GRAPH THEORY L3, L4 and L5

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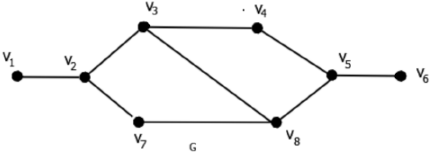
Let G be a connected graph and let u, v be two vertices in G. Shortest path between u and v in G is a $(u, v)-$ path with minimum number of edges in it.

Definition 0.1.

The distance between u and v in G is denoted by $d(u, v)$ is the length of shortest path between them.



$d(v_1, v_2) = 1$

$d(v_1, v_8) = 3$

$d(v_4, v_7) = 3$

$d(v_3, v_6) = 3$

$d(v_7, v_6) = 3$

$d(v_7, v_5) = 2$

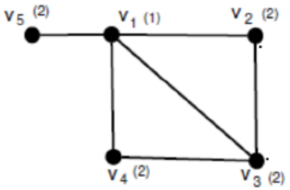
Definition 0.2.

Eccentricity of a vertex v in connected graph G is defined as follows.
$e(v) = max_{u,v \in G} d(u, v)$.

Definition 0.3.

Minimum and maximum of the eccentricities of vertices of G are radius and diameter of the graph G.

A vertex v in G with minimum eccentricity is called a central vertex and set of all central vertices in G is called the centre of G



$e(v_1) = 1$

$e(v_2) = 2$

$e(v_3) = 2$

$e(v_4) = 2$

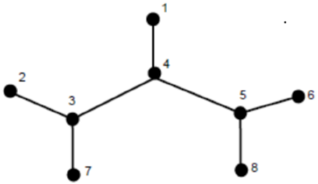
$e(v_5) = 2$

$diam(G) = 2$

$radius(G) = 1$

centre : v_1

Find radius, diameter and centre of the graph.



$e(1) = 3$
$e(2) = 4$
$e(3) = 3$
$e(4) = 2$
$e(5) = 3$
$e(6) = 4$
$e(7) = 4$
$e(8) = 4$

radius = 2

diameter = 4

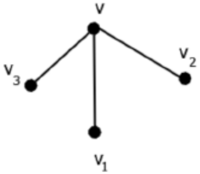
centre of G : vertex 4

Theorem

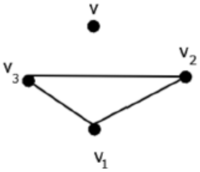
For any Graph G with six vertices, G or $\overline{G}$ contains a triangle.

Proof.

Let G be a graph with six vertices. Let v be any vertex in G. Since v is adjacent to other five vertices either in G or in $\overline{G}$. We assume that, v is adjacent with v_1, v_2, v_3 in G. If any 2 of these vertices say v_1, v_2 are adjacent then v_1, v_2, v form a triangle in G. If no two of them are adjacent in G then v_1, v_2, v_3 are the vertices of a triangle in $\overline{G}$. $\square$



G



$\overline{G}$

Theorem

For any graph G, show that either G or $\overline{G}$ is connected.

Proof.

If G itself is connected, there is nothing to prove. Suppose that the graph G is disconnected and has two components C_1 and C_2. Let u and v be any two vertices, we have the following cases.

1. If u and v are in different components and are not adjacent in G.Then u and v are adjacent in $\overline{G}$. We have, uv path, hence $\overline{G}$ is connected.
2. If u and v belong to the same component but they are not adjacent in G. Hence, they are adjacent in $\overline{G}$. Hence, we have uv path.
3. Suppose that u and v are adjacent in G(Obviously, they belong to the same component).Then we can find w in another component (which does not contain u and v). We have a uv path via w in $\overline{G}$. That is, $u \sim w$ and $v \sim w$.

