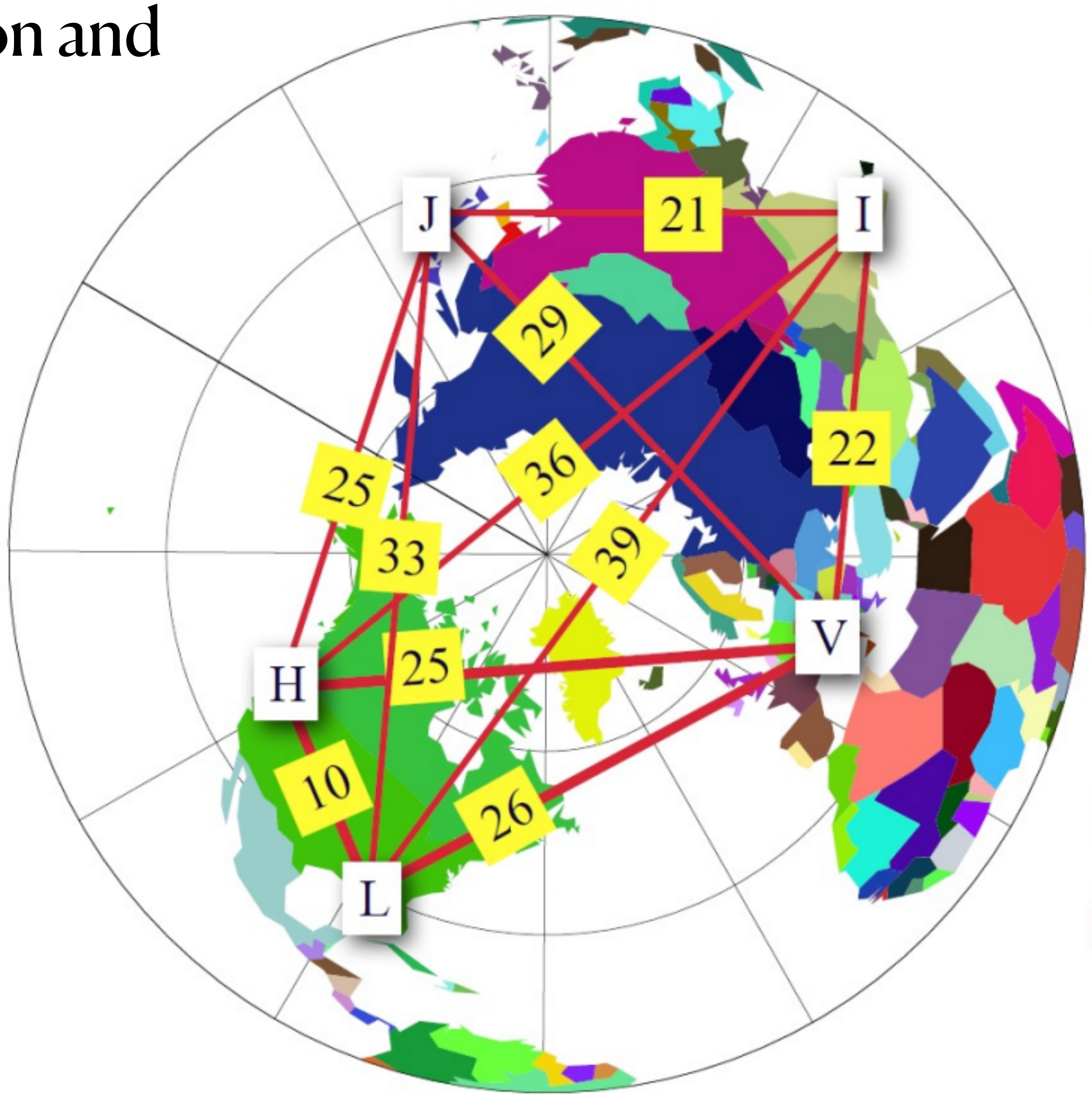
Gravitational waves travel at the speed of light.

Arrival time differences at detectors depend on source location and inter-detector baseline.

Used to check astrophysical origin & assist in sky localization.

The arrival time-delay between the detectors can be written as:

$$\Delta t_{IJ} = t_J - t_I = \frac{(\vec{r}_J - \vec{r}_I)\,.\,\hat{n}}{c}$$



Credit: V. Fafone

Veto by SNR re-weighting

Down-weight the times where the data does not appear as either Gaussian noise or Gaussian noise + our template.

$$\chi_r^2 \equiv \chi_r^2(\rho_1, \rho_2, \ldots \rho_p) = \frac{p}{2p-2} \sum_{j=i}^{p} (\Delta\rho_j)^2$$

Can be done by combining the SNR time series and χ_r^2 time-series

$$\hat{\rho}(t) = \frac{\rho}{\frac{1}{2}\left[1 + (\chi_r^2)^3\right]^{\frac{1}{6}}}$$,where $\chi_r^2 > 1, \rho$ otherwise

