Department of Electrical and Electronic Engineering

Embedded Systems Project 2023-24

DESIGN REPORT #1

Title: Motor Characterisation and Gear Ratio

Group Number: 48

Group members:	ID Number	I confirm that this is the group's own work.
Mohamad Amierul Hakeem	11098269	☒
Teethabhumi Tripatarasit	11114360	☒
Jiexi Lu	11488522	☒
Yanhua Zheng	11047457	☒
Sarah El-Mahmoudi	10864355	☒

Tutor: Yin, Wuliang

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Contents

1. Introduction

The specification of the motor and gearbox will have a significant impact on the project overall as it will determine the maximum mass and speed of the buggy whilst able to complete the track. It is therefore important for us to calculate the requirements with an additional safety margin, as the rest of the buggy's design will be limited to this capacity. A gearbox adds additional weight, inefficiency, and complexity to the design however it may be necessary to increase the output toque. Gearboxes enable precise control of speed and torque, enhance load handling capabilities. Torque requirement was calculated based on an estimation of the total mass of the buggy, experimental data for the motor's characteristics, the static and rolling friction due to the wheels, losses within the gearbox, the range of output voltages to the motor and the resultant force required to drive it up an 18° ramp. A final weight buffer of 10% was added as there are multiple factors in the experiment that may introduce inaccuracies and errors, this is further discussed within the report. Based on this, gearbox 2 was chosen.

The motor driver board receives a pulse-width-modulated (PWM) signal from the Nucleo board, as well as digital signals to set the configuration to unipolar or bipolar and possibly direction. This is done within the motor controller chip, where the configuration determines which combination of switches are used in an H-Bridge. This outputs to the motor terminals – the duty cycle controls the motor speed. The driver board may also be used to monitor the voltage and current supplied to the motor. However, further experimenting with the board are required before deciding on the configuration and any additional features.

K_E	Back EMF constant	ω	Angular velocity
K_T	Torque constant	R_a	Armature resistance
V_b	Brush voltage	μ	Coefficient of friction
μ_{static}	Coefficient of static friction	μ_{roll}	Coefficient of rolling friction
m	Buggy's mass	g	Gravitational acceleration
N	Normal force	$F_{friction}$	Frictional force
T_{min}	Minimum Torque per Motor	T	Torque

Figure 1.1: Table of used symbols in the report

2. Motor Characterisation

Understanding the parameters and characteristics of the motor will allow us to design the rest of the system according and control input voltage depending on this. Specifically, K_T, K_E, V_b and R_a. The data is interpreted with caution as the experiment used introduces several potentials for error. One main source of error is the heat generated from the motor [1]. As the temperature between measurements was not monitored, there was no way of accounting for these fluctuations in the calculations. Another source of error is the method of measuring force used; two strain gauges with a string attaching them to the shaft of the motor. Offset in the initial calibration and setup of these gauges can introduce a systematic error. Slack in the string tension, the string slipping from the motor shaft and human error when reading measurements also have the potential to skew the results. In industrial applications a digital dynamometer or a DC machine with known characteristics mechanically coupled to the shaft of the motor is used to accurately collect measurements for DC motor constant calculations [2].

The voltage is constrained by the maximum current. The experiment carried out follows the guidance in the technical handbook. Therefore, a maximum current of 1.4 A and a voltage of 5 V is used during the lab tests, although the parameters specified by the motor drive board are higher at 1.8 A and 10 V [3].

The resistance of the PSU leads was measured resulting in Red Lead = 57.1 mΩ and Black Lead = 97.9 mΩ. This can have an impact on the voltage supplied to the motor as a high current was used. From this, the voltage drop across them can be calculated using Ohm's Law and subtracted from the supply voltage to find the voltage across the motor. To make the calculations easier, the voltage across the motor was measured directly using a multimeter, which is reflected in the results.