□

Regular graph

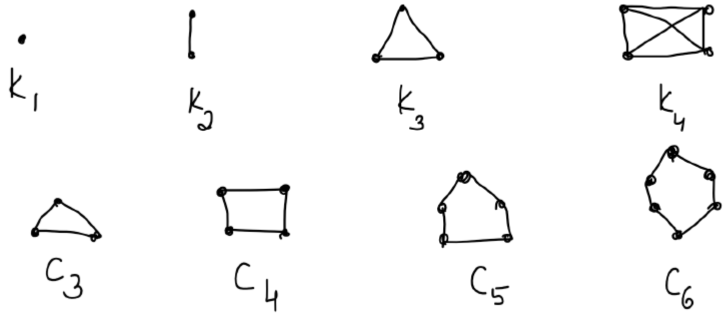
- A graph G in which every vertex is of same degree is called a regular graph.
- When G is regular, $\delta(G) = \triangle(G)$ and the common value is called regularity of G.
- Regular graph with degree 3 is called cubic graph.
- A cubic graph has always even number of vertices.

Cycle

Connected regular graph with regularity 2 is called cycle. Cycle on n vertices is denoted by C_n.

Complete graph

A graph on n vertices in which every two vertices are adjacent is called complete graph and is denoted by K_n.



Note

1. Complement of a complete graph on n vertices is called totally disconnected graph.
2. A graph G is said to be self centered if every vertex of G has the same eccentricity. In such a graph, radius is equal to the diameter.
3. The cycle graph C_n is a self centered graph and is the complete graph K_n.

Home work

Question 1: Draw a regular graph on regularity 4 and number of vertices 6.
Question 2: Draw a complete graph on 6 vertices.
Question 3: Draw a cycle graph on 8 vertices.
Question 4: Draw the complement of cycle graph C_8.
Question 5: The complete graph Kp has ______edges.

Question 6: The cycle graph Cn has _____edges.

Question 7: The complete graph Kp has diameter= ________

Question 8: Draw a regular graph on 6 vertices with regularity 1.

Theorem 0.4.

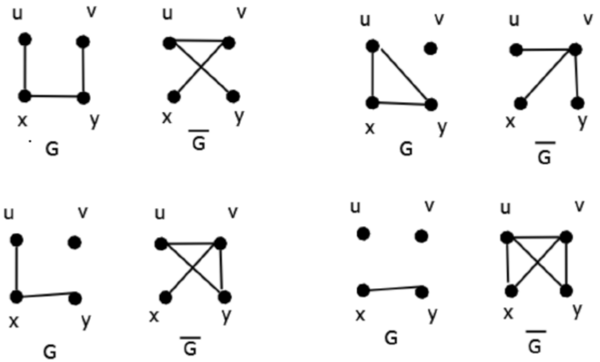
If $diam(G) \geq 3$, then $diam(\overline{G}) \leq 3$.

Proof.

Let G be the graph with $diam \geq 3$.
There are two vertices u and v in G, such that $d(u, v) \geq 3$.
$\implies$ u and v are not adjacent in G.
$\implies$ u and v are adjacent in $\overline{G}$.
Consider any 2 vertices x and y which are adjacent in G. (They are not adjacent in $\overline{G}$.)
To prove $d(x, y) \leq 3$ in $\overline{G}$.
We note that x can not be adjacent to both u and v in G ($\because d(u, v) \geq 3$).
x is adjacent to at most one of u or v.

1. x is adjacent to u, y is adjacent to v.
2. x is adjacent to u, y is not adjacent to v.
3. x is adjacent to v, y is adjacent to u.
4. x and y are not adjacent to u or v.

Proof continues...