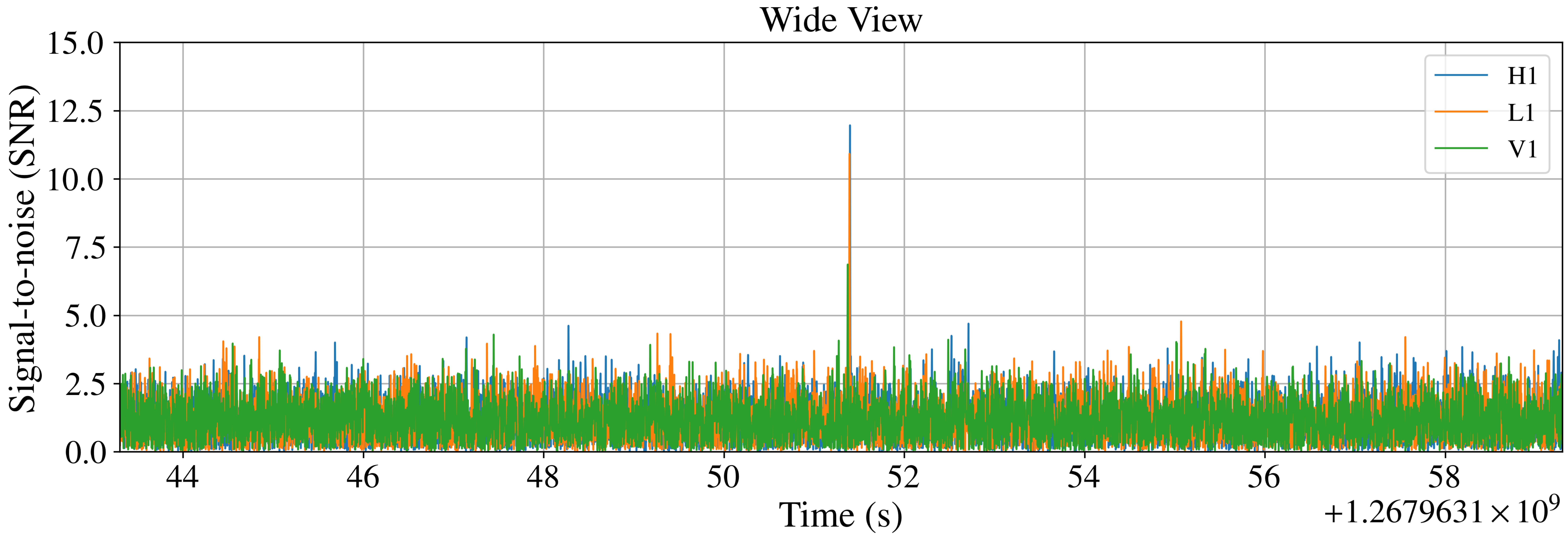
Using these re-weighted SNRs, most of the non-gaussian triggers are removed

Calculating the significance in Virgo Data

Question: estimate the significance of signal-to-noise peak observed in the Virgo instrument coincident with the large peaks observed in the LIGO-Hanford and LIGO-Livingston observatories, for a particular event.

Let's do it for GW200311_115853 !

1. we calculate the SNR time-series (do it for the best template)



Calculating the significance in Virgo Data

Question: estimate the significance of signal-to-noise peak observed in the Virgo instrument coincident with the large peaks observed in the LIGO-Hanford and LIGO-Livingston observatories, for a particular event.

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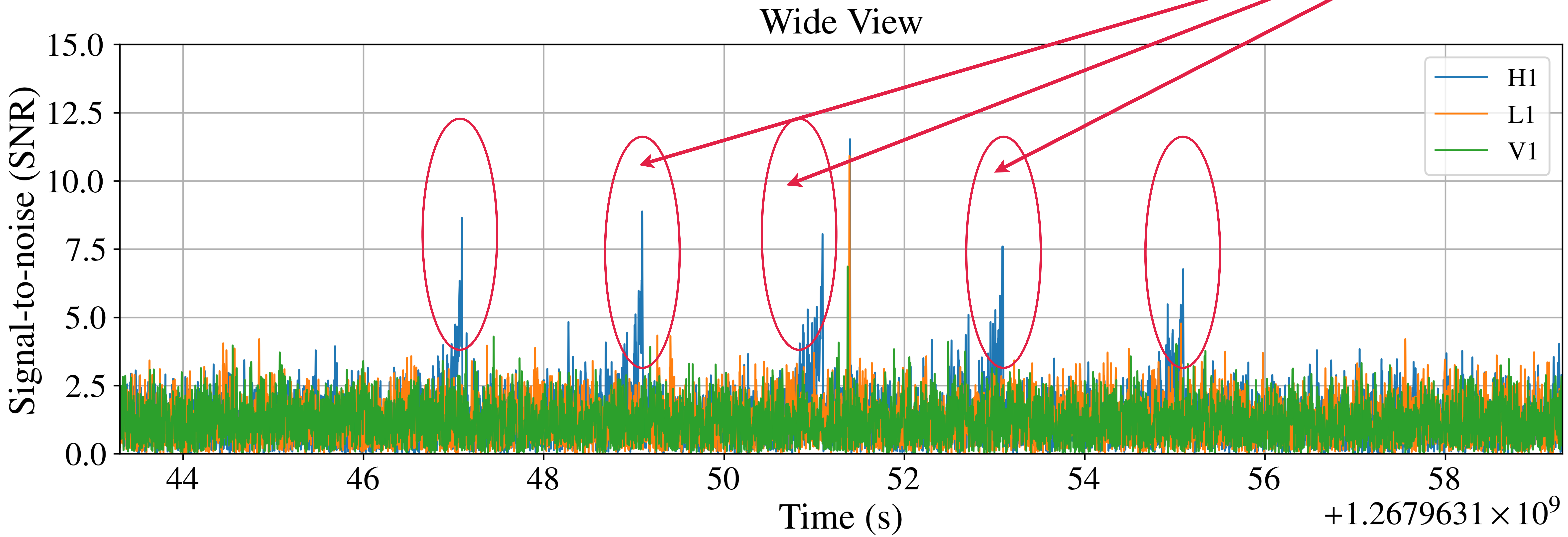
Glitch added by hand