2.1. Test 1 – Armature Resistance and Brush Voltage Measurement

To calculate armature resistance and brush voltage, the current and voltage at stall was measured. The RPM is at zero therefore the torque is at its maximum. Using the equation:

$$V = I R_a + V_b \tag{1}$$

The armature resistance and brush voltage correspond to the gradient of the graph and Y-intercept respectively. The brush voltage is negative which is likely due to the motor overheating. Due to time constraints, only one set of data was obtained. To improve the validity of the results, the test can be repeated multiple times with an average and standard deviation calculated.

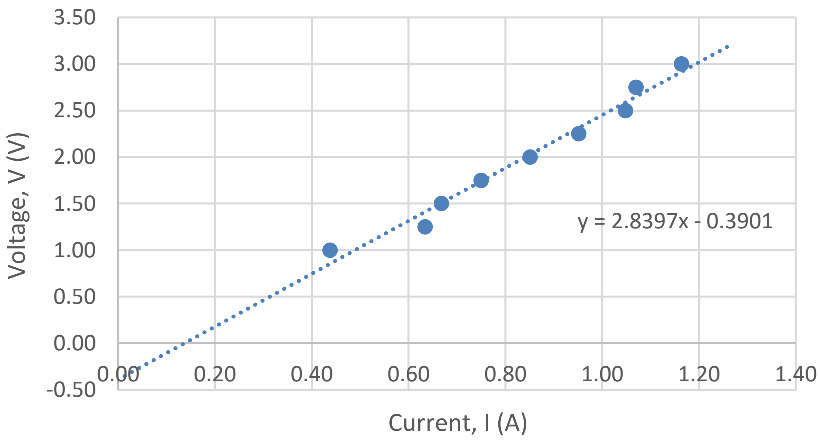


Figure 2.1: V against I (at stall)

V_b = -0.3901 V, R_a = 2.840 Ω

2.2. Test 2 – K_E and K_T Calculations

For this test, the motor torque, RPM and current was measured at a constant voltage of 5 V to determine K_E and K_T. This is done by connecting the shaft of the motor to strain gauges using a string to determine the force in N mm and the RPM using a tachometer.

To determine K_E the relationship between Angular velocity and Motor EMF was used which is shown below:

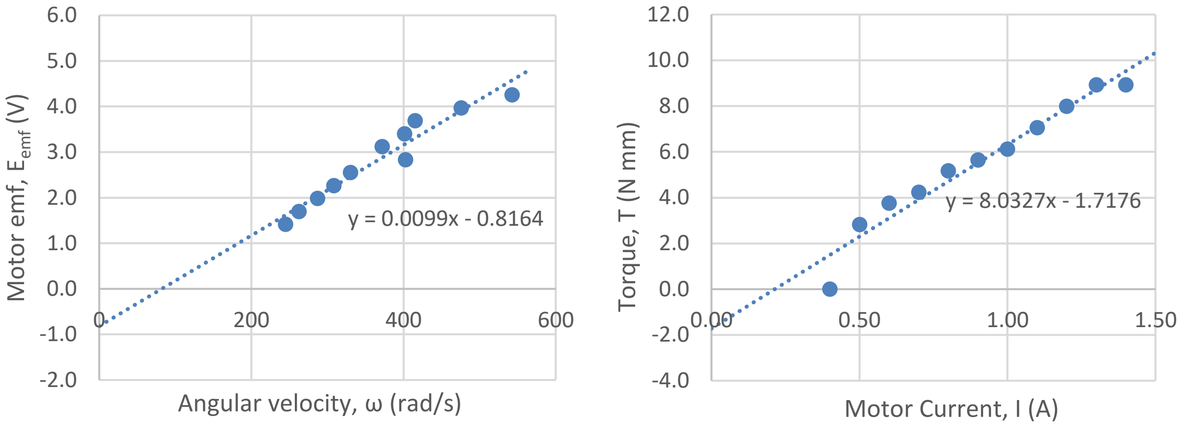
$$V = K_E \omega - V_{friction} \tag{2}$$

This corresponds to the gradient in Figure 2.2(a).

To determine K_T, the relationship between torque and current was used which is shown below:

$$T = K_T I - T_{friction} \tag{3}$$

This corresponds to the gradient in figure 2.2(b).



