Lab 3

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Question 1: E-Commerce Retail Sales as a Percentage of Total Sales

We have a time series that describes the percentage of sales attributed to e-commerce. Modelling this series will require three major steps. First, we will explore and visualize the series to understand potential transformations as well as AR and MA orders. Second, we will find the SARIMA specification with the optimal AIC value. Finally, we will compare various model specifications (informed either through our EDA or our AutoArima results) on the basis of model performance in a hold-out data set. Data up until 2015 will be used for training, with 2015 and 2016 data being used for model selection. We generate a forecast for 2017 data using the best-performing specification trained over all data available.

Exploration and Visualization

Although it has been dictated that we are to create a SARIMA model, we will use exploratory analysis to show that this is suitable. First, we load the data, convert it to a time series, split the time series into training and testing data, display the first few rows, check for missing values and display the summary.

```
ecom = read.csv(file = "ECOMPCTNSA.csv")
ts_ecom = ts(ecom$ECOMPCTNSA, start = c(1999, 4), frequency = 4)
ts_ecom_train = ts(ts_ecom[time(ts_ecom) < 2015], start = c(1999, 4), freq = 4)
ts_ecom_test = ts(ts_ecom[time(ts_ecom) >= 2015], start = c(2015, 1), freq = 4)
head(ts_ecom, 13); sum(is.na(ts_ecom_train)); summary(ts_ecom_train)
```

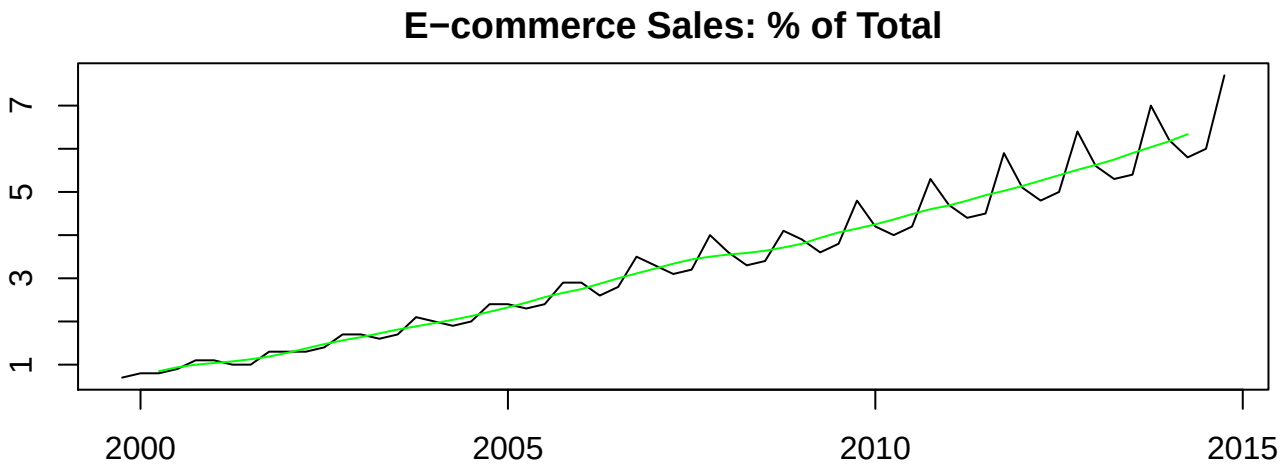
```
##      Qtr1 Qtr2 Qtr3 Qtr4
## 1999                 0.7
## 2000  0.8  0.8  0.9  1.1
## 2001  1.1  1.0  1.0  1.3
## 2002  1.3  1.3  1.4  1.7
```

```
## [1] 0
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   0.700   1.700   3.300   3.331   4.700   7.700
```

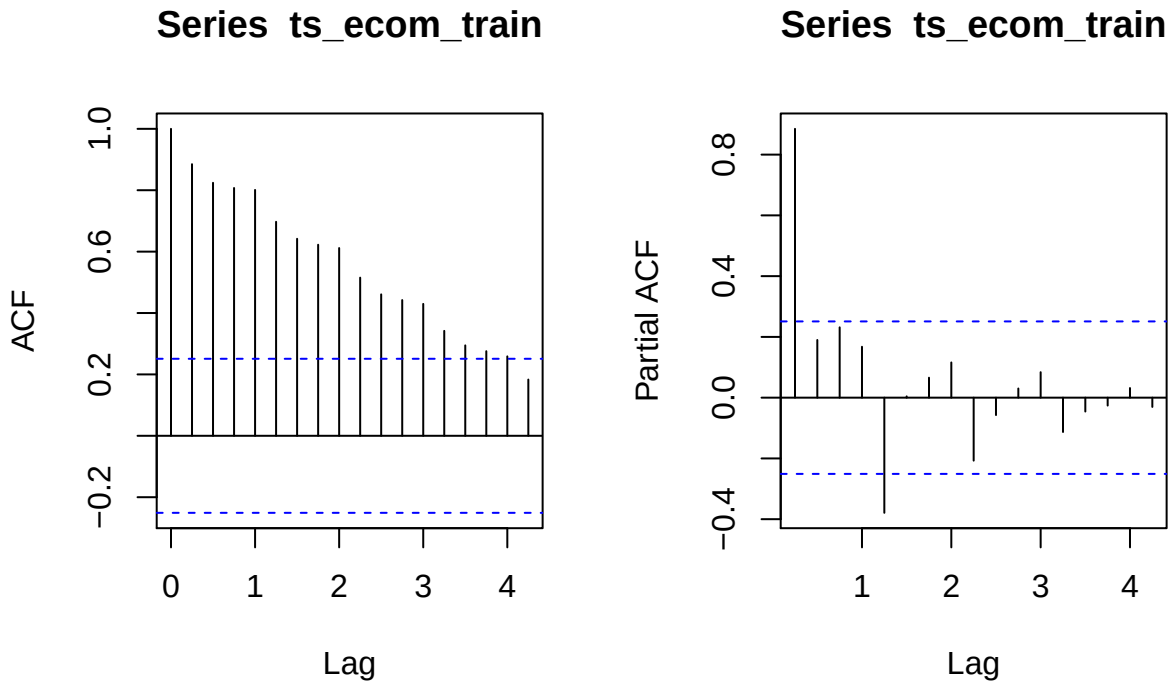
We have no missing values in this series. The series ranges from 0.7 to 7.7 with a mean of 3.3 and a median of 3.3. Next, we plot the training time series.

```
par(mfrow=c(1,1), mai = c(.4, 0.4, 0.4, 0.4))
plot(ts_ecom_train, xlab = 'Date', ylab = 'E-Commerce Retail Sales', main = 'E-commerce Sales: % of Total')
lines(ma(ts_ecom_train, order = 4, centre = T), col = 'green')
```



The plot shows both a deterministic trend as well as strong seasonality. A MA(4) model of this series smoothens out the variance: reinforcing the observation that we have annual seasonality in this series. We next look at autocorrelation and partial autocorrelation functions of the training series.

