The rationale behind voluntary underpricing of equity issuances

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Introduction

Many actors of the financial sector, including mutual funds, pension funds, fundamental hedge funds and retail investors use the link between a company's market capitalization and its fundamental value as a starting point in their investment decisions.In the 2010 decade, however, the well documented underperformance of value investing challenges the effectiveness of corporate valuation in predicting future stock returns.

With ever increasing information flows on prospective earnings, risks and growth outlooks, markets have arguably grown a lot more efficient than what they used to be. Having an edge on other market participants is harder to achieve, and that translates into a surge of passive investing through index funds and ETFs. Passive investment funds now account for north of 20% of aggregate investment fund assets, versus 10% just a decade ago. Furthermore, 16% of actively managed assets in the US are invested through quantitative strategies, which attempt to enhance portfolio return through a variety of statistical methods.

The paper presented here proposes a framework to account for the growing importance of those market participants in the field of equity valuation (I). We will focus more specifically on the IPO process, and offer a view on how it may be optimal for a company to issue shares below its alleged fundamental value rather than on par: dividing the investors into three categories (fundamental investors, noise traders/signal traders and passive investors), we will see that the dispersion of opinions among the first can be overcome by voluntarily setting a low price. As fundamental investors gradually plough in the under-priced company (1), the positive trend attracts trend chasers who drive the stock even higher (2). When a critical market capitalization is reached, the company enters an index and benefits from capital inflows of passive investors (3).

In a second step, we will numerically simulate our model using several trends, weights and sizes of populations and illustrate how non-rational market participants can have a persistent impact on a stock price after IPO (II).

1 Model

We first start by describing the investors. In reality, this population might represent market participants who are likely to invest in a company because of its sector, geographical location, size... or simply because they do not have any constraint regarding their investment universe (that will be the case for retail investors). We assume that investors each have the same purchasing power.
All random variables used in this model are defined on a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\in\mathbb{N}}, \mathbb{P})$.

1.1 Fundamental investors

α individuals of the total population L are fundamental investors who receive an i.i.d signal $\delta_i \sim \mathcal{N}(\mu, \sigma^2)$, where μ represents the average estimate for the company's fundamental value and σ its standard deviation.

We consider the share price $S(0)$ set by the company to be a choice variable. Assuming that the number of shares is constant over time (we exclude stock splits/primary issuances after IPO), then we can simply normalize that number to 1. The share price $S(t)$, however, evolves, and investors have to compare it with δ_i, i.e. the market cap at which the company decides to price itself, versus their perception of its value.

In this situation, they face two alternatives:

- If $S(t) < \delta_i$: they decide to buy the stock as they think it is undervalued
- If $S(t) > \delta_i$, they decide to sell the stock

Eventually, the entire fraction α of fundamental investors will have taken a decision on whether to invest in the company or not. We call that time T, and it can be interpreted as the amount of time required for the share price to stabilize in a range and follow typical stochastic trends that hold in normal market conditions, such as the geometric Brownian motion. During the discrete time interval $[\![0, T]\!]$, fundamental investors gradually make their choice depending on δ_i.

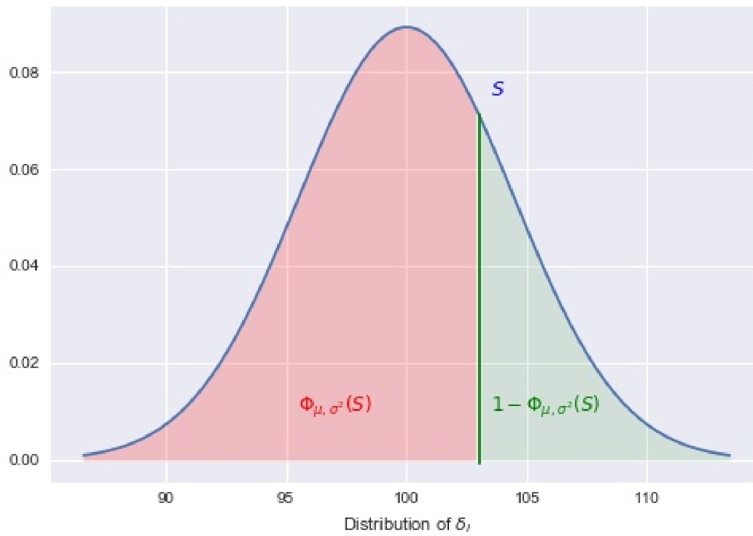

Fig 1: The area highlighted in green represents the proportion of fundamental investors $(1 - \phi_{\mu,\sigma}(S))$ willing to buy the stock when its price is set at S ($\mu = 100$).

However, in reality, it is more likely for investors who have the most extreme views (optimistic or pessimistic) to position themselves first, as they believe they have identified a serious mispricing that they can profit from. To account for that fact, we express the decision to buy or to sell as a probability both increasing in time and in the difference $\delta_i - S$. Indeed, as time passes, it is more likely that stock markets participants have made their choices.
An exponential distribution is convenient to model that process, as it is the distribution of the time between events in a Poisson process: a large cohort of investors gradually position themselves, and while most will buy or sell rather quickly, some will require a very long time to think.