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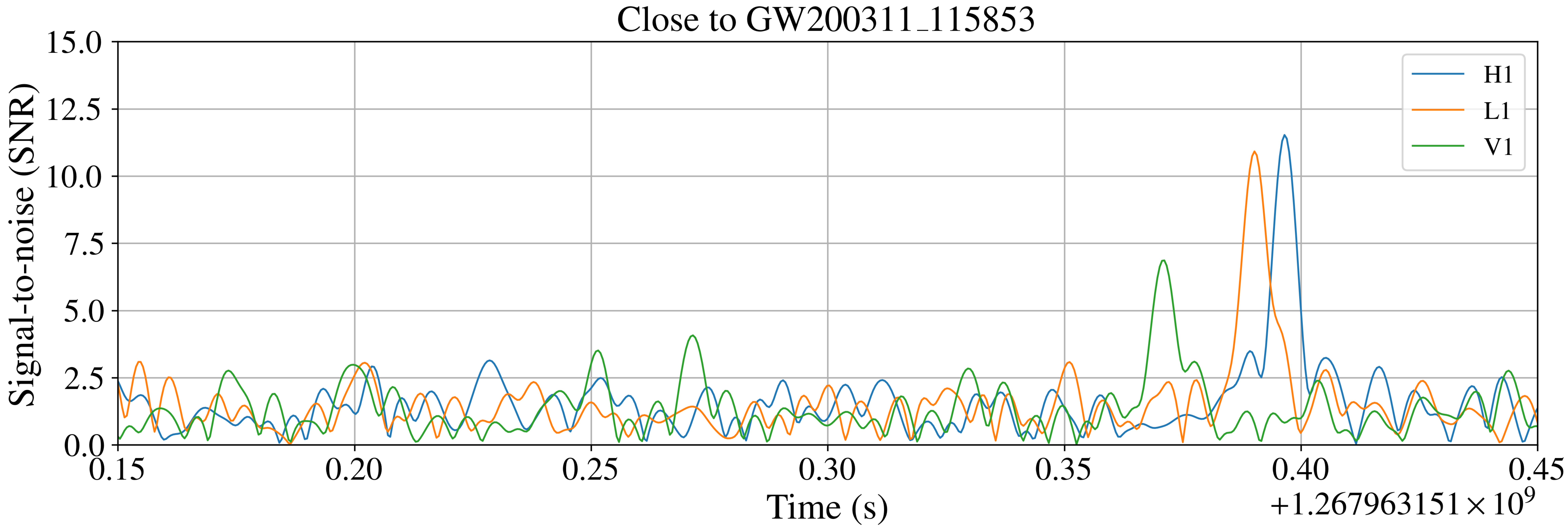


Calculating the significance in Virgo Data

2. We calculate the χ_r^2 time-series

Calculating the significance in Virgo Data

3. compute the reweighed SNR time-series!

$$\hat{\rho}(t) = \frac{\rho}{\frac{1}{2}\left[1 + (\chi_r^2)^3\right]^{\frac{1}{6}}}$$,where $\chi_r^2 > 1$, ρ otherwise

