Inflation Dynamics and Adaptive Expectations in an Estimated DSGE Model*

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Abstract

We estimate a "hybrid expectations" version of the Smets and Wouters (2007) model in which a subset of agents employ simple moving-average forecast rules that place a significant weight on the most recent data observation. We show that the overall fit is improved relative to an otherwise similar version in which all agents have fully rational expectations. In-sample and out-of-sample analyses show the superiority of the hybrid expectations model in generating an expected inflation series that more closely tracks expected inflation from the Survey of Professional Forecasters.

Keywords: inflation expectations, Bayesian estimation, local identification, adaptive expectations, survey of professional forecast expectations.

JEL codes: D84, E32, E70

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1 Introduction

A wide variety of survey evidence suggests that agents do not form their expectations rationally. Survey data on inflation and from both stock and real estate markets provide strong empirical support for considering extrapolative or moving average type forecast rules.

On the basis of that evidence, this paper estimates a "hybrid expectations" version of the Smets and Wouters (2007) model in which a subset of agents employ simple moving-average forecast rules that place a significant weight on the most recent data observation. In this respect those agents have adaptive expectations. Our aim is to quantitatively evaluate the performance of the model relative to an otherwise similar version in which all agents have fully rational expectations. Our analysis builds on the work of Gelain, Lansing, and Mendicino (2013) who introduce hybrid expectations into the model of Iacoviello (2005).

We estimate two versions of the Smets and Wouters (2007) model, one with hybrid expectations and one with rational expectations. We find that the data strongly and statistically support the hybrid expectations version from a goodness-of-fit point of view. Nevertheless, there is not strong evidence in favor of one model or the other when it comes to a moments matching comparison. But in contrast, in-sample and out-of-sample forecast analyses clearly indicate the superiority of the hybrid expectations model in generating an expected inflation series that more closely tracks expected inflation from the Survey of Professional Forecasters.

Our estimation indicates that about 70% of agents uses moving-average forecast rules to form their expectations. For most of the forecasted variables, we estimate a large weight on most recent observations. This allows the hybrid expectations model to generate more endogenous persistence and rely less on the structural parameters that the rational expectations model typically relies on to account for the persistence in the data. The overall fit, as measured by marginal data density, points out a better fit of the hybrid expectations model.

Local identification analysis suggests that the observable variables that are more informative regarding the estimates of the fraction of households who employ the moving-average forecast rule are consumption and the short-term interest rate. The identification tests do not provide results that strongly favour one of the two models. Indeed, despite the larger number of free parameters, the model with hybrid expectations does not perform necessarily worst than the rational expectation model in terms of parameters identification analysis.

A clear rank can be determined on the basis of two forecast exercises. First, we compare the estimated implied inflation expectations from both models with the expected inflation from the Survey of Professional Forecasters. We find that at all horizons the presence of backward looking agents helps in tracking survey data. This is especially true for the continuous decline of inflation expectations in the second half of the 1990s. Second we compute out-of-sample inflation point and density forecasts and show that there is a statistical dominance of the hybrid expectations model.

In what follows we present a review of the related literature. We then describe the model and how we introduce adaptive expectations. Finally, before providing some concluding remarks, we show and discuss the estimation results.

1.1 Related literature

Considering the possibility of departures from rational expectations is justified by empirical evidence from surveys that seek to directly measure investor expectations. With regard to macroeconomic variables (inflation, output growth, the unemployment rate, and housing starts), Coibion and Gorodnichenko (2015) find strong evidence of predictability in the mean ex post forecast errors of professional forecasters—a feature that is not consistent with rational expectations. Numerous studies document evidence of bias and inefficiency in survey forecasts of U.S. inflation.[1] Greenwood and Shleifer (2014) and Adam, Marcet, and Beutel (2017) show that measures of investor expectations about future stock returns are strongly correlated with past stock returns and the price-dividend ratio. Interestingly, even though a higher price-dividend ratio in the data empirically predicts lower *realized* stock returns (Cochrane 2008), the survey evidence shows that investors fail to take this relationship into account; instead they continue to forecast high future returns on stocks following a sustained run-up in the price-dividend ratio. In a review of the time series evidence on housing investor expectations from 2002 to 2008, Case, Shiller, and Thompson (2012, p. 282) find that "1-year expectations [of future house prices changes] are fairly well described as attenuated versions of lagged actual 1-year price changes." Jurgilas and Lansing (2013) show that the balance of households in Norway and Sweden expecting a house price increase over the next year is strongly correlated with nominal house price growth over the preceding year. Ling, Ooi, and Le (2015) find that past house price changes help to predict future house price changes even after taking into account every conceivable fundamental variable that the theory says should matter. In a study of data from the Michigan Survey of Consumers, Piazzesi and Schneider (2009, p. 407) report that "starting in 2004, more and more households became optimistic after having watched house prices increase for several years."

The moving-average forecast rules employed in our model embed a unit root assumption which tends to be partially self-fulfilling. As shown originally by Muth (1960), a moving-average forecast rule with exponentially-declining weights on past data will coincide with rational expectations when the forecast variable evolves as a random walk with permanent and temporary shocks. But even if this is not the case, a moving-average forecast rule can be viewed as boundedly-rational because it economizes on the costs of collecting and processing information. As noted by Nerlove (1983, p. 1255): "Purposeful economic agents have incentives to eliminate errors up to a point justified by the costs of obtaining the information necessary to do so...The most readily available and least costly information about the future value of a variable is its past value."

An empirical study by Chow (1989) finds that an asset pricing model with adaptive expectations out-

[1] See, for example, Roberts (1997), Mehra (2002), Carroll (2003), and Mankiw, Reis, and Wolfers (2004).

performs one with rational expectations in accounting for observed movements in U.S. stock prices and interest rates. Huh and Lansing (2000) show that a model with backward-looking expectations is better able to capture the temporary rise in long-term nominal interest rates observed in U.S. data at the start of the Volcker disinflation in the early-1980s. Some recent research that incorporates adaptive expectations or moving-average type forecast rules into otherwise standard models include Sargent (1999, Chapter 6), Evans and Ramey (2006), Lansing (2009), Huang et al. (2009), and Gelain and Lansing (2014), among others. Lansing (2009) shows that survey-based measures of U.S. inflation expectations are well-captured by a moving average of past realized inflation rates, i.e., adaptive expectations. The study by Huang et al. (2009) relying on a standard RBC model conclude that "adaptive expectations can be an important source of frictions that amplify and propagate technology shocks and seem promising for generating plausible labor market dynamics." Levine et al. (2012) also find significant empirical support in favor of adaptive expectations in a small-scale model.

Constant-gain learning algorithms of the type described by Evans and Honkapoja (2001) are similar in many respects to adaptive expectations; both formulations assume that agents apply exponentially-declining weights to past data when constructing forecasts of future variables. Along these lines, Sargent (1996, p. 543) remarks "[A]daptive expectations has made a comeback in other areas of theory, in the guise of non-Bayesian theories of learning." A few papers conclude that adaptive learning models are more successful than rational expectations models in capturing several quantitative properties of U.S. macroeconomic data. Among others, Orphanides and Williams (2003, 2005a, 2005b) document that adaptive learning can help to match the increased persistence of inflation. Eusepi and Preston (2011), using forecast survey data to discipline the learning mechanism, show that the calibrated real business cycle learning model performs better than the rational expectation model, in matching second-order moments of output, hours worked and investment growth.

An increasing number of papers quantify the importance of the adaptive learning mechanism by private agents by relying on estimated DSGE models. Milani (2007), estimating a three-equation New Keynesian model show that adaptive learning requires a lower degree of habit formation and price indexation to match business cycle features and improves upon the rational expectation version of the same model in terms of overall goodness of fit. In contrast, Slobodyan and Wouters (2012a, 2012b) show that in an estimated medium-scale model, learning by private agents improves the fit of the model only if agents use a reduced information set in forming expectations. Further, the constant-gain learning mechanism introduces some additional persistence to the model but it does not significantly alter the estimates of the nominal and real structural frictions. Beqiraj et al. (2017) estimate and compare behavioral New-Keynesian DSGE models derived under two alternative ways to introduce heterogeneous expectations, i.e. agents may be either short-sighted or long-horizon forecasters. Bayesian estimations show that a behavioral model based on short-sighted forecasters fits the data better than one based on long-horizon forecasters. Hommes et al.

(2018) estimate DSGE models (small and medium scale) under a simple misspecification learning equilibrium that arises from expectational frictions. They show that the empirical fit and forecasting performance of both models improve compared to the rational expectations versions of the same models.

2 The Model

Our analysis is based on a DSGE model that features nominal and real rigidities and a large set of shocks as developed by Smets and Wouters (2007). The economy is populated by a continuum of households indexed by j, each maximizing the following utility function

$$E_t\left[\sum_{s=0}^{\infty}\beta^s \frac{1}{1-\sigma_c}\left((C_{t+s}(j)-\lambda C_{t+s-1}(j))^{1-\sigma_c}\right)\exp\left(\frac{\sigma_c-1}{1+\sigma_l}L_{t+s}(j)^{1+\sigma_l}\right)\right], \tag{1}$$

where $C_{t+s}(j)$ is consumption, $L_{t+s}(j)$ is hours worked.

Households supply homogeneous labor services to labor unions indexed by l. Labor services are differentiated by a union, and sold to labor packers. Wage setting is subject to nominal rigidities with a Calvo mechanism whereby each period a union can set the nominal wage to the optimal level with constant probability equal to $1-\xi_w$. Unions that cannot adjust their nominal wage optimally change it according to the following indexation rule

$$W_{t+s}(l) = \gamma W_{t-1}(l)\pi_{t-1}^{\iota_w}\pi_*^{(1-\iota_w)}, \tag{2}$$

where γ is the deterministic growth rate, ι_w measures the degree of wage indexation to past inflation, and π_* is the steady state rate of inflation.

Labor packers buy differentiated labor services $L_t(l)$ from unions, package and sell composite labor L_t, defined implicitly by

$$\int_0^1 \mathcal{H}\left(\frac{L_t(l)}{L_t};\lambda_{w,t}\right)dl = 1, \tag{3}$$

to the intermediate good sector firms. The function $\mathcal{H}$ is increasing, concave, and satisfies $\mathcal{H}(1)=1$; $\lambda_{w,t}$ is a stochastic exogenous process changing the elasticity of demand, and the wage markup over the marginal disutility from work.

In addition to supplying labor, households rent capital to the intermediate goods producers at rate $R_t^K(j)$. Households accumulate physical capital according to the following law of motion:

$$\bar{K}_t(j) = (1-\delta)\bar{K}_{t-1}(j) + \varepsilon_t^i\left[1-\mathcal{S}\left(\frac{I_t(j)}{I_{t-1}(j)}\right)\right]I_t(j), \tag{4}$$

where δ is the rate of depreciation, I_t is gross investment, and the investment adjustment cost function $\mathcal{S}$ satisfies $\mathcal{S}'>0$, $\mathcal{S}''>0$, and in steady state $\mathcal{S}=0$, $\mathcal{S}'=0$; ε_t^i represents the current state of technology for producing capital, and is interpreted as investment-specific technological progress.