We denote X_i the time taken by investor $i \in [\![1, \alpha]\!]$ to make a choice. We suppose that $X_i \hookrightarrow \varepsilon(\lambda_i)$, where $\lambda_i = (\delta_i - S)^+ = max(\delta_i - S, 0)$.

$$\forall t \in \mathbb{N}, \pi_\alpha(t) = \sum_{\{i \in [\![1,\alpha]\!], \delta_i > S\}} \mathbb{1}_{[X_i \leq t]}$$

Here, $\pi_\alpha(t)$ denotes the population of optimists who have bought the stock before time t. As $t \to \infty, \mathbb{1}_{[X_i \leq t]} \xrightarrow{a.s} 1$.
Hence, $\pi_\alpha(t) \xrightarrow{a.s} card(\{i \in [\![1, \alpha]\!], \delta_i > S\})$ and that is exactly the asymptotic result we want: in a population where all individuals are fundamental investors, those who will buy/sell the stock at some point in the future are those who

think it is under/overvalued.
If the population is now a continuum of fundamental investors $i \in [0, \alpha]$, we can express $\mathbb{E}(\pi_\alpha(t))$ in the following way:

$$\mathbb{E}(\pi_\alpha(t)) = \alpha \int_S^\infty \frac{1}{\sigma\sqrt{2\pi}}.e^{\frac{-(\delta_i-\mu)^2}{2\sigma^2}}.(1-e^{-\lambda_i t})\,d\delta_i$$

where under the integral, the normal density represents the mass of investors with valuations above S, and the exponential probability $(1-e^{-\lambda_i t})$ represents individual i's probability of buying before time t (N.B: $\mathbb{E}(\mathbb{1}_{[X_i \le t]}) = \mathbb{P}(X_i \le t)$).

$$\begin{aligned}
\mathbb{E}(\pi_\alpha(t)) &= \int_S^\infty \frac{1}{\sigma\sqrt{2\pi}}.e^{\frac{-(\delta_i-\mu)^2}{2\sigma^2}}.(1-e^{-(\delta_i-S)^+t})\,d\delta_i \\
&= \alpha(1-\Phi_{\mu,\sigma^2}(S)) - \frac{\alpha}{\sigma\sqrt{2\pi}}\int_S^\infty e^{-(\delta_i-S)^+t}e^{\frac{-(\delta_i-\mu)^2}{2\sigma^2}}\,d\delta_i \\
&= \alpha(1-\Phi_{\mu,\sigma^2}(S)) - \frac{\alpha}{\sigma\sqrt{2\pi}} exp\left(t(\mu-S)-\frac{\sigma^2t^2}{2}\right)\int_S^\infty e^{\frac{(\delta_i-(\mu-\sigma^2t))^2}{2\sigma^2}}\,d\delta_i
\end{aligned}$$

where $\Phi_{\mu,\sigma^2}(S)$ is the CDF of $\mathcal{N}(\mu,\sigma^2)$ evaluated at S. Noting that we have transformed the expression in the second integral to display a Gaussian density, we finally have the following result:

$$\mathbb{E}(\pi_\alpha(t)) = \alpha(1-\Phi_{\mu,\sigma^2}(S)) - \alpha exp\left(t(\mu-S)-\frac{\sigma^2t^2}{2}\right)\left(1-\Phi_{\mu-\sigma^2t,\sigma^2}(S)\right)$$

As $t \to \infty$, we notice once again that $\mathbb{E}(\pi_\alpha(t)) \to \pi_\alpha^\infty = \alpha(1-\Phi_{\mu,\sigma^2}(S))$. If the market were constituted only of fundamental investors and if we waited long enough, the proportion of buyers would be identical to the proportion of optimists who think the stock is undervalued at $t = 0$.

To determine the offer, i.e. the set of market participants willing to sell the stock at price S because they consider it too high, we can proceed in a completely symmetric way by considering $\mu_i = (S-\delta_i)^+ = max(S-\delta_i, 0)$.
If $\pi_{\overline{\alpha}}(t)$ denotes the number of persons who have sold before date t, we have: $\pi_{\overline{\alpha}}^\infty = \alpha(\Phi_{\mu,\sigma^2}(S))$ (the red area in Fig 1). Furthermore:

$$\forall t \in \mathbb{N}, \mathbb{E}(\pi_{\overline{\alpha}}(t)) = \alpha\Phi_{\mu,\sigma^2}(S) - \alpha exp\left(t(\mu-S)-\frac{\sigma^2t^2}{2}\right)\Phi_{\mu-\sigma^2t,\sigma^2}(S)$$

Remark: While $\pi_{\overline{\alpha}}^\infty + \pi_\alpha^\infty = \alpha$, for all $t \in \mathbb{N}, \pi_{\overline{\alpha}}(t) + \pi_\alpha(t) < \alpha$ (among all market participants, if a finite time has passed, some undecided investors still exist in the overall population α).

These two conflicting selling and buying pressures will yield an equilibrium price that we must characterize. At a given time t, if there are as many buyers as sellers, i.e $\pi_\alpha(t) = \pi_{\overline{\alpha}}(t)$, S obviously doesn't move (recall that all investors have the same purchasing power).

However, if $\pi_\alpha(t) > \pi_{\overline{\alpha}}(t)$, S must increase to balance supply and demand. For $t > 0$, where it is defined, we denote $q_\alpha(t) = \frac{\pi_\alpha(t)}{\pi_{\overline{\alpha}}(t)+\pi_\alpha(t)}$, representing the buyers' weight in the entire population.

$q_\alpha^t \in [0,1]$, and since the δ_i's are normally distributed with mean μ and variance σ^2, we can interpret $q_\alpha(t)$ as the area under the curve of $\mathcal{N}(\mu,\sigma^2)$ for $x \leq S$, representing the level of demand for price S at time t. If S were an equilibrium price, the selling and buying pressures would cancel each other out, and q_α would be equal to $\frac{1}{2}$.

If S is not an equilibrium, the demand for $S(t)$ yields a certain proportion of buyers $q_\alpha(t) \neq \frac{1}{2}$. To satisfy the demand, $S(t+1)$ shifts from its central value $S(t)$ and becomes the price for which exactly $q_\alpha(t)$ persons would be willing to sell.