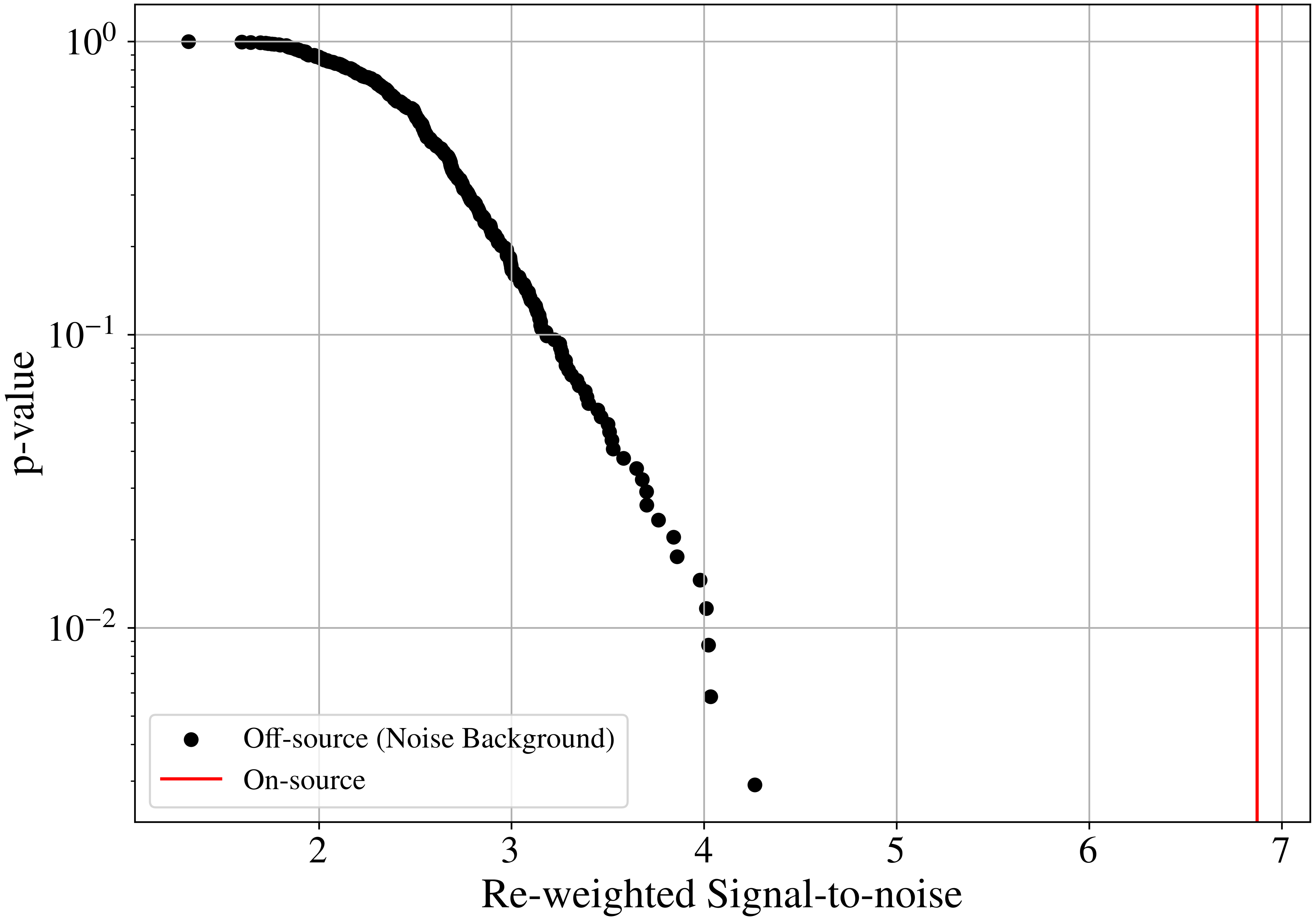
Calculating the significance in Virgo Data

