::: ## Lambda calculus

environ

vocabularies LAMBDA,

NUMBERS,

NAT\_1, XBOOLE\_0, SUBSET\_1, FINSEQ\_1, XXREAL\_0, CARD\_1,

ARYTM\_1, ARYTM\_3, TARSKI, RELAT\_1, ORDINAL4, FUNCOP\_1;

:: etc...

begin

reserve D for DecoratedTree,

p,q,r for FinSequence of NAT,

x for set;

definition

let D;

attr D is LambdaTerm-like means

(dom D qua Tree) is finite &

::> \*143,306

for r st r in dom D holds

r is FinSequence of {0,1} &

r^<\*0\*> in dom D implies D.r = 0;

end;

registration

cluster LambdaTerm-like for DecoratedTree of NAT;

existence;

::> \*4

end;

definition

mode LambdaTerm is LambdaTerm-like DecoratedTree of NAT;

end;

::: Then we extend this ordinary one-step beta reduction, that is,

::: any subterm is also allowed to reduce.

definition

let M,N;

pred M beta N means

ex p st

M|p beta\_shallow N|p &

for q st not p is\_a\_prefix\_of q holds

[r,x] in M iff [r,x] in N;

end;

theorem Th4:

ProperPrefixes (v^<\*x\*>) = ProperPrefixes v \/ {v}

proof

thus ProperPrefixes (v^<\*x\*>) c= ProperPrefixes v \/ {v}

proof

let y;

assume y in ProperPrefixes (v^<\*x\*>);

then consider v1 such that

A1: y = v1 and

A2: v1 is\_a\_proper\_prefix\_of v^<\*x\*> by TREES\_1:def 2;

v1 is\_a\_prefix\_of v & v1 <> v or v1 = v by A2,TREES\_1:9;

then

v1 is\_a\_proper\_prefix\_of v or v1 in {v} by TARSKI:def 1,XBOOLE\_0:def 8;

then y in ProperPrefixes v or y in {v} by A1,TREES\_1:def 2;

hence thesis by XBOOLE\_0:def 3;

end;

let y;

assume y in ProperPrefixes v \/ {v};

then A3: y in ProperPrefixes v or y in {v} by XBOOLE\_0:def 3;

A4: now

assume y in ProperPrefixes v;

then consider v1 such that

A5: y = v1 and

A6: v1 is\_a\_proper\_prefix\_of v by TREES\_1:def 2;

v is\_a\_prefix\_of v^<\*x\*> by TREES\_1:1;

then v1 is\_a\_proper\_prefix\_of v^<\*x\*> by A6,XBOOLE\_1:58;

hence thesis by A5,TREES\_1:def 2;

end;

v^{} = v by FINSEQ\_1:34;

then

v is\_a\_prefix\_of v^<\*x\*> & v <> v^<\*x\*> by FINSEQ\_1:33,TREES\_1:1;

then v is\_a\_proper\_prefix\_of v^<\*x\*> by XBOOLE\_0:def 8;

then y in ProperPrefixes v or y = v & v in ProperPrefixes (v^<\*x\*>)

by A3,TARSKI:def 1,TREES\_1:def 2;

hence thesis by A4;

end;