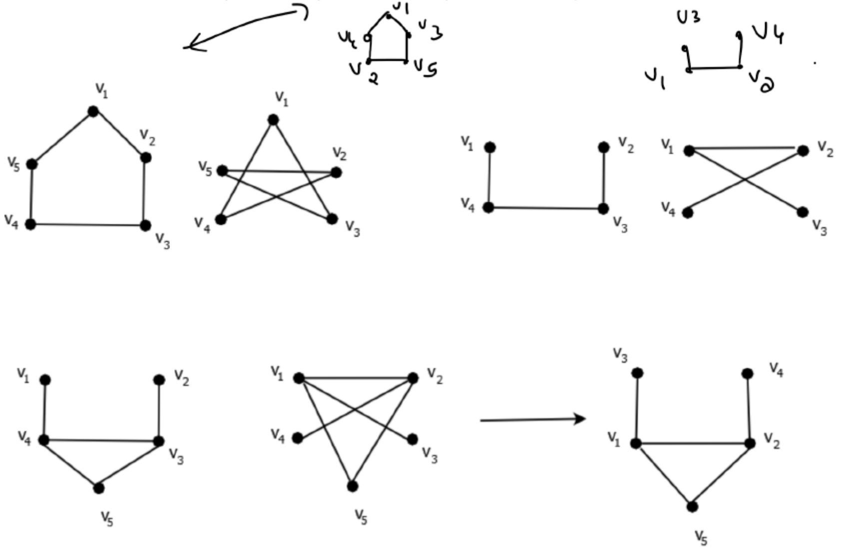


Since u and v have common neighbor in G, both x and y are adjacent to u or v in $\overline{G}$.

$\implies d(x, y) \leq 3$ in $\overline{G}$.

$\implies diam(\overline{G}) \leq 3$.

A graph G is said to be self complementary if G is isomorphic to its complement.



Theorem 0.5.

Every nontrivial self complementary graph has diameter 2 or 3.

Proof.

Let G be a self complementary graph. Clearly, G cannot have diameter 1. If G is graph of diameter 1, then $G \cong K_n$ which is not self complementary graph.

Hence, self complementary graphs have diameter at least 2.

Suppose that $diam(G) \geq 3$. By the above theorem, $diam(\overline{G}) \leq 3$.

Hence, diameter of every self complementary graph is either 2 or 3. $\square$

Theorem

Let G be a self complementary graph. Show that the number of vertices in G is of the form $4n$ or $4n+1$.

Proof.

Let G be a (p, q) graph. Number of edges in $K_p = p(p-1)/2 = pC_2$
Since G is self complementary, number of edges in G = number of edges in $\overline{G}$ = q
Number of edges in K_p = number of edges in G + number of edges in $\overline{G}$.
$\Rightarrow$ Number of edges in $\overline{G} = p(p-1)/2$ - q
$\Rightarrow q = p(p-1)/2 - q, \Rightarrow 4q = p(p-1)$
Therefore, q = p(p-1)/4
$\Rightarrow 4/p \;\; or \;\; 4/(p-1)$
$\Rightarrow p = 4n \;\; or \;\; p-1 = 4n$
$\Rightarrow p = 4n \;\; or \;\; p = 4n+1$ $\square$

Extra questions

1. There exists a self complementary graph on —— vertices.

(i) 3 (ii) 8 (iii) 11 (iv) 14

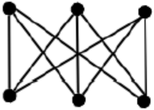
2. Let G be a simple graph with 6 vertices. The degrees of 5 vertices are (2,3,3,3,5). Then the degree of 6th vertex is ——-.

(i) 0 (ii) 1 (iii) 2 (iv) 4

3. Let G be a simple graph with 6 vertices. The degrees of 5 vertices are (2,3,3,3,5). Then the number of edges is equal to ——-.

(i) 7 (ii) 8 (iii) 9 (iv) 10.

A **bipartite graph** is one whose vertex set can be partitioned into 2 subsets X and Y so that each edge has one end vertex in X and one end vertex in Y. Such a partition (X, Y) is called a bipartition of the graph G. A complete bipartite graph is a bipartite graph with bipartition (X, Y) in which each vertex of X is joined to each vertex of Y; if $|X| = m$ and $|Y| = n$, such a graph is denoted by $K_{m,n}$.
The graphs (a) and (b) below are complete bipartite and bipartite graphs respectively.