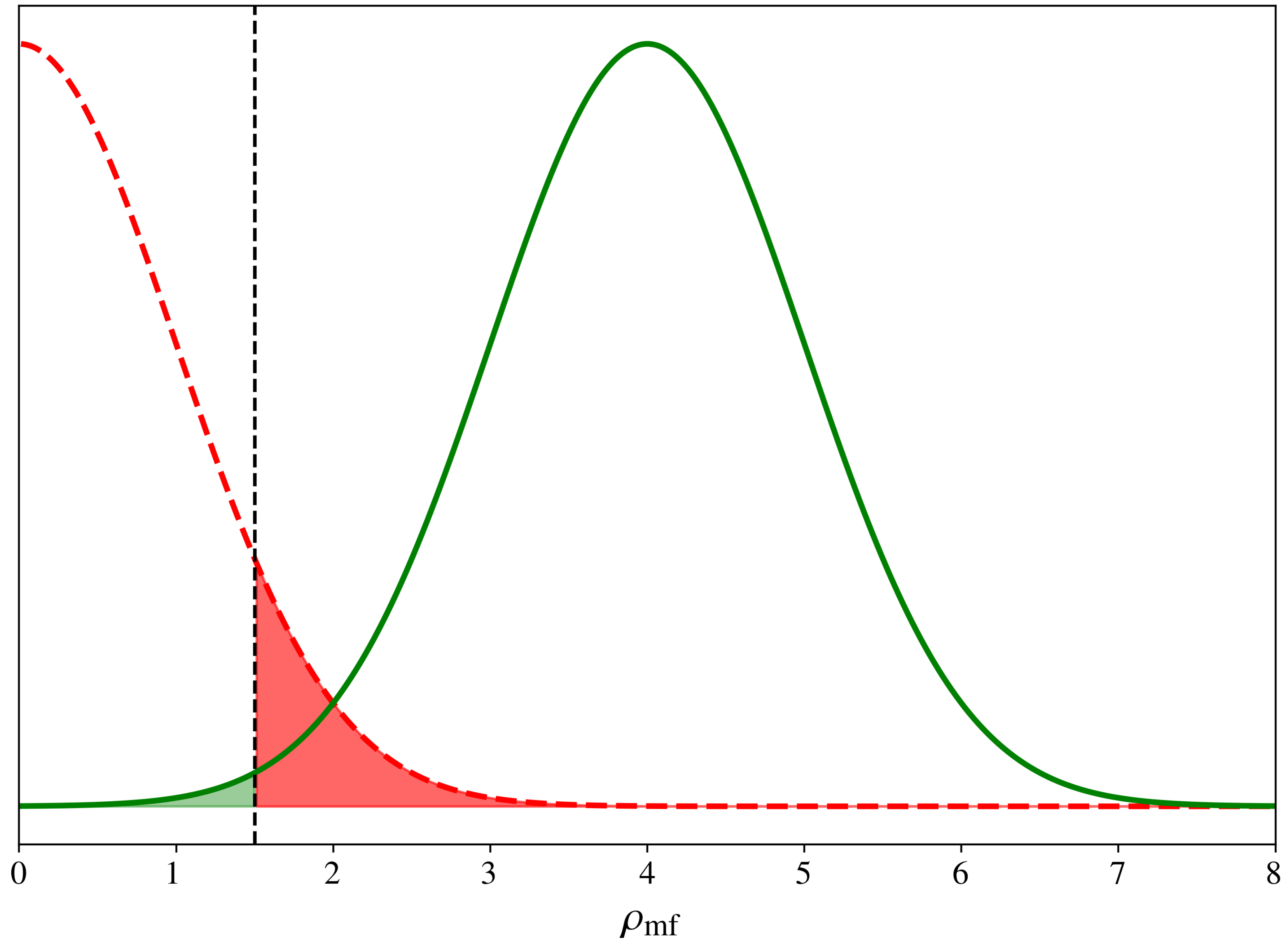
3. compute the reweighed SNR time-series!

Calculating the significance in Virgo Data

4. We construct the time-window, where we should expect a peak in the virgo detector. This will help us to estimate the *background*.

Calculating the significance in Virgo Data





Back-up Slides

Calculating the significance in Virgo Data

Question: estimate the significance of signal-to-noise peak observed in the Virgo instrument coincident with the large peaks observed in the LIGO-Hanford and LIGO-Livingston observatories, for a particular event.

Let's do it for GW200220_061928 !

Calculating the significance in Virgo Data

Question: estimate the significance of signal-to-noise peak observed in the Virgo instrument coincident with the large peaks observed in the LIGO-Hanford and LIGO-Livingston observatories, for a particular event.

Let's do it for GW200220_061928 !

Calculating the significance in Virgo Data

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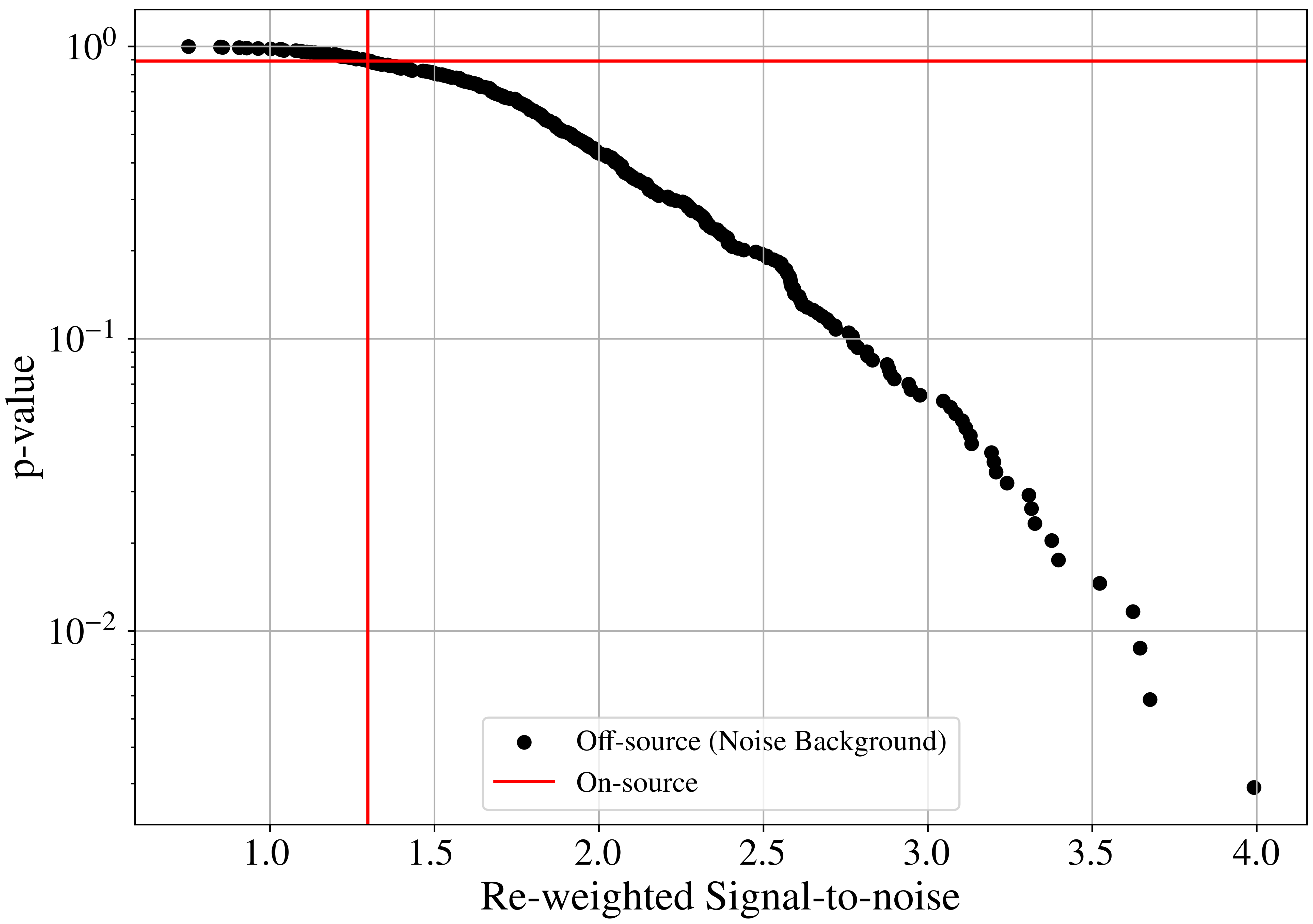

