2019

Project 3 : Derivatives Finance

Objective : How a stochastic volatility process can generate model prices that exhibit a Black-Scholes implied volatility smile

Submitted by-
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Volatility Smiles and the Stochastic Volatility Process

Data: We will price a call option on the S&P 500 index. The index level at the close of yesterday was equal to 1,065. Assume an annualized long term volatility for the index of 20% per year. The 1-year LIBOR rate is at 1.25%. For the stochastic volatility process take a = 0:95 and c = 0:85.

Objective: We want to price several European call options on the S&P 500 index with maturity equal to 1 year (250 trading days) and strike prices between 100 and 2000. We will do this under different correlation scenarios. The purpose is to show how a stochastic volatility process can generate model prices that exhibit a Black-Scholes implied volatility smile.

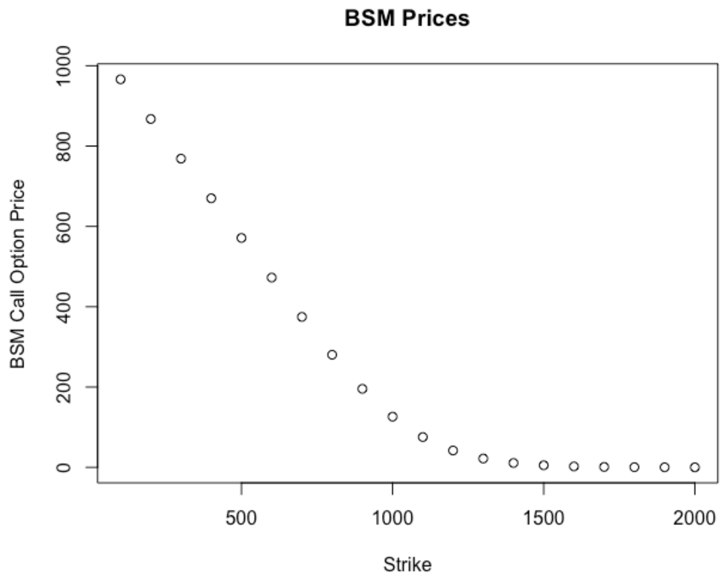
Methodology: Monte Carlo simulation method-simulate 1000 paths of the underlying and the volatility/variance under the risk neutral probability. Using the following formula recursively(simulate path starting with S0 = $1065 , Vl= (0.2)^2 ,simulating the paths at a daily frequency(Δt =1/250), continue till S250,repeat the procedure for 1000 paths) :

$$
\begin{aligned}
S_{(t+\Delta t)} &= S_t e^{(r-0.5V_t)\Delta t+\sqrt{\Delta t\ V_t}\ \epsilon^1_{(t+\Delta t)}} & (1)\\
V_{(t+\Delta t)} &= V_t + a(V_L - V_t)\Delta t + c\sqrt{\Delta t\ |V_t|}\left[\rho\epsilon^1_{(t+\Delta t)} + \sqrt{(1-\rho^2)}\epsilon^2_{(t+\Delta t)}\right] & (2)
\end{aligned}
$$

Result

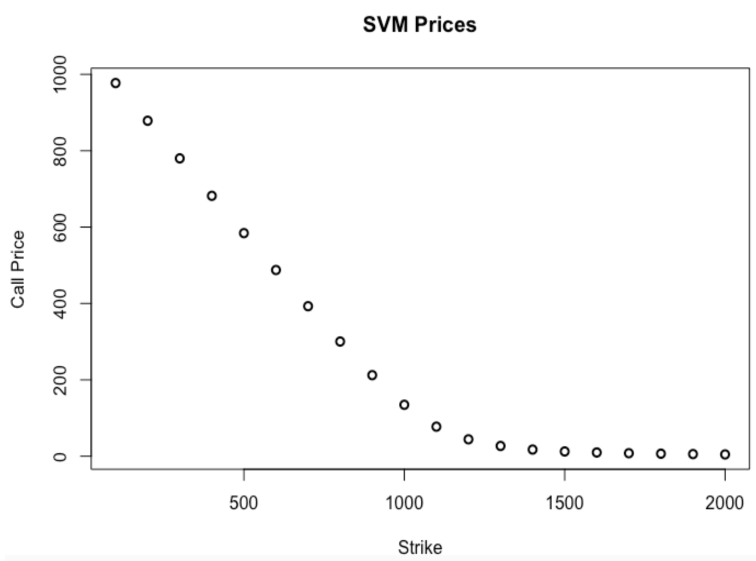
PART 1 -- BSM Prices :

X	BSM
100	966.24221995
200	867.48443990
300	768.72665985
400	669.96888840
500	571.21314455
600	472.53016528
700	374.62891893
800	280.37986717
900	195.51390259
1000	126.19121389
1100	75.43488098
1200	42.00615226
1300	21.97262657
1400	10.89483484
1500	5.16555626
1600	2.36043923
1700	1.04672874
1800	0.45310355
1900	0.19241868
2000	0.08050169