(a) E_{emf} against ω (b) T against I

Figure 2.2: Test 2 results (at 5V)

Emf loss = 0.8164 V
K_E = 9.934 mV s rad^{-1},

Torque loss = 1.718 N mm
K_T = 8.033 N mm A^{-1}

2.3. Test 3 - Alternative KT calculation

An alternative method of estimating K_T is when the motor is stalled. This is done by again measuring the current and the force between the strain gauges at regular voltage intervals. This gives the maximum torque at the given voltage input. Using the relationship between current and torque given in (3), K_T was determined again.

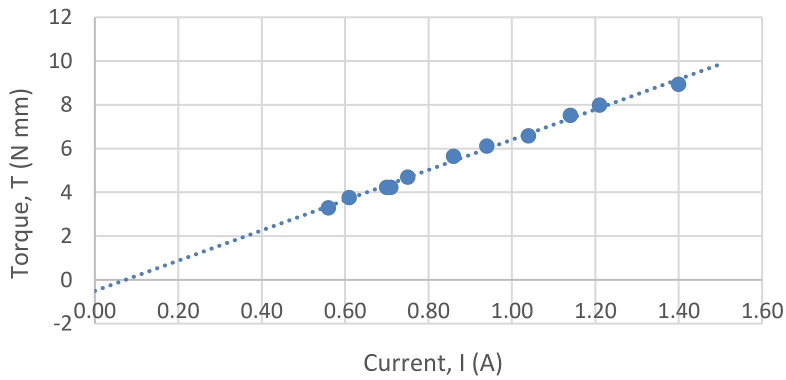


Figure 2.3: T against I (at stall)

Based off Figure 2.3: Torque loss to friction = 0.5067 N mm, K_T = 6.911 N mm A^{-1}

The values for V_b = 0.2175 V and R_a = 2.420 Ω was also calculated from this test by finding the offset and gradient of a I/V graph derived from this data.

Although K_T and K_E were expected to be the same, there was a 21% difference between the two values in Test 2, likely due to inaccuracies discussed previously. The value calculated at stall (Test 3) was used as the human error aspect is minimised making it the most robust, because the motor is completely stationary, which is easier to observe than the previous two tests. It is also the lowest value, therefore the safest one to use when choosing the gearbox as it will ensure the lowest estimate for K_T will meet the minimal torque required. The value for V_b and R_a from Test 3 was also used for future calculations for the same reason. Using these characteristics, the maximum speed and torque of the gearbox can be determined.

3. Load Measurements

The Force Measurements Experiment was conducted to estimate the coefficients of static and rolling friction. The primary goal is to then calculate the force required for the buggy to move on a flat surface and ascend an 18° slope. This is crucial for ensuring that the buggy has sufficient torque to climb the slope successfully. The experiment involves pulling the buggy at a constant velocity with a force gauge while varying the buggy's weight.

The experimental setup used has some inherent flaws. As an analogue strain gauge was used to manually pull the buggy two main issues arise: firstly, it is difficult for the individual pulling to maintain a constant velocity, and secondly the scale is difficult to read when the buggy is moving, and the gauge is bouncing between values. To mitigate this, one individual practiced with the buggy a few times on a trial run with the second member reading the scale. The real test was then conducted multiple times.

3.1. Coefficient of Friction

The formula for frictional force is given by,

$$F_{friction} = \mu N = \mu m g \tag{4}$$

Therefore, by plotting $F_{friction}$ against N, the gradient will be μ. The graph of the experiment result on a flat plane is plotted below:

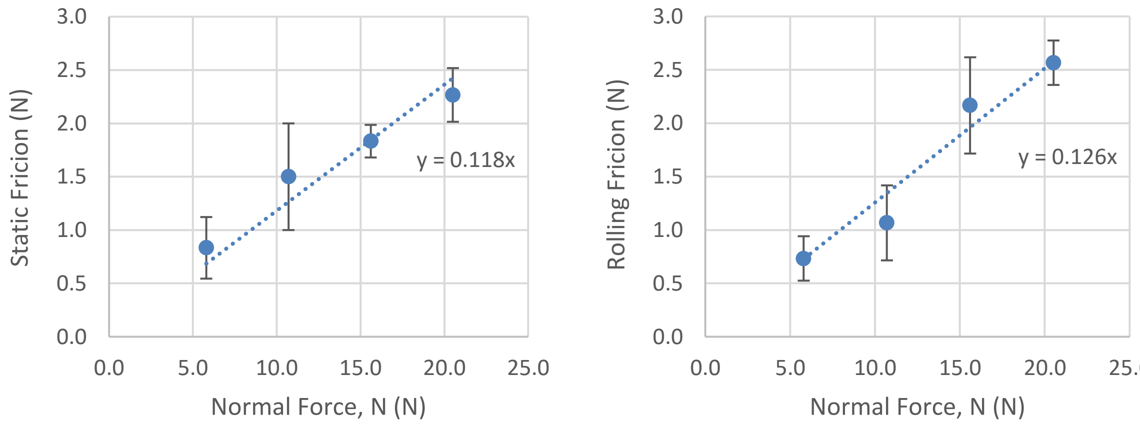


Figure 3.1.1: Static friction(left), Rolling friction (right) on a flat plane.

For each weight, the friction force was measured three times and averaged, with vertical error bars indicating the standard deviation. Static friction was measured as it starts to move, while rolling friction was measured as it's moving with constant velocity. The trendline's y-intercept was set to zero since friction is zero when weight is zero.

From the gradient of the graphs, μ_{static} = 0.118 and μ_{roll} = 0.126. The first observation is that the μ_{roll} is higher than μ_{static} which theoretically should be the other way around. This suggests that there might be a net force by the pulling hand. Additionally, the error bars are large for some points showing that the deviation is quite significant, and more repeats are needed to reduce the uncertainty. The value for μ_{roll} will be used for μ for future calculations, given its higher value and the buggy will be moving most of the time.

The minimum force required to move up the slope, F_{slope} is the sum of the frictional force and gravitational force. This can be calculated using the equation from the handbook [3] and is shown below:

$$F_{slope} = F_{gravity} + F_{friction}$$
$$F_{slope} = mg\sin(\theta) + \mu\, m\, g\cos(\theta)$$
$$F_{slope} = m\,(9.81)\sin(18°) + (0.126)\,m\,(9.81)\cos(18°)$$
$$F_{slope} = 0.4207\,m \tag{5}$$

Equation (5) is then plotted along with the result of the experiment on the slope:

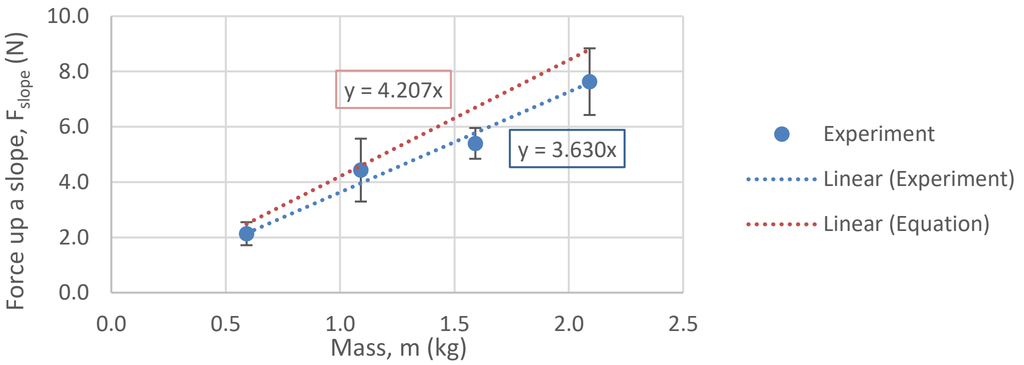


Figure 3.1.1: Force required for the buggy to move up an 18° slope.

Equation (5) predicts a higher frictional force than the trendline of the experiment. To be safe, (5) was used instead of the trendline of the experiment result to estimate the force required to move up the slope.