Other searches

- Non-templated search such as coherent wave-burst or oLIB

  Klimenko et al. CQG 33 (2016) 21, 215004

- Morphology independent manner, using a sum of sine-Gaussian waveforms e.g. : Bayeswave

  Lynch et al. Phys. Rev. D 95, 104046

- Continuous GWs

- Time-delay Interferometry

Forming null streams

Assume we have more than one detector. Effective projected waveform on the i^{th} detector is:

$$\tilde{h}_i(f) = e^{-2\pi i f \tau_i(\hat{n})} \left[F_{+,i}(\hat{n},\psi)\, \tilde{h}_+(f) + F_{\times,i}(\hat{n},\psi)\, \tilde{h}_\times(f) \right]$$

Strain-calibrated readout from the i^{th} detector contains: $\tilde{s}_i(f) = \tilde{n}_i(f) + \tilde{h}_i(f)$

For a detector network containing N detectors:

$$\begin{bmatrix} s_1(t+\tau_1) \\ s_2(t+\tau_2) \\ \vdots \\ s_N(t+\tau_N) \end{bmatrix} = \begin{bmatrix} n_1(t+\tau_1) \\ n_2(t+\tau_2) \\ \vdots \\ n_N(t+\tau_N) \end{bmatrix} + \begin{bmatrix} F_{1,+} & F_{1,\times} \\ F_{2,+} & F_{2,\times} \\ \vdots & \vdots \\ F_{N,+} & F_{N,\times} \end{bmatrix} \begin{bmatrix} h_+(t) \\ h_\times(t) \end{bmatrix}$$

$$\vec{s} = \vec{n} + F\,.\,\vec{s}$$

Find $\vec{c} = [c_0, c_1, \ldots, c_N]$ such that $s_0(t) = \vec{c}\,.\,\vec{s}$ contains no signal

$$\vec{c}F = 0 \implies F^{\mathrm{T}}\vec{c}^{\mathrm{T}} = 0$$

Null stream: 2 Co-aligned, Co-located detectors

Find $\vec{c} = [c_0, c_1, \ldots, c_N]$ such that $s_0(t) = \vec{c} \,.\, \vec{s}$ contains no signal

$$\vec{c}F = 0 \implies F^{\mathrm{T}}\vec{c}^{\mathrm{T}} = 0$$

$$\begin{bmatrix} F_+ & F_+ \\ F_\times & F_\times \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = 0 \quad \implies \quad c_1 + c_2 = 0$$

As we anticipate, $s(t) = s_1(t) - s_2(t)$, is the null stream

Null stream: Three detector Network

Find $\vec{c} = [c_0, c_1, \ldots, c_N]$ such that $s_0(t) = \vec{c}\,.\,\vec{s}$ contains no signal

$$\vec{c}F = 0 \implies F^{\mathrm{T}}\vec{c}^{\mathrm{T}} = 0$$

$$\begin{bmatrix} F_{+,\mathrm{H}} & F_{+,\mathrm{L}} & F_{+,\mathrm{V}} \\ F_{\times,\mathrm{H}} & F_{\times,\mathrm{L}} & F_{\times,\mathrm{V}} \end{bmatrix} \begin{bmatrix} c_\mathrm{H} \\ c_\mathrm{L} \\ c_\mathrm{V} \end{bmatrix} = 0$$