This allows us to write the price adjustment process for S:

$$\begin{aligned} S(t+1) &= \Phi^{-1}_{S(t),\sigma^2}(q_\alpha(t)) \\ &= \Phi^{-1}_{S(t),\sigma^2}\left(\frac{\pi_\alpha(t)}{\pi_{\overline{\alpha}}(t)+\pi_\alpha(t)}\right) \end{aligned}$$

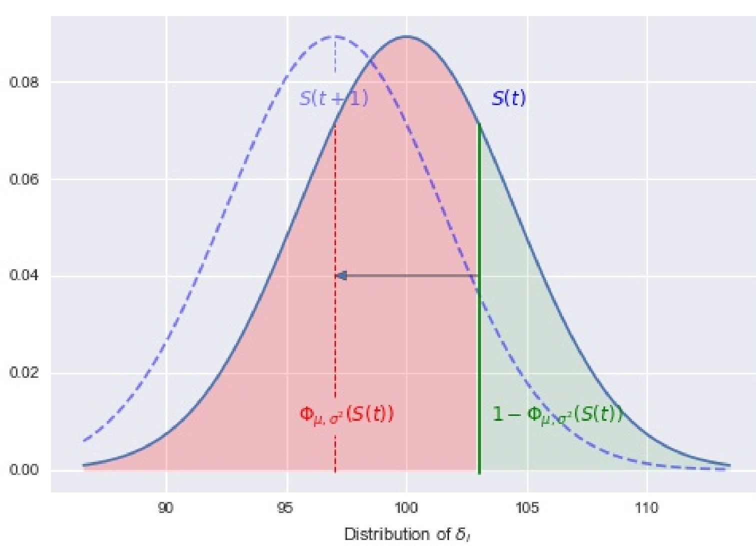

Fig 2: Price adjustment process: the price S is set at the level that supplies the current demand, resulting in an oscillating behaviour around the mean μ .

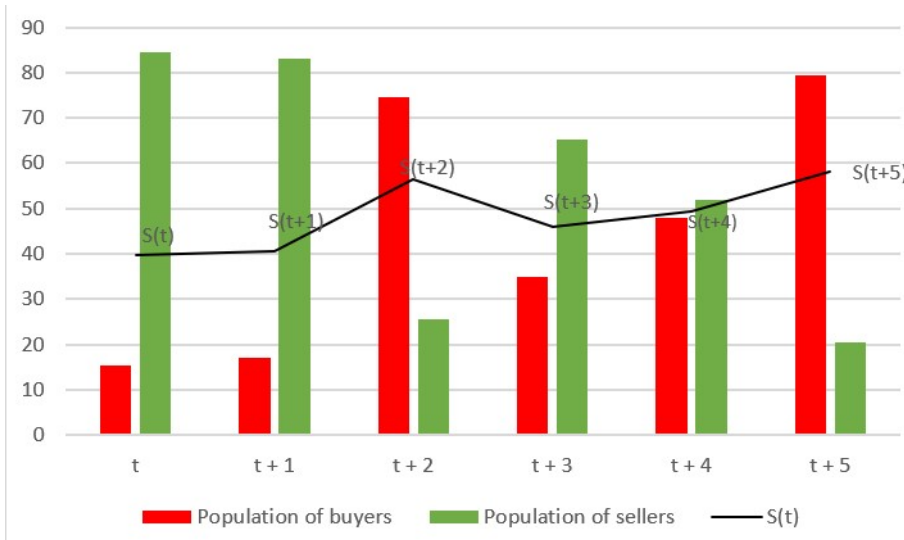

Fig 2bis: Price adjustment process illustrated (Walrasian auction system) .

If no new population entered the market, S would endlessly oscillate symmetrically between an initial value and its symmetrical point with respect to the normal density. But since new fundamental investors who are closer to the mean value μ enter the market at an exponential decay rate, S approaches its equilibrium value $S^* = \mu$, which we can verify analytically:

$$\begin{aligned} S^*(t+1) &= S^*(t) \\ \Leftrightarrow \Phi^{-1}_{S^*(t),\sigma^2}\left(\frac{\pi_\alpha(t)}{\pi_{\overline{\alpha}}(t)+\pi_\alpha(t)}\right) &= S^*(t) \\ \Leftrightarrow \frac{\pi_\alpha(t)}{\pi_{\overline{\alpha}}(t)+\pi_\alpha(t)} &= \frac{1}{2} \end{aligned}$$

Since $\frac{\pi_\alpha(t)}{\pi_{\overline{\alpha}}(t)+\pi_\alpha(t)} \xrightarrow[t\to\infty]{} \frac{\pi_\alpha^\infty}{\pi_{\overline{\alpha}}^\infty+\pi_\alpha^\infty} = 1 - \Phi_{\mu,\sigma^2}(S^*)$, we retrieve the equality:

$$\frac{1}{2} = 1 - \Phi_{\mu,\sigma^2}(S^*) \Leftrightarrow S^* = \mu$$

As we expected, if only fundamental investors were participating in the market, S would asymptotically stabilize around the mean of the distribution. We now have to model the rest of investors and see their impact on S.

1.2 Momentum traders

At time t, investor $i \in [\![1, \beta]\!]$ makes the decision to sell, buy or wait. As β denotes the population of trend-chasers / noise traders, this decision is motivated both by the trajectory of S between time 0 and $t-1$, and by randomness at the individual level. In our model, at the aggregate level, the trend part will be given a weight $w(t) \in [0,1]$, and the idiosyncratic randomness will have a weight $1-w(t)$. This weight may indeed change as time passes, because a trend is easier to identify when significant data have been gathered on S rather than directly at $t = 0$.

We respectively denote $v(t)$ and $u_i(t)$ the deterministic and random signals at time t for investor i. They both take their values in the segment $[-1, 1]$.

NB: We can in fact argue that the specificity of investor i should be captured by the random term u_i. Indeed, if we aim to model a herd whose members react in a similar way, with some noise due to each individual's degree of freedom, then we should adjust $w(t)$, the weight attributable to that freedom, rather than v which should be the same for all.

In this case, $((1-w(t))u_i + w(t)v(t))(\Omega) = [-1, 1]$. In absolute value, this represents the probability that investor i positions herself.

If $(1-w(t))u_i(t) + w(t)v(t) \geq 0$, i buys S with a probability $(1-w(t))u_i(t) + w(t)v(t)$. Else, i sells S with a probability $-((1-w(t))u_i(t) + w(t)v(t))$.

We denote $\pi_\beta(t)$ the population of optimistic trend-chasers who buy the stock at price $S(t)$ at time t.

$$\forall t \in \mathbb{N}, \pi_\beta(t) = \sum_{i=1}^{\beta} X_i(t) \mathbb{1}_{[(1-w(t))u_i(t)+w(t)v(t)\geq 0]}$$

, where $X_i(t) \hookrightarrow \mathcal{B}((1-w(t))u_i(t) + w(t)v(t))$.