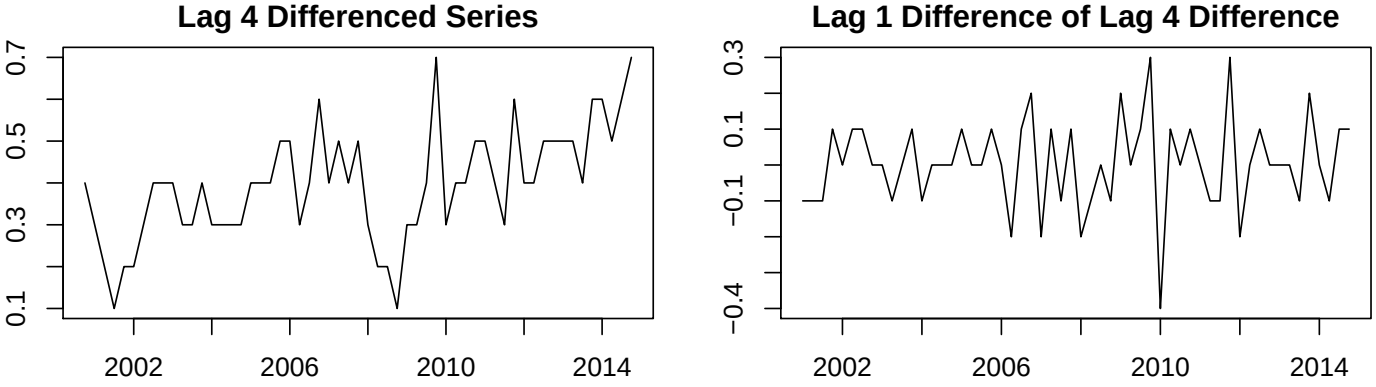
```
par(mfrow=c(1,2))
acf(ts_ecom_train)
pacf(ts_ecom_train)
```



We see that the autocorrelations are significant for a large number of lags (16 quarters), which is further evidence of non-stationarity. The gradual decay in the acf without any spikes at seasonal intervals tells us that we will likely need a non-seasonal AR term p but may not need a seasonal AR term P.

Imposing stationarity will require differencing the series. We take first and fourth order differences as well as the first order difference of the fourth order differences. For the time series that look stationary in the differenced plots, we conduct Augmented Dickey Fuller (ADF) Tests with null hypothesis: The series contains unit roots (non-stationary stochastic trend).

```
par(mfrow=c(1,2), mai = c(.4, 0.4, 0.4, 0.4))

plot(diff(ts_ecom_train, lag = 4),xlab=NULL,ylab=NULL,main = "Lag 4 Differenced Series")
plot(diff(diff(ts_ecom_train, lag = 4),lag=1),xlab=NULL,ylab=NULL,
     main = "Lag 1 Difference of Lag 4 Difference")
```



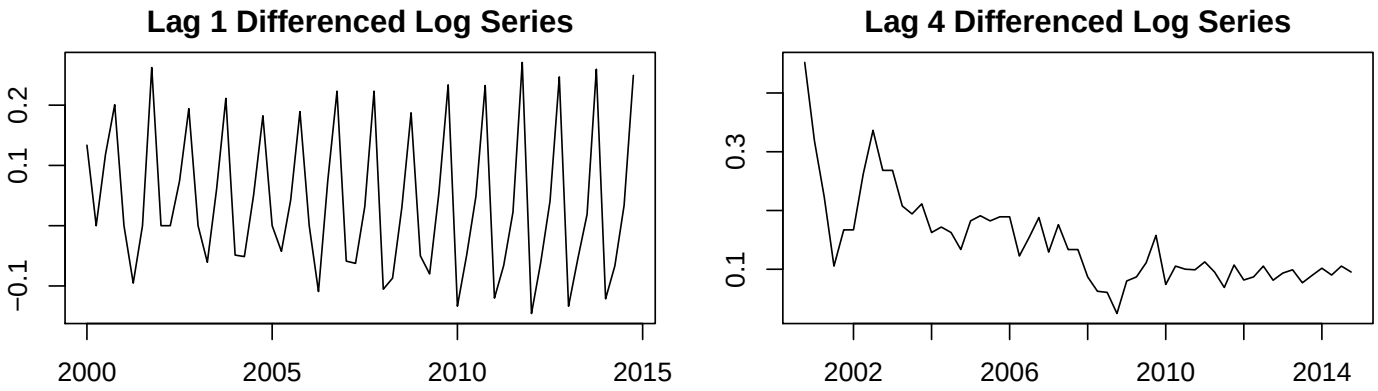

```
adf.test(diff(diff(ts_ecom_train, lag = 4),lag=1))$p.value
```

```
## [1] 0.01
```

We can see that our difference of differences approach leads to stationarity, which is corroborated by the ADF test result. As this series increases in magnitude, the associated seasonal variance also increases, and none of the differences taken above are able to attain visually consistent variance.

We thus examine the effect of taking the log transform of this series prior to differencing.

```
par(mfrow=c(1,2), mai = c(.4, 0.4, 0.4, 0.4))

log_ts_ecom_train = log(ts_ecom_train); log_ts_ecom_test = log(ts_ecom_test)
plot(diff(log_ts_ecom_train, lag = 1),xlab=NULL,ylab=NULL,main = "Lag 1 Differenced Log Series")
plot(diff(log_ts_ecom_train, lag = 4),xlab=NULL,ylab=NULL,main = "Lag 4 Differenced Log Series")
```




```
adf.test(diff(log_ts_ecom_train))$p.value
```

```
## [1] 0.01
```

We can see that the first order difference of the logarithm of the training series has near-constant variance, and also achieves stationarity by the ADF test.

Modelling

First, we will specify two models purely informed by our exploratory data analysis. One model each for the log transformed and the raw (non-transformed) series.

The spike in the ACF at a lag of 1 quarter suggests a nonseasonal MA(1) (q=1) component and the spikes at intervals of 4 quarters of lag out to approximately lag 16 suggest a seasonal MA(4) (Q=4). Additionally, the spike at at a lag of 1 quarter in the PACF and the spikes at intervals of 4 quarters of lag tells us that a a nonseasonal AR(1) (p=1) component and a seasonal AR(1) (P=1) component are appropriate for our initial model. Therefore, our initial model will be of the form $ARIMA(1,1,1)(1,1,4)_4$

In the log transformed case - without accounting for seasonality - we observed (plot not shown in this document) a somewhat sinusoidal ACF with a PACF showing no significant spikes beyond lag 4. We furthermore achieved steady variance through differencing with lag 1. Hence, an $ARIMA(0,1,4)$ might be appropriate.

```
manual_model_raw <- Arima(ts_ecom_train, order=c(1,1,1), seasonal=c(1,1,4))
manual_model_log <- Arima(ts_ecom_train, order = c(0,1,4), seasonal=c(0,0,0), lambda=0)
```

Next, We will use the auto.arima function to identify the SARIMA parameters that minimize the corrected AIC. Again, we will explore modelling both, the log transformed time series (handled by the argument "lambda=0") and the non-transformed series.

```
auto_model_log <- auto.arima(ts_ecom_train, ic = 'aicc', lambda = 0)
auto_model_raw <- auto.arima(ts_ecom_train, ic = 'aicc')
```