Households also choose the utilization rate $Z_t(j)$ of the physical capital they own, and pay $P_t a(Z_t(j))\bar{K}_{t-1}(j)$ in terms of consumption good when the capital intensity is $Z_t(j)$. The income from renting capital to firms is $R_t^k K_t(j)$, where $K_t(j) = Z_t(j)\bar{K}_{t-1}(j)$ is the flow of capital services provided by the existing stock of physical capital $\bar{K}_{t-1}(j)$. The utility function (1) is maximized with respect to consumption, hours, investment, and capital utilization, subject to the capital accumulation equation (4), and the following budget constraint:

$$
\begin{aligned}
&C_{t+s}(j) + I_{t+s}(j) + \frac{B_{t+s}(j)}{\varepsilon_{t+s}^b R_{t+s} P_{t+s}} - T_{t+s} = \frac{W_{t+s}(j)}{P_{t+s}} L_{t+s}(j) \\
&+ \left(\frac{R_{t+s}^k Z_{t+s}(j)}{P_{t+s}} - a(Z_{t+s}(j)) \right) \bar{K}_{t+s-1}(j) + \frac{B_{t+s-1}(j)}{P_{t+s}} + \frac{\Pi_{t+s}(j)}{P_{t+s}},
\end{aligned} \tag{5}
$$

where B_{t+s} is a one-period nominal bond expressed on a discount basis, ε_t^b is an exogenous premium on the bond return, T_{t+s} is lump-sum taxes or subsidies, and Π_{t+s} is profit distributed by the labor union.

There is a perfectly competitive sector producing a single final good used for consumption and investment. The final good is produced from intermediate inputs $Y_t(i)$ using technology defined implicitly by

$$
\int_0^1 \mathcal{G}\left(\frac{Y_t(i)}{Y_t}; \lambda_{p,t} \right) di = 1, \tag{6}
$$

where $\mathcal{G}$ is increasing, concave, and $\mathcal{G}(1) = 1$; $\lambda_{p,t}$ is an exogenous stochastic process affecting the elasticity of substitution between different intermediate goods, also corresponding to a markup over marginal cost for intermediate good firms. Firms maximize profits given by

$$
P_t Y_t - \int_0^1 P_t(i) Y_t(i) di, \tag{7}
$$

where $P_t(i)$ is the price of intermediate good $Y_t(i)$.

Intermediate goods are produced in a monopolistically competitive sector. Each variety i is produced by a single firm using the following production technology:

$$
Y_t(i) = \varepsilon_t^a K_t(i)^\alpha (\gamma^t L_t(i))^{1-\alpha} - \Phi \gamma^t, \tag{8}
$$

where Φ is a fixed cost of production, and ε_t^a is the total factor productivity. As with wages, every period only a fraction $1 - \xi_P$ of intermediate firms can set optimally the price of the good they produce. The remaining ξ_p firms index their prices to past inflation according to

$$
P_t(t) = \gamma P_{t-1}(i) \pi_{t-1}^{\iota_p} \pi_*^{(1-\iota_p)}, \tag{9}
$$

where ι_p measures the degree of price indexation to past inflation.

The central bank sets the nominal interest rate according to the following rule

$$\frac{R_t}{R^*} = \varepsilon_t^r \left(\frac{R_{t-1}}{R^*}\right)^{\rho} \left[\left(\frac{\pi_t}{\pi^*}\right)^{r_\pi} \left(\frac{Y_t}{Y_t^*}\right)^{r_y}\right]^{1-\rho} \left(\frac{Y_t/Y_{t-1}}{Y_t^*/Y_{t-1}^*}\right)^{r_{\triangle y}} \tag{10}$$

where R^* is the steady state level of the gross nominal interest rate, r_t is a monetary policy shock, and Y^* is potential output, defined as the output in a flexible price and wage economy; ε_t^r represents exogenous deviations from the interest-rate rule.

The government also collects lump-sum taxes in order to finance its consumption so as to respect the following budget constraint

$$P_t G_t + B_{t-1} = T_t + \frac{B_t}{R_t}, \tag{11}$$

where G_t is government consumption in terms of final good.

There are seven exogenous shocks in the model. Shocks to the risk premium (ε_t^b), total factor productivity(ε_t^a) investment-specific technology (ε_t^q), wage markup (ε_t^w) and price markup (ε_t^p) and monetary policy (ε_t^r), follow in log-linear form AR(1) processes

$$ln\varepsilon_t^x = \rho^x ln\varepsilon_{t-1}^x + u_{x,t}.$$

where $x = \{a, b, q, w, p, r\}$, ρ^x is the persistence parameter and $u_{x,t}$ is i.i.d. white noise process with mean zero and standard deviation σ_x.[2]

We allow the government spending shock, ε_t^g, to depend on the productivity shock

$$ln\varepsilon_t^g = \rho^g ln\varepsilon_{t-1}^g + u_{g,t} + \rho^{ga} u_{a,t}.$$

The economy is assumed to evolve along a deterministic growth path, driven by the deterministic labor-augmenting technological progress, γ. All growing variables - consumption, investment, capital, real wages, output and government spending, are made stationary and then all equilibrium conditions are log-linearized around the deterministic steady state of the stationary variables. Table 1 reports the log-linear system of equations.

2.1 Expectation Formation

Rational expectations are built on strong assumptions about agents' information. In actual forecasting applications, real-time difficulties in observing stochastic shocks, together with empirical instabilities in the

[2]The specification of mark-up shocks differs from the original Smets and Wouters (2007) specification. We assume AR(1) instead of ARMA(1,1) processes. This does not affect our results.

underlying shock distributions could lead to large and persistent forecast errors. These ideas motivate consideration of a boundedly-rational forecasting algorithm, one that requires substantially less computational and informational resources. A long history in macroeconomics suggests the following adaptive (or error-correction) approach:

$$F_t\, X_{t+1} = F_{t-1}\, X_t \;+\; \lambda_x \left(X_t - F_{t-1}\, X_t\right), \qquad 0 < \lambda_x \le 1,$$

$$= \lambda_x \left[X_t + (1-\lambda_x)\; X_{t-1} + (1-\lambda_x)^2\; X_{t-2} + ...\right], \tag{12}$$

where X_{t+1} is the object to be forecasted and $F_t\, X_{t+1}$ is the corresponding subjective forecast. In this model, X_{t+1} is typically a nonlinear combination of endogenous and exogenous variables dated at time $t+1$. The term $X_t - F_{t-1}X_t$ is the observed forecast error in period t. The parameter λ_x governs the forecast response to the most recent data observation X_t.

Equation (12) implies that the forecast at time t is an exponentially-weighted moving average of past observed values of the forecast object, where λ_x governs the distribution of weights assigned to past observed values—analogous to the gain parameter in the adaptive learning literature. When $\lambda_x = 1$, households employ a simple random walk forecast. By comparison, the "sticky-information" model of Mankiw and Reis (2002) implies that the forecast at time t is based on an exponentially-weighted moving average of past rational forecasts. A sticky-information version of equation (12) could be written recursively as $F_t\, X_{t+1} = F_{t-1}\, X_t \;+\; \mu \left(E_t\, X_{t+1} - F_{t-1}\, X_t\right)$, where μ represents the fraction of households who update their forecast to the most-recent rational forecast $E_t\, X_{t+1}$.

For each of the model's first order conditions, we nest the moving-average forecast rule (12) together with the rational expectation $E_t\, X_{t+1}$ to obtain the following "hybrid expectation" which is a weighted-average of the two forecasts

$$\widehat{E}_{j,t}\, X_{t+1} = \omega F_t\, X_{t+1} + (1-\omega)\, E_t\, X_{t+1}, \qquad 0 \le \omega \le 1, \qquad j = 1, 2, \tag{13}$$

where ω can be interpreted as the fraction of households who employ the moving-average forecast rule (12). For simplicity, we assume that ω is the same for all agents. In equilibrium, the fully-rational forecast $E_t X_{t+1}$ takes into account the influence of households who employ the moving-average forecast rule. In this way, the influence of the moving-average forecast rule on the behavior of endogenous variables is leveraged up.[3]

[3] A simple example with $\lambda = 1$ can illustrate what we mean with that. Suppose that the Phillips curve is given by $\pi_t = \beta E_t \pi_{t+1} + \gamma y_t$, where y_t follows an AR(1) process with persistence ρ and $\widehat{E}_t \pi_{t+1} = \omega F_t \pi_{t+1} + (1-\omega) E_t \pi_{t+1}$. When $F_t \pi_{t+1} = \pi_t$, the equilibrium law of motion is $\pi_t = \gamma y_t / [1 - \beta\omega - \rho\beta(1-\omega)]$, which implies $Var(\pi_t) = \gamma^2 Var(y_t)/[1 - \beta\omega - \rho\beta(1-\omega)]^2$. When $\rho < 1$, an increase in ω increases both $Var(\pi_t)$ and the variance of the rational forecast $Var(E_t \pi_{t+1})$. The magnification of the volatility of the rational forecast $E_t \pi_{t+1}$ is an important channel for leveraging up the influence of the backward-looking agents. If the rational, forward-looking agents ignored the presence of the backward-looking agents, then we would have $\pi_t = \gamma y_t (1 - \rho\beta\omega)/[(1-\rho\beta)(1-\omega\beta)]$, which implies a smaller $Var(\pi_t)$ for any $0 < \omega < 1$.

Our setup has rational agents whose decision rules and forecasts are influenced by the presence of the non-rational agents. This is the same setup as in the sticky information model of Mankiw and Reis (2002). In both setups, the lagged forecast of the non-rational agents becomes a state variable for the rational agent's decision rule. The law of motion for this state variable is used to construct the rational agent's conditional forecast. For a simple analytical example of the sticky information setup, see Lansing (2009), Appendix C.2.

To sidestep issues about the long-term survival of agents who employ moving-average forecast rules, we rule out direct asset trading between these agents and agents with fully-rational expectations. Alternatively, we could interpret ω as the probability weight that a single agent type assigns to the moving-average rule when constructing a one-period-ahead forecast, along the lines of De Grauwe (2012).

Table 1 shows that the log-linearized version of the model features seven objects to be forecasted: inflation, $\widehat{E}_t\,\widehat{\pi}_{t+1}$, consumption, $\widehat{E}_t\widehat{c}_{t+1}$, investment, $\widehat{E}_t\widehat{i}_{t+1}$, real wages, $\widehat{E}_t\widehat{w}_{t+1}$, the value of capital, $\widehat{E}_t\,\widehat{q}_{t+1}$, the rental rate of capital, $\widehat{E}_t\hat{r}^k_{t+1}$, and hours worked, $\widehat{E}_t\widehat{l}_{t+1}$. We allow agents to weight current and past observed values of the alternative variables differently. Thus, λ_x is allowed to be different across the forecasted objects, $X_t = \left\{\widehat{\pi}_{t+1}, \widehat{c}_{t+1}, \widehat{i}_{t+1}, \widehat{w}_{t+1}, \widehat{q}_{t+1}, \hat{r}^k_{t+1}, \widehat{l}_{t+1}\right\}$.