(a)

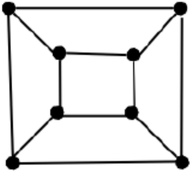
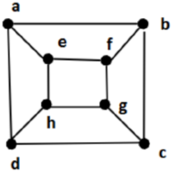
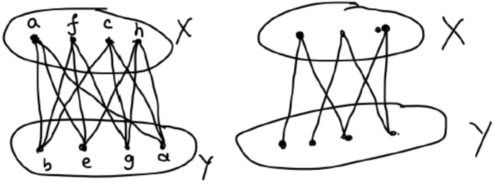
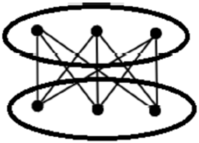

(b)

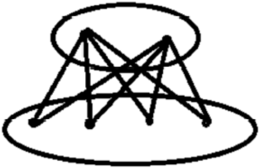
Figure: Complete bipartite and bipartite graphs

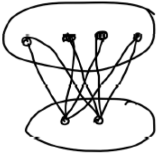




Bipartite graph on 8 vertices with partition X = {a, f , c, h} and Y = {b, e, g, d}.



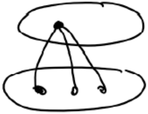




Complete bipartite graph $K_{3,3}$

$K_{2,4}$

K_{42}



K_{13}

A complete bipartite graph Km,n has mn edges.

Check whether C_8 and C_7 are bipartite or not.

Solution: C_8 is bipartite but C_7 is not bipartite.

Theorem 0.6.

A graph is bipartite if and only if all its cycles are even.

Proof.

Let G be a connected bipartite graph. Then its vertex set V can be partitioned into two sets V_1 and V_2 such that every edge of G joins a vertex of V_1 with a vertex of V_2.
Thus, every cycle $v_1, v_2, \ldots, v_n, v_1$ in G necessarily has its oddly sub-scripted vertices in V_1(say). i.e,$v_1, v_3, ... \in V_1$ and other vertices $v_2, v_4, ... \in V_2$.
In a cycle $v_1, v_2, \ldots, v_n, v_1$: v_n, v_1 is an edge in G.
Since, $v_1 \in V_1$ we must have $v_n \in V_2$. This implies n is even.
Hence,the length of the cycle is even. $\square$

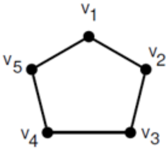
Proof continues....

Conversely, suppose that G is a connected graph with no odd cycles.
Let $u \in G$ be any vertex.
Let $V_1 = \{v \in V | d(u, v) = even\}$, $V_2 = \{v \in V | d(u, v) = odd\}$. Then, $V = V_1 \bigcup V2, V1 \bigcap V2 = \phi$.
We must prove that no two vertices in V_1 and V_2 are adjacent.
Suppose that $x, w \in V_1$ be adjacent. $w \in V_1 \implies d(u, w) = 2k$ and $x \in V_1 \implies d(u, x) = 2l$.
Thus, the path $u - w - x - u$ forms a cycle of length $2k + 2l + 1$, odd a contradiction.
Therefore, x and w cannot be adjacent.
That is, no two vertices in V_1 are adjacent.
Similarly we can prove no two vertices in V_2 are adjacent.
Hence, the graph is bipartite.

- A cut vertex of a graph is one whose removal increases the number of components and bridge is such an edge.
- A non separable graph is connected, nontrivial and has no cut vertices.
- A block of a graph is a maximal non separable sub graph.
- We note that every non trivial connected graph has at least two vertices which are not cut vertices.

Adjacency matrix of a Graph

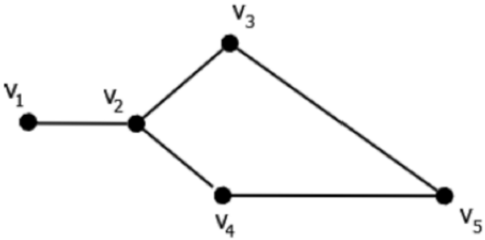
For a graph G with $V(G) = \{v_1, v_2, ..., v_n\}$, the adjacency matrix of G, denoted by $A(G)$ is the $n \times n$ matrix defined as follows.
The rows and columns of $A(G)$ are indexed by $V(G)$. If $i \neq j$ then the $(i,j)th-$ entry of $A(G)$ is 0 for vertices v_i and v_j non adjacent, and $(i,j)th-$ entry of $A(G)$ is 1 for vertices v_i and v_j adjacent. The $(i,i)th-$ entry of $A(G)$ is 0 for $i = 1, 2, ..., n$. We often denoted by $A(G)$ or simply A.