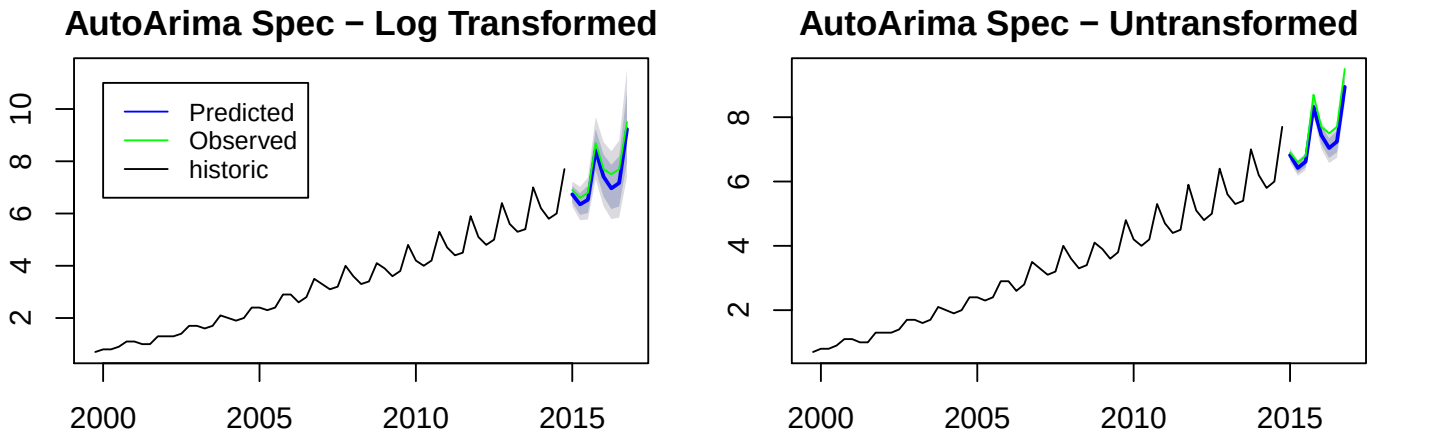
For the log transformed series, the auto.arima algorithm returns a model of order (0,1,0)(2,1,0)[4] and for the non-transformed series it returns a model of order (0,1,1)(0,1,0)[4].

We display and discuss AICc and BIC for all four models in the table in the 'Residual diagnostics' section below.

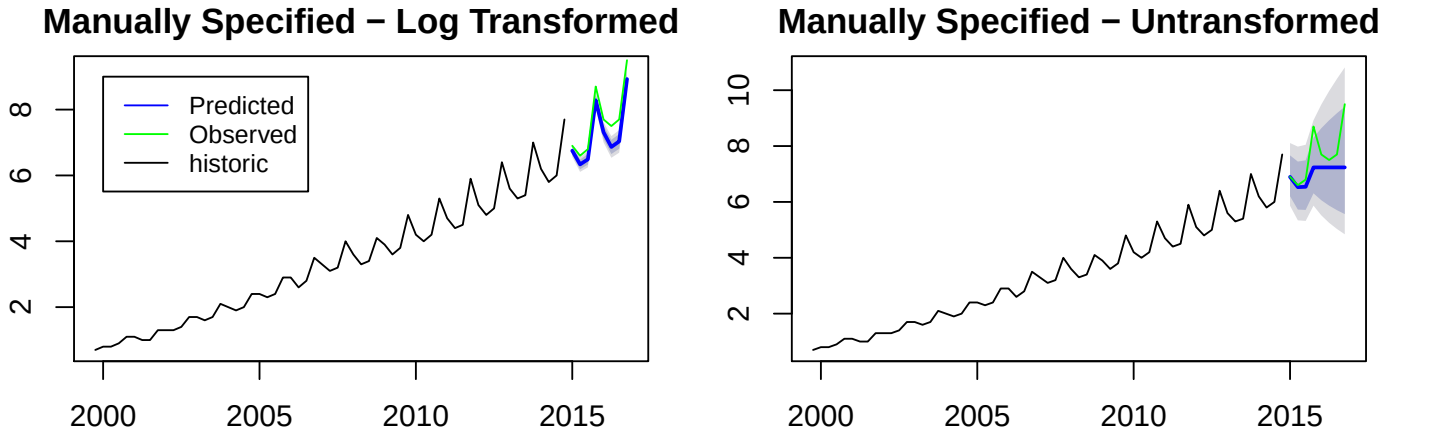
Forecast for 2015 and 2016

Next, we calculate and visualize forecasts for the years 2015 and 2016, for which we have validation data. Visuals of predicted versus actual values help us understand the forecasts.

```
forecast_log_auto = forecast(auto_model_log, h = 8, lambda=0)
forecast_auto = forecast(auto_model_raw, h = 8)
forecast_log_man = forecast(manual_model_raw, h = 8)
forecast_man = forecast(manual_model_log, h = 8, lambda=0)

par(mfrow=c(1,2), mai = c(.4, 0.4, 0.4, 0.4))
plot(forecast_log_auto, main="AutoArima Spec - Log Transformed"); lines(ts_ecom_test, col='green')
legend(2000, 11, legend=c('Predicted','Observed', 'historic'), col=c('blue','green', 'black'),
       lty=1, cex=0.8)
plot(forecast_auto, main="AutoArima Spec - Untransformed"); lines(ts_ecom_test, col='green')
```




```
par(mfrow=c(1,2), mai = c(.4, 0.4, 0.4, 0.4))
plot(forecast_log_man, main="Manually Specified - Log Transformed"); lines(ts_ecom_test, col='green')
legend(2000, 9, legend=c('Predicted','Observed', 'historic'), col=c('blue','green', 'black'),
       lty=1, cex=0.8)
plot(forecast_man, main="Manually Specified - Untransformed"); lines(ts_ecom_test, col='green')
```