3.2. Minimum Torque Required

The final buggy's weight was estimated to calculate the minimum torque required. 10% safety margin are added to include the smaller components that are not considered for example wires, screws, and adhesives.

Component	Approximate Mass (g)
Microcontroller board	70
Breakout Board	38
Motor Drive Board	53
Battery Pack	265
Chassis + Motors + Gearbox + Wheels + Ball Castor	591
Encoders	20
Total:	**1037**
Total + 10% safety margin:	**1141**

Figure 3.2.1: Component Mass Estimation Table

By using m = 1.141 kg, the frictional force on a flat plane can be calculated using (3). For the force required on the slope, (5) was used. Then, by using the formula for torque, $T = \frac{Fd}{2}$ where d is the diameter. The wheel diameter was measured to be 78 mm. Since the load will be shared with two motors, torque required per motor is only half, $T_{min} = \frac{T}{2}$. The results of the calculations are shown below:

	Minimum Force (N)	Minimum Torque (N mm)	Minimum Torque per Motor, T_{min} (N mm)
Flat	1.407	54.87	27.44
Slope	4.797	187.08	93.54

Figure 3.2.2: Minimum Torque Calculation Results

4. Gear Ratio Selection

Using a gearbox can increase the max amount of torque produced at the cost of max rotational speed. Therefore, a gearbox that can produce just enough torque will be chosen so that the maximum buggy speed will not be reduced as much. First, the current and voltage requirements without a gearbox was calculated to ensure that a gearbox is actually required. Then, the minimum gear ratio required was calculated so that the best gearbox can be chosen. Finally, with a chosen gearbox, the final speed of the buggy was estimated for both on a flat surface and on an 18° slope.

4.1. Current and Voltage Required Without Gearbox

Without a gearbox, the current required by the motor is related only to the minimum torque on the motor and the torque constant. This is given by:

$$I = \frac{T_{min,slope}}{K_T} = \frac{0.09354}{0.00691} = 13.54\ A$$

For the voltage, it can be calculated by this equation:

$$V = V_b + I\ R_a + K_e\ \omega \tag{6}$$

In this case, the minimum speed possible was used which means the buggy is not moving, $\omega = 0$. The equation becomes:

$$V = V_b + I\ R_a$$
$$V = 0.2175 + (13.54)(2.840)$$
$$V = 38.66\ V$$

Therefore, the current and voltage needed without a gearbox is 13.54 A at 38.66 V. This is impossible due to the limits of the motor driver board and the motor itself, so a gearbox is needed to produce enough torque.

4.2. Gearbox Selection

In this case, a current limit to the motor, I = 1.4 A was set to avoid overheating of the motor. Thus, the maximum torque that the motor can produce, T_{max} is:

$$T_{max} = K_t\ I = 0.00691 \times 1.4 = 9.674\ N\ mm$$

In an ideal case, the minimum gear ratio can be calculated:

$$GR_{min} = \frac{T_{min}}{T_{max}} = \frac{0.09354}{0.009674} = 9.669 : 1$$

However, there will be loss between gears, and for this project, each gear stage has a gear efficiency, η of 0.85 [3]. All gearbox option has two stages, so the total gear efficiency is η^2 or 0.7225. Thus, GR_{min} with losses can be calculated:

$$GR_{min\ with\ losses} = \frac{GR_{min}}{\eta^2} = \frac{9.669}{0.7225} = 13.38 : 1$$

Then, by using the formula derived from the handbook [3] the gearbox ratio, GR for each gearbox option can be calculated:

$$GR = \frac{T_3}{T_1} = \frac{N_3\ N_{2A}}{N_{2B}\ N_1} \tag{7}$$

Gearbox	No. of teeth			Gearbox Ratio, GR
	Input Gear, N_1	Intermediate Gear, N_2	Output Gear, N_3	
1	16	48/12	48	12:1
2	16	50/10	48	15:1
3	16	50/10	60	18.75