$$c_\mathrm{H} = -\,c_\mathrm{V}\frac{F_{+,\mathrm{V}}F_{\times,\mathrm{L}} - F_{\times,\mathrm{V}}F_{+,\mathrm{L}}}{F_{+,\mathrm{H}}F_{\times,\mathrm{L}} - F_{\times,\mathrm{H}}F_{+,\mathrm{L}}},$$
$$c_\mathrm{L} = +\,c_\mathrm{V}\frac{F_{+,\mathrm{V}}F_{\times,\mathrm{H}} - F_{\times,\mathrm{V}}F_{+,\mathrm{H}}}{F_{+,\mathrm{H}}F_{\times,\mathrm{L}} - F_{\times,\mathrm{H}}F_{+,\mathrm{L}}},$$

$$\begin{aligned} c_\mathrm{H} &= F_{+,\mathrm{L}}F_{\times,\mathrm{V}} - F_{\times,\mathrm{L}}F_{+,\mathrm{V}}, \\ c_\mathrm{L} &= F_{+,\mathrm{V}}F_{\times,\mathrm{H}} - F_{\times,\mathrm{V}}F_{+,\mathrm{H}}, \\ c_\mathrm{V} &= F_{+,\mathrm{H}}F_{\times,\mathrm{L}} - F_{\times,\mathrm{H}}F_{+,\mathrm{L}}\,. \end{aligned}$$

$$s_0(t) = c_\mathrm{H}s_\mathrm{H}(t+\tau_\mathrm{H}) + c_\mathrm{L}s_\mathrm{L}(t+\tau_\mathrm{L}) + c_\mathrm{V}s_\mathrm{V}(t+\tau_\mathrm{V})\,.$$

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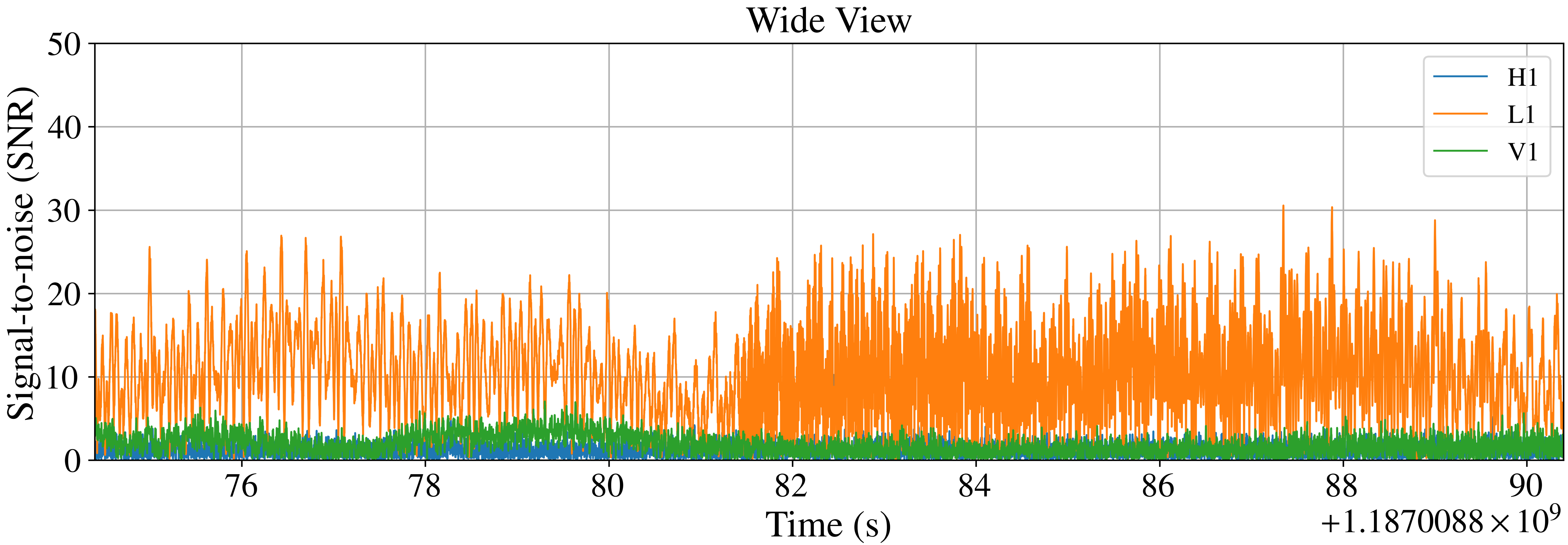

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Let's do it for GW170817 !