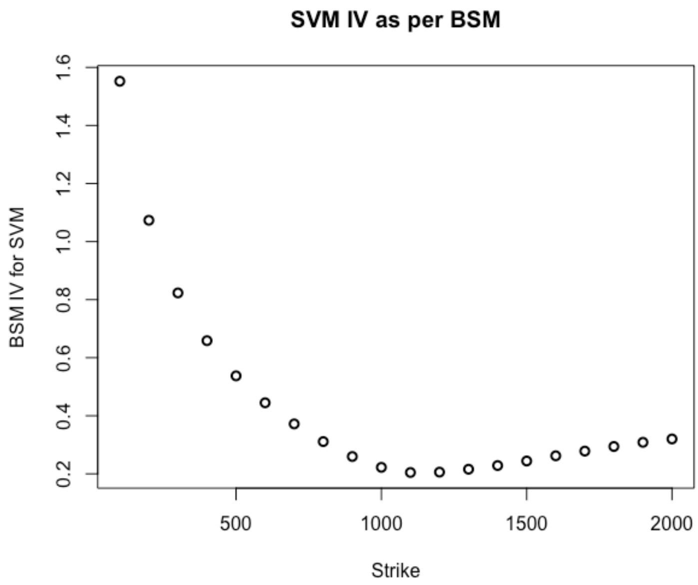
PART 2 --- Stochastic Volatility Model :

Strike	SVM Call Price
100	977.270345
200	878.538780
300	779.965602
400	681.850652
500	584.242223
600	487.653120
700	392.820221
800	300.363823
900	212.300850
1000	134.686329
1100	77.372146
1200	44.273394
1300	26.873572
1400	17.401848
1500	12.440155
1600	9.776832
1700	7.881773
1800	6.622628
1900	5.635050
2000	4.696951



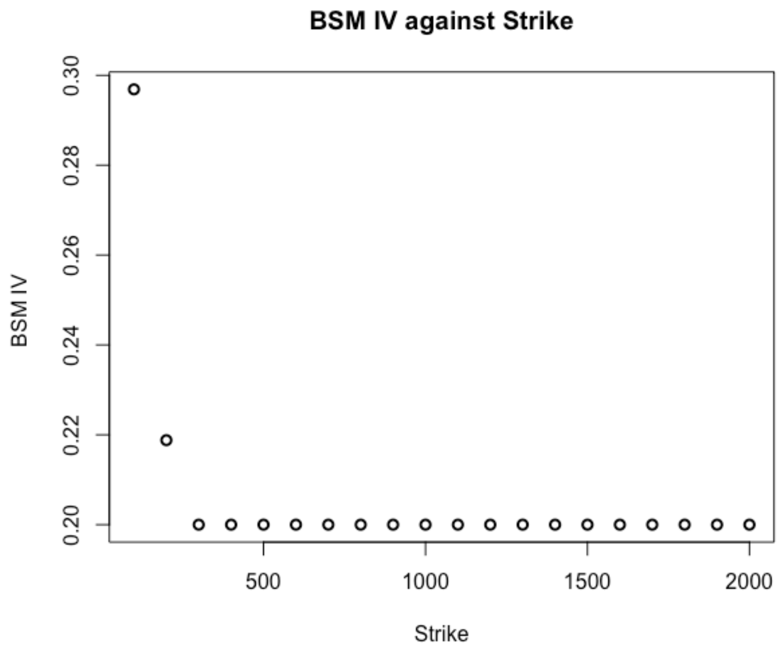
PART 3 --- Implied Volatility for SVM Calls :

Strike	IV for SVM Call
100	1.5522362
200	1.0733228
300	0.8231001
400	0.6586494
500	0.5374615
600	0.4446916
700	0.3719177
800	0.3109736
900	0.2595893
1000	0.2222610
1100	0.2045597
1200	0.2058410
1300	0.2158279
1400	0.2284444
1500	0.2441821
1600	0.2620529
1700	0.2782105
1800	0.2941058
1900	0.3085198
2000	0.3200302