Figure 4.2.1: Table of gearbox ratio for each gearbox option

Based on Figure 4.2.1, Gearbox 2 is the best gearbox since it is the next gearbox that has a GR larger than $GR_{min\ with\ losses}$. Next, the position of the intermediate shaft in the gearbox needs to be calculated. The pitch circle diameter, PCD of each gear was first calculated using the equation:

$$PCD = No.of\ teeth \cdot MOD \tag{8}$$

Since all gears of the gearbox are 0.5 mm module, MOD = 0.5. The centre distance between the two gears is then calculated using the following function:

$$centre\ \ distance = \frac{(PCD_A + PCD_B)}{2} + 0.1\ mm \tag{9}$$

The 0.1 mm in the formula is the clearance allowance, which is necessary if the gears are to function properly. Using (8) and (9), the centre distance between intermediate and motor shaft is 16.6 mm, while intermediate and final drive shaft is 14.6 mm. The centre distance between motor shaft and final shaft is fixed, known to be 31.1 mm. These three sides form a triangle. The motor shaft is set at the origin (0, 0) and the final shaft centre at (0, 31.1) where 1 unit is 1 mm. The triangle is then divided into two smaller right-angled triangles. By equating both triangle's height using Pythagorean theorem, the x-coordinate of the intermediate shaft, x can be calculated:

$$height\ calculated\ from\ left\ triangle = height\ calculated\ from\ right\ triangle$$
$$14.6^2 - x^2 = 16.6^2 - (31.1 - x)^2$$
$$x = 16.55\ mm$$

Then, by using the Pythagorean theorem again, the y-coordinate of intermediate shaft (height of the triangle) was calculated to be, y = 1.246 mm. Thus, the coordinate position of the intermediate shaft is (16.55,1.246).

4.3. Expected Buggy Speed

The torque required on the motor shaft, T_{motor} can be calculated by the equation:

$$T_{motor} = \frac{T_{min}}{\eta^2 \times GR} \tag{10}$$

Therefore, $T_{motor,flat}$ = 2.532 N mm and $T_{motor,slope}$ = 8.631 N mm. Then, the angular speed of motor can be calculated by the equation:

$$\omega = \frac{V_{motor} - \frac{T_{motor}}{K_T} R_a - V_b}{K_E} \tag{11}$$

V_{motor} = 6V was chosen to prevent the motor from overvoltage and overheating. Thus, $\omega_{motor,flat}$ = 477.1 rad/s and $\omega_{motor,slope}$ = 225.0 rad/s. Equation (12) shows the relationship between motor angular speed and wheel speed.

$$\omega_{wheel} = \frac{\omega_{motor}}{Gearbox\, ratio} \tag{12}$$

By using (12), $\omega_{flat\, of\, wheel}$ = 31.24 rad/s and $\omega_{slope\, of\, wheel}$ = 15.00 rad/s. Then by using the formula for linear velocity, $v = \omega \times r$, where r is the radius of the wheel, the final buggy speed, v was calculated. On a flat surface, $v_{flat} =$ 1.218 m/s and on an 18° slope, v_{slope} = 0.585 m/s.

5. Summary

The report aims to allow the buggy to overcome given conditions whilst optimised to deliver best performance. For motor characterisation, the fundamental values are K_E = 9.934 mV s rad^{-1} and K_T = 8.033 N mm A^{-1}. The critical value for the load measurements section is the rolling friction coefficient is 0.126, the static friction coefficient is 0.118. Before the tests, the team was certain that gearbox 3 would work due to it providing the highest torque while speculating that gearbox 2 could possibly clear the conditions as well at higher speed (higher performance). In the calculations above, it is deduced that the required gear ratio is 13.38 : 1 (including the torque loss due to efficiency) which is less than 15 : 1 of gearbox 2; therefore, team has decided to use gearbox 2 in the buggy since it provides more than the minimum torque for going up the slope while also being the fastest option when compared to gearbox 3.

6. References

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[3] The University of Manchester (2023), *Embedded Systems Project Technical Handbook* [Online]. Available: https://online.manchester.ac.uk/webapps/blackboard/execute/content/file?cmd=view&content_id=_15044516_1&course_id=_78308_1.