

**MTH-326 MATH MODELING SPRING 2025**

**HW 6 Due Sunday 04/13/2025**

1. (a) Find all fixed points of  $f(f(x))$  for  $f(x) = x^2 - 1$ .

(b) Find all non-negative fixed points of  $f_r(x) = xr^{1-x}$ .

(c) Find the fixed point of the given system.

$$\begin{aligned}P_{n+1} &= P_n - 2Q_n + 100 \\Q_{n+1} &= -5P_n + Q_n + 200\end{aligned}$$

2. Complex numbers and linear algebra

(a) Suppose that  $A = \begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix}$ , find  $A^{-1}$ , eigenvalues of  $A$ .

(b) Write the expression in the form  $\alpha + \beta i$  for  $\left[ \sqrt{2} \left( \cos \frac{\pi}{10} + i \sin \frac{\pi}{10} \right) \right]^5$ .

3. Consider the homogeneous system:

$$\begin{aligned}P_{n+1} &= 2P_n - Q_n \\Q_{n+1} &= P_n + Q_n\end{aligned}$$

(a) Find the general form of all solutions:

(b) Find the unique solution that satisfies  $P_0 = 1000, Q_0 = 1000$ .

(c) Determine the stability (sink, source, spiral to  $(0, 0)$  or spiral to  $\infty$ ).

4.

$$\begin{cases} x_{n+1} = 2x_n - 4y_n \\ y_{n+1} = 4x_n - 6y_n \end{cases} \quad x_0 = 2, y_0 = 1.$$

(a) Find the general form of all solutions.

(b) Find the unique solution that satisfies the given initial values.

(c) Compute its limit as  $n \rightarrow \infty$  if it exists.