Symmetrically, the population of pessimistic trend-chasers who sell the stock at price $S(t)$ at time t is:

$$\forall t \in \mathbb{N}, \pi_{\overline{\beta}}(t) = \sum_{i=1}^{\beta} X_i(t) \mathbb{1}_{[(1-w(t))u_i(t)+w(t)v(t)\leq 0]}$$

, where $X_i(t) \hookrightarrow \mathcal{B}(-((1-w(t))u_i(t) + w(t)v(t)))$

N.B: In particular, we have no guarantee that $\pi_\beta(t) + \pi_{\overline{\beta}}(t) \xrightarrow[t\to\infty]{} \beta$: trend-chasers enter and exit the market at every instant and there is no reason for them to be all positioned asymptotically.

Before making any further assumption on the shape of the trend $v(t)$, we can draw some conclusions about the average value that both π^t_β and $\pi^t_{\overline{\beta}}$ take. By definition of the conditional expectation,

$$\mathbb{E}(X_i(t)\mathbb{1}_{[(1-w(t))u_i(t)+w(t)v^t\geq 0]}) = \mathbb{E}(X^t_i|(1-w(t))u^t_i+w(t)v(t) \geq 0) = (1-w(t))u_i(t)+w(t)v(t)$$

for all i such that $(1-w(t))u_i(t) + w(t)v(t) \geq 0$ (indeed, X_i follows a Bernoulli distribution of parameter $(1-w(t))u_i(t) + (1-w(t))v(t)$, so its expected value is just the value of the parameter).
Hence,

$$\mathrm{E}(\pi_{\overline{\beta}}(t)) = \textstyle\sum_{\{i\in[\![1,\beta]\!]|(1-w(t))u_i(t)+w(t)v(t)\leq 0\}} |(1-w(t))u_i(t) + w(t)v(t)|$$

In this case,

$$\begin{aligned}
\mathbb{E}(\pi_\beta(t)) &= w_t\beta v(t) + \sum_{\{i\in[\![1,\beta]\!]|(1-w(t))u_i(t)+w(t)v(t)\geq 0\}} (1-w(t))u_i(t) \\
\mathbb{E}(\pi^t_{\overline{\beta}}) &= \sum_{\{i\in[\![1,\beta]\!]|(1-w(t))u_i(t)+w(t)v(t)\leq 0\}} |(1-w(t))u_i(t) + w(t)v(t)| \\
&= -w(t)\beta v(t) - \sum_{\{i\in[\![1,\beta]\!]|(1-w_t)u_i(t)+w(t)v(t)\leq 0\}} (1-w(t))u_i(t)
\end{aligned}$$

If the $u_i(t)$ are i.i.d, then denoting $B_t = card\{i \in [\![1,\beta]\!]|(1-w(t))u_i(t) + w(t)v(t) \geq 0\}$, on average, $\pi_\beta(t)$ will be the sum of $w(t)\beta v(t)$ (weighted value of the common signal) and of a sum of i.i.d uniformly distributed variables, which yield a support $[-B_t, B_t]$, a mean 0, and a variance $2 * \frac{B_t}{12} = \frac{B_t}{6}$ (for details,it is a shifted Irwin-Hall distribution). Symmetrically, if $\overline{B_t} = card\{i \in [\![1, \beta]\!] | (1 - w(t))u_i(t) + w(t)v(t) \leq 0\}$, on average, $\pi_{\overline{\beta}}(t)$ will be the sum of $-w(t)\beta v(t)$ and of an Irwin-Hall random variable with support $[-B_t, B_t]$, mean 0 and variance $\frac{\overline{B_t}}{6}$.

The higher the population β, the closer this random term will be to a sample of a normal distribution with mean 0 and variance $\frac{\overline{B_t}}{6}$. When we move to a continuum of trend-chasers, this is in fact exactly the result:

$$\mathbb{E}(\pi_\beta(t)) = w_t \beta v(t) + (1 - w(t))Z$$

where $Z \hookrightarrow \mathcal{N}(0, \frac{\beta}{6})$.

This leads to the following observation: Since Z is centered at 0, on average, the value of the population of trend-chasers is decreasing with the degree of freedom $1 - w$ of each member of the herd. If more members begin to interpret in the same way the signal $S(t-1), S(t-2), ..., S(0)$, trend-chasers will participate more in the market, whether that involves buying or selling S. Depending on the nature of that common signal $v(t)$, sheep-like behaviour may work in a self-reinforcing way, as a positive/negative signal attracts more buyers/sellers.

At this stage, we propose several combinations of signals $v(t)$ and weights $w(t)$ that we will try in our numerical simulation:

- $v(t) = (S(t) > S(0)), w(t) = w$ for some fixed $w \in [0, 1]$: At each time, the deterministic part of the decision is due to the comparison between the current price and the price at issuance. This first example is perhaps not the most interesting, as a constant weight w fails to describe the fact that a trend is more identifiable with much data on the past than with very few. Typically, we would like $w(t)$ to increase in t.

- $v(t) = \frac{1}{t-1} (\#\{i \in [1, t-1] | S(i) > S(i-1)\} - \#\{i \in [1, t-1] | S(i) \leq S(i-1)\})$ and $w(t) = 1 - e^{-\lambda t}$ for some fixed $\lambda \in \mathbb{R}_+^*$. This is simply a form of voting rule: counting the number of days when stock drops and when stock rises and rescaling this between -1 (the stock has kept dropping) and +1 (it has kept rising).

- $v(t) = 2\frac{\pi_\alpha(t)}{\pi_\alpha(t) + \pi_{\overline{\alpha}}(t)} - 1$ with the same $w(t)$. This trend rule could be interpreted as semi-rational: trend-chasers are here supposed to be aware of the behaviour of α 's: their decisions will be partly derived from fundamental investors communicating on their positions.