3 Estimation

The solution to the log-linear approximation of the DSGE model described above can be characterized by a transition equation

$$s_t = A(\theta)s_{t-1} + B(\theta)u_t. \tag{14}$$

and a measurement equation

$$y_t = d(\theta) + C(\theta)x_t \tag{15}$$

where $u_t = (\varepsilon^a_t, \varepsilon^b_t, \varepsilon^g_t, \varepsilon^q_t, \varepsilon^r_t, \varepsilon^p_t, \varepsilon^w_t)$ is the vector of structural shocks, s_t is the vector of stationary variables and y_t is the vector of observables, $d(\theta)$ is a vector and $A(\theta)$, $B(\theta)$ and $C(\theta)$ are matrices of the reduced-form parameters $\phi(\theta)$ and

$$\theta = [\sigma^c, h, \sigma^l, \xi^w, \xi^p, \iota^w, \iota^p, \varphi, \psi, \alpha, \phi, \rho, r_\pi, r_y, r_{\Delta y}, l_{ss}, \pi_{ss}, \bar{\beta}, \bar{\gamma}, \omega, \lambda_i, \lambda_p, \lambda_{r^k}, \lambda_q, \lambda_w, \lambda_c, \lambda_l,$$
$$\rho^a, \rho^b, \rho^g, \rho^q, \rho^r, \rho^p, \rho^w, \rho^{ga}, \mu^p, \mu^w, \sigma^a, \sigma^b, \sigma^g, \sigma^q, \sigma^r, \sigma^p, \sigma^w]'$$

is the 44-dimensional vector of deep parameters of the model describing price and wage stickiness, investment adjustment cost, risk premium elasticities, the monetary policy rule, the expectation parameters and the shocks. The vector of observables

$$y_t = [\Delta\ln(GDP_t), \Delta\ln(c_t), \Delta\ln(I_t), \Delta\ln(w_t), \pi_t, \ln(h_t), r_t]'$$

includes the log difference of real GDP, $\Delta \ln(GDPt)$, real consumption, $\Delta \ln(c_t)$; real investment, $\Delta \ln(I_t)$; real wages, $\Delta \ln(w_t)$; the GDP deflator, π_t; the log hours worked, $\ln(h_t)$; and the federal fund rate, r_t; where Δ denotes the difference operator. See Data Appendix for details on the data used.[4]

We estimate the vector of parameters θ using seven time series for the US with quarterly frequencies over the sample from 1981:Q4 to 2007:Q4 and Bayesian techniques.[5] The Kalman filter is used to compute the likelihood $L(\Gamma_t \mid\theta)$ for the given sample of data Γ_t, as in Hamilton (1994). We add some informative priors, $\varphi(\theta)$, to downweight regions of the parameter space that are widely accepted to be uninteresting. Using Bayes's rule, the posterior distribution can be written as the product of the likelihood function of the data given the parameters, $L(\Gamma_t \mid\theta)$, and the prior, $\varphi(\theta)$ such that $P(\theta|\Gamma_t) \propto L(\Gamma_t \mid\theta)\varphi(\theta)$.

First, we obtain the mode of the posterior distribution, which summarizes information about the likelihood of the data and the priors on the parameters' distributions, by numerically maximizing the log of the posterior. We then approximate the inverse of the Hessian matrix evaluated at the mode. Second, we use the random-walk Metropolis-Hastings algorithm to simulate the posteriors distribution, where the covariance matrix of the proposal distribution is proportional to the inverse Hessian at the posterior mode computed in the first step. After checking for convergence, we use the posterior means of the parameters to draw statistical inference on the parameters themselves or functions of the parameters, such as second moments.[6] For recent surveys of Bayesian methods, see An and Schorfheide (2007) and Fernandéz-Villaverde (2010).[7]

As in Smets and Wouters (2007) we calibrate the depreciation rate, δ, at 0.025 and the exogenous government spending to GDP ratio, g_y, at 18 per cent and the steady state labor market mark-up, λ_w, equal to 1.5. We also fix the curvature of the labor and good market aggregator at 10, i.e. ζ^w and ζ^π.

Priors distributions are summarized in the first block of Tables 2 and 3. The priors on the parameters and stochastic processes of the shocks follow Smets and Wouters (2007). Regarding the expectation parameters, we use a beta distribution with mean 0.5 and standard deviation 0.2 as in Levine et al. (2012).

Prior to estimating the model, we check whether the free parameters can be identified from the data. Lack of identification would suggest either problems in the structure of the model, or that the set of observed variables we use does not provide sufficient information about certain parameters. In particular, if some parameters do not affect the equilibrium conditions of the model, or if different parameters have identical impact, then those parameters would be unidentified regardless of which variables are used in the estimation. On the other hand, identification may also fail because the effect of some parameters on the likelihood function of the observed variables we use is either undetectable or indistinguishable from the effect of other parameters. In that case, the model may still be identified from a different set of observables. We know that the rational

[4]All series are from FRED Database maintained by the Federal Reserve Bank of St. Louis.

[5]The choice of the sample period reflects the desire of estimating the model over a period of stable conduct of monetary policy.

[6]We obtain a random draw of size 1.000.000 from the posterior distribution using the random-walk Metropolis-Hastings algorithm. To perform inference we discard the first 10 per cent of observations. For further details on the estimation and the convergence of the algorithm see Estimation Appendix.

[7]We use Dynare. See Adjemian et al. (2011).

expectations version of the model, as originally estimated in Smets and Wouters (2007), is identified (see Iskrev (2010)). The hybrid expectations version of the model introduces several additional parameters and that could cause problems with identification. A necessary and sufficient condition for identifiability is that the Jacobian matrix with derivatives of first and second order moments which enter the likelihood function has a full rank. Following Iskrev (2010)), we check the rank condition at 100,000 random draws from the prior distribution of the parameter described in Tables 2 and 3. We find that the Jacobian matrix has a full rank at all points, and therefore conclude that the hybrid expectations version of the model can be identified. Further identification analysis is reported in Section 4.3.

4 Estimation Results

In this section we present the estimation results. We first starts with a comparison of the estimated parameters for the two estimated models. We then provide an evaluation of the fit, first in general terms and second from the point of view of moment matching. Third we proceed with the identification, determinacy, and stability analysis. Finally we compare model implied inflation expectations with expectations from the survey of professional forecasters.

4.1 Parameters Estimates

In the following, we report the parameters estimates for the two nested models: the model that allows for moving-average forecast rules (HYBRID), i.e. $0 \leq \omega \leq 1$ and $\lambda_x \geqslant 0$, and the restricted version of the model where expectations are completely rational (RE), i.e. $\omega = \lambda_x = 0$. Tables 2 and 3 display the priors chosen for the parameters model and the processes for the exogenous shocks, as well as the posterior mean, standard deviations and the 5 and 95 percent probability intervals for both versions of the model. We report both estimated models, i.e. RE and HYBRID.

Several are the differences in the parameter estimates between the RE and HYBRID model. The posterior estimates of the HYBRID model's parameters feature a substantially lower degree of wage and price stickiness, ξ^w. The intertemporal elasticity of substitution, σ^c, is also remarkably lower in the HYBRID model. The response of investment to changes in the value of capital is faster in the HYBRID model, as indicated by the lower estimates of the elasticity cost of changing investment, φ. The estimated capital share in production, α, and the fixed cost, ψ, also turn out to be somewhat lower in the HYBRID model.

Regarding the estimated processes for the exogenous disturbances, the HYBRID model requires a substantially lower persistence of the investment-specific shocks, ρ^q, and the price markup shock, ρ^p. In contrast, the persistence of the risk premium shock, ρ^b, is substantially larger in the HYBRID model. The HYBRID estimate of the standard deviations of the investment specific shock, σ^q, and the wage markup shock, σ^w, are larger.

Most of those changes can be explained by the mechanism at play when adaptive expectations are considered. In fact, the HYBRID model provides an endogenous mechanism to account for part of the persistence in the data. As a consequence it does not have to rely as much as the rational expectations model on those structural parameters that are responsible to capture the persistence.

In Table 4 we report the estimates of the eight adaptive expectations parameters. They all point out that data strongly support the existence of a large number of backward looking agents applying moving-average forecast rule with more or less weight on the most recent observations. The estimate of ω at the posterior mean shows that about 70 per cent of the agents employing the error correction rule, while 30 per cent is the percentage of rational agents. The 5- to 95-per cent probability interval exclude the possibility of expectations formed in a purely adaptive way. As for the adaptive forecasting rules, the lowest weight to the most recent data observation is estimated for the forecast of inflation, λ_p. In contrast, the highest weight is estimated for the forecast of the relative price of capital, λ_q, followed by the forecast response to current investment, λ_i. The 95 per cent probability interval of both parameters reach values above 0.90 indicating the possibility of almost-random-walk forecasting rules.

4.2 Fit

Results presented in Table 5 suggest that the data strongly favor the inclusion of moving-average forecast rules. Table 5 reports the log marginal data density of each model, the difference with respect to the log marginal data density of the model without moving-average forecast rule, and the implied Bayes factor.[8] The HYBRID version of the model that allow for moving-average forecast rules displays a significantly higher log data density compared to the *RE* model.[9] Accordingly, the Bayes factor indicates decisive evidence in favor of the HYBRID model and implies a posterior odds ratio of $e^{4.88}$. Thus, in order for the RE version of the model to be preferred, we would need a priori probability over this model 131.63 larger than the prior belief about the HYBRID model.[10] The ranking among the alternative specifications of the model is not affected by the priors, as reported by the log marginal likelihood at the posterior mean.

We want to investigate further where the increased ability of the model with hybrid expectations to fit the data comes from. To do that we present in Table 6 the theoretical and empirical values of several key second moments. The theoretical moments are computed at the posterior means of the RE and HYBRID models. Both models do a reasonably good job in matching most of the key business cycle moments. In particular, they replicate the observed volatility and persistence of output, investment and wages, as well as the correlation with output of consumption, investment and interest rates. At the same time, both models fail to capture the negative correlation of output and inflation in the data. The RE model does a better job at matching the observed levels of volatility in hours and inflation, while the HYBRID model performs better

[8] Given that *a priori* we assign equal probability to each model, the Bayes factor equals the posterior odds ratio.
[9] The same results hold if we use data over the sample 1981:Q1 to 2007:Q4.
[10] See Jeffreys (1961) and Kass and Raftery (1995)

at replicating the interest rate volatility. The difficulty of the models to match well all empirical moments in Table 6 is a consequence of the fact that our full information estimator tries to mimic the full covariance structure of the data, not just a few second order-moments.

Further information on the models' performance is provided in Figures 1 and 2, where we show all empirical and theoretical cross-correlations up to lag 10. It is clearly a much more demanding task for the models to match this larger number of second moments. However, with a few exceptions, the models' fit is very good. In the case of the correlation between output and inflation, for instance, we see that when we account for leads and lags, the models fit the data very well, with the observed cross-correlations being well within the 95% bounds around the respective theoretical moments[11] In terms of relative performance, we see that the main trade-off between the two models is in matching the cross-correlations of inflation and interest rates, where the RE model performs better, and those of hours with either wages or interest rates, where the fit of the HYBRID model is better.

These results show that neither model dominates in terms of its ability to match the unconditional second order moments of the data. So other tests are needed to establish a clear rank. For this propose, in the next section we start with the identification analysis and we continue afterwards with in- and out-of-sample forecasts comparison.

Before doing that it is worth inspecting what are the implications of having moving average expectations in the model dynamics. Given the unit root implicitly embedded in the model, it is reasonable to expect that this mechanism delivers an endogenous persistence to the system, without relying exclusively and excessively on other channels regulated by the value of the estimated parameters, like for instance those generated by high indexation, strong habits formation or high adjustments costs. To test that we report in Figure 3 the impulse response function of some selected variables to all shocks in the model. As expected, in most cases there is an increased persistence of the variables to shocks. This is much in line with literature exploiting this type of deviations from rational expectations, e.g. Gelain, Lansing, and Mendicino (2013), Gelain and Lansing (2014), and Gelain, Lansing, and Natvik (2018), or similar types like learning, e.g. Milani (2007), Eusepi and Preston (2011), and Hommes et al. (2018.)

4.3 Expectation Formation: Identification

From the a priori analysis (see Section 3), we know that the parameters of the hybrid expectations model are identifiable on the basis of the log-likelihood function. We also check that the rank condition for identification is satisfied at the posterior mean, confirming that the estimated parameters are identified at that point as well. Here we present additional analysis of the identification properties of the model with a particular focus on the adaptive expectations parameters. We follow the methodology proposed in Iskrev (2014), the main

[11] The confidence bounds were constructed by evaluating the sample cross-correlations on the basis of artificial samples generated using the two models.

idea behind which we describe next.