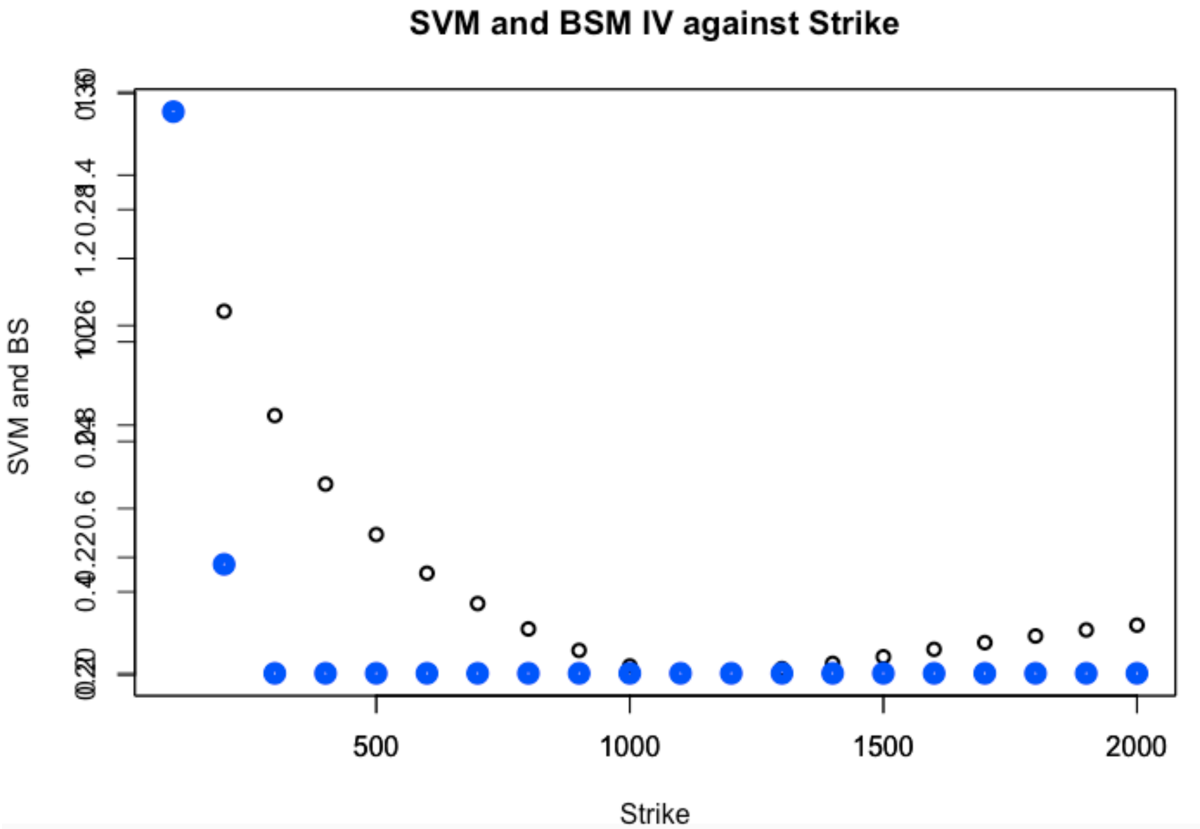


PART 3 --- Implied Volatility for BSM Calls :

Strike	IV for BSM
100	0.2969
200	0.2188
300	0.2000
400	0.2000
500	0.2000
600	0.2000
700	0.2000
800	0.2000
900	0.2000
1000	0.2000
1100	0.2000
1200	0.2000
1300	0.2000
1400	0.2000
1500	0.2000
1600	0.2000
1700	0.2000
1800	0.2000
1900	0.2000
2000	0.2000

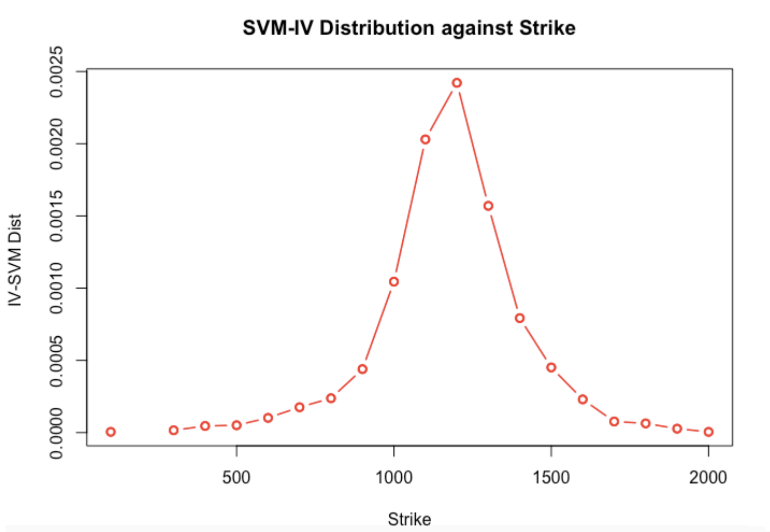


PART 3 --- Overlay Chart for Implied Volatilities :



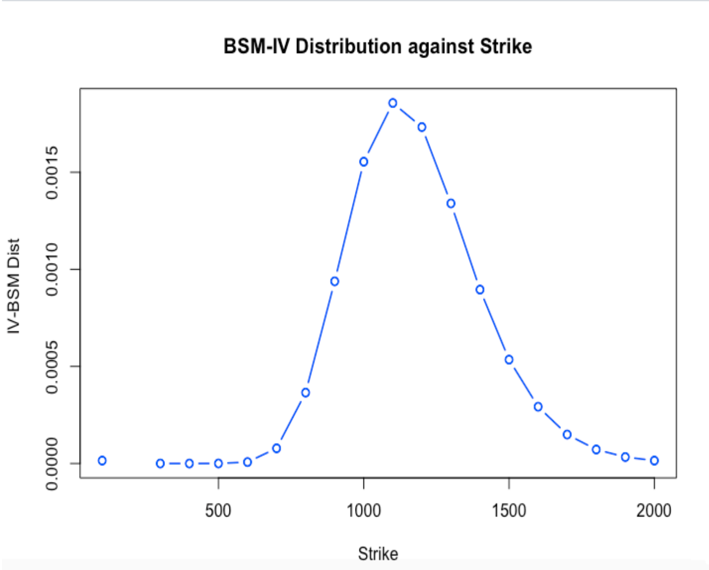
PART 4 ---- Implied Volatility Distribution for IV's of SVM Call Prices:

Strike	IV for SVM Call	ivsvm.dist
300	0.8231001	0.00001584
400	0.6586494	0.00004582
500	0.5374615	0.00005065
600	0.4446916	0.00010193
700	0.3719177	0.00017562
800	0.3109736	0.00023765
900	0.2595893	0.00043934
1000	0.2222610	0.00104485
1100	0.2045597	0.00203003
1200	0.2058410	0.00242154
1300	0.2158279	0.00156989
1400	0.2284444	0.00079281
1500	0.2441821	0.00045100
1600	0.2620529	0.00022984
1700	0.2782105	0.00007683
1800	0.2941058	0.00006359
1900	0.3085198	0.00002716
2000	0.3200302	0.00000495