1.3 Passive investors

This is the simplest category to model: The full cohort of passive investors $i \in [\![1, \gamma]\!]$ will enter the market and purchase the stock if it stays above a critical level $\underline{S}$ for a fixed period D.This category is meant to represent long only passive investors participating into the market through index funds and ETFs. As the latter usually target indices, being part of an index can be critical for a company to benefit from passive capital inflows. Simple though it may seem, this category's importance has grown to be considerable: 3 of the largest asset managers in the world (BlackRock, Vanguard and Fidelity), accounting for 17 trillion dollars of AUM, have reached this size through index investing.
Thus, we simply define $\pi_\gamma(t)$ in the following way:

$$\forall t \in \mathbb{N}, \pi_\gamma(t) = \begin{cases} \gamma & \text{if } \exists t_0 \in [0, t-D] \text{ s.t } S(k) \geq \underline{S} \text{ for all } k \in [t_0, t_0 + D] \\ 0 & \text{otherwise} \end{cases}$$

It will be most interesting to set that critical stock price $\underline{S}$ strictly above the mean value of the fundamental investors' distribution μ, to see if there is a way for the firm to benefit from the behaviour of trend chasers to reach a market capitalization that it wouldn't be able to sustain if it were only facing fundamental investors.

1.4 Aggregate equilibrium pricing

We retain the same price adjustment process we used in section 2.1 to describe fundamental investors only, except that we include now the role of both trend chasers and passive investors. Using the same notations,

$$q_{\alpha\beta\gamma}(t) = \frac{\pi_\alpha(t) + \pi_\beta(t) + \pi_\gamma(t)}{\pi_{\overline{\alpha}}(t) + \pi_\alpha(t) + \pi_{\overline{\beta}}^t + \pi_\beta^t} + \pi_\gamma(t)$$

$$\begin{aligned} S(t+1) &= \Phi^{-1}_{S(t),\sigma^2}(q_{\alpha\beta\gamma}(t)) \\ &= \Phi^{-1}_{S(t),\sigma^2}\left(\frac{\pi_\alpha(t) + \pi_\beta(t) + \pi_\gamma(t)}{\pi_{\overline{\alpha}}(t) + \pi_\alpha(t) + \pi_{\overline{\beta}}(t) + \pi_\beta(t) + \pi_\gamma(t)}\right) \end{aligned}$$

2 Numerical Simulation

2.1 Fundamental investors only

Population: $\alpha = 10\,000$
Fundamental investors distribution: $\delta_i \sim \mathcal{N}(\mu, \sigma^2)$ with $\mu = 50, \sigma = 20$
T = 4 days
Frequency = 500 price samples
IPO initial price: $S(0) = 60$

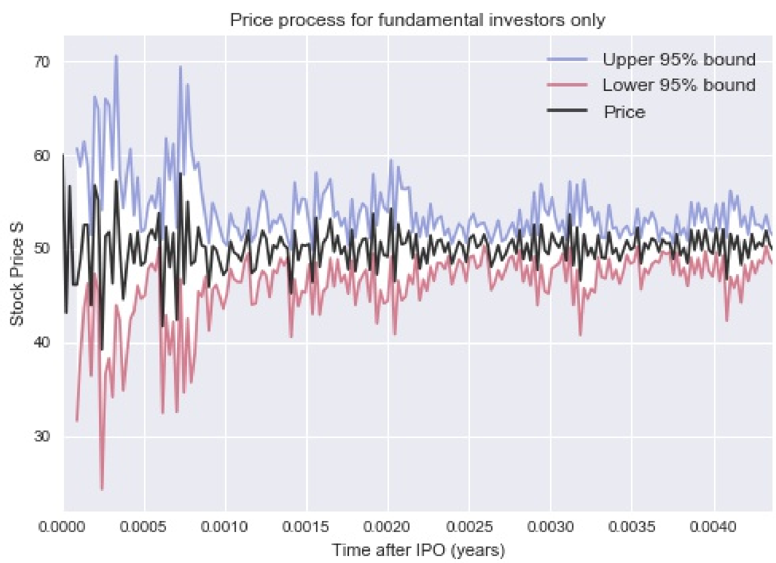

Fig 3: Price trajectory for fundamental investors only: the price $S(0)$ is set at a different level than the mean μ. S oscillates and eventually converges towards μ, with a dispersion proportional to σ.

2.2 Fundamental investors + trend-chasers

We keep every parameter from the previous simulation unchanged: same α with same distribution, same T, same price frequency, same $S(0)$.
We add our new population β and numerically simulate the price trajectory while changing the size of β, the degree of coordination w_t, and the signal $v(t)$

2.2.1 Binary signal: $v(t) = (S(t) > S(0))$

Fig 4: low β ($\frac{1}{6}$ of the total population), low $w = 0.1$: trend-chasers account for a small amount of the total population and are poorly coordinated. They can't dictate the asymptotic tendency of the stock. On the contrary, since they enter the market all at the same time, they shift the price further beyond the fundamental consensus μ and help investors α coordinate faster: the price reaches its equilibrium level faster than with α investors only.

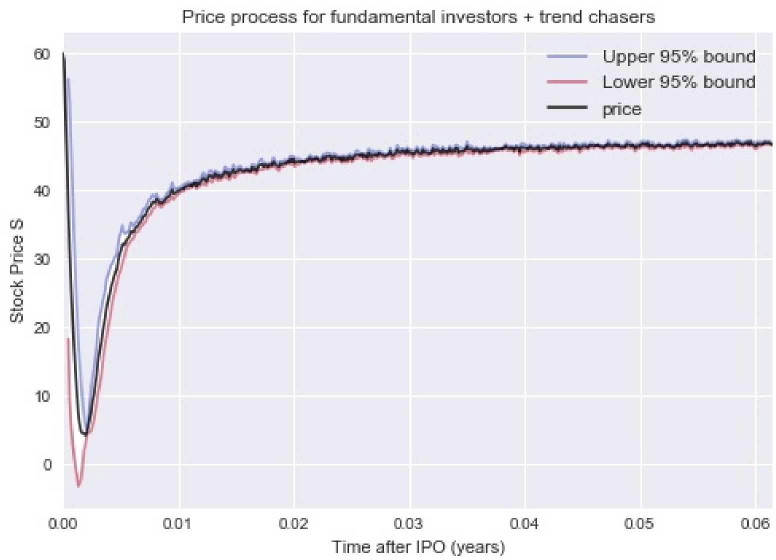

Fig 5: low β ($\frac{1}{6}$ of the total population), strong $w = 0.45$: Now trend-chasers are more coordinated. Since the price is initially set too high, they follow the first fundamental investors to short the stock and cause a sharp drop. However, since they account for a minority of the population, eventually, the arrival of new fundamental investors on the market will help the price stabilize at its normal level μ.