We start with the observation that for a parameter to be well identified from the data its effect on the log-likelihood function must be both strong and distinct from the effects of the other estimated parameters. A violation of either one of these conditions would result in a flat likelihood and lack of identification. A useful way to quantify these conditions is in terms of sensitivity of the log-likelihood function, denoted with $\ell_T(\boldsymbol{\theta})$, with respect to a parameter θ_i, namely

$$\triangle_i = \sqrt{\mathrm{E}\left(\frac{\partial \ell_T(\boldsymbol{\theta})}{\partial \theta_i}\right)^2},$$

and collinearity between the effects of different parameters on the log-likelihood[12]

$$\boldsymbol{\varrho}_i = \operatorname{corr}\left(\frac{\partial \ell_T(\boldsymbol{\theta})}{\partial \theta_i}, \frac{\partial \ell_T(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}_{-i}}\right)$$

If either $\triangle_i(\boldsymbol{\theta}) = 0$ or $\boldsymbol{\varrho}_i(\boldsymbol{\theta}) = 1$ the log-likelihood is flat and some of the parameters are unidentified. Even if the log-likelihood is not exactly flat, identification may be weak due to very low curvature with respect to some parameters. As a result the values of those parameters are difficult to pin on the basis of the data alone, and the prior distribution becomes very influential. The link between the measures of sensitivity and collinearity, on one hand, and estimation uncertainty, on the other, becomes more transparent if we consider the following relationship (see Iskrev (2014)):

$$\mathrm{CRLB}(\theta_i) = \frac{1}{\sqrt{\triangle_i}} \times \frac{1}{\sqrt{(1-\boldsymbol{\varrho}_i^2)}}, \tag{16}$$

where $\mathrm{CRLB}(\theta_i)$ is the Cramér-Rao lower bound for θ_i, i.e. the lower bound on the standard deviation of an unbiased estimator of that parameter. Both small $\triangle_i$ and large $\boldsymbol{\varrho}_i$ imply a large value of $\mathrm{CRLB}(\theta_i)$, suggesting greater estimation uncertainty for that parameter. Note also that $\boldsymbol{\varrho}_i = 0$ when θ_i is the only freely-estimated parameter. Thus $\frac{1}{\sqrt{\triangle_i}}$ can be interpreted as the Cramér-Rao lower bound for θ_i conditional on knowing the values of all other parameters, and we can express $\mathrm{CRLB}(\theta_i)$ as follows:

$$\mathrm{CRLB}(\theta_i) = \mathrm{CRLB}(\theta_i|\boldsymbol{\theta}_{-i}) \times \frac{1}{\sqrt{(1-\boldsymbol{\varrho}_i^2)}}, \tag{17}$$

Figure 4 reports the conditional and unconditional CRLB for the estimated parameters in our two models. To make the values of the bounds comparable across parameters and between the two models, we normalize them with the parameter values, i.e. the figures shows $\mathrm{CRLB}(\theta_i)/\theta_i$ and $\mathrm{CRLB}(\theta_i|\boldsymbol{\theta}_{-i})/\theta_i$. As can be expected, because to the larger number of parameters (42 vs 34), identification in the HYBRID model tends

[12]That is, $\boldsymbol{\varrho}_i$ is the multiple correlation coefficient between the derivative of the log-likelihood with respect to θ_i and the derivative with respect to $\boldsymbol{\theta}_{-i} = \{\theta_j | j \neq i\}$.

to be worse that in the RE model. A larger number of free parameters means that the ones which are present in both models are likely to have a less distinct impact on the log-likelihood function of the HYBRID model compared to the RE model. In the figure this can be seen from the fact that the values of the conditional CRLB are generally similar for the two models, while the part of the unconditional bounds due to parameter collinearity (shown in grey) tends to be larger in the HYBRID model. A notable exception to this pattern is $\bar{l}$, where identification is much worse in the HYBRID model due mainly to very weak sensitivity. The largest impact stemming from stronger parameter collinearity is on φ, σ_c, ρ_p, and σ_p, whose CRLB are between 3 and 8 times larger in the HYBRID model. Further analysis along the lines of Iskrev (2014) shows that, among the adaptive expectations parameters, the ones which contribute the most to weaken the identification of these 4 parameters are λ_I and λ_q - in the case of φ, λ_l - in the case of σ_c, and λ_π - in the case of ρ_p, and σ_p. We also find a significant negative impact of λ_I on the identification of ρ_I and σ_I, of λ_w – on the identification of ρ_w and σ_w, while both ω and λ_q contribute to weaken the identification of σ_b.

Note that, in spite of having more free parameters, identification in the HYBRID model is not always worse than in the RE model. This can be explained with the fact that, first, due to different expectation formation, the models are not strictly nested, and second, the parameter values at which the CRLB are evaluated are different. This is the reason why in several cases the conditional bounds are quite different, and the overall bounds for several parameters are smaller in the HYBRID model. In particular, identification is relatively stronger in the HYBRID model for ι_p, ρ, r_y, ρ_{ga}, $\bar{\beta}$, ρ_b, σ_a, and σ_p.

Another question we address concerns the main sources of information about the parameters we estimate. Specifically, we examine the contribution of information each observed variable makes with respect to the adaptive expectations parameters. Our measure of information is again based on the CRLB, which we use to compute efficiency gains, which we define as the reduction of CRLB due to observing a variable as a percent of the CRLB when that variable is excluded from the set of observables. The difference between the values of CRLB with and without a given variable reflect the information content of the model-impled restrictions on the joint distribution of that variable and the other observables. Therefore, the efficiency gains are larger with respect to parameters for which these restrictions are more informative. The efficiency gains with respect to the 8 expectations parameters are reported in Table 7.[13] The results show that while all variables contribute some information for all parameters, in most cases there is one or two variable making the largest contributions. In the case of λ_I and λ_{r^k}, investment provides the most information, while for λ_π and λ_w the most informative variables are inflation and wages, respectively. For λ_c, λ_l, and ω, both consumption and interest are the most informative variables with about equal contributions. Finally, in the case of λ_q consumption is most informative, with investment and interest rate also providing significant contributions of equal sizes.

[13]Results for all estimated parameters are reported in the Appendix.

4.4 Expectations Formation: Stability and Determinacy

In addition to verifying the identifiability of the parameters, we also investigate the stability and determinacy properties of the two models. Since our estimation procedure restricts the parameter space to the region where the model has a unique stable solution, it is interesting to know how the stability and determinacy properties of the model with hybrid expectations compare to those of the rational expectations model.[14] To that end, we apply the Monte Carlo filtering procedure suggested by Ratto (2008), wherein one separates the acceptable and unacceptable regions of the parameter space by generating draws from the prior distribution and checking the uniqueness and stability of the model solution for each draw. Parameters whose draws are systematically different across regions are considered to be important drivers of the stability and determinacy of the model solution. Following Ratto (2008), the key parameters are identified by testing, using the Kolmogorov-Smirnov test, whether the marginal distributions of the values in each region are significantly different from each other. Based on a sample of 100,000 draws from the prior distribution (see Tables 4-6), the analysis of the model with hybrid expectations shows that about 16% of the prior support produces a unique stable solution, while the remaining 84% yield unstable dynamics. In contrast, in the model with rational expectations more than 99% of the prior draws give a unique stable solution. Not surprisingly, the main driver of the acceptable model behavior is ω, and smaller values of that parameter tend to produce a unique stable solution.[15] This is consistent with Beqiraj et al. (2017) who, in a model with heterogenous expectations, find that a stable model solution is more likely when the fraction of rational agents is greater. However, ω is not the only parameter affecting the stability of the model solution, and there are acceptable regions of the parameter space where ω is large. Among the other parameters whose marginal distributions in the stable and unstable regions are significantly different, the most important ones are λ_c, h, and λ_i.[16] As with ω, smaller values of h and λ_i tend to be associated with stable dynamics, while in the case of λ_c stability is more likely for larger values of that parameter. Our results are in line with previous findings in the literature as described in Branch and McGough (2009), Massaro (2013), and Beqiraj et al. (2017).

4.5 Inflation Expectations: Models vs Survey of Professional Forecasters

Most of the tests run in previous subsections do not allow us to establish whether or not the model with hybrid expectations is better than the model with rational expectations. In this subsection we investigate if our way of accounting for agents boundedly-rational behavior puts the model in the condition to generate plausible series for inflation expectations. In our context, plausible means those expectations are consistent

[14] We thank an anonymous referee for alerting us to the possibility that introducing hybrid expectations may shrink the acceptable region of the parameter space.

[15] Remember that $\omega = 0$ implies fully rational expectations.

[16] There are 16 parameters in total that show statistically significant differences, at a critical value of 1% for the test statistics, between the marginal distributions in the stable and unstable regions. For most of them, however, the differences are very small and would be insignificant at a critical value of 5%. In general, the number of parameters that are found to be significant increases with the number of draws from the prior distribution.

with survey evidence on the expectations of real-world professional forecasters.

In Figure 5 we show the model-implied and observed inflation expectations for horizons from one to four quarters ahead. The model-implied expectations are obtained by applying the Kalman smoother to the RE and HYBRID versions of the model evaluated at the respective means of the posterior distributions. The empirical expectations are taken from the Survey of professional forecasters (SPF) as the mean of the aggregate distribution of the forecasts.[17] Overall, results show that the HYBRID model does a much better job at matching the evolution of the observed expectations series. In particular, it captures the continuing decline of expectations in the second half of the 1990s, and the gradual increase starting in the early 2000s. The RE model, in contrast, implies a sharp increase of inflation expectations in the late 1990s followed by a downward shift in the 2000s. The RE model also fails to capture the persistent decline of expectations in the earlier part of the sample, indicating instead low and relatively constant values, especially at the longer horizons. This testifies once more that the unit root brought into the model by hybrid expectations guarantees the necessary persistence to capture the observed survey of professional forecast expectations persistence.

This test can be seen as in-sample forecast exercise. In fact, the implied expectations series are obtained by the Kalman smoother. At every point in time it computes the smoothed values for the variable of interest by exploiting the whole information set, from the beginning to the end of the sample. In other words, it exploits information in the future, as well as in the past. This results can be seen as an unfair comparison with the survey of professional forecasters forecast, because at each point in time they can only rely on information from the past and up to the time when they compute their forecasts. Surely not from the future.

In order to cope with that unfairness, we run an out-of-sample forecast analysis to evaluate whether or not the model with hybrid expectations is superior also in this dimension. The comparison is once again between the forecast (or expectations) from the two models and the expectations from the survey for horizon from one to four.

We produce both point and density forecasts. We estimate the models using a 10-year rolling window recursive estimation. The first window spans from the beginning of the sample to $1992q1$. Hence the first forecasts are for $1992q2$, $1992q3$, $1992q4$, $1993q1$ for $t+1$, $t+2$, $t+3$, and $t+4$ respectively. To compute the predictive density we draw 5000 sets of parameters from the posterior distributions and for each of them we simulate 7 path of future shocks. In that way we account for the two main sources of forecast uncertainty: parameters and future shocks uncertainty.[18]

We perform formal statistical tests to judge the differences in the ability of the model to fit the data. For the point forecasts we run the Diebold and Mariano test, which does not require much formal explanations.

[17] We use the Survey of Professional Forecasters maintained by the Federal Reserve Bank of Philadelphia.

[18] Adolfson et al. (2007) identify tow other sources of uncertainty: current state and measurement errors uncertainty. Here we abstract from them because we do not have measurement errors and because the two sources that we considered are generally the most relevant.