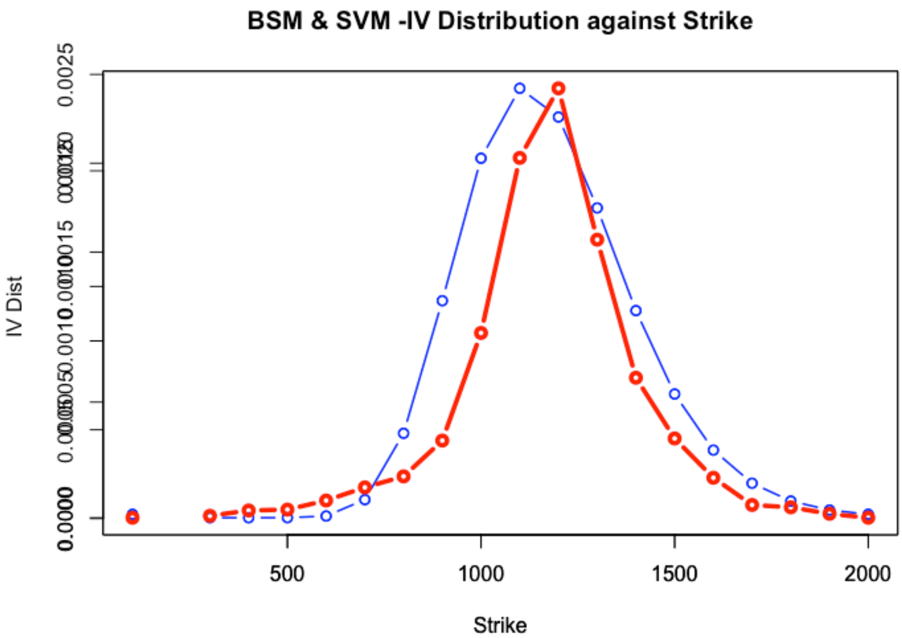


PART 4 --- Implied Volatility Distribution for IV's of BSM Call Prices:

Strike	IV for BSM	ivBSM.dist
300	0.2	0.00000000
400	0.2	0.00000000
500	0.2	0.00000020
600	0.2	0.00000728
700	0.2	0.00007817
800	0.2	0.00036522
900	0.2	0.00093831
1000	0.2	0.00155433
1100	0.2	0.00185664
1200	0.2	0.00173276
1300	0.2	0.00133952
1400	0.2	0.00089557
1500	0.2	0.00053485
1600	0.2	0.00029242
1700	0.2	0.00014914
1800	0.2	0.00007201
1900	0.2	0.00003329
2000	0.2	0.00001488



PART 4 --- Overlaying Plots of Implied Volatility Distribution derived from SVM and BSM Call Prices:

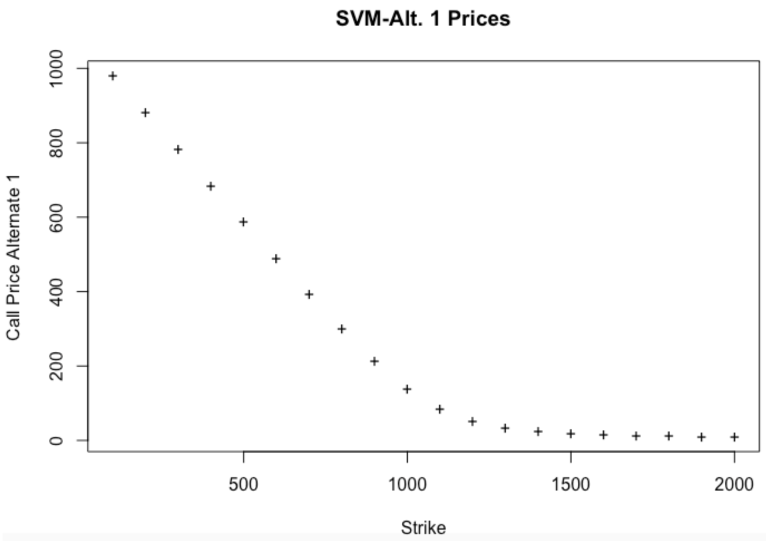


PART 5 -- In which way does a stochastic volatility model helps solving the implied volatility puzzle?
BSM cannot explain negative skewness and leptokurtosis commonly observed in the market returns. This may be attributable to the fact that in practice, the assumption(behind BSM) of constant volatility does not hold.

Volatility as a stochastic process is modeled into the SVM model of option pricing. The “c” in the model ,which is the volatility of volatility, helps to model clustering and produce heavier tails(which in turn create leptokurtic distribution of returns) ,while the correlation “p” between shocks to returns and shocks to volatility adds skew to the distribution(p<0 gives negative skew as observed in base case).Hence, SVM are useful since they help explain why options with different strikes and expirations have different BS implied volatilities(Volatility Smiles and Skew).