$$A(G)=\begin{bmatrix}0&1&0&0&1\\1&0&1&0&0\\0&1&0&1&0\\0&0&1&0&1\\1&0&0&1&0\end{bmatrix}$$

Figure: Graph G and its adjacency matrix $A(G)$

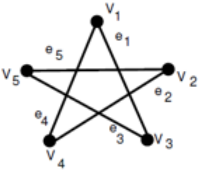
Write adjacency matrix of graph



$$A(G) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

Incidence matrix

For a graph G with $V(G) = \{v_1, v_2, \ldots, v_n\}$ and $E(G) = \{e_1, e_2, \ldots, e_m\}$, the (vertex-edge) incidence matrix of G, which we denote by $B(G)$ is the $n \times m$ matrix defined as follows. The $(i, j)th-$entry of $B(G)$ is 0 if vertex v_i and edge e_j are not incident, and otherwise $(i, j)th-$ entry of $B(G)$ is 1. This is often referred to as the $(0, 1)-$ incidence matrix.



$$B(G) = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

Figure: Graph G and its incidence matrix $B(G)$

Trees

An acyclic is one that contains no cycles. It is also called a forest.
A **tree** is a connected acyclic graph. In a tree, any two vertices are connected by a unique path.
All the trees on six vertices are given below.

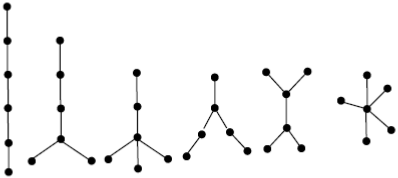


Figure: Trees

- If G is a (n, m) tree then $m = n - 1$.
- Every nontrivial tree has at least two vertices of degree one (pendant vertices).
- A tree with exactly two vertices of degree one is a **path**.
- A tree in which all the vertices except one is of degree one is called a **star**.
- If G is a tree with $\triangle(G) \geq k$, then G has at least k vertices of degree 1.
- Centre of a tree contains either a single vertex or two adjacent vertices. Accordingly, a tree is called uni-central or bi-central.
- A **spanning tree** of G is a spanning subgraph of G that is a tree. We note that every connected subgraph has a spanning tree. Hence, if G is a connected (n, m) graph then $m \geq n - 1$.

Theorem 0.7.

A graph G is a tree if and only if between every pair of vertices there exist a unique path.

Proof.

Let G be a tree then G is connected. Hence, there exist at least one path between every pair of vertices.

Suppose that between two vertices say u and v, there are two distinct paths then union of these two paths will contain a cycle; a contradiction.

Thus, if G is a tree, there is at most one path joining any two vertices.

Conversely,

suppose that there is a unique path between every pair of vertices in G.

Then G is connected.

A cycle in the graph implies that there is at least one pair of vertices u and v such that there are two distinct paths between u and v. Which is not possible because of our hypothesis.

Hence, G is acyclic and therefore it is a tree. $\square$

Theorem 0.8.

A tree with p vertices has $p-1$ edges.

Proof.

The theorem will be proved by induction on the number of vertices.
If $p=1$, we get a tree with one vertex and no edge.
If $p=2$, we get a tree with two vertices and one edge.
If $p=3$, we get a tree with three vertices and two edges.
Assume that the statement is true with all tree with k vertices ($k<p$).
Let G be a tree with p vertices.
Since G is a tree, there exist a unique path between every pair of vertices in G.
Thus, removal of an edge e from G will disconnect the graph G.
Further, $G-e$ consists of exactly two components with number of vertices say m and n with $m+n=p$. Each component is again a tree.
By induction, the component with m vertices has $m-1$ edges and the component with n vertices has $n-1$ edges. Thus, the number of edges in $G=m-1+n-1+1=m+n-1=p-1$.