On the contrary, for the tests to evaluate density forecasts we need to provide some details. In particular, comparison based on the Kullback-Leibler divergence or Kullback-Leibler Information Criterion (KLIC) is particularly suitable to our context.[19] The KLIC distance is defined as follows:

$$\begin{aligned} KLIC_i &= \int f_t(y_t) \ln \frac{f_t(y_t)}{f_{t,i}(y_t)} dy_t, \\ &= E\left[\ln f_t(y_t) - \ln f_{t,i}(y_t)\right], \end{aligned} \tag{18}$$

where f_t is the true density of a random variable y_t with domain and range $\mathbb{R}$, the integral is taken over $\mathbb{R}$, and E denotes expectations. A set of $f_{t,i}$, $i = 1, 2$, from different models is available. It is assumed that $f_t(y_t) > 0$ and $f_{t,i}(y_t) > 0$ for all $y \in \mathbb{R}$. In order to compare the KLIC distance between f_1 and f_2, only the last term of expression (18), that is, the expected logarithmic score, $E \ln S_i$, can be considered:

$$E \ln S_i = E\left[\ln f_{t,i}(y_t)\right]. \tag{19}$$

Note that, when $E \ln S_1 > E \ln S_2$ then $KLIC_1 < KLIC_2$. Under some regularity conditions a consistent estimate of (19) can be obtained from the average of the sample information, $y_1, ..., y_T$:

$$\ln S_i = \frac{1}{T} \sum_{t=1}^{T} \ln f_{i,t}(y_t).$$

This last expression defines the so called average logarithmic score or simply log score. The model selection criterion is such that the model that maximizes the log score is the best model.

Finally let us denote $f_{t+h,t,i}$ as a prediction of the density Y_{t+h}, conditional on information up to date t. Also denote y_{t+h} as the realization of Y_{t+h} and assume the h-step ahead density forecasts are available from a starting date T^s based on a total number of T observations. A measure of out-of-sample forecasting performance is the out-of-sample log score given by:

$$\ln S_{i,h} = \frac{1}{T - h - T^s + 1} \sum_{t=T^s}^{T-h} \ln f_{t+h,t,i}(y_{t+h}).$$

This is our chosen measure to judge the forecast performance of our models.

We also consider an alternative metric for evaluating density forecasts – the Continuous Ranked Proba-

[19] The advantages of using the KLIC are: it chooses the model that on average gives higher probability to events that have actually occurred; it provides a unified framework in the density forecast framework; it can be easily related to other measures of ex-post forecast evaluation, like the probability integral transforms (PITs); it has a Bayesian interpretation as the KLIC-best model is the model with the highest posterior probability. See for example Fernandez-Villaverde and Rubio-Ramirez (2004), Mitchell and Hall (2005), Kascha and Ravazzolo (2010), Christoffel et al. (2010) and Geweke and Amisano (2011), Wolters (2015).

bility Score:

$$CRPS = \int \left(F\left(z\right) - I_{[y_{t+h},+\infty)}\left(z\right)\right)^2 dz,$$

where F is the cumulative distribution function of $f_{t+h,t,i}$. The CRPS for a specific model measures the average absolute distance between the empirical cumulative distribution function (CDF) of y_{t+h} and the empirical CDF that is associated with the model predictive density. Differently from the log score, the CRPS rewards values from the predictive density that are close but not necessarily equal to the realizations. Smaller CRPS values imply a higher precision of the model.[20]

This three tests measure the relative performances. For the density forecasts we also provide a test for the absolute forecast accuracy, relative to the "true" but unobserved density. We utilize the probability integral transforms, PITS, of the realisation of the variable with respect to the forecast densities as in Rosenblatt (1952) and Diebold et al. (1998). A forecast density is preferred if the density is correctly calibrated, regardless of the forecasters loss function. The PITS should be both uniformly distributed, and independently and identically distributed if the forecast densities are correctly calibrated. Hence, calibration evaluation requires the application of tests for goodness of fit and independence. The goodness of test employed is the Likelihood Ratio test proposed by Berkowitz (2001).[21] It is a three degrees of freedom variant, with a test for independence.

In table 8 we report the mean squared forecast errors (MSFE) for both models and the results of the Diebold and Mariano test. We unequivocally show that the model with hybrid expectations is better at any horizon. In fact its MSFE are always lower. Moreover, for the first three horizons they are also statistically lower (at more than 1% significance level) than those of the model with rational expectations.

Density forecasts, reported in table 9, convey a similar message. The log-score is always higher and the CRPS always lower for the hybrid model. For the first three horizons the difference in those measure is highly statistically different, while for 4 quarters ahead it is not.

Finally, in table 10 we report the results of the Berkowits test. We report only the p-values. Values bigger that 0.05 indicate that the predictive density is correctly calibrated, i.e. PITS are *iid* $N(0, 1)$, so the model is accurate in absolute sense. The test reveals that none of the models has that property for all horizons, but the model with hybrid expectations is slightly better.

We can then conclude that the forecast analysis provides us with strong arguments in favor of the hybrid expectations specification.

[20]For more details and applications related to the CRPS, see, among others, Gneiting and Raftery (2007), Panagiotelis and Smith (2008), Billio et al. (2013), Ravazzolo and Vahey (2014), Aastveit et al. (2014), and Clark and Ravazzolo (2015).

[21]We also run the Knuppel test.

5 Conclusions

This paper investigates the extent to which a departure from fully rational expectations can improve the empirical fit and performance of the Smets and Wouters (2007) model. Specifically, we consider a "hybrid expectations" version of the model in which a subset of agents employ simple moving-average forecast rules that place a significant weight on the most recent data observation.

We employ a wide variety of tests that rank the performance of the hybrid expectations model versus an otherwise similar model in which all agents are fully rational. From an overall fit perspective, the data strongly favor the hybrid expectations model. Nevertheless, a moments matching exercise does not favor one model over the other.

We also compare the expected inflation series from each model to the expected inflation series from the Survey of Professional Forecasters. This comparison strongly favors the hybrid expectations model at all horizons. Moreover, in- and out-of-sample point and density forecast exercises, show that the hybrid expectations model can always achieve (statistically significantly) lower forecast errors.

Our analysis shows and confirms the quantitative relevance of the existence of boundedly-rational agents employing simple moving-average forecast rules. We estimate a share of backward looking agents of about 70%. Moreover, those rules place a significant weight on the most recent data observation for most of the forecasted variables. This is particularly interesting also from a policy point of view. DSGE models are extensively used in policy institutions for policy and forecast analysis. Moreover, managing expectations is one of the main tasks and channels through which central banks implement their policies. Hence, using models able to correctly account for expectations formation like our hybrid expectations model, seems of paramount importance to us. Moreover, as showed for instance by Gelain, Lansing, and Mendicino (2013), conducting policy analysis with models with rational expectations might lead to very different policy conclusions than with models in which a subset of agents are not fully rational.

Our framework has also some limitations. For instance we assume that boundedly-rational expectations among households do not differ from boundedly-rational expectations among firms, while that is potentially not true in reality. Moreover, we do not allow for parameters to be time varying. Especially those related to adaptive expectations. A regime switching model could account for such time variation and give a nice description of times when the share of rational forecasters might be bigger than the share of backward looking agents. Finally, our models do not feature some relevant frictions, such as financial frictions, that have been shown to be very important heading to, during, and after the Great Recession. These are all interesting extensions that we leave for future research.

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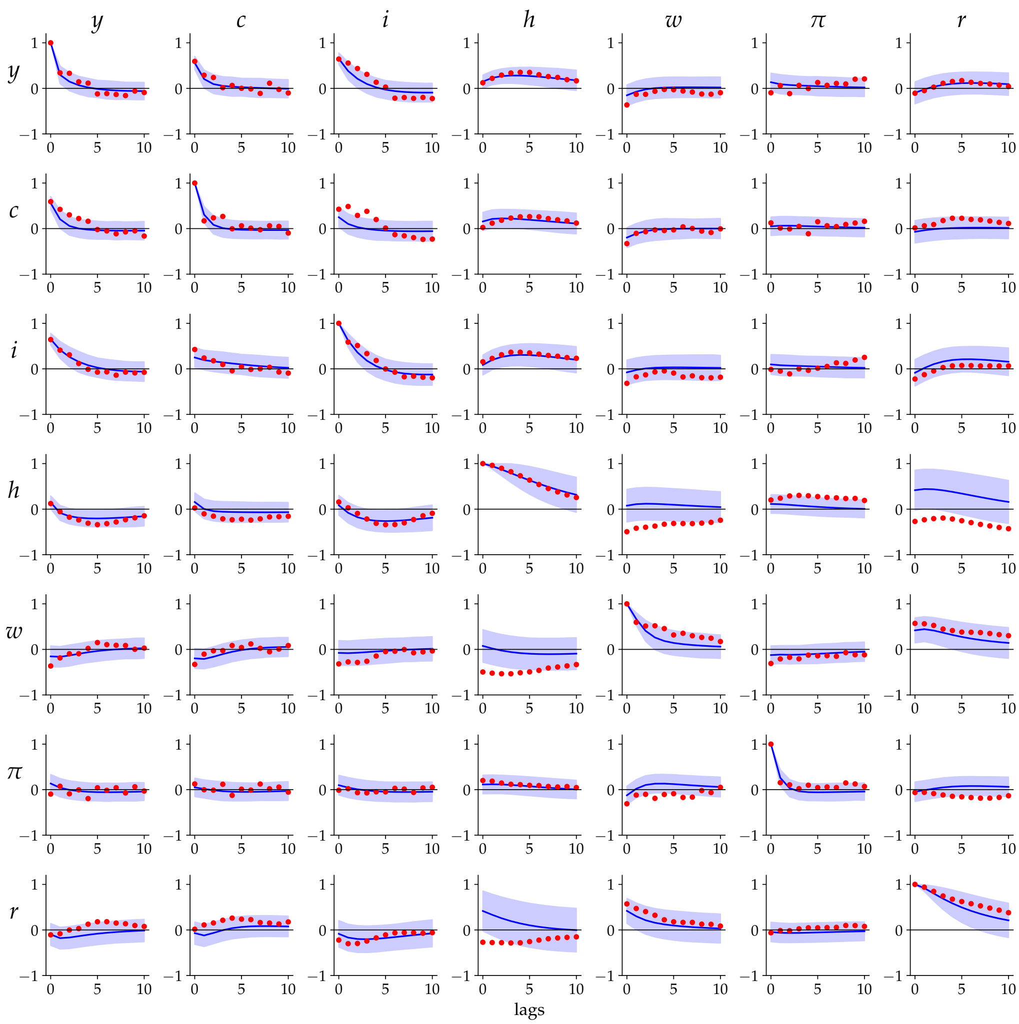


Figure (1) **Cross-correlations, RE model**. The figure shows empirical (red dots) and model-implied (solid line) cross-correlations up to lag 10. The shaded area represents 95% confidence bands.