Residual diagnostics

We check each of these four models for validity: We use the Shapiro-Wilk and Box-Ljung tests. The former tests for normality of residuals within the training set, while the latter tests for autocorrelation within these same residuals. The null hypothesis of the Shapiro-Wilk test states that the residuals come from a normal distribution. The null hypothesis of the Box-Ljung test states that the autocorrelation is zero. We further compare these models on the basis of root mean square error (RMSE) of predictions in the validation holdout.

```
calculate_rmse = function(fcast, test){
  rmse = sqrt(mean((fcast - test)^2))
  return(rmse)
}
auto_model_log_sw <- shapiro.test(auto_model_log $resid)
auto_model_log_box <-Box.test(auto_model_log $resid, type = "Ljung-Box")
auto_model_raw_sw <-shapiro.test(auto_model_raw $resid)
auto_model_raw_box <-Box.test(auto_model_raw $resid, type = "Ljung-Box")
manual_model_raw_sw <-shapiro.test(manual_model_raw $resid)
manual_model_raw_box <-Box.test(manual_model_raw $resid, type = "Ljung-Box")
manual_model_log_sw <-shapiro.test(manual_model_log $resid)
manual_model_log_box <-Box.test(manual_model_log $resid, type = "Ljung-Box")
results <- data.frame(cbind(
  rbind(auto_model_log$aicc, auto_model_raw$aicc, manual_model_raw$aicc, manual_model_log$aicc),
  rbind(auto_model_log$bic, auto_model_raw$bic, manual_model_raw$bic, manual_model_log$bic),
  rbind(auto_model_log_sw$p.value,auto_model_raw_sw$p.value,
        manual_model_raw_sw$p.value, manual_model_log_sw$p.value),
  rbind(auto_model_log_box$p.value, auto_model_raw_box$p.value,
        manual_model_raw_box$p.value, manual_model_log_box$p.value),
  rbind(calculate_rmse(forecast_log_auto$mean,ts_ecom_test),
        calculate_rmse(forecast_auto$mean,ts_ecom_test),
        calculate_rmse(forecast_log_man$mean, ts_ecom_test),
        calculate_rmse(forecast_man$mean, ts_ecom_test))))

colnames(results) <- c('AICc', '       BIC', '  Shapiro-Wilk', '  Box-Ljung', '   RMSE')
rownames(results) <- c('Log, Auto','Raw, Auto','Log, Manual', 'Raw, Manual')
round(results,3)
```

```
##                 AICc        BIC   Shapiro-Wilk   Box-Ljung    RMSE
## Log, Auto   -206.038   -200.424          0.003       0.095   0.349
## Raw, Auto    -80.571    -76.747          0.170       0.609   0.356
## Log, Manual  -84.093    -70.954          0.079       0.269   0.459
## Raw, Manual -116.332   -106.972          0.104       0.434   0.990
```

Aside of the auto.arima specification over the log, all models show no statistical significance for the Shapiro-Wilk or the Box-Ljung Test. Hence the residuals are likely to be normally distributed and not autocorrelated for every model except the auto.arima model with log transformation.

The auto.arima specification over the log transformed model ('Log,Auto') has the best AICc, BIC and RMSE in out of sample comparisons. However, this specification also leads to non-normal residuals and potential autocorrelation in the residuals (as identified by the Shapiro-Wilk and Box-Ljung test). We therefore decided to not consider this model any longer. Between the remaining models, we have to decide if we prefer to go with the model with the lowest AICc/BIC ('Raw,Manual') or the model with the lowest RMSE ('Raw,Auto'), or the model that exhibits a compromise between those two ('Log,Manual'). We chose to continue with the auto arima specification over the untransformed series because it has the best RMSE without failing tests for residual normality and autocorrelation.

Forecast for 2017

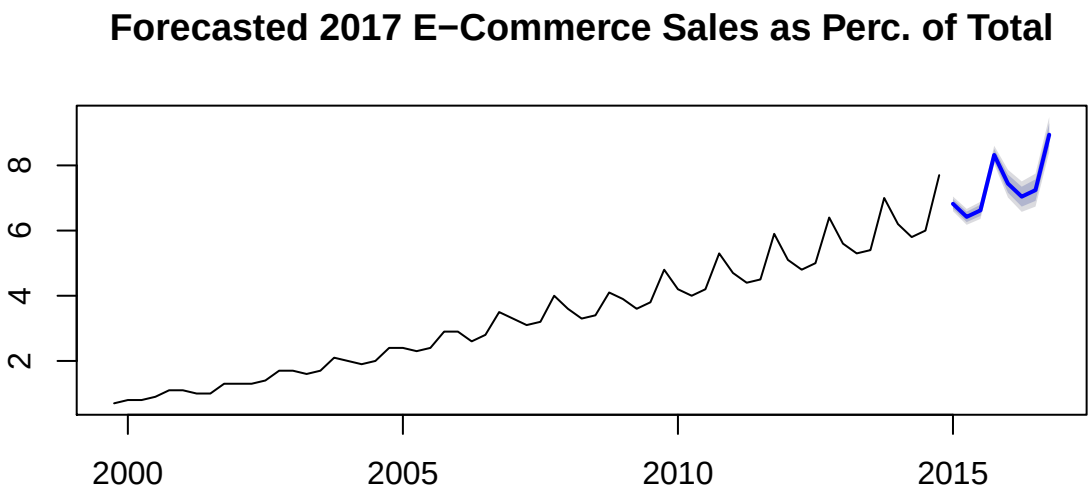
To produce our final forecast, we will extend our forecast from our best performing model to include 2017. We will not retrain the final model using 2015-2016 data.

Our final model has the following form:

$$(1 - B) \cdot (1 - B^4) \cdot y_t = (1 - 0.5680103 \cdot B) \cdot e_t$$

We visualize the 2017 forecast in the below plot.

```
plot(forecast(auto_model_raw, h = 8), main='Forecasted 2017 E-Commerce Sales as Perc. of Total')
```



Question 2:

We will build a vector autoregressive (VAR) model to forecast four time series. Neither the names, nor the units of measures are given for the time series. We will therefore have to rely exclusively on statistical metrics, tests and graphical analysis to specify the order of the VAR model and to measure the model fit. We will also discuss possible enhancments of the model that might provide a better fit.

Exploration and Visualization

First, we load the time series, split the data into test and train data, display the first few rows, check for missing values and print a summary of the train data.

```
mts = read.csv(file = "data_2018Spring_MTS.txt", sep=' ')
ts_mts = ts(mts[ , c("series1", "series2", "series3", "series4")], start = c(1947, 1), freq = 12)
ts_mts_train = ts(ts_mts[time(ts_mts) < 1993, ], start = c(1947, 1), freq = 12)
ts_mts_test = ts(ts_mts[time(ts_mts) >= 1993, ], start = c(1993, 1), end = c(1993,12), freq = 12)
head(ts_mts); sum(is.na(ts_mts)); summary(ts_mts_train)
```

```
##          series1 series2 series3 series4
## Jan 1947 11.1764 21.5500  5.8735 13.5172
## Feb 1947 11.5193 21.2241  5.9369 13.6239
## Mar 1947 11.7707 21.1833  5.9685 14.0771
## Apr 1947 11.8621 21.0204  6.0531 13.7305
## May 1947 11.7478 20.8575  6.0847 13.7838
## Jun 1947 11.8392 20.8575  6.1270 13.6505
```