Theorem 0.9.

Every tree has a center consisting of either one vertex or two adjacent vertices.

Proof.

The result is obvious for the trees K_1 and K_2.
We show that any other tree T has the same central vertices as the tree T_1 obtained by removing all end vertices of T.
Clearly, the maximum of the distances from a given vertex u of T to any other vertex v of T will occur only when v is an end vertex.
Thus, the eccentricity of each vertex in T_1 will be exactly one less than the eccentricity of the same vertex in T. Hence, the vertices of T which possess minimum eccentricity in T are the same vertices having minimum eccentricity in T_1.
That is, T and T_1 have the same centre.

□

Proof continues..

If the process of removing end vertices is repeated, we obtain successive trees having the same centre as T.
Since T is finite, we eventually obtain a tree which is either K_1 or K_2.
In either case all vertices of this ultimate trees constitute the centre of T which consists of just a single vertex or of two adjacent vertices.

Eulerian graph

A walk that traverses every edge of G exactly once, goes through all vertices and ends at the starting vertex is called **Eulerian circuit** or **Eulerian cycle**. A graph G is said to be **Eulerian graph**, if it has an Eulerian cycle.

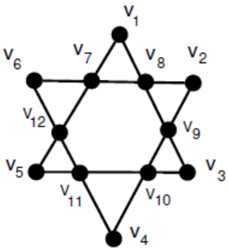


Figure: *An Eulerian graph*

Theorem 0.10.

A non empty connected graph is Eulerian if and only if all of its vertices of even degree.

Proof.

Suppose that G is connected and Eulerian.
Since G has a Eulerian circuit which passes through each edge exactly once, goes through all vertices and all its vertices are of even degree.
Conversely,
Let G be a connected graph such that every vertex of G is of even degree.
Since, G is connected, no vertex can be of degree zero.
Thus, every vertex of degree ≥ 2, so G contains a cycle.
Let C be a cycle in a graph G.
Remove edges of the cycle C from the graph G.
The resulting graph (Say G_1) may not be connected, but every vertex of the resulting graph is of even degree.

$\square$

Proof continues....

Suppose G consists only of this cycle C, then G is obviously Eulerian. Otherwise, there is another cycle C_1 with a vertex v in common with C. The walk beginning at v and consisting of the cycles C and C_1 in succession is a closed trial containing the edges of these two cycles. By continuing this process, we can construct a closed trial containing all edges of G, hence G is Eulerian.

Hamiltonian graph

A path that contains every vertex of G is called a **Hamilton path** of G. Similarly, a **Hamiltonian cycle** of G is a cycle that contains every vertex of G. A graph is **Hamiltonian graph**, if it contains a Hamilton cycle.

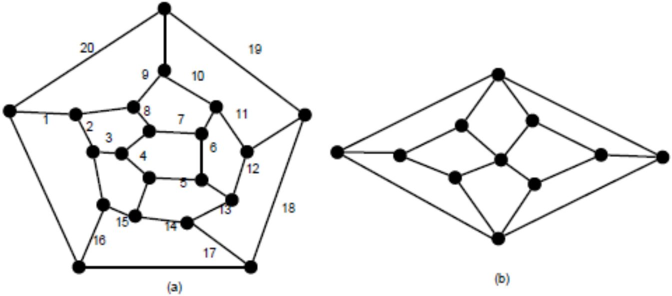


Figure: Hamiltonian and Non hamiltonian graphs

Directed graph

A **directed graph** or **digraph** D consists of a finite nonempty set V of points together with a prescribed collection X of ordered pairs of distinct points. The elements of X are directed lines or arcs. By definition, a digraph has no loops or multiple arcs.

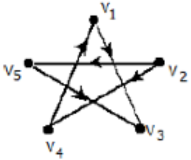


Figure: A directed graph

Dijkstra's algorithm.

