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①

$$L(\theta_1, \theta_2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_i-\mu)^2}{2\sigma^2}}$$

$$\ln L(\theta_1, \theta_2) = \sum_{i=1}^{n} \left(-\frac{(x_i-\mu)^2}{2\sigma^2} - \frac{1}{2} \ln(2\pi\sigma^2) \right)$$

$$\frac{\partial}{\partial \theta_1} \ln L(\theta_1, \theta_2) = \sum_{i=1}^{n} \left(\frac{x_i-\mu}{\sigma^2} \right) = 0$$

$$\sum_{i=1}^{n} x_i - n\mu = 0$$

$$\frac{\theta_1}{\mu} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

$$\frac{\partial}{\partial \theta_2} \ln L(\theta_1, \theta_2) = \sum_{i=1}^{n} \left(\frac{-(x_i-\theta_1)^2}{2(\theta_2)^2} + \frac{1}{2\theta_2} \right) = 0$$

$$\sum_{i=1}^{n} \left(\frac{(x_i-\theta_1)^2}{\theta_2} \right) - \frac{n}{\theta_2} = 0$$

$$\frac{\theta_2^2}{\theta_2} = \frac{1}{n} \sum_{i=1}^{n} (x_i-\theta_1)^2$$

$$\theta_2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \theta_1)^2$$

Sample variance

(2)

$$L(\theta) = \prod_{i=1}^{n}\binom{m}{x_i}\theta^{x_i}(1-\theta)^{m-x_i}$$

$$\ln(L(\theta)) = \sum_{i=1}^{n}\left(\ln\binom{m}{x_i} + x_i \ln(\theta) + (m - x_i)\ln(1-\theta)\right)$$

$$\frac{\partial}{\partial\theta}\ln(L(\theta)) = \sum_{i=1}^{n}\left(\frac{x_i}{\theta} - \frac{m-x_i}{1-\theta}\right) = 0$$

$$\sum_{i=1}^{n}\frac{x_i}{\theta} = \sum_{i=1}^{n}\frac{m-x_i}{1-\theta}$$

$$\sum_{i=1}^{n} x_i(1-\theta) = \sum_{i=1}^{n}(m-x_i)\theta$$

$$\theta\sum_{i=1}^{n} x_i = m\sum_{i=1}^{n}\theta$$

$$\theta = \frac{1}{m}\sum_{i=1}^{n} x_i$$

∴ MLE of θ is sample mean of observations.