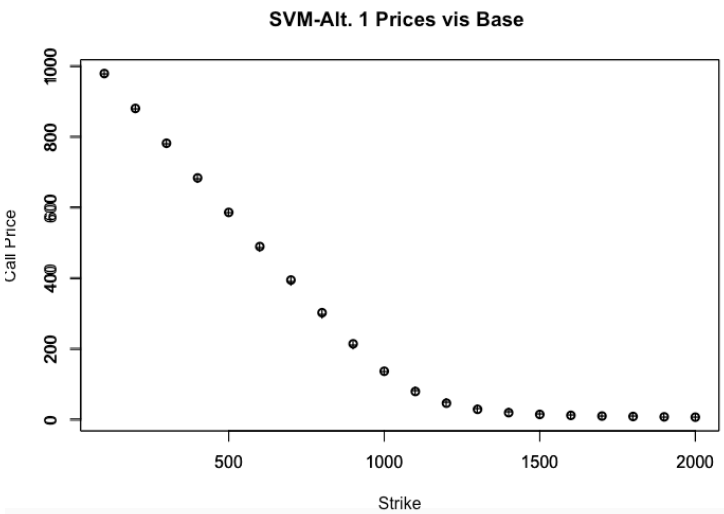
PART 6 --- Alternate 1 : When Correlation = 0
Call Prices under Alternate 1 vis-à-vis Base Case

Strike Price	Alternate 1	Base Case
100	980.941345	977.270345
200	882.183565	878.538780
300	783.443536	779.965602
400	684.991843	681.850652
500	586.872276	584.242223
600	489.553660	487.653120
700	394.105110	392.820221
800	301.110871	300.363823
900	213.541928	212.300850
1000	137.059494	134.686329
1100	83.619617	77.372146
1200	51.778583	44.273394
1300	33.462623	26.873572
1400	24.053231	17.401848
1500	18.673699	12.440155
1600	15.311918	9.776832
1700	12.900748	7.881773
1800	11.152195	6.622628
1900	9.862219	5.635050
2000	8.757150	4.696951



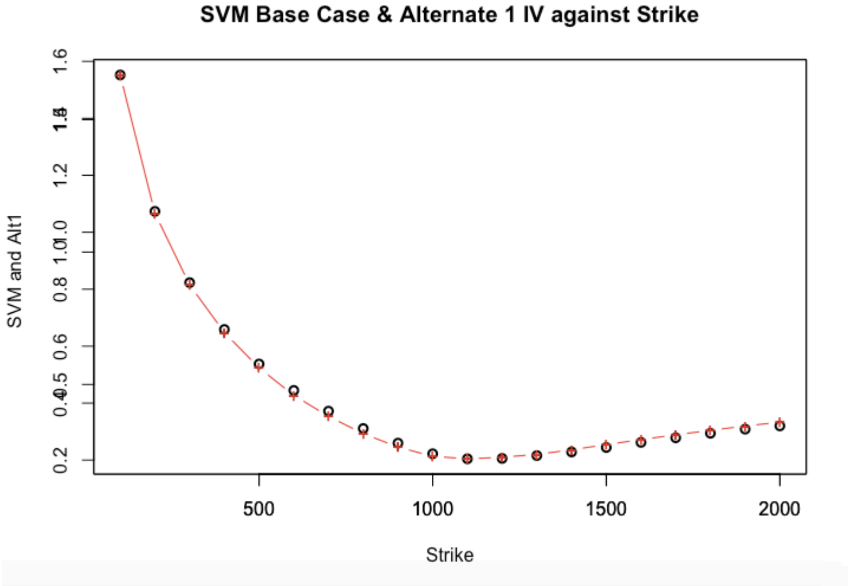
Comments : The options of the model derived from alternate 2 are priced slightly more than the base case

Overlaying Call Prices obtained in Alternative 1 with Base Case:



Implied Volatilities from SVM Call Prices under Alternative 1 compared with Base case:

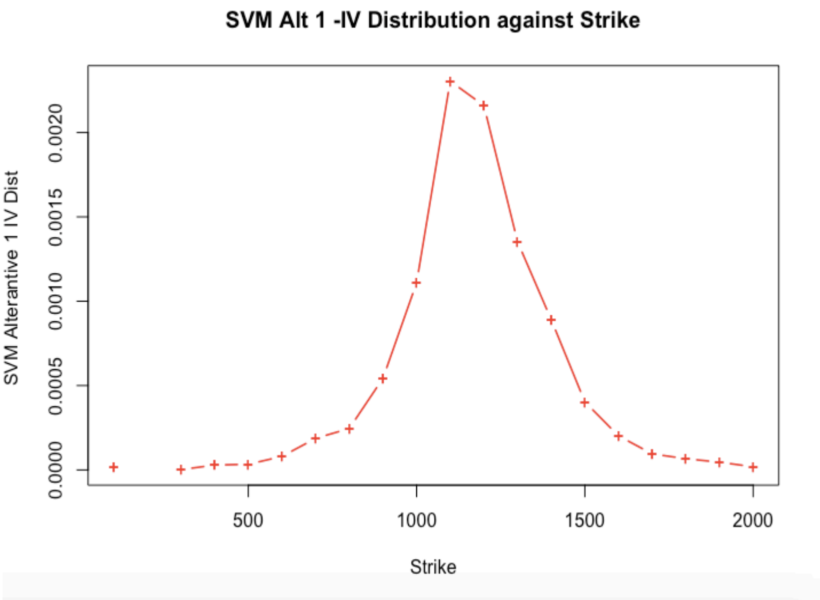
Strike	Alternate 1	Base Case
100	1.6690703	1.5522362
200	1.1468901	1.0733228
300	0.8759219	0.8231001
400	0.6962041	0.6586494
500	0.5625349	0.5374615
600	0.4589703	0.4446916
700	0.3793231	0.3719177
800	0.3142349	0.3109736
900	0.2636843	0.2595893
1000	0.2284352	0.2222610
1100	0.2192653	0.2045597
1200	0.2248938	0.2058410
1300	0.2358580	0.2158279
1400	0.2533583	0.2284444
1500	0.2725709	0.2441821
1600	0.2918672	0.2620529
1700	0.3097627	0.2782105
1800	0.3267223	0.2941058
1900	0.3429605	0.3085198
2000	0.3575299	0.3200302