Shortest paths in graphs: The graph G has n vertices and a distance associated with each edge of the graph G (such a graph is often called a network). The representation of the network will be as a distance matrix D.

The distance matrix $D = (d_{ij})$ where,

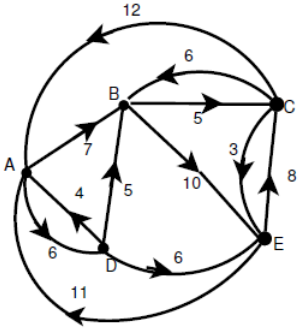
$$d_{ij} = \begin{cases} 0, \text{if } i = j \\ \infty, \text{if } i \text{ is not joined to } j \text{ by an edge} \\ \text{distance associated with an edge from } i \text{ to } j, \text{ if } i \text{ is joined to } j \\ \qquad \text{by an edge.} \end{cases}$$

We shall find the shortest distance between the vertices of a graph G using **Dijkstra's algorithm.**

Let us define two sets K and U, where K consists of those vertices which have been fully investigated and between which the best path is known, and U of those vertices which have not yet been processed. Clearly, every vertex belongs to either K or U but not both. Let a vertex r be selected from which we shall find the shortest paths to all the other vertices of the network. Let the array $bestd(i)$ hold the length of the shortest path so far formed from r to vertex i, and another array $tree(i)$ the next vertex to i on the current shortest path.

Dijkstra's algorithm:
Step 1: Intialise $K = \{r\}$, U={ all other vertices of G except r}. Set bestd(i)= d_{ri} and tree(i)= r.
Step 2: Find the vertex s in U which has the minimum value of bestd. Remove s from U and put it in K.
Step 3: For each vertex u in U, find bestd(s) + d_{su} and if it is less than bestd(u) replace bestd(u) by this new value and let tree(u)= s.(a shorter path to u has been found by going via vertex s.)
Step 4: If U contains only one vertex then stop the process or else go to step 2. The array bestd(i) contains the length of shortest path from r to i.

Example: Implement Dijkstra′s algorithm to find shortest path from the vertex B to all other vertices of following graph G.



$$D(G) = \begin{bmatrix} 0 & 7 & \infty & 6 & \infty \\ \infty & 0 & 5 & \infty & 10 \\ 12 & 6 & 0 & \infty & 3 \\ 4 & 5 & \infty & 0 & 6 \\ 11 & \infty & 8 & \infty & 0 \end{bmatrix}$$

Figure: Graph G and its distance matrix $D(G)$

Step 1: Intialise $K = \{B\}$, U={ A,C,D,E}.

	A	**C**	**D**	**E**
best d	∞	**5**	∞	**10**
tree	**B**	**B**	**B**	**B**

Therefore, bestd(C)= 5 (minimum distance) and tree(C)= B.
Step 2: Remove C from U and put it in K. Now, U={ A,D,E}. K={ B,C}. Find minimum distance from B to A, D, E via C. Therefore, bestd(A) = $17 < \infty$, tree(A)=C and bestd(E)= $8 < 10$, tree(E)=C.

	A	**D**	**E**
best d	**17**	∞	**8**
tree	**C**	**B**	**C**

Therefore, bestd(E)= 8 and tree(E)= C

Step 3: Remove E from U having minimum distance and put it in K. Now, U={ A,D}. K={ B, C, E}. Find minimum distance from B to A, D via C and E.The distances from B to A are 17 (B *to* C *to* A), 30 (B *to* E *to* C *to* A), 21 (B *to* E *to* A). Therefore, bestd(A) $= 17 <$ (30 , 21), tree(A)=C and bestd(D)= 8 $<$ 10, tree(E)=C.