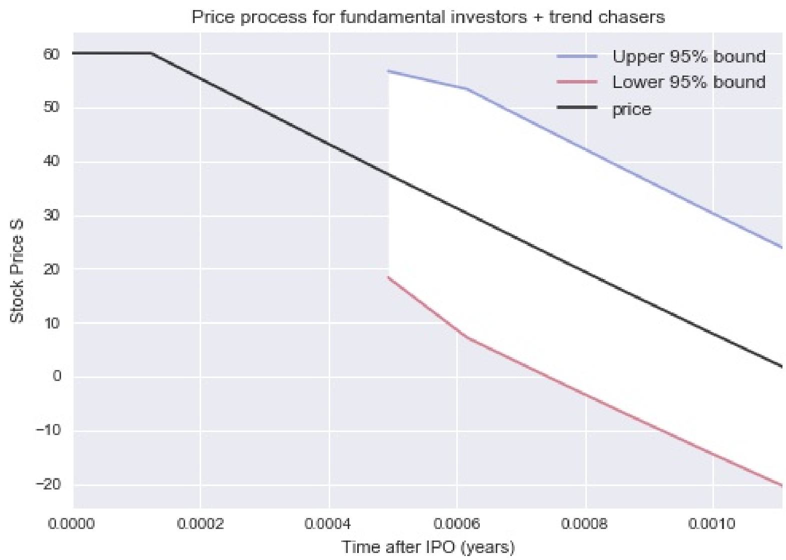

Fig 6: high β ($\frac{3}{4}$ of the total population), strong $w = 0.45$: trend-chasers are both coordinated and numerous with respect to the entire population. As before, they follow the first fundamental investors to short the stock, but the importance of the price attack causes the stock to drop to 0: the price never reaches its fundamental equilibrium value μ.

Fig 7: high β ($\frac{3}{4}$ of the total population), strong $w = 0.45$, $S(0) < \mu$: This is the same dynamic as in Fig 6 working in the opposite way: trend-chasers keep following the tendency set by the first fundamental investors and create a price bubble.

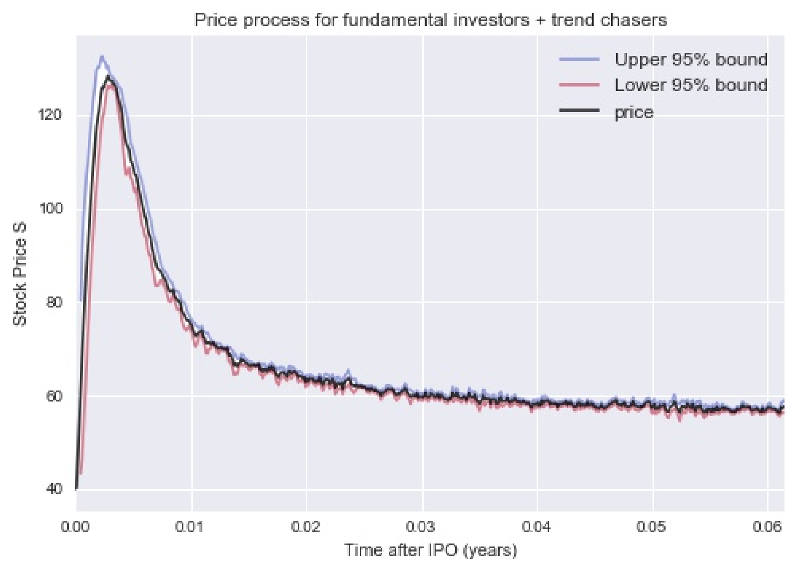

Fig 8: low β ($\frac{1}{6}$ of the total population), strong $w = 0.45$, $S(0) < \mu$: This is the same dynamic as in Fig 5 working in the opposite way: trend-chasers temporarily set the tendency but are eventually corrected by fundamental investors.

2.2.2 Voting signal:

$$v(t) = \frac{1}{t-1}\left(\#\{i \in [1, t-1] | S(i) > S(i-1)\} - \#\{i \in [1, t-1] | S(i) \leq S(i-1)\}\right)$$
$$w(t) = 1 - e^{-\lambda t} \quad \text{(increasing coordination)}$$

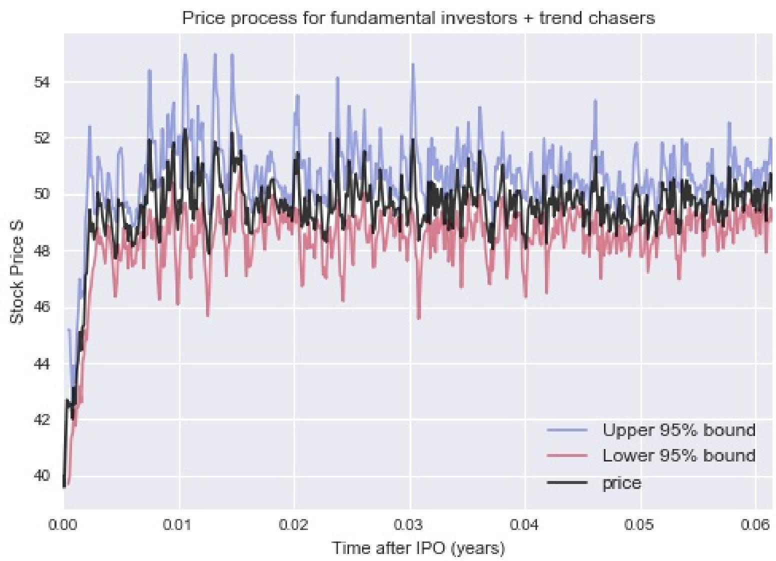

Fig 9: low β ($\frac{1}{6}$ of the total population), , $S(0) < \mu$: if trend-chasers are few and follow the voting rule to evaluate the strength of the trend, they simply add noise but don't dictate the asymptotic price.