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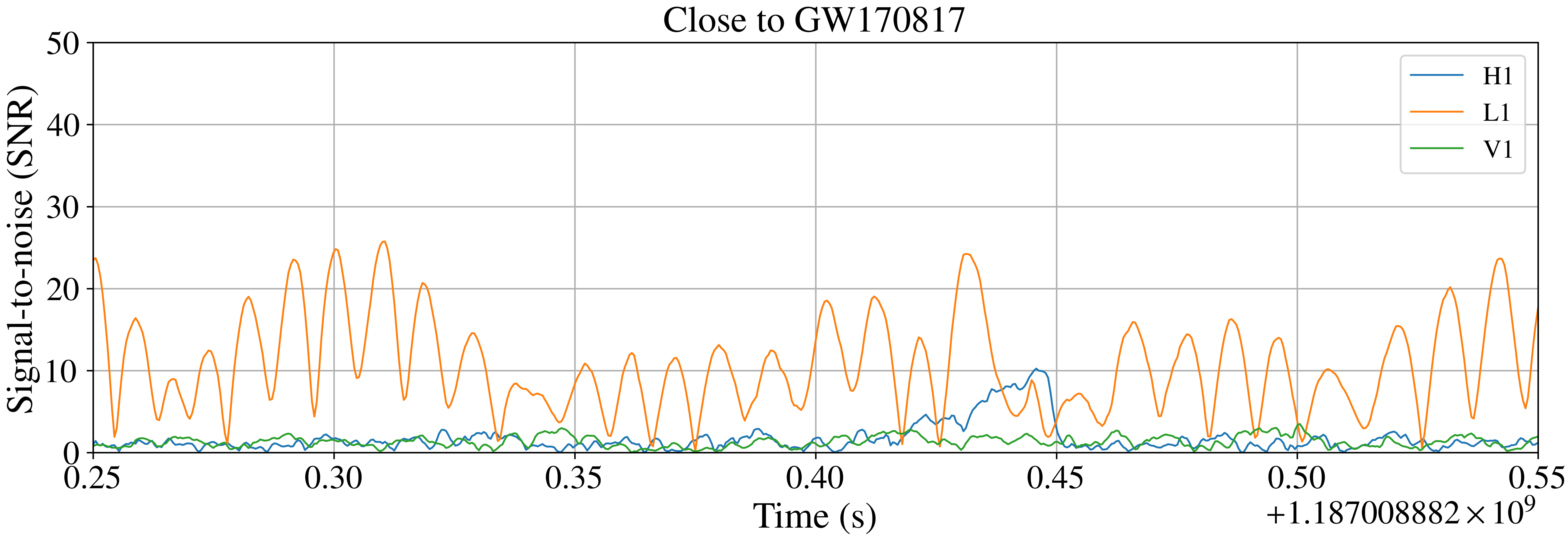

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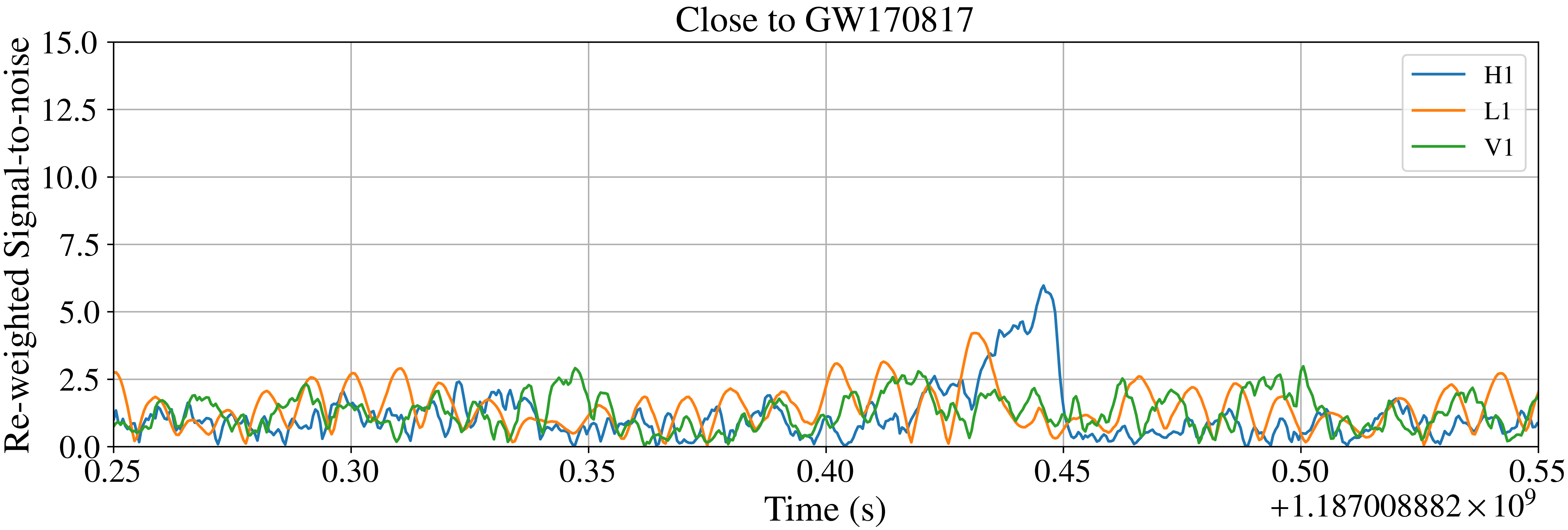

Calculating the significance in Virgo Data

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Calculating the significance in Virgo Data