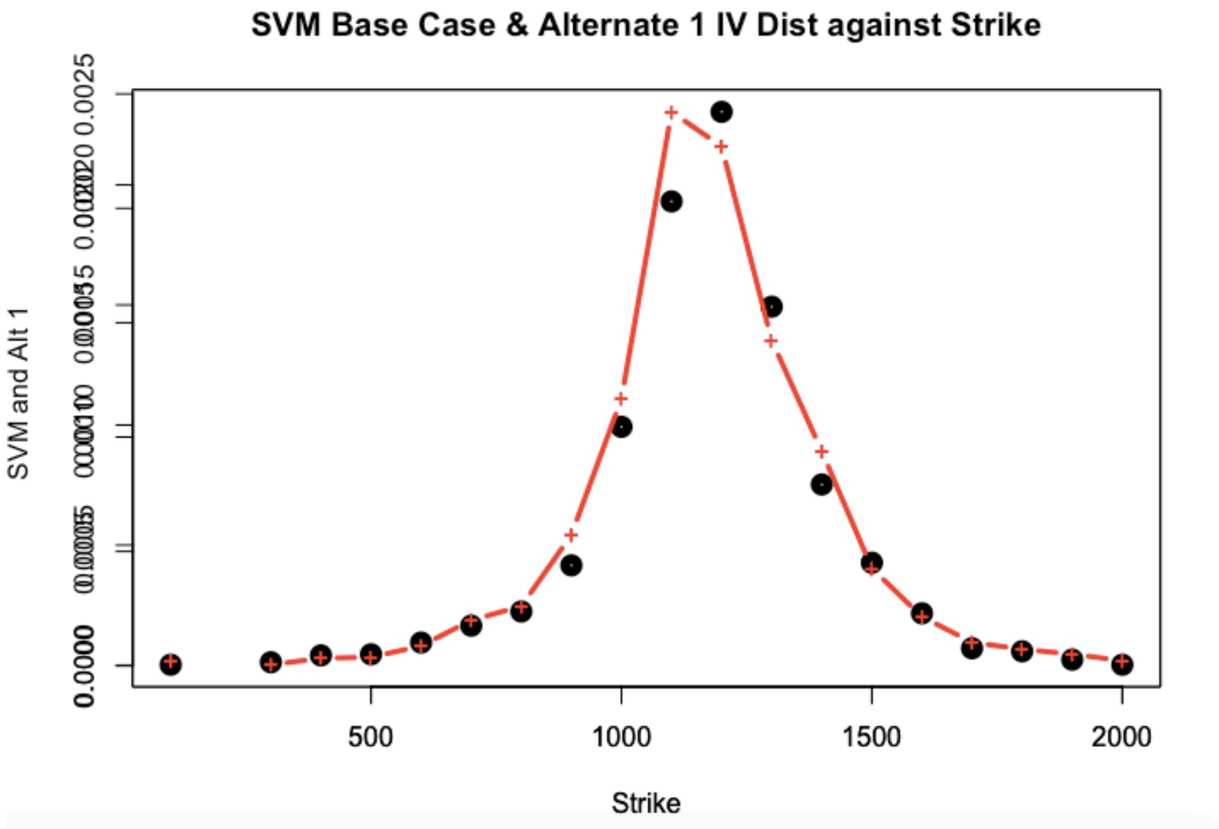
Comments : The Implied Volatilities for alternate 1 are higher than the IVs derived from Base Case SVM Call Prices

Distribution of Implied Volatility derived from SVM Call Prices under Alternate 1:

Strike Price	Base Case	Alternate 1
300	0.00001584	0.00000000
400	0.00004582	0.00000000
500	0.00005065	0.00000347
600	0.00010193	0.00002309
700	0.00017562	0.00012694
800	0.00023765	0.00034301
900	0.00043934	0.00081243
1000	0.00104485	0.00128566
1100	0.00203003	0.00188740
1200	0.00242154	0.00195006
1300	0.00156989	0.00143888
1400	0.00079281	0.00093471
1500	0.00045100	0.00052843
1600	0.00022984	0.00029060
1700	0.00007683	0.00013532
1800	0.00006359	0.00005095
1900	0.00002716	0.00002240
2000	0.00000495	0.00002522



Overlaying Distribution of Implied Volatility derived from SVM Call Prices under Alternate 1 with Base

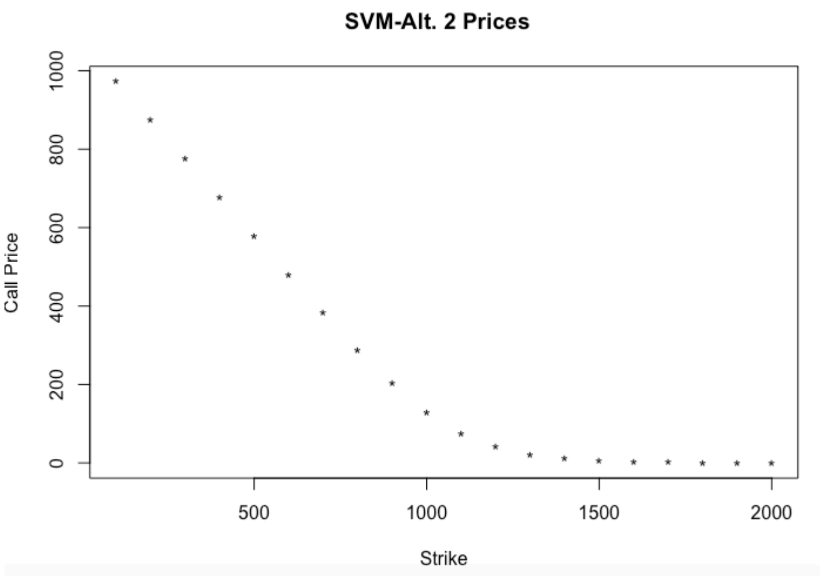


Comments :