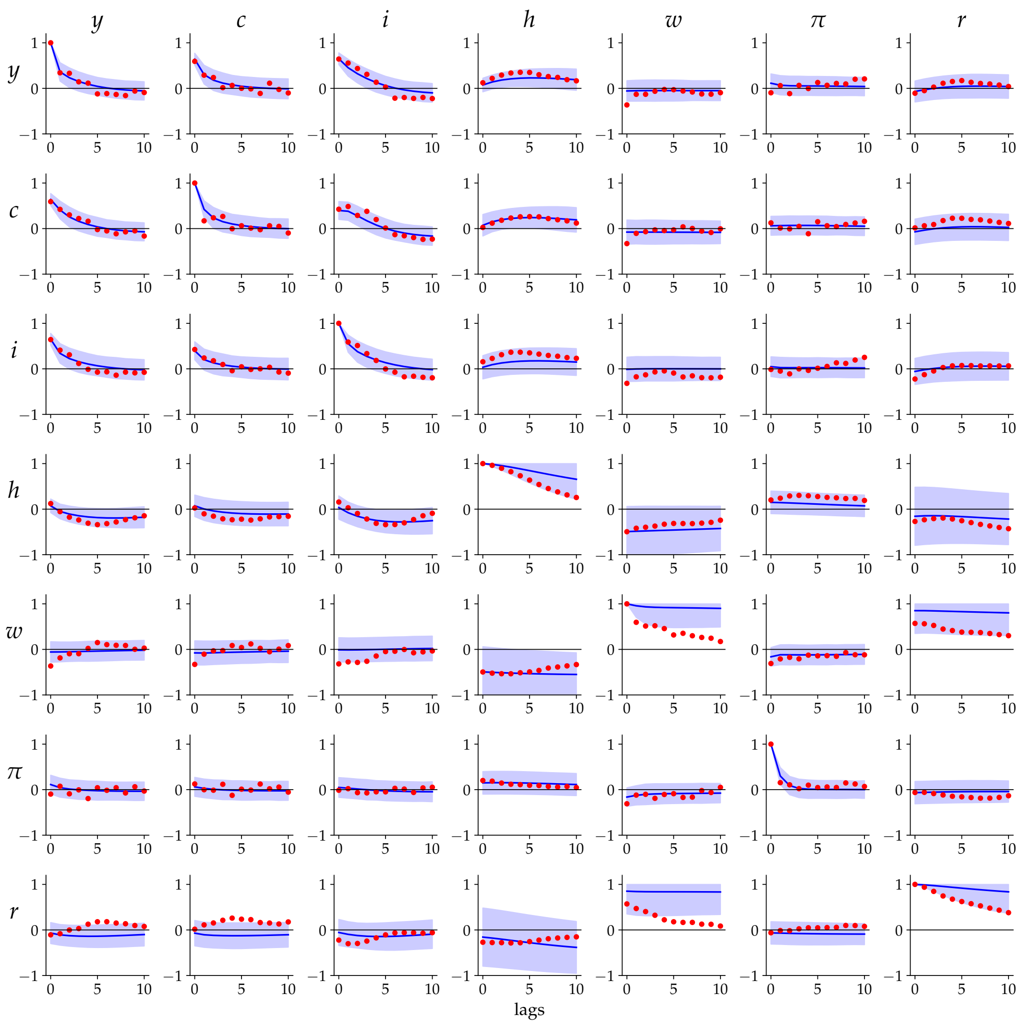


Figure (2) **Cross-correlations, Hybrid model**. The figure shows empirical (red dots) and model-implied (solid line) cross-correlations up to lag 10. The shaded area represents 95% confidence bands.

Table (1) Log-Linearized Equations

$$(1)\quad \widehat{\pi}_t = \frac{\iota^P}{1+\beta\gamma^{(1-\sigma_c)}\iota^P}\widehat{\pi}_{t-1} + \frac{\beta\gamma^{(1-\sigma_c)}}{1+\beta\gamma^{(1-\sigma_c)}\iota^P}\widehat{E}_t\widehat{\pi}_{t+1} - \frac{1}{1+\beta\gamma^{(1-\sigma_c)}\iota^P}\left[\left(1-\beta\gamma^{(1-\sigma_c)}\xi^P\right)\frac{1-\xi^P}{\xi^P((\phi_p-1)\zeta^p+1)}\right]\widehat{\mu}_t^P + \widehat{\varepsilon}_{p,t}$$

$$(2)\quad \widehat{c}_t = \frac{h/\gamma}{1+h/\gamma}\widehat{c}_{t-1} + \left(1-\frac{1}{1+h/\gamma}\right)\widehat{E}_t\widehat{c}_{t+1} + \frac{(\sigma_c-1)\left(w^h l/c\right)}{\sigma_c(1+h/\gamma)}\left(\widehat{l}_t - \widehat{E}_t\widehat{l}_{t+1}\right) - \frac{1-h/\gamma}{\sigma_C(1+h/\gamma)}\left(\widehat{R}_t + \widehat{\varepsilon}_{b,t}\right)$$

$$(3)\quad \widehat{i}_t = \frac{1}{1+\beta\gamma^{(1-\sigma_c)}}\widehat{i}_{t-1} + \left(1-\frac{1}{1+\beta\gamma^{(1-\sigma_c)}}\right)\widehat{E}_t\widehat{i}_{t+1} + \frac{1}{1+\beta\gamma^{(1-\sigma_c)}}\frac{1}{\gamma^2\varphi}\widehat{q}_t + \widehat{\varepsilon}_{q,t}$$

$$(4)\quad \widehat{w}_t = \frac{1}{1+\beta\gamma^{(1-\sigma_c)}}\widehat{w}_{t-1} + \frac{\beta\gamma^{(1-\sigma_C)}}{1+\beta\gamma^{(1-\sigma_C)}}\left(\widehat{E}_t\widehat{w}_{t+1} + \widehat{E}_t\widehat{\pi}_{t+1}\right) - \frac{1+\beta\gamma^{(1-\sigma_c)}\iota^W}{1+\beta\gamma^{(1-\sigma_c)}}\widehat{\pi}_t + \frac{\iota^W}{1+\beta\gamma^{(1-\sigma_c)}}\widehat{\pi}_{t-1} +$$

$$(5)\quad -\left[\frac{1-\beta\gamma^{(1-\sigma_c)}\xi^W}{1+\beta\gamma^{(1-\sigma_c)}}\frac{1-\xi^W}{\xi^W((\phi_W-1)\zeta^W+1)}\right]\widehat{\mu}_t^W + \widehat{\varepsilon}_{w,t}$$

$$(6)\quad \widehat{\mu}_t^W = \widehat{w}_t - \sigma_l\widehat{l}_t + \frac{1}{1-h/\gamma}\left(\widehat{c}_t - (h/\gamma)\,\widehat{c}_{t-1}\right)$$

$$(7)\quad \widehat{\mu}_t^P = \alpha\left(\widehat{k}_t^s - \widehat{l}_t\right) - \widehat{w}_t + \widehat{\varepsilon}_{a,t}$$

$$(8)\quad \widehat{q}_t = \beta\gamma^{-\sigma_c}(1-\delta)\,\widehat{E}_t\widehat{q}_{t+1} + (1-\beta\gamma^{-\sigma_c}(1-\delta))\,\widehat{E}_t\widehat{r}_{t+1}^k - \widehat{R}_t - \widehat{\varepsilon}_{b,t}$$

$$(9)\quad \widehat{k}_t = ((1-\delta)/\gamma)\,\widehat{k}_{t-1} + (1-(1-\delta)/\gamma)\,\widehat{i}_t + \left[(1-(1-\delta)/\gamma)\left(1+\beta\gamma^{(1-\sigma_c)}\right)\gamma^2\varphi\right]\widehat{\varepsilon}_{i,t}$$

$$(10)\quad \widehat{y}_t = \phi_p\left[\ \alpha\widehat{k}_t^s + (\ 1-\alpha)\ \ \widehat{l}_t + \widehat{\varepsilon}_{a,t}\right]$$

$$(11)\quad \widehat{k}_t^s = \widehat{k}_{t-1} + \widehat{z}_t$$

$$(12)\quad \widehat{z}_t = \frac{1-\psi}{\psi}\widehat{r}_t^k$$

$$(13)\quad \widehat{r}_t^k = -\left(\widehat{k}_t - \widehat{l}_t\right) + \widehat{w}_t$$

$$(14)\quad \widehat{y}_t = c_y\widehat{c}_t + i_y\widehat{i}_t + z_y\widehat{z}_t + \widehat{\varepsilon}_{g,t}$$

$$(15)\quad \widehat{r}_t = \rho\widehat{r}_{t-1} + (1-\rho)\left[r_\pi\widehat{\pi}_t + r_y\left(\widehat{y}_t - \widehat{y}_t^P\right)\right] + r_{\Delta y}\left[\left(\widehat{y}_t - \widehat{y}_t^P\right) - \left(\widehat{y}_{t-1} - \widehat{y}_{t-1}^{fP}\right)\right] + \widehat{\varepsilon}_{r,t}$$

$$(16)\quad \widehat{\varepsilon}_{a,t} = \rho^a\widehat{\varepsilon}_{a,t-1} + u_{a,t}$$

$$(17)\quad \widehat{\varepsilon}_{b,t} = \rho^b\widehat{\varepsilon}_{b,t-1} + u_{b,t}$$

$$(18)\quad \widehat{\varepsilon}_{x,t} = \rho^x\widehat{\varepsilon}_{x,t-1} + u_{x,t}$$

$$(19)\quad \widehat{\varepsilon}_{x,t} = \rho^x\widehat{\varepsilon}_{x,t-1} + u_{x,t}$$

$$(20)\quad \hat{\varepsilon}_{w,t} = \rho^w\hat{\varepsilon}_{w,t-1} + u_{w,t}$$

$$(21)\quad \hat{\varepsilon}_{p,t} = \rho^p\hat{\varepsilon}_{p,t-1} + u_{p,t}$$

$$(22)\quad \widehat{\varepsilon}_{g,t} = \rho^g\widehat{\varepsilon}_{g,t-1} + u_{g,t} + \rho^{ga}u_{a,t}$$

Measurement Equations

$$(23)\quad \Delta\ln(GDP_t) = \widehat{y}_t - \widehat{y}_{t-1} + \bar{\gamma}$$

$$(24)\quad \Delta\ln(c_t) = \widehat{c}_t - \widehat{c}_{t-1} + \bar{\gamma}$$

$$(25)\quad \Delta\ln(I_t) = \widehat{i}_t - \widehat{i}_{t-1} + \bar{\gamma}$$

$$(26)\quad \Delta\ln(w_t) = \widehat{w}_t - \widehat{w}_{t-1} + \bar{\gamma}$$

$$(27)\quad \pi_t = \widehat{\pi}_t + \pi_{ss}$$

$$(28)\quad ln(h_t) = \widehat{l}_t + l_{ss}$$

$$(29)\quad r_t = \widehat{r}_t + \bar{r}$$

$$\bar{\gamma} = 100(\gamma-1);\pi_{ss} = 100(\Pi_*-1);\bar{r} = 100(\beta^{-1}\gamma^{\sigma_c}\Pi_*-1)$$