	A	**D**
best d	**17**	∞
tree	**C**	**B**

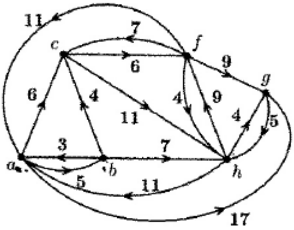
Therefore, bestd(A)= 17 and tree(A)= C

Step 4: Remove A from U having minimum distance and put it in K. Now, $U = \{$ D$\}$, K=$\{$ B, C, E, A$\}$. Find minimum distance from B to D via A, E, C. The distances from B to D are 23 (B *to* C *to* A *to* D), 36 (B *to* E *to* C *to* A *to* D), 27 (B *to* E *to* A D). Therefore, bestd(D) = 23 $<$ (36 *and* 27), tree(D)=A. Now, U contains only one vertex and stop the process.Therefore, bestd(D)= 23 and tree(D)= A.

The array bestd contains the length of shortest path from B to all other vertices of G. The shortest path from the vertex B to all other vertices of a graph G is given by

B	**A**	**C**	**D**	**E**
best d	**17**	**5**	**23**	**8**
tree	**C**	**B**	**A**	**C**

Example: Implement Dijkstra's algorithm to find shortest path from c to all other vertices of the following network.



$$D(G) = \begin{bmatrix} 0 & 5 & 6 & \infty & 17 & \infty \\ 3 & 0 & 4 & \infty & \infty & 7 \\ \infty & \infty & 0 & 6 & \infty & 11 \\ 11 & \infty & 7 & 0 & 9 & 4 \\ \infty & \infty & \infty & \infty & 0 & 5 \\ 11 & \infty & \infty & 9 & 4 & 0 \end{bmatrix}$$

Figure: Graph$G$ and its distance matrix $D(G)$

Step 1: Intialise $K = \{c\}$, $U = \{a, b, f, g, h\}$.

	a	b	f	g	h
best d	∞	∞	**6**	∞	**11**
tree	**c**	**c**	**c**	**c**	**c**

Therefore, bestd(f)= 6 (minimum distance) and tree(f)= c.

Step 2: Remove f from U and put it in K.
Now, U={ a, b, g, h}. K={ c, f}.
Find minimum distance from c to a, b, g, h via f.
Therefore, bestd(a) $= 17 < \infty$, tree(a)=f, bestd(g)= 15 $<\infty$, tree(g)=f and bestd(h)= $10 < 11$, tree(h)=f

	a	**b**	**g**	**h**
best d	**17**	∞	**15**	**10**
tree	**f**	**c**	**f**	**f**

Therefore, bestd(h)= 10 and tree(h)= f.

Step 3: Remove h from U having minimum distance and put it in K. Now, U={ a, b, g}. K={ c, f, h}. Find minimum distance from c to a, b, g via h and f.The distances from c to a are 17 , 21 , 22. Therefore, bestd(a) = 17, tree(a)=f and bestd(g)= 14 < 15, tree(g)=h.

	a	**b**	**g**
best d	**17**	∞	**14**
tree	**f**	**c**	**h**

Therefore, bestd(g)= 14 and tree(g)= h

Step 4: Remove g from U having minimum distance and put it in K. Now, U={ a, b}, K={c, f, g, h}. Find minimum distance from c to a and b via c, f, g, h. The distances from c to a and b are 17 and ∞. Therefore, bestd(a) = 17, tree(a)=f.

	a	b
best d	**17**	∞
tree	**f**	**c**

Therefore, bestd(a)= 17 and tree(a)= f
Step 5: Remove a from U having minimum distance and put it in K. Now, U={ b}, K={c, f, g, h, a}. Find minimum distance from c to b via c, f, g, h and a. The distances from c to b are 22, 26,27... Therefore, bestd(b) = 22, tree(b)=a. Now, U contains only one vertex and stop the process.

The array bestd contains the length of shortest path from c to all other vertices of G. The shortest path from the vertex c to all other vertices of a graph G is given by

	a	b	f	g	h
best d	17	22	6	14	10
tree	f	a	c	h	f

References:

[1] Frank Harary, *Graph Theory*, Addison-Wesley Publishing Company, Inc, 1969.

[2] E S Page and L B Wilson, *An introduction to Computational Combinatorics*, Cambridge University Press, Inc, 1979.

THANK YOU