The distribution is still leptokurtic but due to zero correlation between shocks, it is more symmetric than Base Case

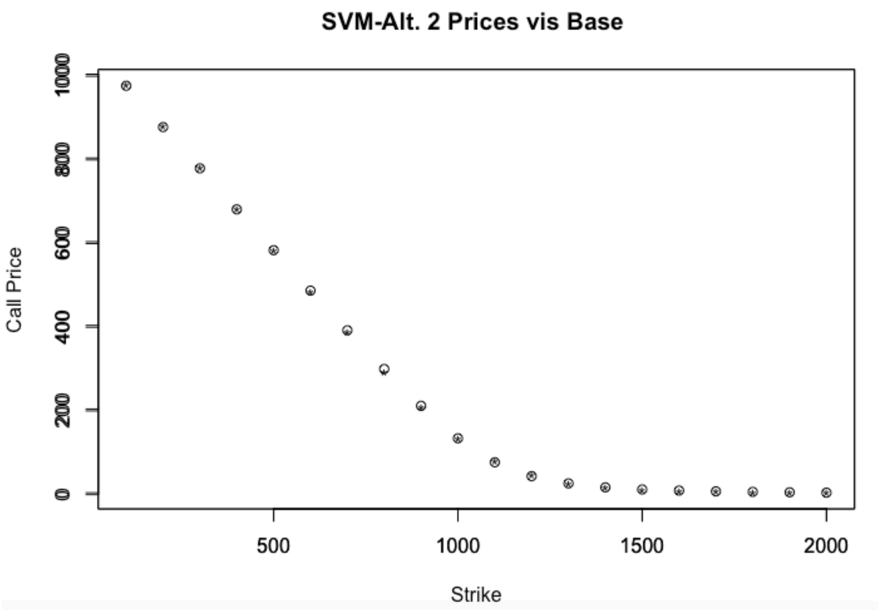
PART 7 --- Alternate 2 : c = 0.15
Call Price under Alternate 2 when compared with Base Case

Strike Price	Alternate 2	Base Case
100	972.49120410	977.270345
200	873.73342405	878.538780
300	774.97564400	779.965602
400	676.21786396	681.850652
500	577.49479065	584.242223
600	479.00259341	487.653120
700	381.77984544	392.820221
800	287.98721917	300.363823
900	202.31887451	212.300850
1000	129.50713569	134.686329
1100	75.56941154	77.372146
1200	41.13224908	44.273394
1300	21.08386853	26.873572
1400	10.38263773	17.401848
1500	4.96575006	12.440155
1600	2.45487354	9.776832
1700	1.29722369	7.881773
1800	0.64907641	6.622628
1900	0.22493153	5.635050
2000	0.05297877	4.696951



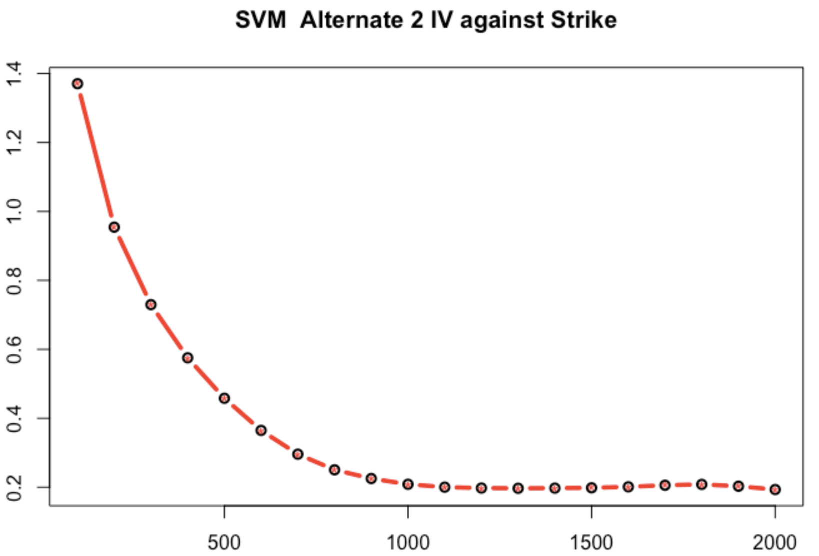
Comments: The options of the model derived from alternate 2 are priced slightly less than the base case

Overlay Plot for Call Prices obtained in Alternative 2 vis-a-vis Base Case:

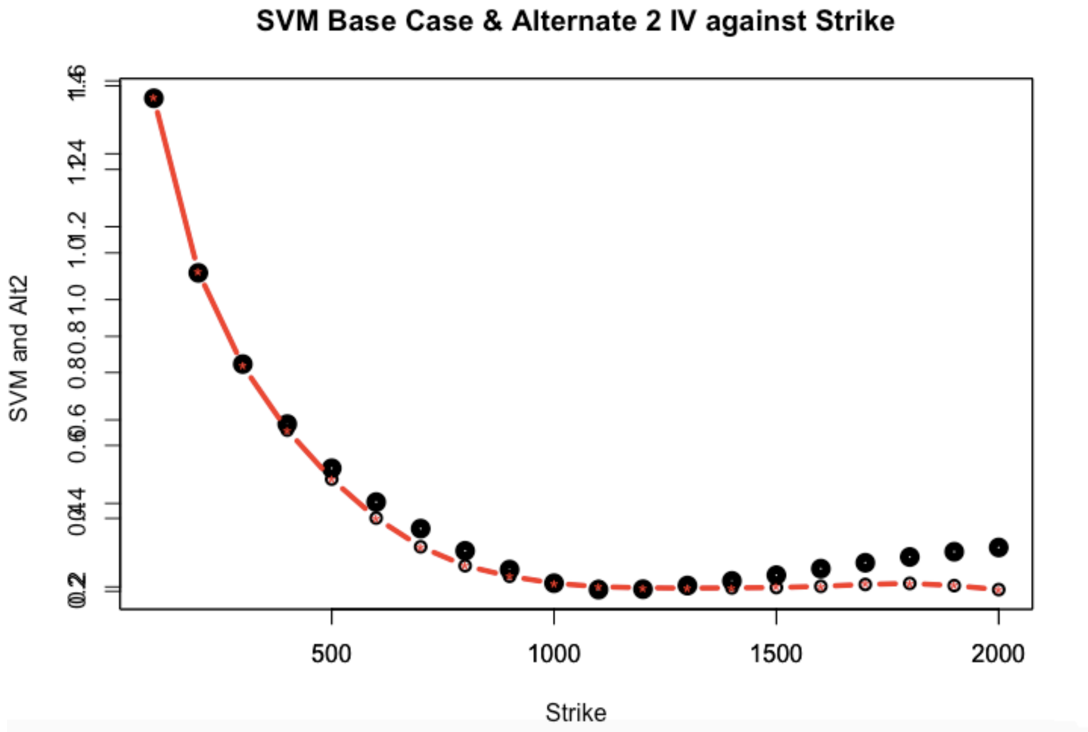


Implied Volatilities from SVM Call Prices under Alternative 2 compared with Base case:

Strike	Alternate 2	Base Case
100	1.3704481	1.5522362
200	0.9543477	1.0733228
300	0.7296530	0.8231001
400	0.5754797	0.6586494
500	0.4580591	0.5374615
600	0.3648928	0.4446916
700	0.2959869	0.3719177
800	0.2505927	0.3109736
900	0.2253355	0.2595893
1000	0.2087223	0.2222610
1100	0.2003167	0.2045597
1200	0.1977362	0.2058410
1300	0.1970176	0.2158279
1400	0.1974814	0.2284444
1500	0.1984197	0.2441821
1600	0.2012676	0.2620529
1700	0.2058384	0.2782105
1800	0.2083465	0.2941058
1900	0.2030088	0.3085198
2000	0.1934053	0.3200302

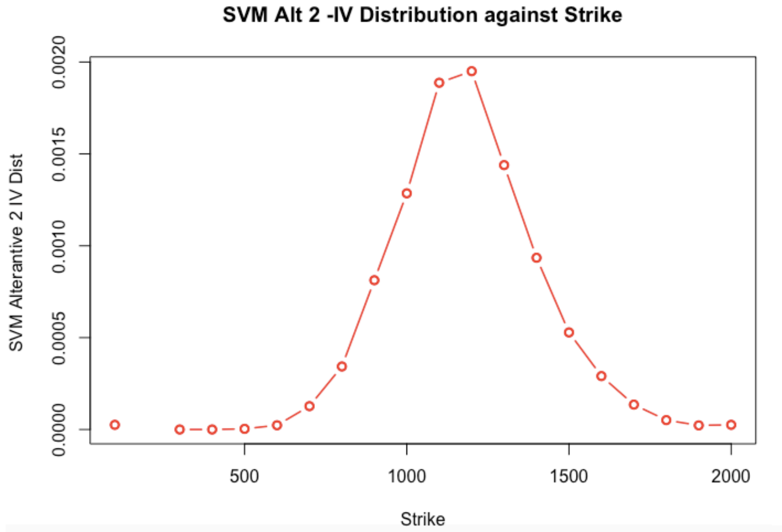


Comments: The Implied Volatilities for alternate 2 are lower than the IVs derived from Base Case SVM Call Prices



Distribution of Implied Volatility derived from SVM Call Prices under Alternate 2:

Strike Price	Base Case	Alternate 2
300	0.00001584	0.00000000
400	0.00004582	0.00000000
500	0.00005065	0.00000347
600	0.00010193	0.00002309
700	0.00017562	0.00012694
800	0.00023765	0.00034301
900	0.00043934	0.00081243
1000	0.00104485	0.00128566
1100	0.00203003	0.00188740
1200	0.00242154	0.00195006
1300	0.00156989	0.00143888
1400	0.00079281	0.00093471
1500	0.00045100	0.00052843
1600	0.00022984	0.00029060
1700	0.00007683	0.00013532
1800	0.00006359	0.00005095
1900	0.00002716	0.00002240
2000	0.00000495	0.00002522



Overlaying Distribution of Implied Volatility derived from SVM Call Prices under Alternate 2 with Base

Comments: The distribution for Alternate 2 is Leptokurtic but tails are thinner than base case