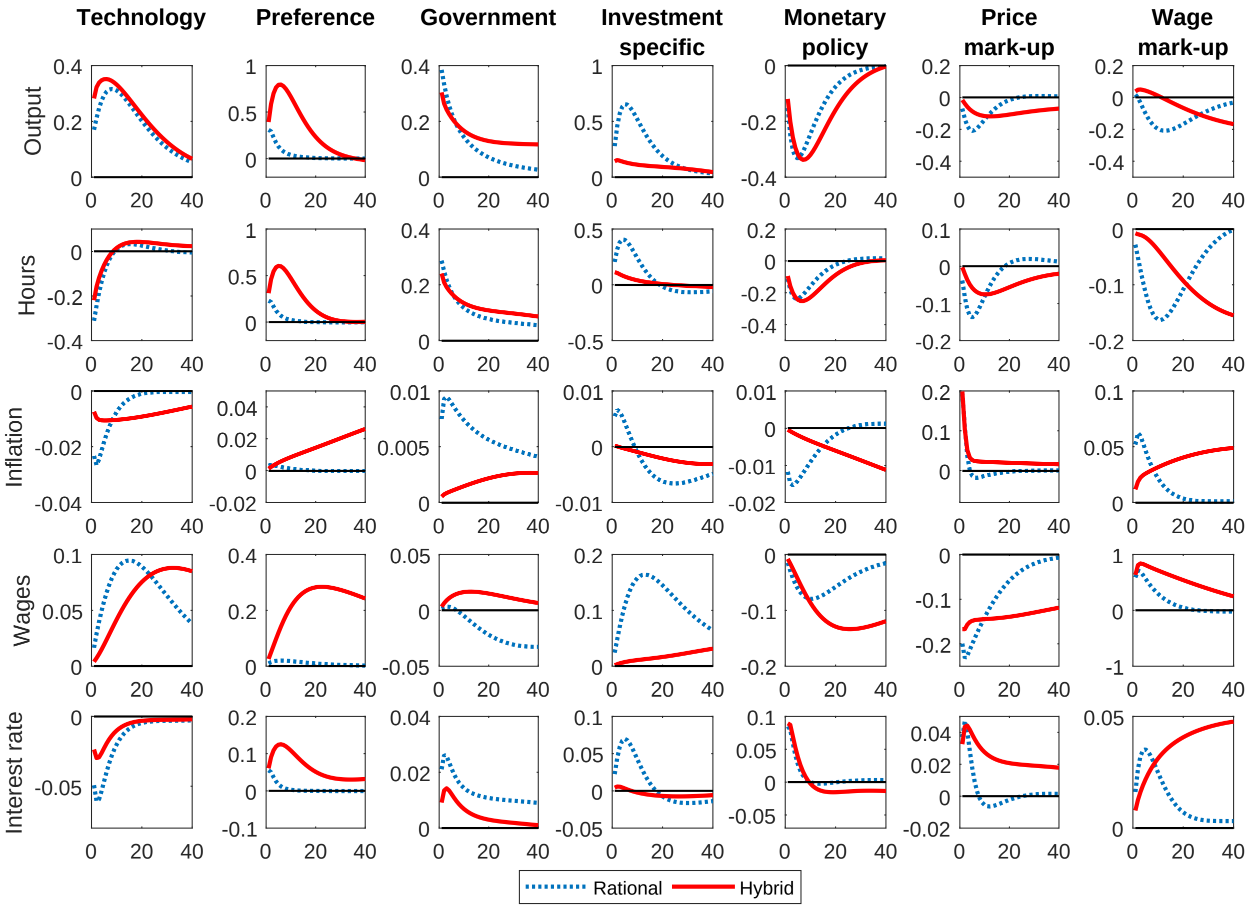
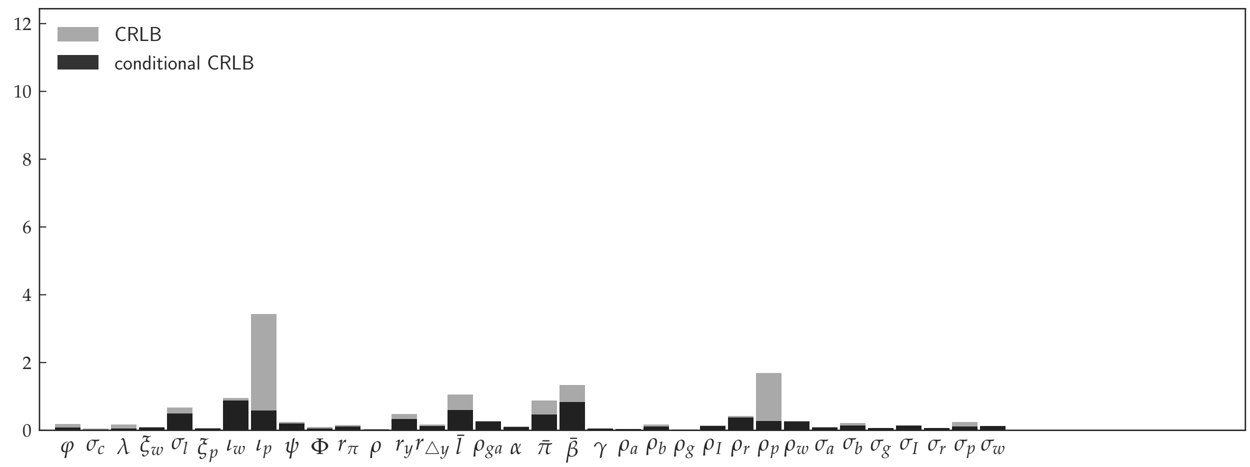
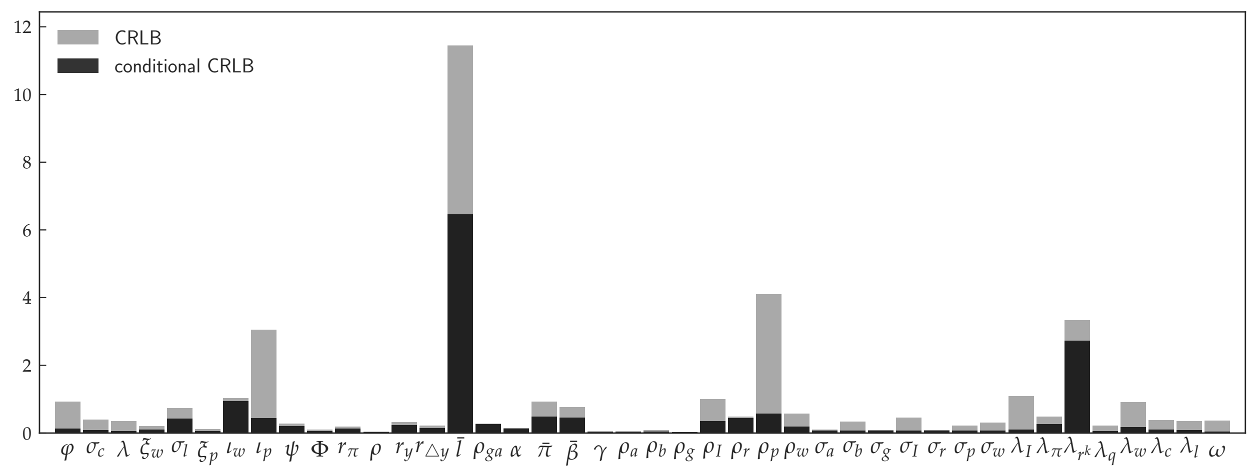


Figure (3) **Impulse response functions**



(a) **Model with rational expectations**



(b) **Model with hybrid expectations**

Figure (4) The Figure shows the CRLB (in grey) and conditional CRLB (in black) for all estimated parameters, normalized by the respective parameter values.

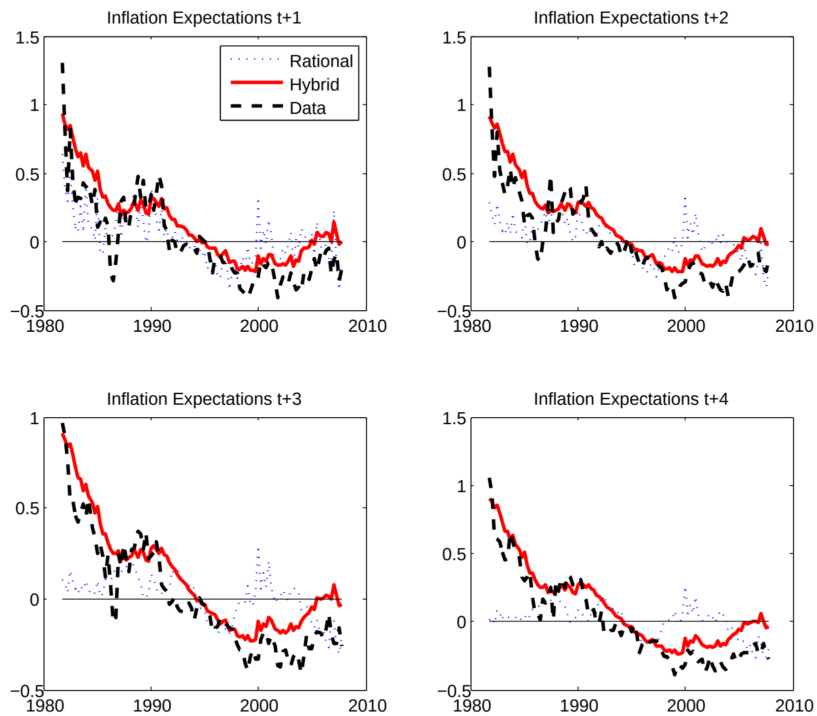


Figure (5) **Inflation expectations**. The figure shows model-implied and empirical inflation expectations for one to four quarters ahead.

Table (2) Estimation results - Parameters

Parameter			**Prior**		**Posterior (RE)**			**Posterior (HYBRID)**		
			Mean	**Std**	**Mean**	**5%**	**95%**	**Mean**	**5%**	**95%**
Intert. elast. substitution	σ^c	$\mathcal{N}$	1.5	0.375	1.4974	1.1617	1.8022	0.3781	0.2500	0.4870
Habits	h	$\mathcal{B}$	0.7	0.1	0.6298	0.5136	0.7354	0.5673	0.4523	0.6862
Labor supply elasticity	σ^l	$\mathcal{N}$	2	0.75	2.3452	1.3654	3.2966	2.6773	1.7050	3.6271
Calvo prob. - wages	ξ^w	$\mathcal{B}$	0.5	0.1	0.8685	0.8193	0.9200	0.7324	0.6437	0.8286
Calvo prob. - prices	ξ^p	$\mathcal{B}$	0.5	0.1	0.7847	0.7149	0.8562	0.7389	0.6201	0.8610
Indexation - wages	ι^w	$\mathcal{B}$	0.5	0.15	0.4113	0.1867	0.6271	0.3820	0.1685	0.5959
Indexation - prices	ι^p	$\mathcal{B}$	0.5	0.15	0.3021	0.0876	0.5064	0.4078	0.1584	0.6477
Investment adjust. cost elast.	φ	$\mathcal{N}$	4	1.5	6.6212	4.5752	8.7068	4.4323	2.8240	5.9886
Capital utiliz. adj. cost	ψ	$\mathcal{B}$	0.5	0.15	0.7097	0.5552	0.8663	0.6621	0.4726	0.8559
Cobb-Douglas	α	$\mathcal{N}$	0.3	0.05	0.2069	0.1714	0.2437	0.1604	0.1280	0.1935
Fixed cost	ϕ	$\mathcal{N}$	1.25	0.125	1.5740	1.4185	1.7273	1.4147	1.2665	1.5655
Taylor rule - smoothing	ρ	$\mathcal{B}$	0.75	0.1	0.8648	0.8326	0.8979	0.8898	0.8554	0.9254
Taylor rule - inflation	r_π	$\mathcal{N}$	1.5	0.25	1.8675	1.5197	2.1999	1.6097	1.2386	1.9952
Taylor rule - output	r_y	$\mathcal{N}$	0.125	0.05	0.1018	0.0490	0.1537	0.1536	0.0901	0.2125
Taylor rule - output growth	$r_{\Delta y}$	$\mathcal{N}$	0.1250	0.0500	0.1639	0.1219	0.2070	0.1374	0.0981	0.1754
Log hours worked	l_{ss}	$\mathcal{N}$	0	2	1.4148	0.1352	2.6780	0.1297	-2.0899	2.3944
Steady-state inflation rate	π_{ss}	$\mathcal{G}$	0.625	0.1	0.6374	0.5349	0.7379	0.6125	0.4538	0.7699
Discount factor	$\bar{\beta}$	$\mathcal{G}$	0.25	0.1	0.1385	0.0527	0.2220	0.2528	0.1045	0.3968
Steady-state growth rate	$\bar{\gamma}$	$\mathcal{N}$	0.4	0.1	0.4481	0.4176	0.4784	0.4706	0.4167	0.5292

B=Beta, N=Normal, G=Gamma, IG= Inverse Gamma. $\bar{\beta} = \beta^{-1} - 1$; $\bar{\gamma} = \gamma_{ss} - 1$