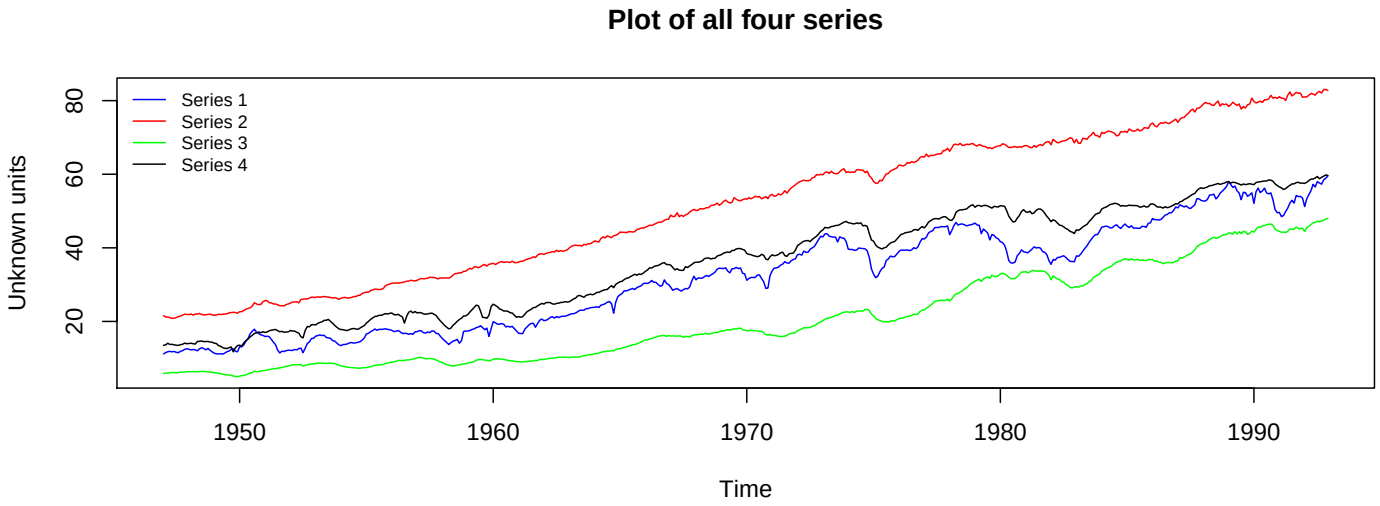
```
## [1] 0
```

```
##     series1         series2         series3          series4
##  Min.   :11.13   Min.   :20.86   Min.   : 5.007   Min.   :11.73
##  1st Qu.:17.51   1st Qu.:32.90   1st Qu.: 9.262   1st Qu.:21.94
##  Median :32.95   Median :53.32   Median :16.938   Median :38.03
##  Mean   :31.99   Mean   :51.62   Mean   :20.712   Mean   :36.38
##  3rd Qu.:43.79   3rd Qu.:68.19   3rd Qu.:31.952   3rd Qu.:49.90
##  Max.   :59.54   Max.   :83.03   Max.   :47.987   Max.   :59.80
```

We have a total of four time series with no missing values. The time series all range between about 5 and 83.

Next, we create a plot of all four time series in one single graph.

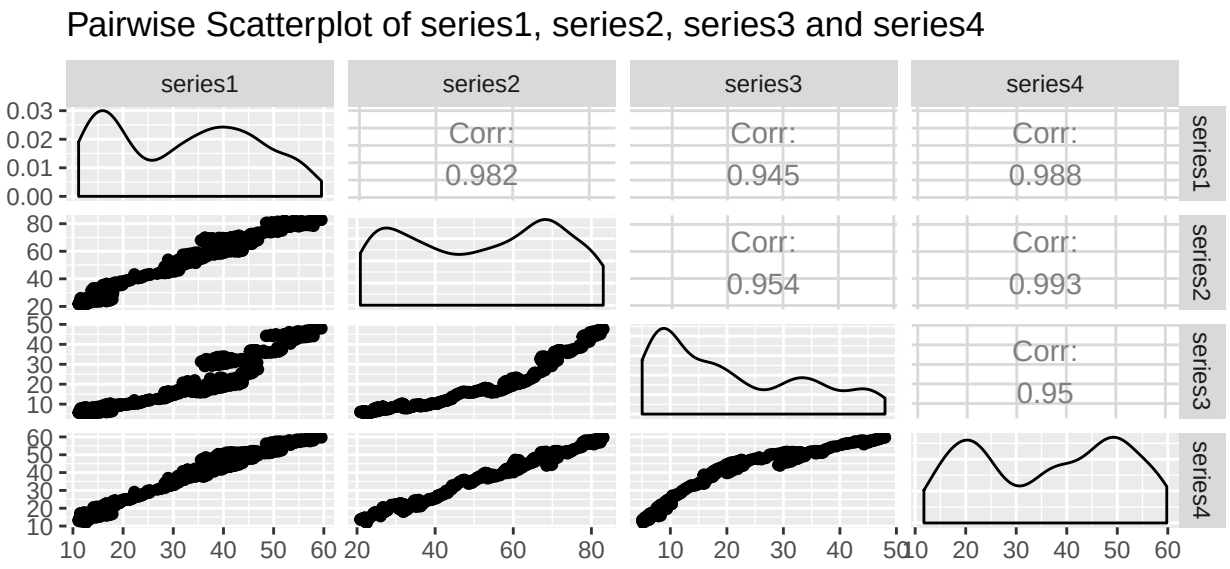
```
ts.plot(ts_mts_train, gpars=list(main="Plot of all four series", ylab="Unknown units",
                                 col=c('blue','red','green','black')))
legend("topleft",c("Series 1","Series 2","Series 3","Series 4"), lty=1,
       col = c('blue','red','green','black'), bty='n', cex=.75)
```



It is pretty obvious that none of the time series is stationary. All time series seem to have a similar deterministic trend. The stochastic trend also seems to be similar between the four time series - just different in magnitude. series1 has the strongest stochastic trend, followed by series4, series3 and series2. There does not appear to be an obvious seasonal trend within any of the four time series.

In the following plot we can see the distribution of the four time series, as well as pairwise scatterplots and correlation coefficients.

```
df_mts_train = data.frame(as.matrix(ts_mts_train), date=time(ts_mts_train))
ggpairs(df_mts_train[c('series1', 'series2', 'series3', 'series4')],
        title='Pairwise Scatterplot of series1, series2, series3 and series4')
```



As expected, the series are highly correlated (> 0.95). High correlation however does not imply a causal relationship between the variables. Since we have no further information about the variables, we are unable to speculate on wheter or not this could be attributed to a common exogenous factor. We also see that the distributions for the series are fairly uniform except for series3, which is closer to following an exponential distribution.

Next, we conduct Dickey-Fuller Tests to test for unit roots. The null hypothesis states that the series contains unit roots

(non-stationary stochastic trend), while the alternative hypothesis states that the series contains no unit roots (stationary stochastic trend).

```
cbind(adf.test(df_mts_train$series1)$p.value, adf.test(df_mts_train$series2)$p.value,
adf.test(df_mts_train$series3)$p.value, adf.test(df_mts_train$series4)$p.value)
```

```
##           [,1]      [,2]      [,3]       [,4]
## [1,] 0.0701259 0.4872257 0.4880169 0.02509535
```

For series4, we reject the null hypothesis of non-stationarity. For all the other time series we fail to reject the null hypothesis. This means series 1 - 3 are likely to contain unit roots.

Next, we check if differencing alleviates this, by conducting Dickey-Fuller Tests on the the differenced series.