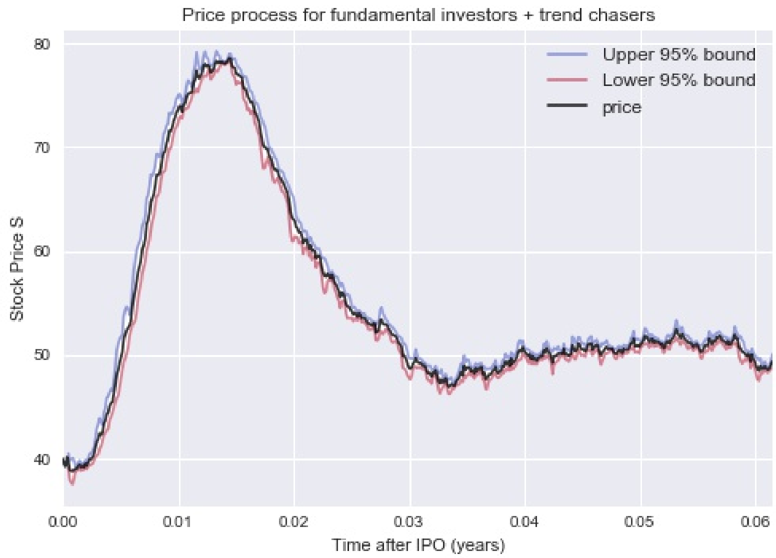

Fig 10: high β ($\frac{3}{4}$ of the total population), , $S(0) < \mu$: if trend-chasers are numerous and follow the voting rule , a quick coordination can create a substantial deviation from the fundamental equilibrium price $\mu = 50$. This is the most interesting framework to investigate when including passive investors.

2.2.3 Semi-rational trend-chasers:

$$v(t) = 2\frac{\pi_\alpha(t)}{\pi_\alpha(t) + \pi_{\overline{\alpha}}(t)} - 1$$
$$w(t) = 1 - e^{-\lambda t} \quad \text{(increasing coordination)}$$

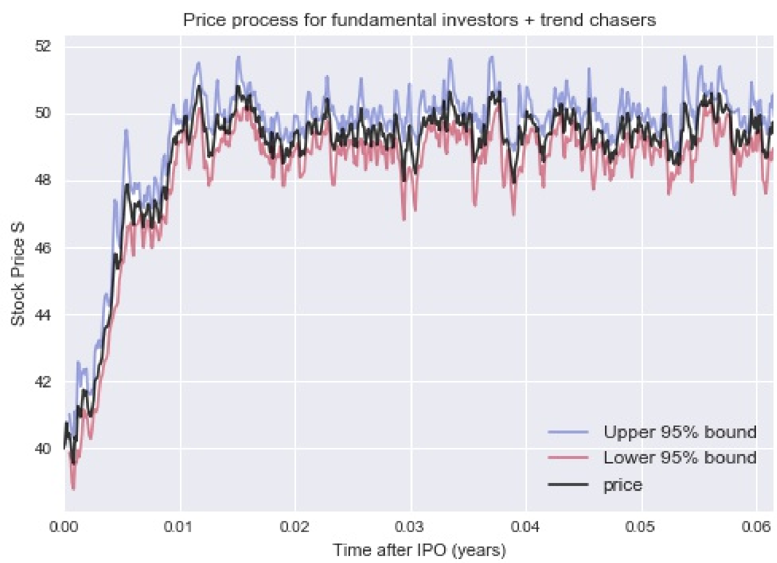

Fig 11: high β ($\frac{3}{4}$ of the total population), , $S(0) < \mu$: Here again, since the β's coordinate rather quickly and make their collective decision based on α's valuations, the price still converges quickly towards its equilibrium value.

2.3 Fundamental investors + trend-chasers + passive investors

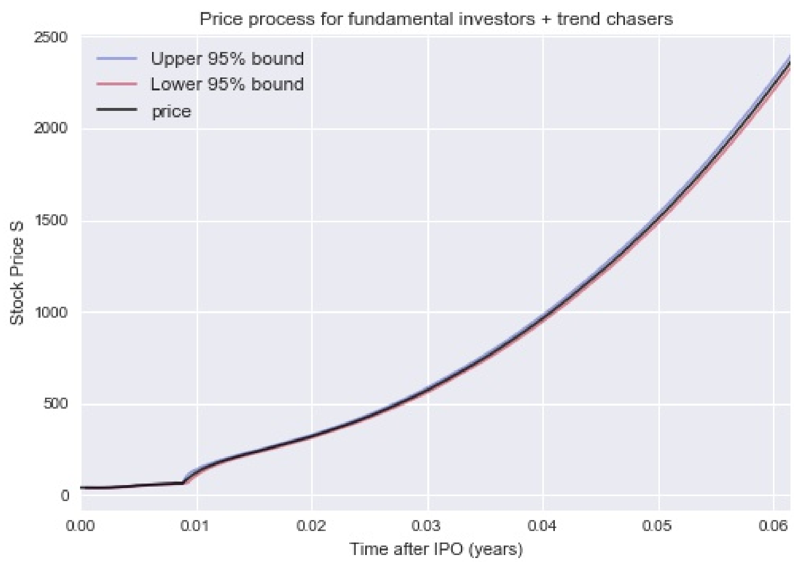

Fig 11: including passive investors (20% of the population), , $S(0) < \mu$
If trend-chasers are numerous and follow the voting rule , a quick coordination can create a substantial deviation from the fundamental equilibrium price $\mu = 50$. However, since passive investors also enter the market (here at $t_0 \approx 0.01$), the sum of γ and β crowds out fundamental short-sellers, who eventually account for the whole population α. The price imperfection, however strong, which could initially be corrected by fundamental investors now work in a self-reinforcing way, as trend-chasers keep buying at a higher price and passive investors don't sell and wait idle with their positions.