Table (3) Estimation results - Shocks

Parameter			**Prior** Mean	Std	**Posterior (RE)** Mean	5%	95%	**Posterior (HYBRID)** Mean	5%	95%
AR coefficients shocks										
Productivity	ρ^a	$\mathcal{B}$	0.5	0.2	0.9123	0.8675	0.9589	0.9309	0.8772	0.9967
Risk-premium	ρ^b	$\mathcal{B}$	0.5	0.2	0.3229	0.0397	0.8282	0.8304	0.7597	0.9046
Government spending	ρ^g	$\mathcal{B}$	0.5	0.2	0.9741	0.9581	0.9914	0.9816	0.9636	0.9986
Investment-specific	ρ^q	$\mathcal{B}$	0.5	0.2	0.7381	0.6254	0.8513	0.2767	0.0780	0.4609
Monetary policy	ρ^r	$\mathcal{B}$	0.5	0.2	0.2541	0.1155	0.3943	0.2195	0.0804	0.3701
Price-markup	ρ^p	$\mathcal{B}$	0.5	0.2	0.3405	0.0793	0.5759	0.1640	0.0238	0.2967
Wage-markup	ρ^w	$\mathcal{B}$	0.5	0.2	0.3218	0.1363	0.4964	0.4558	0.2196	0.6845
Prod. in gov. spending	ρ^{ga}	$\mathcal{N}$	0.5	0.25	0.3945	0.2151	0.5735	0.4046	0.2377	0.5687
St.deviation shocks										
Productivity	σ^a	$\mathcal{IG}$	0.1	2	0.3731	0.3255	0.4192	0.3968	0.3469	0.4464
Risk-premium	σ^b	$\mathcal{IG}$	0.1	2	0.1645	0.0713	0.2181	0.3179	0.2340	0.4064
Government spending	σ^g	$\mathcal{IG}$	0.1	2	0.4177	0.3673	0.4659	0.4086	0.3608	0.4553
Investment-specific	σ^q	$\mathcal{IG}$	0.1	2	0.3489	0.2669	0.4274	0.6806	0.5513	0.8034
Monetary policy	σ^r	$\mathcal{IG}$	0.1	2	0.1154	0.1006	0.1300	0.1084	0.0945	0.1215
Price-markup	σ^p	$\mathcal{IG}$	0.1	2	0.1128	0.0837	0.1408	0.1699	0.1445	0.1949
Wage-markup	σ^w	$\mathcal{IG}$	0.1	2	0.2420	0.1867	0.2970	0.4168	0.3276	0.5043

B=Beta, N=Normal, G=Gamma, IG= Inverse Gamma.

Table (4) Estimation results - Expectation Parameters

Expectation Parameters			**Prior** Mean	**Prior** Std	**Posterior (HYBRID)** Mean	5%	95%
% MA forecast rule	ω	$\mathcal{B}$	0.5	0.2	0.6848	0.5616	0.8154
forecast response i	λ_i	$\mathcal{B}$	0.5	0.2	0.6920	0.4838	0.9110
forecast response π	λ_p	$\mathcal{B}$	0.5	0.2	0.0904	0.0035	0.1691
forecast response r^k	λ_{r^k}	$\mathcal{B}$	0.5	0.2	0.4962	0.1474	0.8164
forecast response q	λ_q	$\mathcal{B}$	0.5	0.2	0.8446	0.7615	0.9331
forecast response w	λ_w	$\mathcal{B}$	0.5	0.2	0.4942	0.2303	0.7628
forecast response c	λ_c	$\mathcal{B}$	0.5	0.2	0.4987	0.3438	0.6494
forecast response l	λ_l	$\mathcal{B}$	0.5	0.2	0.5179	0.3904	0.6423

B=Beta, N=Normal, G=Gamma, IG= Inverse Gamma.

Table (5) Log Marginal Data Density

	(1) RE	(2) HYBRID
Log Marginal Data Density	-446.71	-441.83
difference	-	4.88
implied Bayes factor	-	131.63

Log Marginal Data Density based on the Modified Harmonic Mean Estimator

Table (6) Empirical versus model-implied moments

	$\Delta\ln(GDP)$	$\Delta\ln(c)$	$\Delta\ln(I)$	$\Delta\ln(w)$	$\ln(h)$	π	r
			Standard Deviations				
Data	0.66	0.56	1.82	2.67	0.29	0.65	0.71
RE	0.72	0.53	1.98	1.81	0.29	0.67	0.34
HYBRID	0.70	0.59	1.77	2.99	0.78	0.71	0.92
			Correlation with output growth				
Data	1.00	0.59	0.64	0.12	-0.36	-0.10	-0.11
RE	1.00	0.56	0.65	0.15	-0.15	0.13	-0.10
HYBRID	1.00	0.64	0.65	0.07	-0.06	0.11	-0.07
			Autocorrelation				
Data	0.34	0.17	0.59	0.96	0.59	0.15	0.94
RE	0.30	0.31	0.62	0.94	0.67	0.26	0.92
HYBRID	0.36	0.42	0.55	0.98	0.96	0.30	0.99

Note: The model-implied moments are computed at the posterior means.

Table (7) Efficiency Gains, expectations parameters

	$\Delta\ln(GDP)$	$\Delta\ln(c)$	$\Delta\ln(I)$	$\Delta\ln(w)$	$\ln(h)$	π	r
λ_I	1.4	37.1	53.1	4.6	30.8	4.2	23.5
λ_π	5.1	3.1	1.7	47.3	8.4	56.4	10.3
λ_{r^k}	4.7	25.9	50.4	13.5	20.6	1.9	20.1
λ_q	7.4	42.2	31.2	7.2	24.2	8.8	31.2
λ_w	0.9	1.8	0.4	63.9	2.9	36.6	1.8
λ_c	2.9	47.9	22.4	3.9	20.1	8.1	41.6
λ_l	1.4	32.9	22.3	4.3	26.0	2.8	38.3
ω	11.6	36.5	22.3	8.8	28.5	13.6	37.6

Note: The table reports efficiency gains defined as the reduction of CRLB due to observing a variable as a percent of the CRLB when the variable is excluded.

Table (8) Point forecast evaluation

	MSFE		Diebold-Mariano (p-value)
Inflation expectations given t	RE	HYBRID	
t+1	0.0508	0.0156	6.2067 (0.0000)*
t+2	0.0530	0.0210	5.6670 (0.0000)*
t+3	0.0468	0.0271	4.0745 (0.0000)*
t+4	0.0373	0.0312	0.8548 (0.3927)

Diebold and Mariano (1998) test; null hypothesis: the two models have the same forecast accuracy; the statistics is normally distributed. $***$, $**$, $*$ denote 10%, 5%, and 1% significance respectively.

Table (9) Density forecast evaluation - relative performance

	Log score				CRPS			
	t+1	t+2	t+3	t+4	t+1	t+2	t+3	t+4
RE	0.2881	0.0980	0.0217	-0.0097	0.1016	0.1132	0.1166	0.1147
HYBRID	0.8429	0.7977	0.6628	0.5995	0.0572	0.0622	0.0727	0.0784
p-values	*0.0001*	*0.0035*	*0.0329*	*0.1107*	*0.0000*	*0.0000*	*0.0010*	*0.1031*

p-values are on the statistical difference between the test statistics of the two models.

Table (10) Density forecast evaluation - absolute performance

	Berkowitz			
	t+1	t+2	t+3	t+4
Rational expectations	0.2373	0.0001	0.0000	0.0000
Hybrid expectations	0.4293	0.2740	0.0207	0.0020

Null: pits are iid N(0,1), i.e. values bigger that 0.05 indicate that the predictive density is correctly calibrated.

Boundedly-Rational Expectations in an Estimated DSGE model

Technical Appendix
(NOT FOR PUBLICATION)

A Data

A.1 Observables

In the following we describe in detail the data used in the estimation:

- Real output growth: quarter-on-quarter log difference of real output, defined as real GDP (Billions U.S. Dollar, 2005 prices) divided by the civilian noninstitutional population (aged 16 and over).
- Real consumption growth: quarter on-quarter log difference of real consumption, defined as nominal personal consumption expenditure (Billions U.S. Dollar) divided by the GDP implicit price deflator and then divided by the civilian noninstitutional population (aged 16 and over).
- Real investment growth: quarter on-quarter log difference of real investment, defined as nominal private fixed investment (Billions U.S. Dollar) divided by the GDP implicit price deflator and then divided by the civilian noninstitutional population (aged 16 and over).
- Hours worked: Average hours worked (non-farm business sector) multiplied by total civilian employment and divided by the civilian noninstitutional population (aged 16 and over); log-transformed; 1950:2010=100.
- Inflation: quarter on-quarter log difference in the GDP implicit price deflator.
- Real wage growth: Log difference in real wage, defined as hourly compensation in the non-farm business sector (1992=100) divided by the GDP implicit price deflator.
- Interest rate: Federal-funds rate.(quarterly).

B Efficiency gains

Table (1) Efficiency Gains, all parameters

	$\Delta\ln(GDP)$	$\Delta\ln(c)$	$\Delta\ln(I)$	$\Delta\ln(w)$	$\ln(h)$	π	r
φ	6.0	34.9	52.5	7.6	34.1	5.4	22.6
σ_c	10.2	64.3	14.0	3.9	57.3	5.7	52.4
λ	13.5	31.2	8.8	8.1	39.0	11.1	61.8
ξ_w	7.9	22.9	5.5	55.6	20.3	26.4	21.6
σ_l	24.6	48.5	9.2	6.9	51.1	15.2	49.1
ξ_p	16.4	16.2	9.7	35.6	71.5	56.5	21.5
ι_w	0.5	0.2	0.1	90.6	0.8	85.7	0.2
ι_p	2.5	0.7	0.7	8.2	5.0	80.0	2.6
ψ	42.2	15.9	40.6	30.4	28.8	14.6	19.6
Φ	38.3	7.8	27.8	7.3	88.1	3.9	13.1
r_π	1.4	6.6	6.5	2.4	32.3	47.3	76.0
ρ	1.4	5.5	3.6	6.2	21.1	63.9	75.5
r_y	5.8	9.5	8.0	2.7	17.5	29.1	83.6
$r_{\triangle y}$	11.5	26.1	19.9	6.5	36.2	34.0	89.9
ρ_{ga}	95.0	69.4	37.7	8.9	81.9	0.9	8.1
α	41.9	57.3	76.8	2.1	14.1	29.3	64.4
β	3.3	38.5	4.5	1.9	39.4	73.6	89.9
γ	30.8	5.4	10.4	0.5	31.2	1.9	0.2
ρ_a	64.7	5.5	31.1	4.6	58.6	1.5	5.9
ρ_b	7.3	22.5	18.4	5.3	18.7	17.2	75.5
ρ_g	54.1	40.5	39.3	1.4	25.0	14.4	12.3
ρ_I	1.5	36.4	77.8	4.3	26.5	4.0	21.7
ρ_r	2.4	8.8	0.8	2.0	27.3	40.4	99.0
ρ_p	2.4	0.7	0.7	7.4	5.0	82.5	2.1
ρ_w	1.3	3.7	0.9	92.1	4.1	40.4	3.8
σ_a	92.5	6.7	31.3	9.9	88.9	3.2	8.4
σ_b	12.2	68.5	23.1	8.9	33.3	10.8	79.6
σ_g	93.3	74.9	88.4	6.1	50.1	1.6	3.9
σ_I	3.2	32.7	87.9	5.6	32.1	5.0	24.6
σ_r	7.2	11.9	3.2	4.4	43.6	59.7	99.7
σ_p	5.0	12.5	6.0	10.5	12.7	86.4	16.3
σ_w	4.7	18.5	9.0	95.4	14.5	28.6	19.3
λ_I	1.4	37.1	53.1	4.6	30.8	4.2	23.5
λ_π	5.1	3.1	1.7	47.3	8.4	56.4	10.3
λ_{r^k}	4.7	25.9	50.4	13.5	20.6	1.9	20.1
λ_q	7.4	42.2	31.2	7.2	24.2	8.8	31.2
λ_w	0.9	1.8	0.4	63.9	2.9	36.6	1.8
λ_c	2.9	47.9	22.4	3.9	20.1	8.1	41.6
λ_l	1.4	32.9	22.3	4.3	26.0	2.8	38.3
ω	11.6	36.5	22.3	8.8	28.5	13.6	37.6

Note: The table reports efficiency gains defined as the reduction of CRLB due to observing a variable as a percent of the CRLB when the variable is excluded.