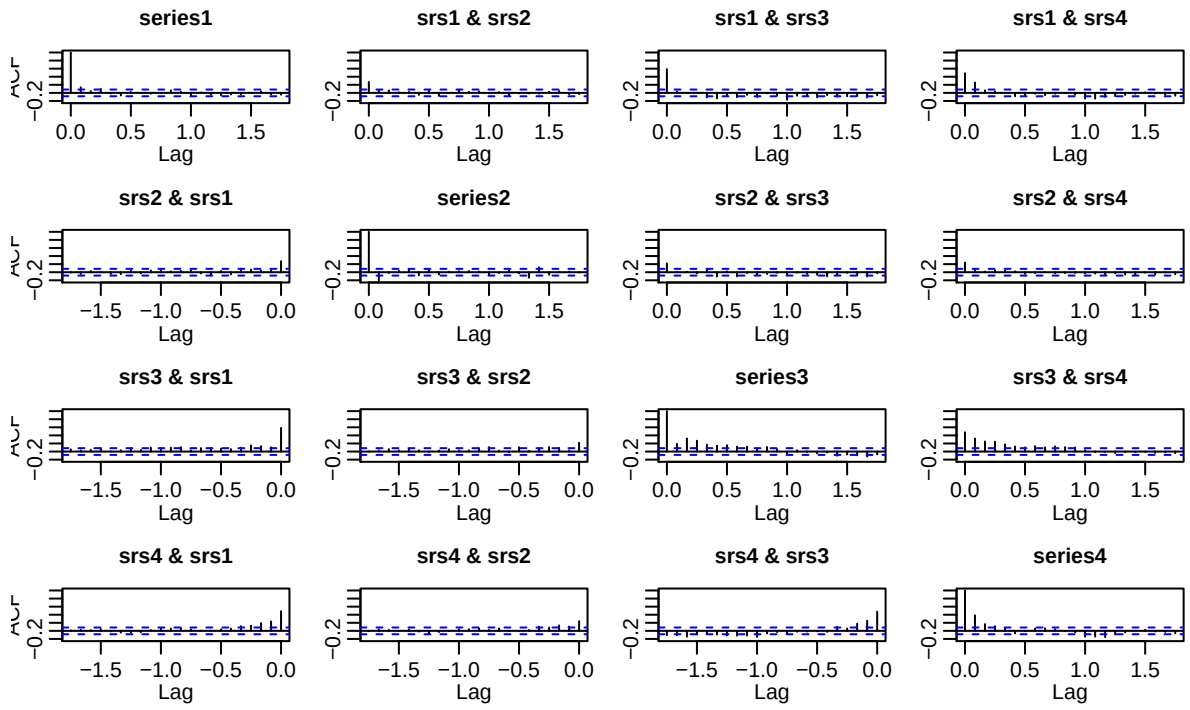
```
cbind(adf.test(diff(df_mts_train$series1))$p.value, adf.test(diff(df_mts_train$series2))$p.value,
adf.test(diff(df_mts_train$series3))$p.value, adf.test(diff(df_mts_train$series4))$p.value)
```

```
##      [,1] [,2] [,3] [,4]
## [1,] 0.01 0.01 0.01 0.01
```

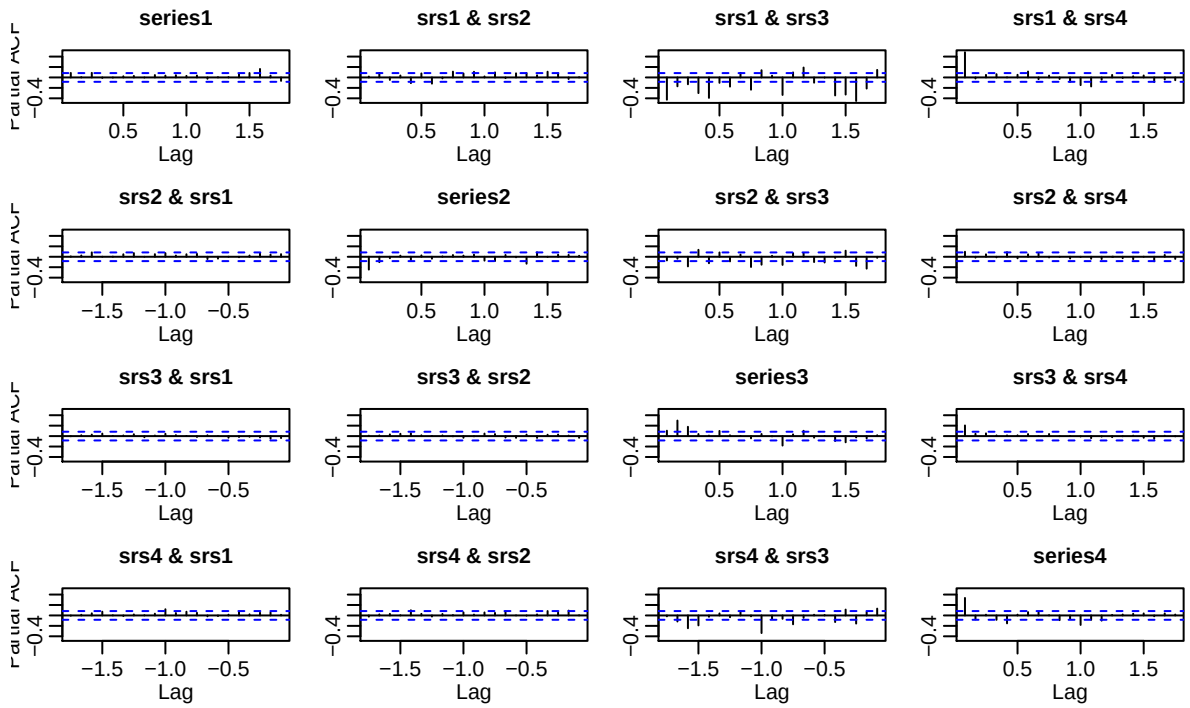
For the lag 1 differenced series, we reject the null hypothesis of non-stationarity for all four series.

We looked at the acf and pacf of the original time series and - as expected - observed that the autocorrelations were significant for all lags and all time series and pairs of time series displayed in the plot. We saw a few significant PACF values for higher lags in select pairs of times series - this suggests that the final VAR model will likely need to have an order of at least 2 (???). We therefore decided to difference all four series. The acf and pacf for the differenced time series are displayed below.

```
acf(diff(ts_mts_train))
```




```
pacf(diff(ts_mts_train))
```



While certainly not perfect, the differenced series' pacf and acf display statistically more convenient behavior. Differencing eliminates the positive drift from each of the series - which results in ACF plots that are more meaningful. We furthermore do not observe any obvious seasonality - evidenced by PACF charts of differenced and non-differenced (not displayed) series. Further differencing did not significantly improve the acf and pacf.

Next, we examine cointegration by conducting Phillips-Ouliaris tests, with the null hypothesis that two series are not cointegrated.

```
cbind(po.test(df_mts_train[c('series1','series2')])$p.value, #column 1
po.test(df_mts_train[c('series1','series3')])$p.value, #column 2
po.test(df_mts_train[c('series1','series4')])$p.value, #column 3
po.test(df_mts_train[c('series2','series3')])$p.value, #column 4
po.test(df_mts_train[c('series2','series4')])$p.value, #column 5
po.test(df_mts_train[c('series3','series4')])$p.value) #column 6
```

```
##          [,1] [,2]       [,3] [,4] [,5] [,6]
## [1,] 0.024708 0.15 0.03785969 0.15 0.01 0.15
```

In a few cases we reject the null hypothesis: series1/series2, series1/series4 and series2/series4. In those cases it is likely that our time series are cointegrated. This means we would be able to fit a linear regression between both time series.