3 Interpretation

In this part, we will provide some economic intuition behind our simulations. We have described a bubble phenomenon created by a company that voluntarily underprices its stocks at IPO, in order to artificially create an upward trend thanks to its knowledge of the different categories of investors entering the market.

The price of the stock is here the major control variable from the company's point of view. On the investor side, the price transmits information about the decisions of other individuals : if it goes up, investors tend to follow the trend and strengthen it. Symmetrically, this very same behaviour may crash the stock if it is initially set too high. As a central element of our analysis, prices can be seen as a "mediated feedback loop" (Robert Shiller, *Speculative Asset Prices*): different types of investors enter the stock market and influence each other's decisions, in a social contagion mechanism that attracts more people in the bubble.

After IPO, fundamental investor's feedback can be positive (invest) or negative (sell). Depending on this, noise traders will adapt their investment strategies. Similarly, passive traders will decide either to invest or not, conditional upon the investment choices of noise investors, which is itself driven by fundamental investors. This process forms a chain dependency, ordered by the quality and density of information. The informed α individuals with extreme views will act first (our fundamental investors). They will be followed by investors with less confidence, and finally, with less information.

The *ordered chain-process* described before rely on the important observation that actions of some investors influence actions of others. This leads to the emergence of "herds" (Christophe P. Chamley, *Rational Herds, Economic models of social learnings*).

Actions of others also convey some information about the state of the world. In our example, β and γ investors derive their information set from past prices, and update their decision rule accordingly.

The notion of "social learning" (Christophe P. Chamley, *Rational Herds, Economic models of social learnings*) refers to the diffusion of private beliefs to all individuals. Through the observation of stock prices and behaviours (buy, sell, wait), individuals learn some information and have in turn an influence on the price that is publicly known. As the number of investors increases, the trend becomes more interpretable and the importance of history in investor's choices increases. They rely less on their initial private information and more on the information conveyed by the price, which is the "mean information", the "information of the majority". Hence, "the majority is always right" (Keynes). In this model, the dominant trend can be both wrong and hard to break. This can be viewed as a *cascade phenomenon* (Chamley, *Rational Herds, Economic models of social learnings*), a failure of social learning due to the weight of

history. If an agent has a private signal that induces him to take a different action after some date, this new piece of information may be crowded out by the herd. Thus, social learning can be very slow.
In our model, we consider that the company can enter an index, which leads to the participation of our last category of investors. Entering an index, like the SP500 or the CAC40 can be seen as a guarantee of quality for the concerned company. Indeed, as explained by Chamley (*Rational Herds, Economic models of social learnings*), reputation has an important impact on the individuals' decisions. Our model views the entry of a company in an index as a sign of good health and as a positive signal.

How could speculators be sufficiently coordinated to dictate the trend? The role of the medias is key in our case thanks to the way they promote the IPO. Before it takes place, β traders are already aware of the event and ready to position themselves. More psychologically, the upward trend of the stock price can be defined as a "psycho-economic phenomenon" because it gets involved in people's view of themselves, (Shiller, *Irrational exuberance*). Due to the medias' pressure, they start to appreciate themselves as the investor they became. Investors have a "Fear of Missing Out" (FOMO) the opportunity of joining the upward trend. What could be a once-in-a-lifetime opportunity spurs more speculation.

Abreu and Brunnermeier (2003) emphasize the role of news event as a key synchronizing tool, as they allow arbitrageurs to accord themselves on when to get out of the stock market. Investors learn about the bubble between its inception and its burst in a uniform fashion. The bubble will burst for sure but may do so for two distinct reasons: the first is that a price attack of significant magnitude is carried out by investors, and the second is due to exogenous reasons (for instance, in 2008, when investment banks had too many undervalued assets, such as CDOs, in their balance sheets, and were unable to cover the default premiums they had to pay to their counterparties in swaps agreements).

In their model, a bubble is sustainable for some time while all agents are aware that the price is driven by bubble, as long as this is not common knowledge. As investors notice a decorrelation between the price of a financial asset and its fundamental value, they often decide to briefly join the party, hoping to get out before the discrepancy becomes too notorious and an attack on prices kills the bubble for good. This of course implies a hope to be among the firsts to spot the discrepancy, as only a limited number of investors hope to disembark before the boat sinks. .

In our case, the very same dynamic is working in the opposite way for speculators. The willingness by fundamental arbitrageurs to enter the market first to benefit from the creation of the bubble could be reflected by the first α investors to buy the stock.

We now have explained the economic intuition behind our model. Our analysis, based on the voluntary underpricing of a company's stock does not fall from the sky. This practice is commonly used in the real economy. Indeed, investment banks systematically survey major asset managers about how much they are willing to pay for a share of the company. After having gathered this piece of information,instead of setting a price that perfectly clears the market (where supply = demand), the price is undervalued in order to artificially generate more orders. Our analysis is grounded into reality. Finally, its result - the situation where an undervaluation creates an bubbly increase of the stock price - could be seen as dangerous as capital gets poorly allocated. We may nuance this conclusion by observing that the gap between the equilibrium price and the actual price could be seen as normal. The price of a stock goes up because higher cash flows are expected from the company. The entry inside an index guarantees a high probability that the company will successfully stay a "major actor" in its sector and pay increasing dividends. So, if the process we describe is actually applied, its consequences could sound legitimate to a certain extent.

4 Conclusion

This simple model highlights how a firm could be taking advantage of systematic trading strategies and passive investing to maximize its market capitalization, resulting in a persistent mispricing. This observation strengthens the need for more fundamental research in the investment process. In particular, we may suggest than in exceptional market conditions, such as an IPO, the generalization of passive investment through index funds and ETFs may give more space for the creation of a bubble than thorough company research does.