Since some of the series are cointegrated, we should be using a VAR specification that includes an error correction term (vector error correction model) and alternative estimation methods to least squares. Since this was not covered in the course, we will have to model a simple VAR model despite detecting cointegration.

Modelling

First, we search for the order of the VAR model by using the VARselect function.

```
VARselect(diff(ts_mts_train), type = 'both')$selection
```

```
## AIC(n)  HQ(n)  SC(n) FPE(n)
##      5      2      2      5
```

For VAR models we preferably use the BIC (SC). The minimum BIC is achieved for the order 2 VAR model. We specify our VAR 2 model and immediatly check if the residuals are well behaved. For this, we are conducting a Portmanteau Test. The null hypothesis of the Portmanteau Test states that there is no serial correlation.

```
var = VAR(diff(ts_mts_train), p = 2, type = 'both')
serial.test(var, type = 'PT.asymptotic')$serial$p.value[1]
```

```
##  Chi-squared
## 4.027162e-05
```

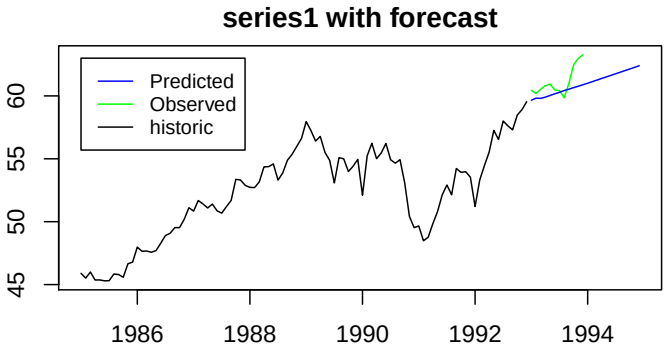
We reject the null hypothesis of no serial correlation for the VAR(2) model. We also conducted this test for VAR(3), VAR(4) and VAR(5). For VAR(3) we received a p-value of around 0.015 and for VAR(4) we received a p-value of 0.08. The p-value for VAR(4) is the first p-value that does not indicate statistical significance. However, it is somewhat of a borderline case. To gain higher confidence we go one step further and chose a VAR(5) model - which also happens to be the one that was selected by the AIC. The p-value for the VAR(5) models is about 0.23 and thus far away from statistical significance.

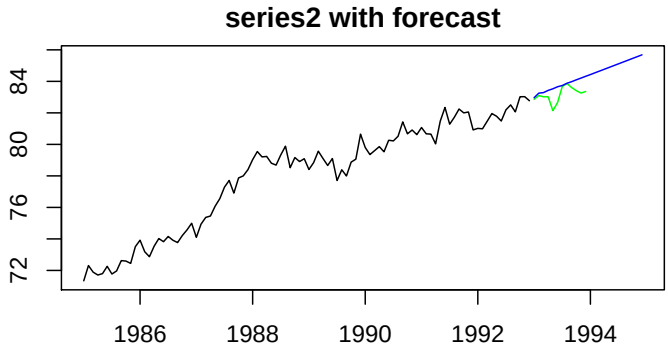
```
var_model = VAR(diff(ts_mts_train), p = 5, type = 'both')
```

Forecasting for 1993 and 1994

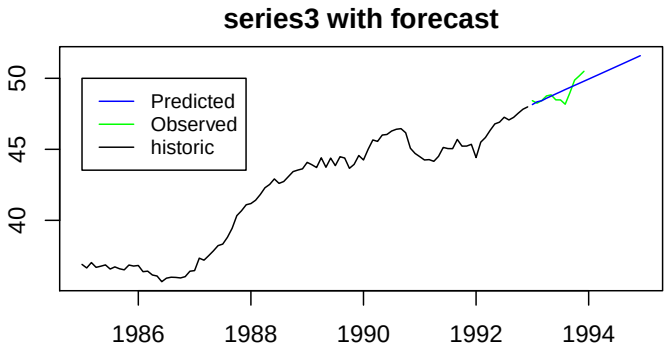
Next, we calculate a two-year forecast based on our model. We plot our forecast for all four time series. To better visualize the forecasts, we only plot the time series from 1985 onwards. The training time series (1/1985 - 12/1992) is plotted in black, while the forecast (1/1993 - 12/1994) is plotted in blue and the test time series (1/1993 - 12/1993) is plotted in green.

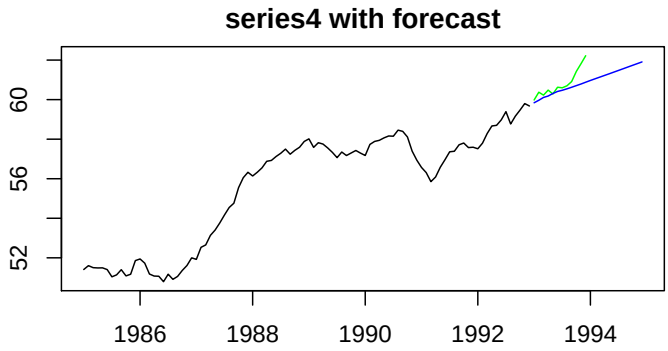
```
calculate_int = function(model, forecast_steps, endvals){
  diff_forecast = forecast(model, h=forecast_steps)
  int_1 = endvals[1] + cumsum(diff_forecast$forecast$series1$mean)
  int_2 = endvals[2] + cumsum(diff_forecast$forecast$series2$mean)
  int_3 = endvals[3] + cumsum(diff_forecast$forecast$series3$mean)
  int_4 = endvals[4] + cumsum(diff_forecast$forecast$series4$mean)
  return(list(int_1, int_2, int_3, int_4))
}
int = calculate_int(var_model, 24, tail(ts_mts_train, n=1))
ts_forecast = ts(cbind(int[[1]], int[[2]], int[[3]], int[[4]]), start=c(1993, 1), freq=12)
ts_mts_train_window = window(ts_mts_train, start = c(1985,1))

par(mfrow=c(1,2), mai = c(.4, 0.4, 0.4, 0.4))
ts.plot(ts_mts_train_window[,1], ts_mts_test[,1], ts_forecast[,1], col=c('black', 'green', 'blue'),
        main = 'series1 with forecast')
legend(1985, 63, legend=c('Predicted','Observed', 'historic'), col=c('blue', 'green', 'black'),
       lty=1, cex=0.8)
ts.plot(ts_mts_train_window[,2], ts_mts_test[,2], ts_forecast[,2], col=c('black', 'green', 'blue'),
        main = 'series2 with forecast')
```






```
par(mfrow=c(1,2), mai = c(.4, 0.4, 0.4, 0.4))
ts.plot(ts_mts_train_window[,3], ts_mts_test[,3], ts_forecast[,3], col=c('black', 'green', 'blue'),
        main = 'series3 with forecast')
legend(1985, 50, legend=c('Predicted','Observed', 'historic'), col=c('blue', 'green', 'black'),
       lty=1, cex=0.8)
ts.plot(ts_mts_train_window[,4], ts_mts_test[,4], ts_forecast[,4], col=c('black', 'green', 'blue'),
        main = 'series4 with forecast')
```





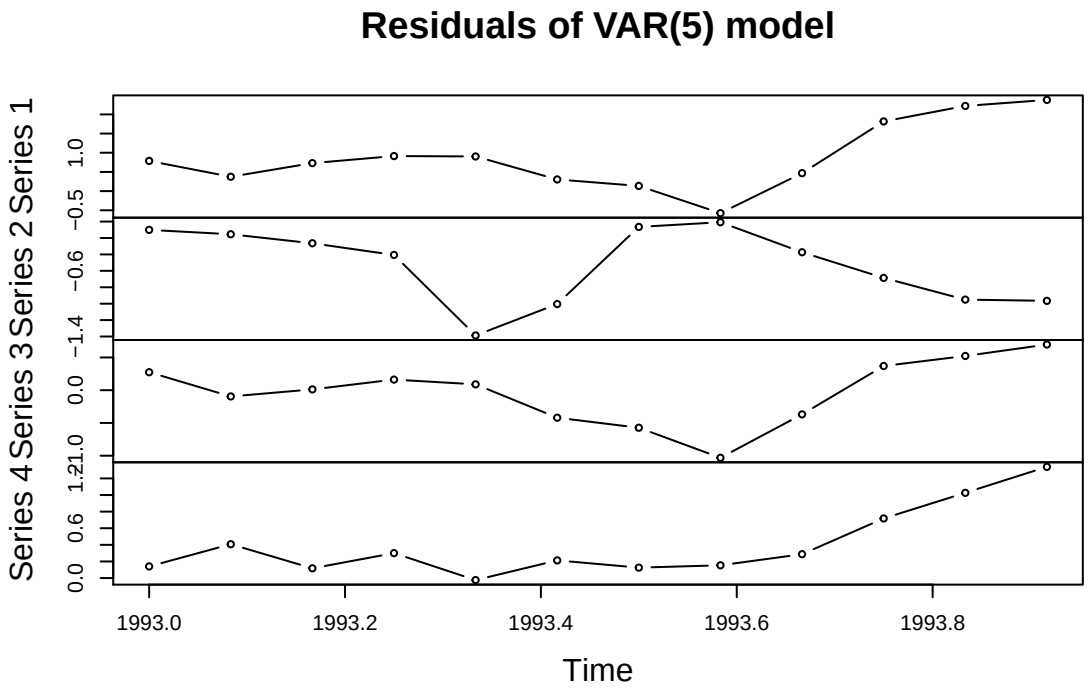
Most of our forecasts look like smoothed linear continuations of the last two years of the training time series.

Residual diagnostics

Finally, we conduct some in-depth residual diagnostics.

First we plot the residuals.

```
residuals = as.numeric(ts_mts_test) - window(ts_forecast, start = c(1993,1), end = c(1993,12))
par(mfrow=c(1,1), mai = c(.2, 0.2, 0.2, 0.2))
plot(residuals, main = 'Residuals of VAR(5) model', ylab = 'Date', type='b')
```



Since we only have 12 test data points, it is hard to determine whether or not the residuals look like white noise.

In our model selection process, we already examined the residuals using an asymptotic Portmanteau Test. We received a p-value of 0.2308 for the VAR(5) model. Next, we conduct a Breush-Godfrey LM test, with null hypothesis that there is no serial correlation

```
serial.test(var_model, type="BG")$serial$p.value
```

```
## Chi-squared
##   0.8203193
```

The Breush-Godfrey gives us a p-value of 0.9054, which means we have statistical significance and the residuals are likely not to be serially correlated.

Finally, we conduct a Jarque-Bera test for normality. We test against the null hypothesis that the residuals are normally distributed.

```
normality.test(var_model, multivariate.only = TRUE)$jb.mul$JB$p.value[1,1]
```

```
## [1] 0
```

We reject the null hypothesis with a highly statistically significant test result, which means that the residuals are not likely normally distributed.

```
int = calculate_int(var_model, 12, tail(ts_mts_train, n=1))
df_RMSE = data.frame(rbind(calculate_rmse(int[[1]], ts_mts_test[,1]),
           calculate_rmse(int[[2]], ts_mts_test[,2]), calculate_rmse(
             int[[3]], ts_mts_test[,3]), calculate_rmse(int[[4]], ts_mts_test[,4])))

colnames(df_RMSE) = c('RMSE')
rownames(df_RMSE) = c('series1','series2','series3', 'series4')
df_RMSE
```

```
##              RMSE
## series1 1.2070008
## series2 0.6863241
## series3 0.4763056
## series4 0.5641822
```

Conclusion

We are aware that our final VAR model is not satisfactory in terms of fit. The following table however shows that different specifications of the VAR model do not lead to a 'better' fit in an objective sense. Rather, we face a trade-off between different statistical metrics. The table below shows an overview of different model specifications from VAR of order 1 to VAR of order 10 (indicated by the row label), with AIC, BIC, three different p-values for statistical tests and the sum of the root mean squared error over all four series.

```
results = data.frame(); orderrange = 1:10
for(i in orderrange) {
  var_iter = VAR(diff(ts_mts_train), p = i, type="both")
  int = calculate_int(var_iter, 12, tail(ts_mts_train, n=1))

  PT = serial.test(var_iter, type="PT.asymptotic")$serial$p.value
  BG = serial.test(var_iter, type="BG")$serial$p.value
  norm = normality.test(var_iter, multivariate.only = TRUE)$jb.mul$JB$p.value
  rmse_sum = calculate_rmse(int[[1]], ts_mts_test[,1]) + calculate_rmse(int[[2]], ts_mts_test[,2]) +
             calculate_rmse(int[[3]], ts_mts_test[,3]) + calculate_rmse(int[[4]], ts_mts_test[,4])
  results = rbind(results, cbind(AIC(var_iter), BIC(var_iter), PT, BG, norm, round(rmse_sum,2)))
}
rownames(results) = orderrange
colnames(results) = c('AIC', '       BIC', ' Port. p-val',
                      ' BG p-val', 'JB p-val', 'sum(RMSE)')
round(results, 3)
```

```
##         AIC        BIC  Port. p-val  BG p-val JB p-val sum(RMSE)
## 1  2112.143   2215.581        0.000     0.000        0      3.06
## 2  2035.322   2207.646        0.000     0.000        0      3.11
## 3  2009.693   2250.845        0.010     0.009        0      2.97
## 4  2004.074   2313.995        0.055     0.121        0      3.03
## 5  1989.995   2368.626        0.166     0.820        0      2.93
## 6  2007.914   2455.196        0.122     0.803        0      2.84
## 7  2024.126   2540.000        0.131     0.076        0      2.85
## 8  2041.797   2626.204        0.072     0.020        0      2.89
## 9  2056.273   2709.154        0.055     0.046        0      2.91
## 10 2068.792   2790.086        0.026     0.052        0      2.44
```